Saimaa University of Applied Sciences
Technology, Lappeenranta
Double Degree Bachelor Programme in Civil and Construction Engineering

Anna Chernova

# Distortional buckling of cold-formed steel member

Bachelor’s Thesis 2019

## ABSTRACT

Anna Chernova

Distortional buckling of cold-formed steel member, 50 pages, 8 appendices

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Faculty of Technology Lappeenranta

Double Degree Bachelor Programme in Civil and Construction Engineering

Bachelor's Thesis 2019

Instructors: Lecturer Petri Himmi, Saimaa University of Applied Sciences
Managing Director Lyudmila Polykova, SMY-17

The main purpose of the thesis was to accumulate and analyze knowledge about the distortional buckling of cold-formed steel member and to determine the behavior of different dimensions of cross-sectional during distortional buckling. The purpose was also to do some calculations of this phenomenon.

The first part of the thesis consists of general information about what distortional buckling actually is, definite paragraphs from Eurocode and some words about other types of buckling. The general part of the thesis contains calculations and analysis about distortional buckling of cold-formed steel member, summarized in a tabular form.

The information was gathered mostly from Eurocode, literature and different titles. Also some articles from the Internet were carried out.

Keywords: Distortional buckling, cold-formed steel structural member, Eurocode

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# 1 Introduction

Nowadays, all calculations of different parts of frame and other structural elements for construction are necessary and always carried out. Essential calculations of them are computation of strength, deflection and buckling. Therefore, it is very important to consider these facts.

There is a lot of knowledge and analyses of those types of buckling like local and overall. But distortional buckling is not very well researched. It can be found only in Eurocode. There is no information about it in Russian norms. However, distortional buckling is also quite important in calculation of construction elements. For many cold-formed steel members distortional buckling may be the predominant buckling mode.

In determining the carrying capacity and stiffness of cold-formed and sheeting members the impact of the local and distortional buckling of cross-sectional shapes should be taken into account.

So the aim of this thesis work is to research what exactly distortional buckling is, how it can affect during working and to do calculation of different dimensions of cross section of cold-formed steel member.

# 2 Cold-formed steel structural members

## 2.1 Definition of cold-formed steel members

In steel construction, there are basically two types of structural members: hot-rolled steel shapes and cold-formed steel shapes. Hot-rolled steel shapes are formed at elevated temperatures while cold-formed steel shapes are formed at room temperature. That why they are named cold-formed steel. Cold-formed steel members are made from structural quality sheet steel and formed into shape, either through press-braking blanks sheared from sheets or coils Figure 1, or more commonly, by roll forming the steel through a series of dies Figure 2. [1]

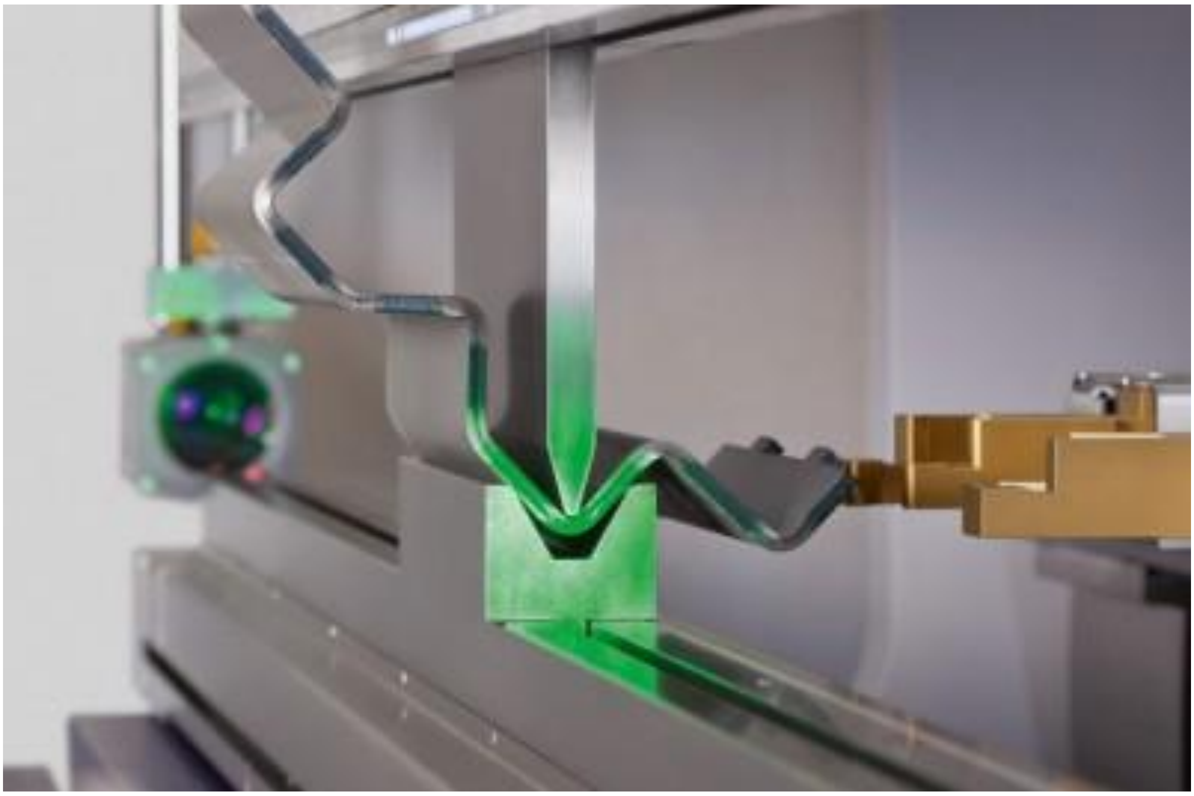


Figure 1.Forming methods for cold-formed steel members - Press braking [1]

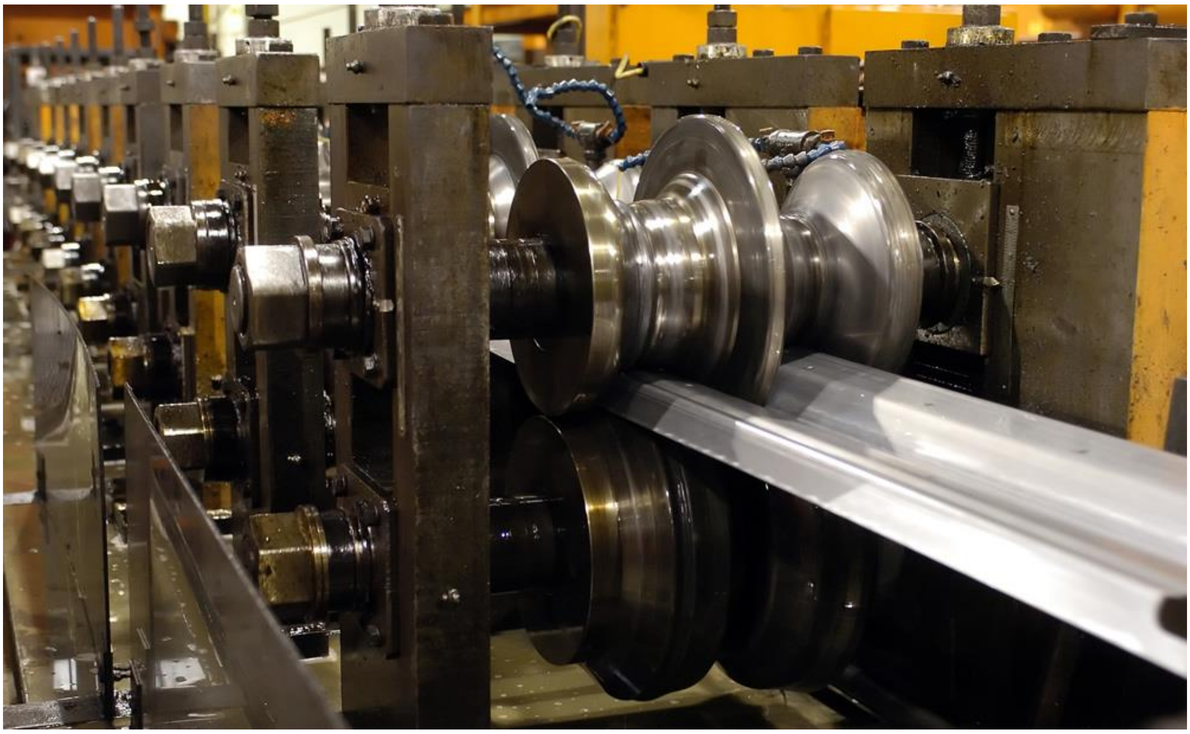


Figure 2.Forming methods for cold-formed steel members -Roll forming [1]

The idea of cold-formed steel members is to use shape rather than thickness to support loads. It has quite an easy method of production, thus a large number of different configurations can be produced. And it fits the requirement of optimized design for both structural and economical purposes. Figure 3 shows typical cold-formed steel shapes.

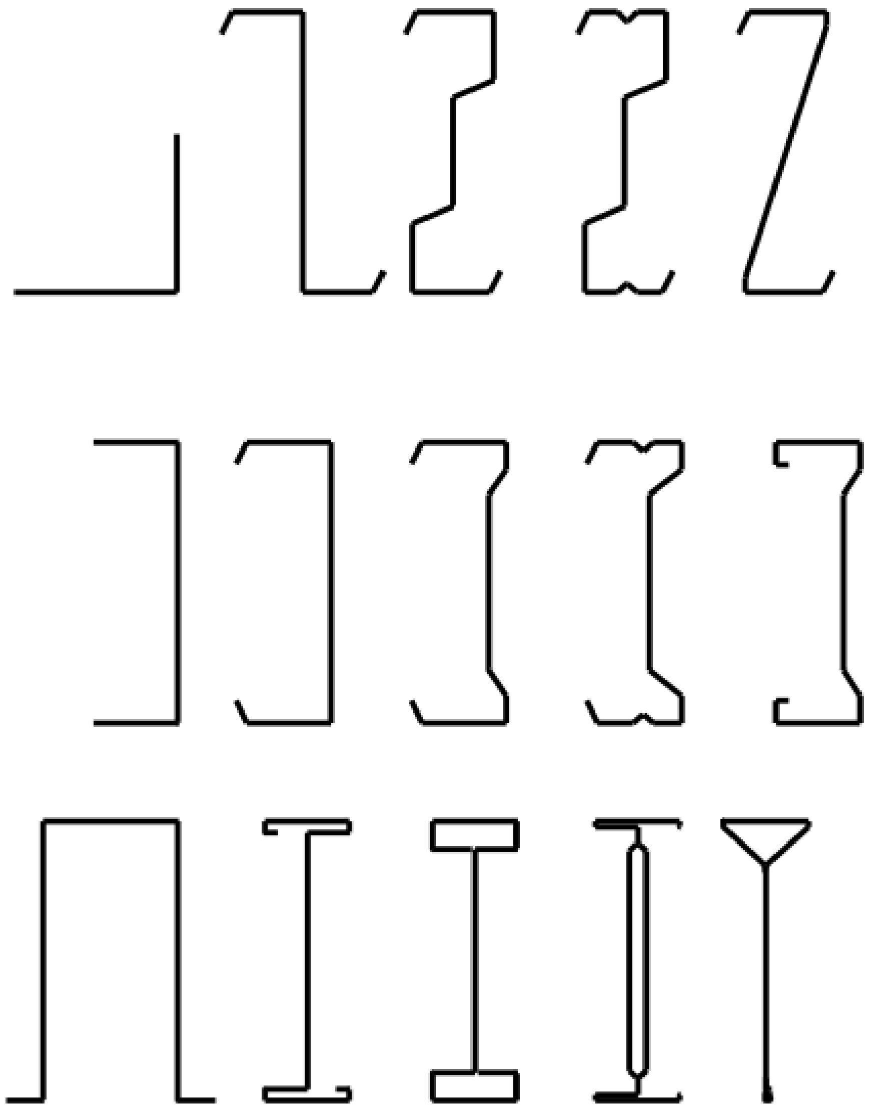


Figure 3.Common cold-formed steel shapes

## 2.2 History of cold-formed steel structural members

Cold-formed steel structural members have a fairly long history. This type of constructions was already known in the middle of the 19 century in the US and the UK. In 1849, during the ”golden rush” in California, a roofer from New York, Peter Naylor, promote and sell “portable iron houses for California” from the corrugated elements.

In the 1920s - 1930s, the development of cold-formed steel structural members slowed due to the lack of standards for these materials and construction standards for the design of this type of structures.

However, different companies in the United States were developing in this direction. Therefore, at the World Fair “Chicago Century of Progress Exposition”, which was held in 1933 in Chicago, architect Howard T. Fisher presented a joint development - the iron house, which can be collected for the four days. During five months of the fair the iron house was visited by about two million visitors.

The progress of cold-formed steel structures continued during the Second World War and after it. Thus the company Lustron Homes developed frame houses (Figure 4). And in the period from 1946 to 1948 the USA built for former soldiers 2498 such houses. In these houses almost all structures were built from steel. Another company, Quonset Huts, developed the projects of storage buildings and hangars with sheet structures. Spans of such structures have reached 36.5 m. [2]

Figure 4.House of Lustron Company built in 1940, modern look [2]

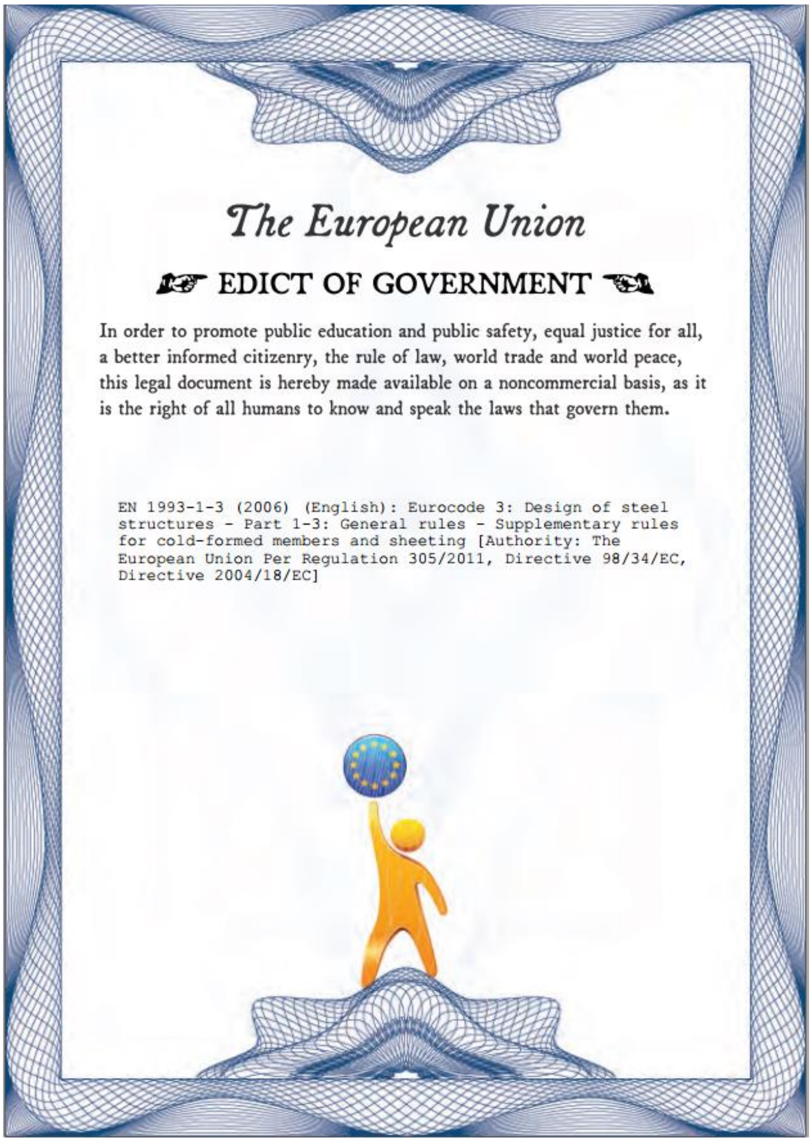
At the same time cold-formed steel structures were used in traditional construction. In 1925, in the city of Lynchburg, Virginia, Baptist Hospital was built. Load-bearing walls of this building were made of bricks, and as floor beams were installed cold-formed steel beams of the two C-section profiles connected by walls as an I-beam. A survey that conducted 80 years after the construction showed that the beams are in a good condition and can be used further.

### 2.3 Standards of cold-formed steel members

Despite of advances in the development of cold-formed structures, the level of its use was still significantly lower than the hot-formed. A considerable factor that corrected this imbalance application was the absence of standards and norms. Therefore, the American Iron and Steel Institute (AISI) decides to develop a standard of design of cold-formed structures. George Winter (Figure 5) led the work on the world's first normative document of the structures of cold-formed steel members, which is called “the father of cold-formed structures. ”Under his leadership in 1946 the first regulatory document was issued - Specification for the Design of Light Gage Steel Structural Members. The standard consists of six sections: general data, design methodology, method of calculation for the allowable stress, connection, design of wall columns and test structures. Subsequently, the standard developed by G. Winter had several editions.[2]

Figure 5. George Winter, “the father of cold-formed structures” [2]

In Europe, the development of standards for design of cold-formed structures initially relied on studies in Cornell University. Further studies were carried out separately in each country. This led to the fact that in the 1980s in almost all European countries there were national standards. In 1955, the organization named European Convention for Constructional Steelwork (ECCS) was established for coordination between research institutes, manufacturers and consumers of structural steel. In ECCS was created and operates till now a technical committee TC7 «Cold-Formed Thin-Walled Sheet Steel in Buildings». TC7 Technical Committee has developed a number of normative documents, which allowed in 1993 to adopt uniform European standards for the design of steel structures. Nowadays in the European Union works standard EN 1993-1-3: 2006 (Figure 6) for the calculation of cold-formed steel structural members. [3]



The European Union

EDICT OF GOVERNMENT

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EN 1993-1-3 (2006) (English): Eurocode 3: Design of steel structures - Part 1-3: General rules - Supplementary rules for cold-formed members and sheeting [Authority: The European Union Per Regulation 305/2011, Directive 98/34/EC, Directive 2004/18/EC]

Figure 6. EN 1993-1-3: 2006 [5]

# 3 Distortional buckling cold-formed steel member

## 3.1 Complex behavior of cold-formed steel member

With the rapidly increasing usage of cold-formed steel structures, research into their applications and understanding of their structural behavior have increased significantly since the first specification of cold-formed steel design was issued by the American Iron and Steel Institute (AISI) in 1946. The light gauge cold-formed steel members encounter some design problems not normally seen in the traditional thicker hot-rolled steel members. For example, buckling is a serious issue leading to their failure modes. [9]

So, the behaviour of cold-formed steel structures is more complex than that of traditional hot-rolled steel structures. Research between 1940 and 1950 has highlighted that cold-formed steel members are subjected to various buckling modes including local, distortional and global modes, and their ultimate strength behaviour is governed by these buckling modes. As an example, a short concentrically loaded C-section almost always fails because of a combination of local buckling of thin plate elements and distortional buckling of the edge stiffeners while the failure mode of longer columns is often governed by a combination of global buckling and local or distortional buckling. Although cold-formed steel members have been researched for a long time, the stability problem is not fully understood. [9]

Previously, most research was devoted to local and global buckling modes and in this area a great amount of knowledge has been accumulated. This knowledge and understanding is reflected in the various design rules in national or international design standards or specifications required to achieve a safe and economical design.

There is one of important thing of structural design - withstand to fire effects. We need to consider that structures can be deliberately set on fire or accidentally catch fire that can cause loss of life and property, not only because of fire but also due to the structural failure of buildings. Light cold-formed steel structures heat up quickly and reduce their stiffness and strength under fire conditions due to the thinness of materials. That why the fire safety design of building structures is very important and has received greater attention in recent times.

Figure 7. Buckling of Columns under Fire [9]

### 3.2 Definition of distortional buckling

Distortional buckling is a mode characterized by the rotation of the member's flange at the flange / web junction, in members with edge stiffeners. [4]

Distortional buckling is associated with the presence of stiffeners. A channel member (cross-section without stiffeners) shows, in general, only one buckling mode: local (plate) mode. The presence of edge stiffeners improves the "performance" of the structural element, but leads to the occurrence of this second mode of instability. Unlike what happens in the local mode, in the distortional mode there is deformation along the walls junctions (flange/lip), as shown on Figure 8. [4]

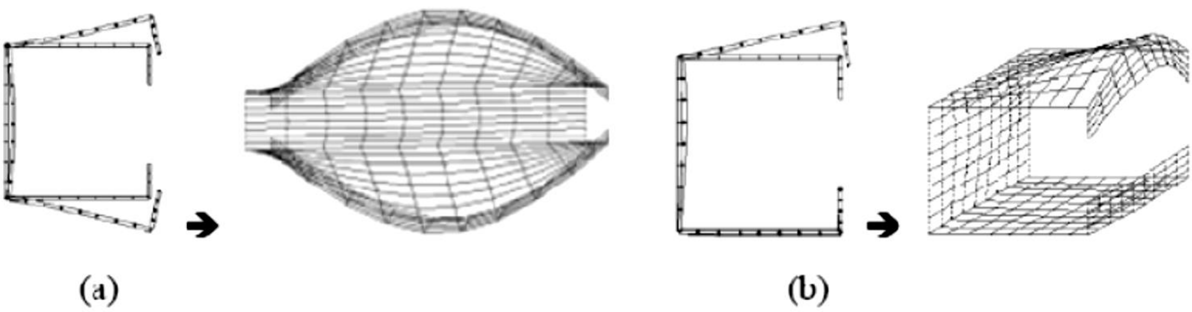



Figure 8. Distortional buckling mode configurations of C-section member subject to (a) compression and (b) bending moment [4]

The study of distortional buckling is still relatively recent. The earliest studies, based on analytical models were the basis to the first design codes that used distortional buckling considerations on the design of thin-walled steel structures.

### 3.2 Conception of distortional buckling

Basically, three types of buckling behaviour can be observed in cold-formed steel members. They are called *local, distortional and flexural torsional* buckling. In addition to these three main types, their interactions can also take place. For example, distortional and flexural, distortional and local, local and flexural, and local distortional and flexural.

The effects of local and distortional buckling should be taken into account in determining the resistance and stiffness of cold-formed members and sheeting. [5]

*Local buckling* occurs in the slender plate elements without changing the position of longitudinal edges of compression members. On the other hand, it occurs due to the buckling of individual plate elements. Local buckling is a common buckling failure in compression members, which are made of slender plate elements. The half-wave-length of the local buckling mode is the shortest one among the other failure modes. Since local buckling has a higher post buckling range, it is not considered as failure of the whole column when columns buckle locally. [9]

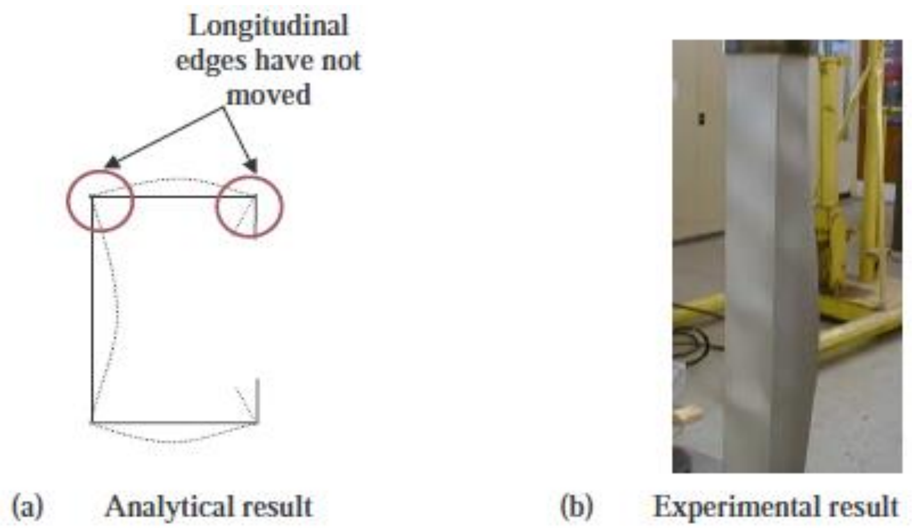



Figure 9. Local Buckling Mode of Compression Members [9]

Most commonly *flexural buckling* mode is known as global buckling. Since the behavior is similar to the behavior of beams this is known as flexural or flexural torsional buckling. In flexural buckling, the cross sectional shape remains unchanged and it has only lateral or lateral torsional movements. This buckling mode is sometimes called rigid-body buckling since the cross section remains the same at any given section after global buckling occurs. The half-wave-length of the flexural mode is the largest among the buckling modes. [9]

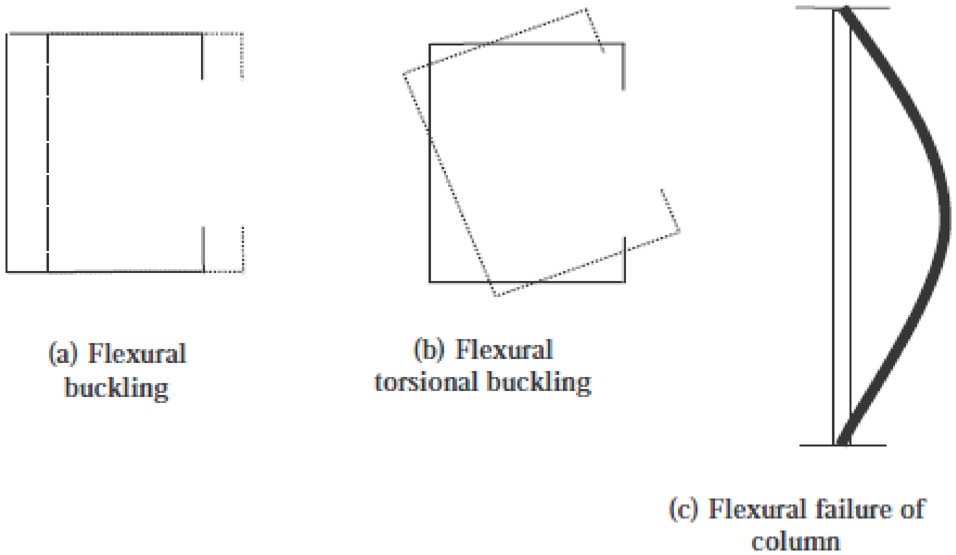



Figure 10. Flexural Buckling Mode of Compression Members [9]

*Distortional buckling* mode is a relatively new and less researched buckling mode compared to the other buckling modes.

Distortional buckling, also known as "stiffener buckling" or "local-torsional buckling", is a mode characterized by rotation of the flange at the flange/web junction in members with edge stiffened elements. In members with intermediately stiffened elements, distortional buckling is characterized by displacement of the intermediate stiffener normal to the plane of the element. It exists at an intermediate half-wave-length between local buckling and flexural or flexural torsional buckling. [6]

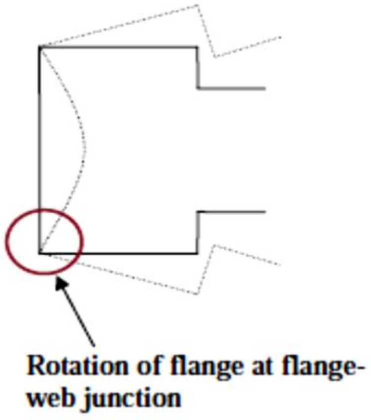




Figure 11. Distortional Buckling Mode of Compression Members [9]

This thesis was focused on investigating the relatively new and less researched distortional buckling mode of cold-formed steel compression members. Thus, the following section describes the previous research on distortional buckling.

However, the distortional buckling can be safely ignored if members are designed to achieve significantly lower local buckling stress than the distortional buckling stress. Most probably, members with narrow flanges fail by local buckling mode since, the web is much slender and buckles locally first, while members with wide flanges buckle distortionally. However, by introducing the stiffeners to the web, narrow flange members can buckle distortionally. [9]

So that this research was focused on members with edge or intermediate stiffeners.

Distortional buckling for elements with edge or intermediate stiffeners is indicated in Figure 12. The effects of distortional buckling should be allowed for in cases such as those indicated in Figure 8a, b and c. In these cases, the effects of distortional buckling should be determined performing linear or non-linear buckling analysis using numerical methods or column stub tests. [5]

As shown on Figure 13 on vertical there is buckling stress. On horizontal - half sine waves (half-wavelengths). Distortional buckling exists at an intermediate half-wavelength, between local buckling and long half-wavelength flexural or flexural-torsional buckling.

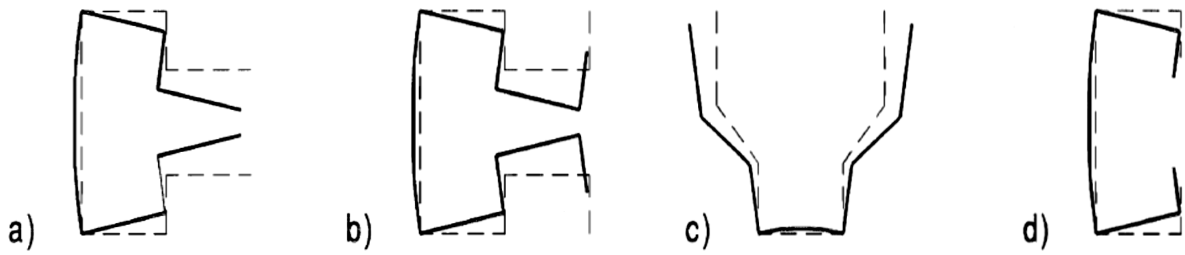



Figure 12. Examples of distortional buckling modes

So pure compression local buckling often occurs at a lower buckling stress than distortional buckling. If the local buckling stress is lower than the distortional buckling stress then it is possible that distortional buckling may be safely ignored. However, many situations exist in which distortional buckling must still be considered, even in routine design. [6]

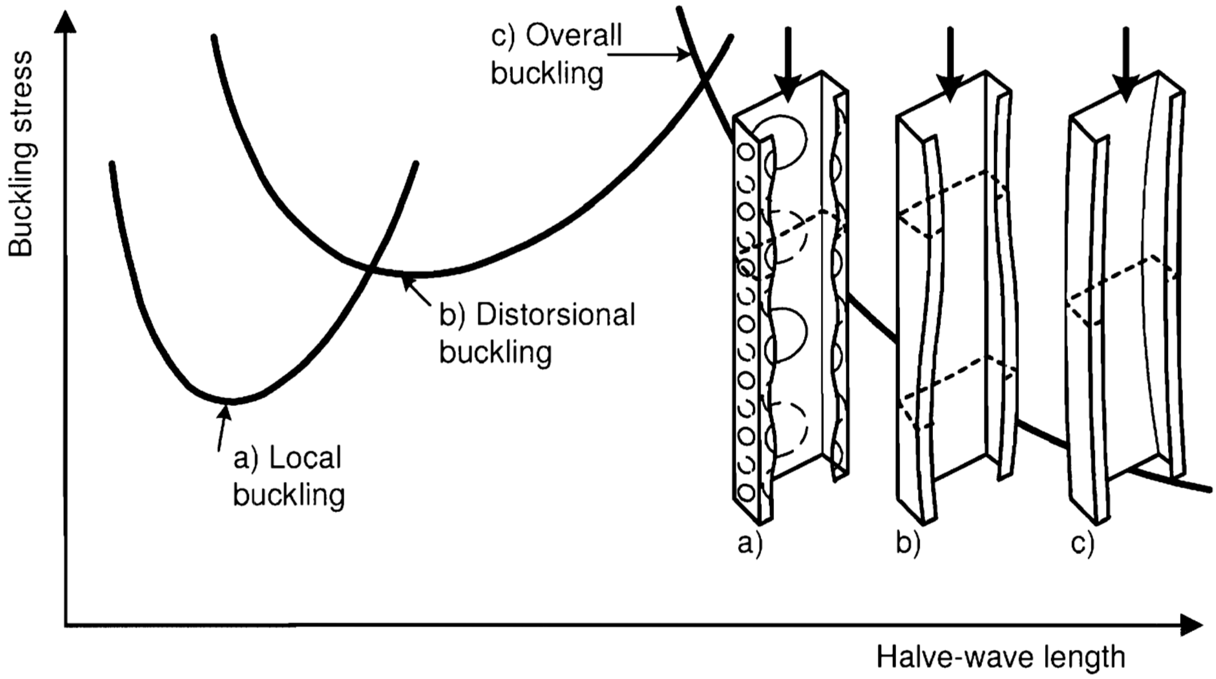



Figure 13. Examples of elastic critical stress for various buckling modes as function of halve-wavelength and examples of buckling modes

### 3.3 Distortional buckling behavior

Intuition for local buckling behavior is a quite simple – widthtothickness ratios increase local buckling stress declines. This fact serves the engineer well in designing for local buckling. Similar intuition for distortional buckling is difficult to arrive at.

A series of examples examining the distortional buckling stress of C-section columns with edge stiffened elements are summarized in Figure 14. The distortional buckling stress was calculated using closed-form expressions derived in Schafer (1997). Figure 14 provides facilities to develop a modest amount of intuition with respect to distortional buckling. [6]

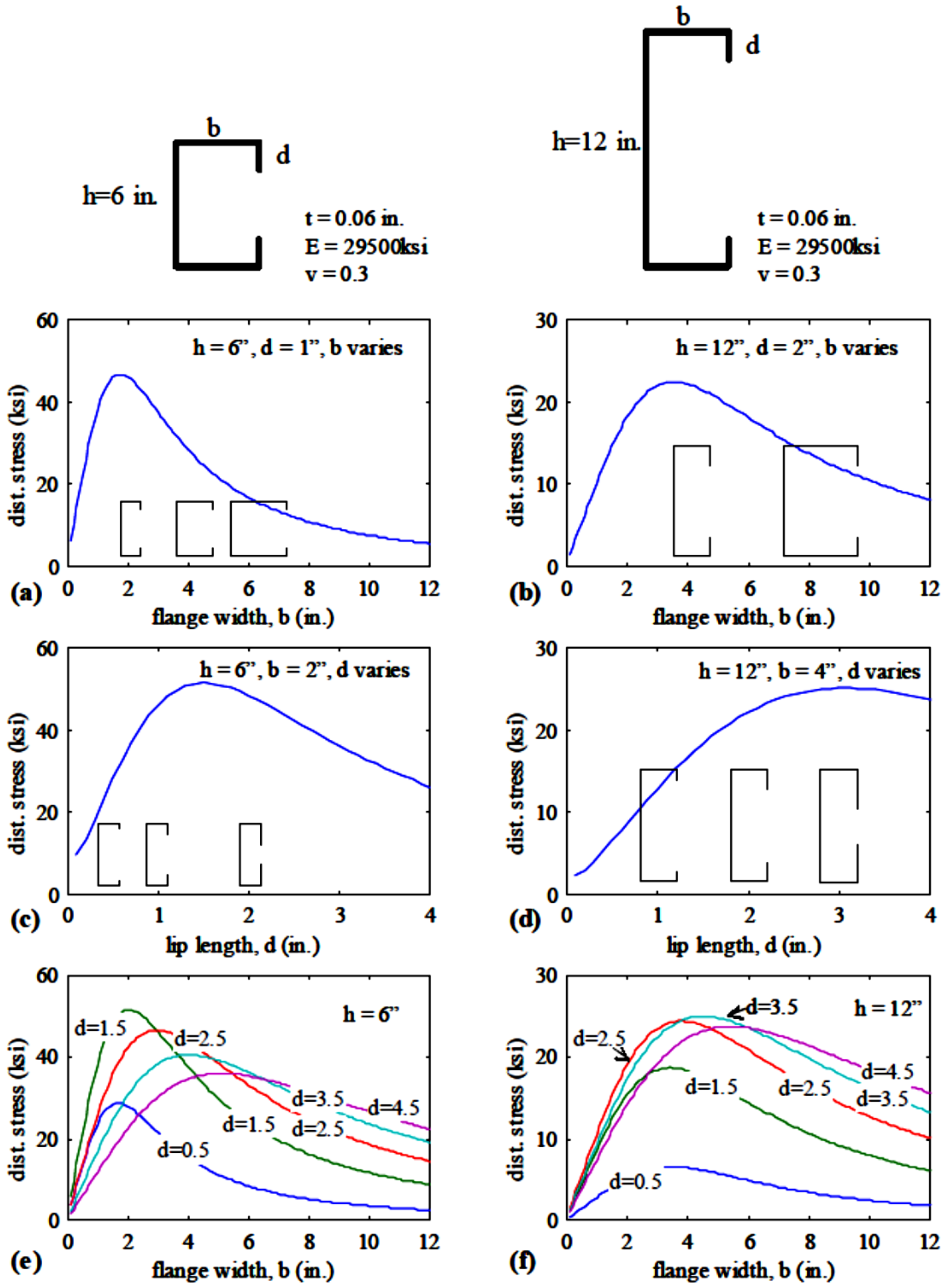



Figure 14. Behavior of Distortional Buckling Stress for C-section Columns with edge stiffened elements: (a-b) variation withrespect to flange width, (c-d) variation with respect to edge length, (e-f) variation with respectto flange width for different edge lengths [6]

Lets consider Figure 14.

In this example all dimension is in inch. b = flange width, h = web height and d = length of edge stiffened elements

Too little or too much flange is not good.
Figure 14 (a) and (b) show results of different flange width. As the width is longer, as the distortional buckling stress is lower. In this example, the highest distortional buckling stresses are achieved around a b/h ratio of 1/3. This conclusion is not general though, as different stiffener lengths give a different optimum b/h ratio. If the flange is too narrow local buckling of the web is at wavelengths near distortional buckling of the flange and the distortional mode easily forms at low stresses. If the flange is very wide local buckling is not the concern, but the problem is to keep the flange in place because of the big size of the stiffener. For practical stiffener lengths, wide flanges also lead to low distortional stresses. [6]

Longer lips are usually better.
Figure 14 (c) and (d) show results of various length of edges. The highest distortional buckling stresses are achieved when the edge length is nearly equal to the flange width (d/b ~ 1). Edges longer than this reduce the distortional buckling stress. From the viewpoint of distortional buckling the edge stiffener lengths should be longer than used in practice now.

Deep webs lead to low stresses.
Comparing the results from the 6 in. deep web to the 12 in., the distortional buckling stress decreases nearly by a factor of 2 when the web depth is doubled. The actual redution in the distortional buckling stress depends on d and b. Distortional buckling is determined by the rotational stiffness at the web/flange joint. Deeper webs are more flexible and thus provide less rotational stiffness to the web/flange joint. This results in earlier distortional buckling for deep webs. However, the trend appears approximately linear, as opposed to local buckling which changes as $(t/h)^2$. And thus local buckling stresses decrease at a faster rate with deeper webs. [6]

As Figure 14 shows, the interaction of the flange, web, and edge in determining the distortional buckling stress is complex. Development of simple general criteria to incorporate this behavior has not proven successful to date.

Ultimate strength of member failing in the distortional mode is worthy of attention because distortional failures have lower post-buckling capacity than local modes of failure. Distortional failure modes have higher imperfection sensitivity. And it may control failure even when the elastic distortional buckling stress is higher than the elastic local buckling stress.

# 4 Distortional buckling calculation

## 4.1 Calculation of cross-section №1

The aim of the calculation is to determine the geometric characteristics of the effective cross-section of cold-formed C-shaped profile working in compression.

Calculations are made in program MathCad.

As an example of calculation, we should take the following dimensions of cross-section. Other calculations are located in Appendix.

$h := 200mm$

$b := 65mm$

$c := 25mm$

$t := 2mm$

$r := 3mm$

$\varphi := 90°$

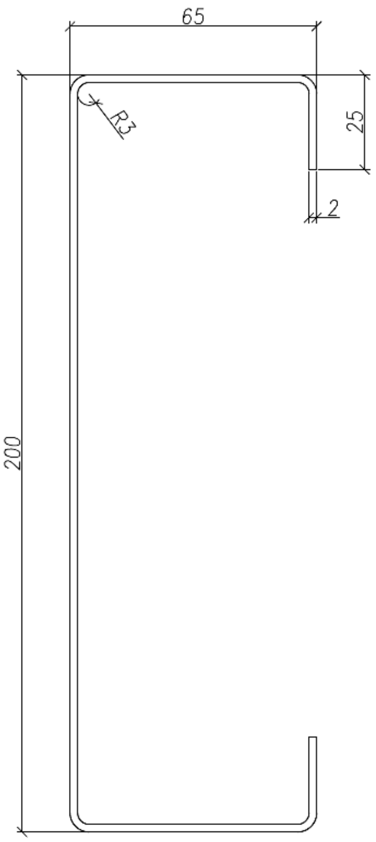



Figure 15.Cross-section dimensions №1

The calculation of compressed elements with edge or intermediate stiffeners is based on the assumption that the stiffener works as compressed with continuous fastening by elastic connections, the stiffness of which depends on the edge conditions and the flexural rigidity of the adjacent flat elements.

1. First of all, we should check element size ratios to determine whether this profile can be applied to the calculation of EN 1993-1-3 (in accordance with par. 5.2 (1) and table 5.1 [5]).

The provisions for design by calculation should not be applied to cross-sections outside the range of width-to-thickness ratios, given in Table 1.

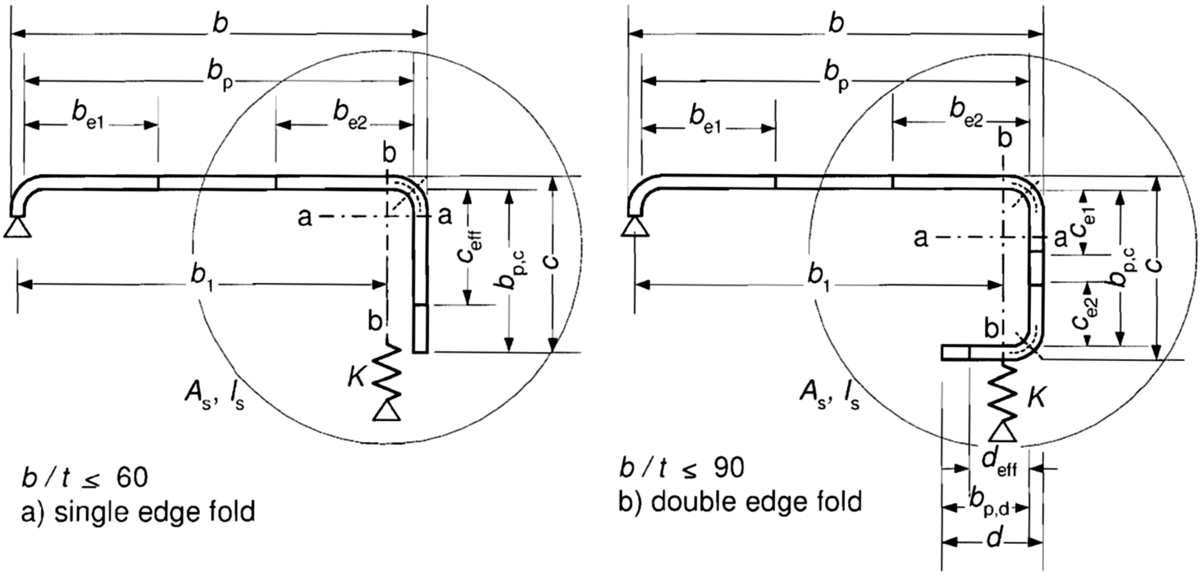



Figure 16. Edge stiffeners [5]

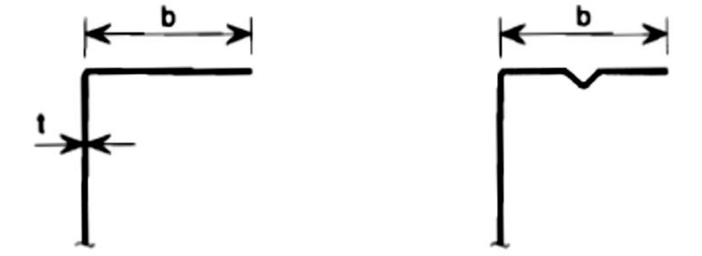
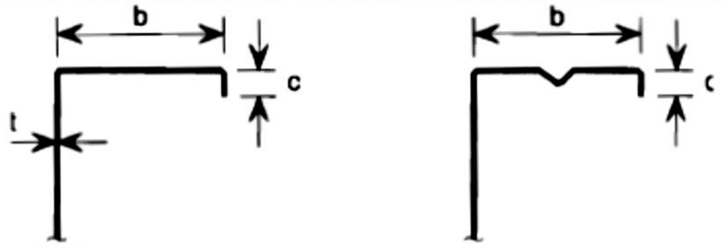
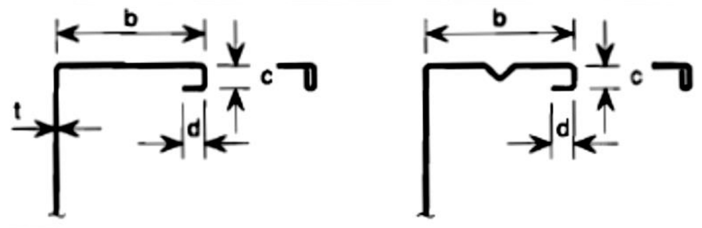
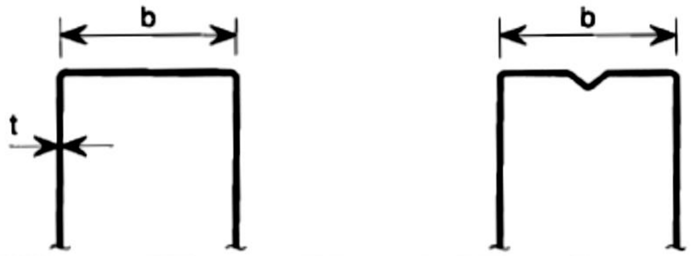
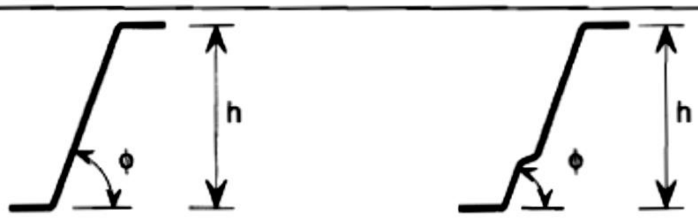
So that determination of the effective geometric characteristics of the edge stiffener elements is possible under the following conditions:

• the angle between the limb and the flat element is in the range from 45 ° to 135 °;

• limb width with not less than 0.2b, where b and c are shown in Figure 3.12;

• b / t ratio does not exceed 60 for the edge bend from one element (single) or 90 for the edge bend from two elements (double).

Table 1. Maximum width to thickness ratio

| Element of cross-section | Maximum value |
|---|---|
|   | $b/t \leq 50$ |
|   | $b/t \leq 60$<br>$c/t \leq 50$ |
|   | $b/t \leq 90$<br>$c/t \leq 60$<br>$d/t \leq 50$ |
|   | $b/t \leq 500$ |
|   | $45° \leq \phi \leq 90°$<br>$h/t \leq 500 \sin\phi$ |

Our situation is the second one.

$$\begin{cases} \text{"Can calculate"} & \text{if } \left(\frac{b}{t} \leq 60\right) \wedge \left(\frac{c}{t} \leq 50\right) \wedge \left(\frac{h}{t} \leq 500 \cdot \sin(\varphi)\right) \\ \text{"Can't calculat"} & \text{otherwise} \end{cases} = \text{"Can calculate"}$$

According to this, we can apply this profile to calculations.

2. In order to provide sufficient stiffness and to avoid primary buckling of the stiffness element itself, the sizes of stiffeners should be within the following ranges (in accordance with par. 5.2 (2) [5]):

$$\begin{cases} \text{"Stiffness is provided" if } 0.2 \le \frac{c}{b} \le 0.6 \\ \text{"Stiffener does't take into account in the calculation" if } \frac{c}{b} < 0.2 \\ \text{"Stiffness is not provided" otherwise} \end{cases} = \text{"Stiffness is provided"}$$

Stiffness is provided.

3. After that we should find theoretical dimensions of flat areas to midlines of elements (in accordance with par. 1.5.3 (2) and 1.6.5 (1) [5]):

$hp := h - t = 198 \cdot mm$

$bp := b - t = 63 \cdot mm$

$cp := c - 0.5 \cdot t = 24 \cdot mm$

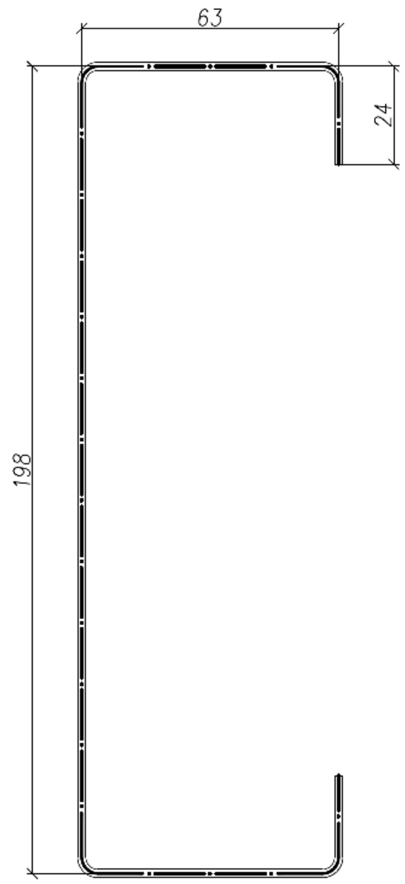



Figure 17. Theoretical dimensions №1 of flat areas to midlines of elements

4. Theoretical widths of flat areas of cross-section (in accordance with par. 5.1 (1) and fig. 5.1 [5]):

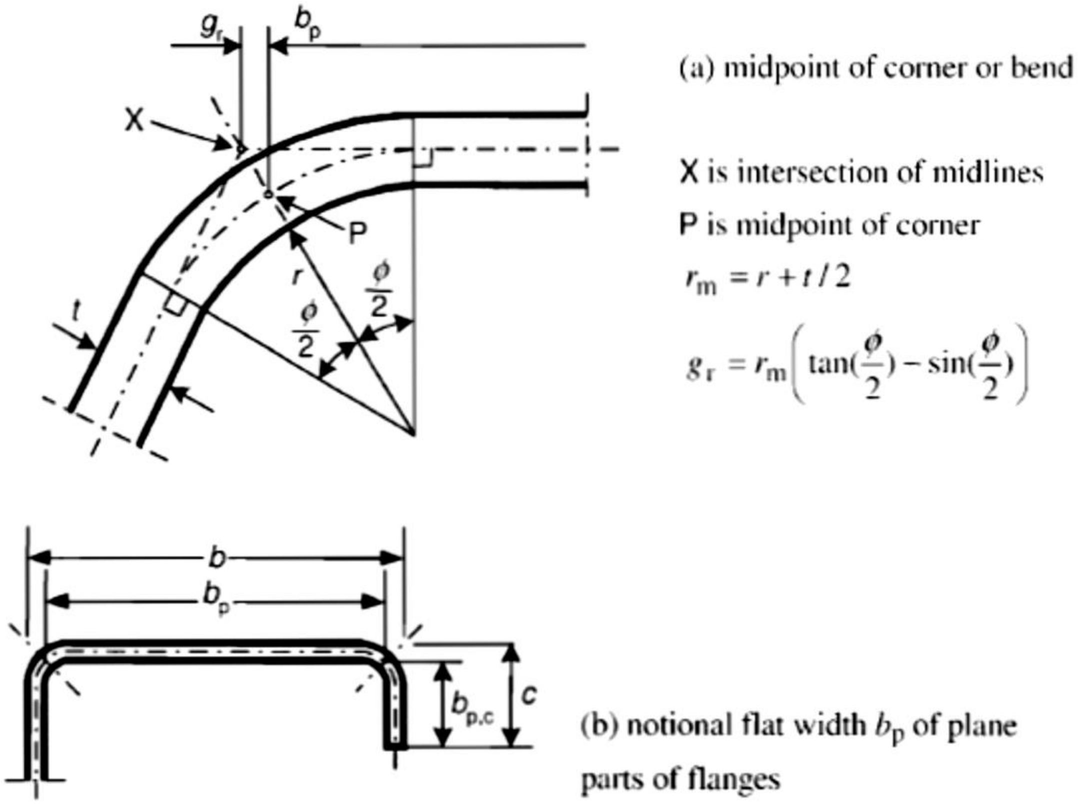



Figure 18. Notional widths of plane cross section parts bp allowing for corner radii [5]

$rm := r + 0.5 \cdot t = 4 \cdot mm$
$gr := rm \cdot (\tan(0.5 \cdot \varphi) - \sin(0.5 \cdot \varphi)) = 1.17 \cdot mm$
$bph := hp - 2 \cdot gr = 195.66 \cdot mm$
$bpb := bp - 2 \cdot gr = 60.66 \cdot mm$
$bpc := cp - gr = 22.83 \cdot mm$

5. There are two methods that are used to determine the geometric characteristics of cross sections with followed calculation of strength of the element: method with taking into account the corner elements and without taking them into account.

Unless more appropriate methods are used to determine the section properties the following approximate procedure may be used. The influence of rounded corners on cross-section resistance may be neglected if the internal radius r:S 5 t and r:S 0, I ° bp and the cross-section may be assumed to consist of plane elements with sharp corners (according to Figure 5.2, note bp for all flat plane elements, inclusive plane elements in tension).

The need to take into account the influence of rounded corners on cross-section resistance (in accordance with par. 5.1 (3) [5]):

$$\begin{cases} \text{"Shouldn't take into account"} & \text{if } (r \le 5 \cdot t) \wedge (r \le 0.1 \cdot bph) \wedge (r \le 0.1 \cdot bpb) \wedge (r \le 0.1 \cdot bpc) \\ \text{"Should take into account"} & \text{otherwise} \end{cases} = \text{"Should take into account"}$$

Notation: For cross-section stiffness properties the influence of rounded corners should always be taken into account.

6.1. The influence of rounded corners on section properties may be taken into account by reducing the propeliies calculated for an otherwise similar cross-section with sharp corners (in accordance with par. 5.1 (4) [5]):

$$\delta := \frac{4 \cdot 0.43 \cdot r}{bph + 2 \cdot bpb + bpc} \cdot \frac{\varphi}{1.571} = 0.015$$

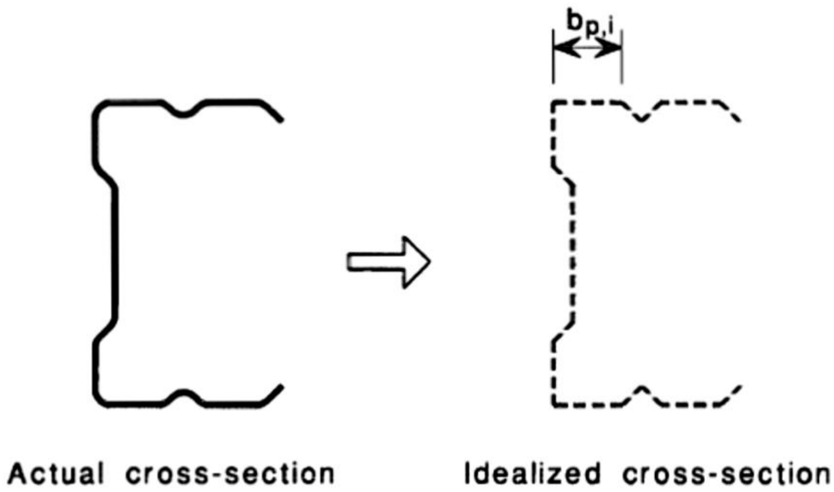



Figure 19. Approximate allowance for rounded corners [5]

6.2. Section properties with sharp corners:

$$Agsh := t \cdot (hp + 2 \cdot bp + 2 \cdot cp) = 744 \cdot mm^2$$

$$Igsh := \frac{t \cdot hp^3}{12} + 2 \cdot \left[\frac{bp \cdot t^3}{12} + \left(\frac{hp}{2}\right)^2 \cdot bp \cdot t\right] + 2 \cdot \left[\frac{t \cdot cp^3}{12} + \left(\frac{hp - cp}{2}\right)^2 \cdot cp \cdot t\right] = 449.49 \cdot cm^4$$

6.3. Section properties with the influence of rounded corners by approximate formulas (in accordance with par. 5.1 (4) [5]):

$$Ag := Agsh \cdot (1 - \delta) = 732.7 \cdot mm^2$$
$$Ig := Igsh \cdot (1 - 2 \cdot \delta) = 435.84 \cdot cm^4$$

7. Now we are going to the direct determination of section properties of the effective section on central compression. There are a few steps of it.

*Step 1a. Finding the effective width of the shelves*

7.1. Setting the nominal value of the basic yield strength (in accordance with table 3.1a [5]):

$$fyb := 350MPa$$

7.2. Then the basic yield strength (in accordance with par. 3.2.2 (1) [5]):

$$fy := fyb = 350 \cdot MPa$$

7.3. Factor ε (in accordance with par. 4.4 (2) [7]):

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

7.4. The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

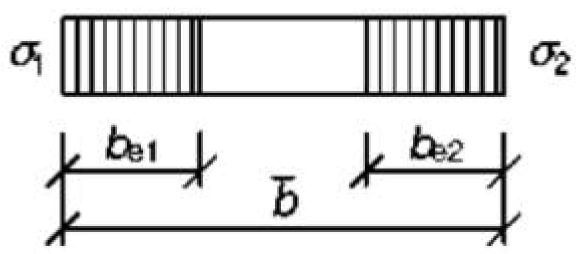
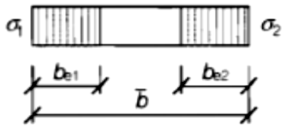
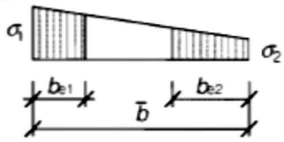
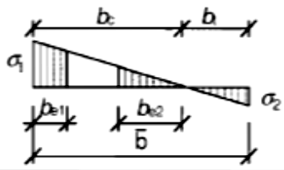



Figure 20. Stresses σ1 and σ2 on central compression

7.5. Buckling factor $k_\sigma$ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Table 2. Internal compression elements

| Stress distribution (compression positive) | Effective[1)] width $b_{eff}$ |
|---|---|
|  | $\psi = 1$:<br>$b_{eff} = \rho\ \bar{b}$<br>$b_{e1} = 0{,}5\ b_{eff}$ $\quad b_{e2} = 0{,}5\ b_{eff}$ |
|  | $1 > \psi \geq 0$:<br>$b_{eff} = \rho\ \bar{b}$<br>$b_{e1} = \frac{2}{5-\psi} b_{eff}$ $\quad b_{e2} = b_{eff} - b_{e1}$ |
|  | $\psi < 0$:<br>$b_{eff} = \rho\ b_c = \rho\ \bar{b} / (1-\psi)$<br>$b_{e1} = 0{,}4\ b_{eff}$ $\quad b_{e2} = 0{,}6\ b_{eff}$ |

| $\psi = \sigma_2/\sigma_1$ | 1 | $1 > \psi > 0$ | 0 | $0 > \psi > -1$ | -1 | AC1 $-1 > \psi \geq -3$ AC1 |
|---|---|---|---|---|---|---|
| Buckling factor $k_\sigma$ | 4,0 | $8{,}2 / (1{,}05 + \psi)$ | 7,81 | $7{,}81 - 6{,}29\psi + 9{,}78\psi^2$ | 23,9 | $5{,}98\ (1 - \psi)^2$ |

7.6. The effective widths of unstiffened elements should be obtained using the notional flatwidth bp for b by determining the reduction factors for plate buckling based on the plate slenderness).

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pb := \frac{bp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.68$$

7.7. Reduction factor for loss of stability of compressed plate plate with double-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{vmatrix} \frac{\lambda pb - 0.055 \cdot (3 + \psi)}{\lambda pb^2} & \text{if } \lambda pb > 0.673 \\ 1 & \text{otherwise} \end{vmatrix} = 1$$

7.8. Effective width of the shelf (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$beff := \rho \cdot bp = 62.8268 \cdot mm$$
$$be1 := 0.5 \cdot beff = 31.4134 \cdot mm$$
$$be2 := 0.5 \cdot beff = 31.4134 \cdot mm$$

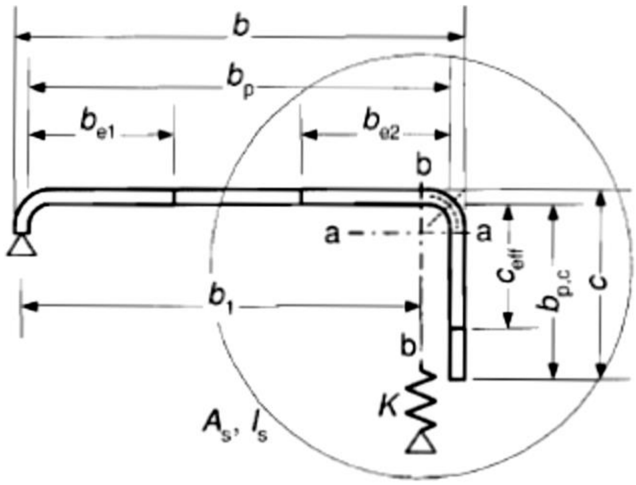



Figure 21. Edge stiffeners [5]

8. *Step 1b. Finding the effective width of the edge stiffener*

8.1. Stability loss factor for single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$k\sigma := \begin{cases} 0.5 & \text{if } \dfrac{bpc}{bp} \le 0.35 \\ 0.5 + 0.83 \cdot \sqrt[3]{\left(\dfrac{bpc}{bp} - 0.35\right)^2} & \text{if } 0.35 < \dfrac{bpc}{bp} < 0.60 \end{cases} = 0.54$$

8.2. Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] andpar. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.7$$

8.3. Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \dfrac{\lambda pc - 0.188}{\lambda pb^2} & \text{if } \lambda pb > 0.748 \\ 1 & \text{otherwise} \end{cases} = 1$$

8.4. Effective width of single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$ceff := \rho \cdot cp = 24 \cdot mm$$

8.5. The effective cross-sectional area of the edge stiffener (in accordance with par. 5.5.3.2 (6) [5]):

$$As := t \cdot (be2 + ceff) = 1.11 \times 10^{-4} m^2$$

$$As1 := As = 1.11 \times 10^{-4} m^2$$

$$As2 := As = 1.11 \times 10^{-4} m^2$$

*9. Step 2. Determination of bearing capacity reduction rate due to distortional buckling*

9.1. Elastic modulus and Poisson's ratio in the elastic stage (in accordance with par. 3.2.6 [8]):

$$E := 210000 MPa$$

$$\nu := 0.3$$

9.2. The distance from the intersection of the wall and shelves to the centers of gravity of the effective area of the edge stiffener:

$$b1 := bp - \frac{0.5 \cdot be2 \cdot t \cdot be2}{t \cdot (be2 + ceff)} = 0.05 m$$

$$b2 := b1 = 0.05 m$$

9.3. Stress factor kf (in accordance with par. 5.5.3.1 (5) [5]):

$k_f = 0$ if flange 2 is in tension (e.g. for beam in bending about the y-y axis);

$k_f = \frac{A_{s2}}{A_{s1}}$ if flange 2 is also in compression (e.g. for a beam in axial compression);

$k_f = 1$ for a symmetric section in compression.

$$kf := 1$$

9.4. Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

$$K1 := \frac{E \cdot t^3}{4 \cdot (1 - \nu^2)} \cdot \frac{1}{b1^2 \cdot hp + b1^3 + 0.5 \cdot b1 \cdot b2 \cdot hp \cdot kf} = 0.45 \cdot \frac{\frac{N}{mm}}{mm}$$

9.5. The moment of inertia of the effective section of the edge stiffener, defined by the effective area As relative to the central axis a-a of the effective cross section:

$$bs := \frac{t \cdot be2 \cdot 0.5 \cdot t + t \cdot ceff \cdot 0.5 \cdot (ceff + t)}{As} = 6.2 \cdot mm$$

$$Is := \left[\frac{be2 \cdot t^3}{12} + t \cdot be2 \cdot (bs - 0.5 \cdot t)^2\right] + \left[\frac{t \cdot ceff^3}{12} + t \cdot ceff \cdot [0.5 \cdot (ceff + t) - bs]^2\right] = 6.24 \times 10^3 \cdot mm^4$$

9.6. The elastic critical buckling stress for an edge stiffener (in accordance with par. 5.5.3.2 (7) [5]):

$$\sigma crs := \frac{2 \cdot \sqrt{K1 \cdot E \cdot Is}}{As} = 437.95 \cdot MPa$$

9.7. Relative slenderness (in accordance with par. 5.5.3.1 (7) [5]):

$$\lambda d := \sqrt{\frac{fyb}{\sigma crs}} = 0.89$$

9.8. The reduction factor for the distional buckling resistance (flexural buckling of a stiffener) (in accordance with par. 5.5.3.1 (7) [5]):

$$\chi d := 1.0 \cdot (\lambda d < 0.65) + (1.47 - 0.723 \cdot \lambda d) \cdot (0.65 < \lambda d < 1.38) + \frac{0.66}{\lambda d} \cdot (\lambda d \geq 1.38) = 0.82$$

9.9. The reduced effective thickness of the stiffener (in accordance with par. 5.5.3.2 (11) and (12) [5]):

$$tred := t \cdot \chi d = 1.65 \cdot mm$$

*10. Step 3. Effective section characteristics*

10.1. The ratio of stresses σ 2 / σ 1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

10.2. Buckling factor $k\sigma$ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

10.3. Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda ph := \frac{hp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 2.13$$

10.4. Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \dfrac{\lambda ph - 0.055 \cdot (3 + \psi)}{\lambda ph^2} & \text{if } \lambda ph > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.42$$

10.5. Effective cross-section wall width (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$heff := \rho \cdot hp = 83.46 \cdot mm$$

$$he1 := 0.5 \cdot heff = 41.73 \cdot mm$$

$$he2 := 0.5 \cdot heff = 41.73 \cdot mm$$

Thus, there are results of calculations of cross-section №1.

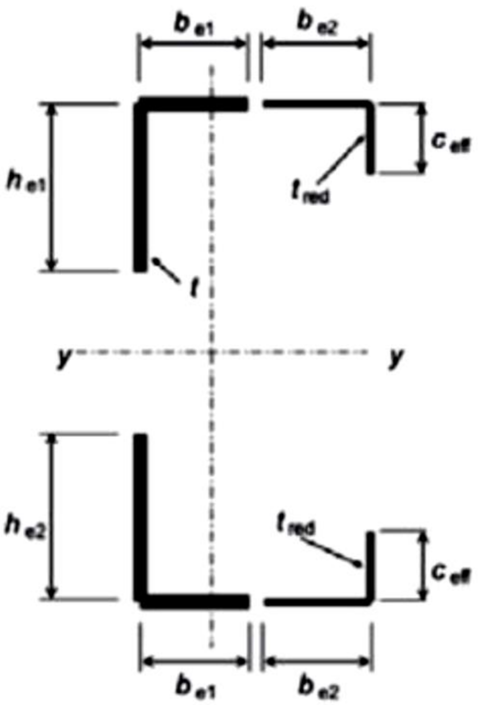



Figure 22. General scheme of effective dimensions

Table 3. Effective dimensions of calculations of cross-section №1

| Feature | Designation | Dimensions of cross-section before buckling | Effective dimensions of cross-section after buckling |
|---|---|---|---|
| Height | h | 200 | 41.73+41.73=83.46 |
| Width | b | 65 | 31.41+31.41=62.82 |
| Length of edge stiffener | c | 25 | 24 |
| Thickness | t | 2 | 1.65 |

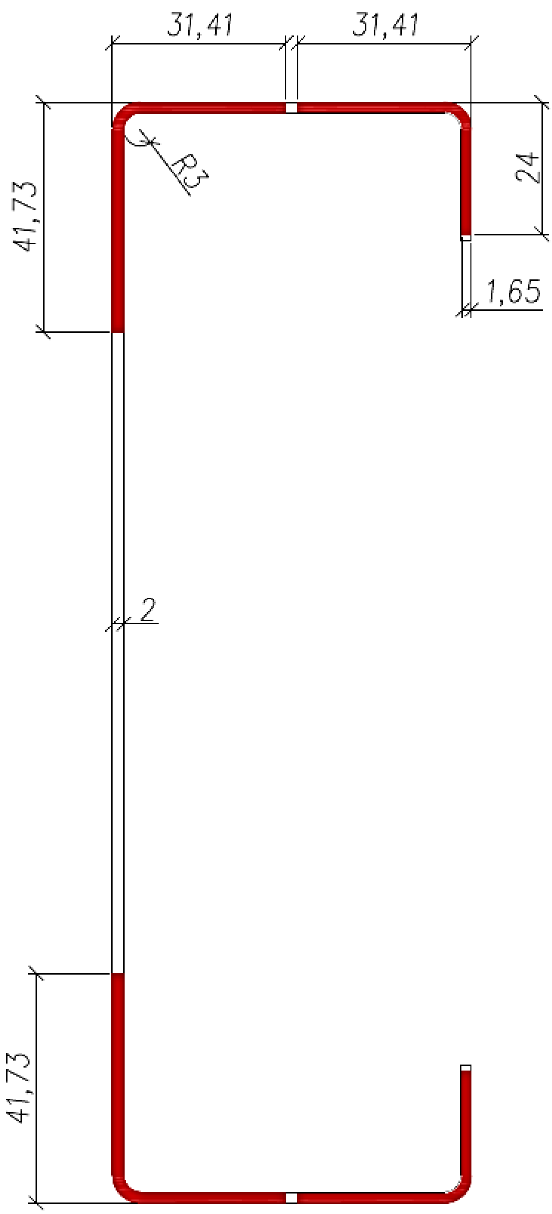



Figure 23. Size comparison of cross-section №1 after distortional buckling

## 4.2. Dimensions that are applied for calculation

Consider in more detail from what dimensions of cross-section we can not use, since it is not applied for calculations.

1. First of all we have such conditions.

$\frac{b}{t} \leq 60\,, \frac{c}{t} \leq 50\ and\ \frac{h}{t} \leq 500$ if the angle equals 90°.

According to this, if we consider it on example №1, we can not use profile with thickness equals and under 1 mm. And it has to be more than 2 mm, if we take width around 120 mm.

Most of all here it all depends on width and thickness, either on height and length of edge stiffener. This is explained by the fact that is impractical to take too high value of height of cross-section and also too large length of edge stiffener.

So that the major condition is that thickness less then width 60 times. And it should be done.

2. Secondly, the stiffness of edge stiffener should be provided. For this the following condition should be done.

$$0.2 \leq \frac{b}{t} \leq 0.6$$

Based on this, the edge cannot be too large or small compared to the width.

And there is one more condition when we do not need to take the edge stiffener into account. If $\frac{b}{t} < 0.2$.

So the length of stiffener is so small compared to width that it is not used. But then there is no meaning in calculations, if the stiffener does not work.

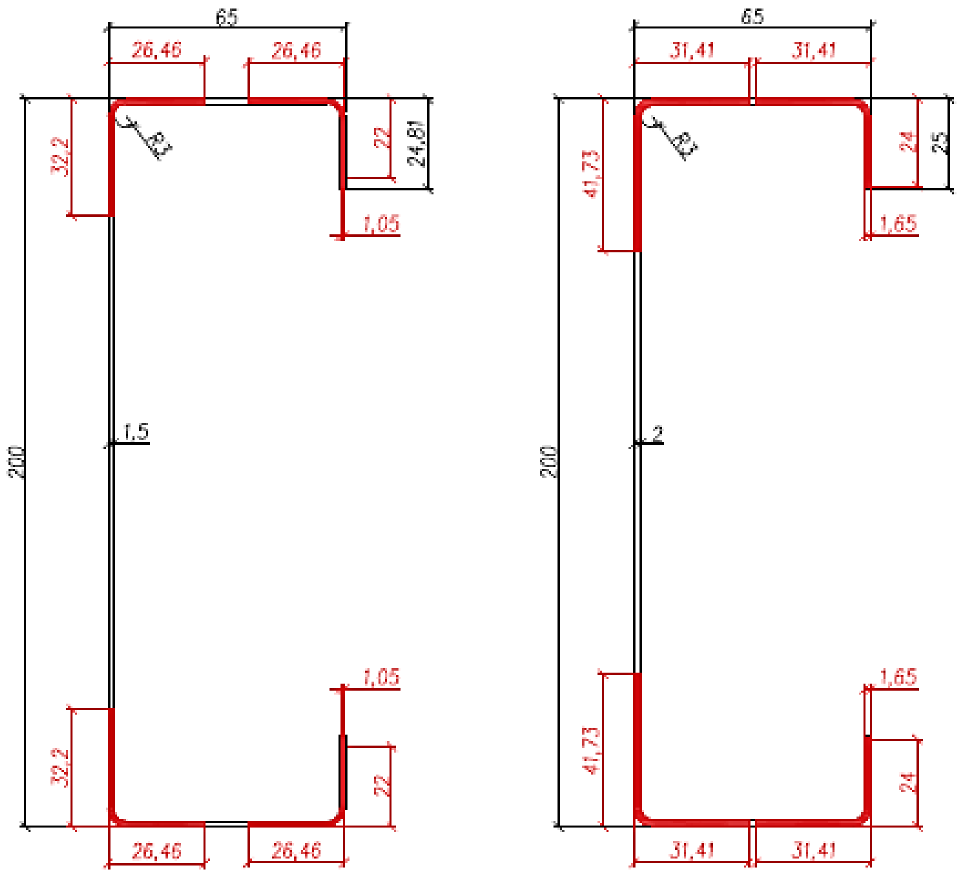
### 4.3. Changes of thickness

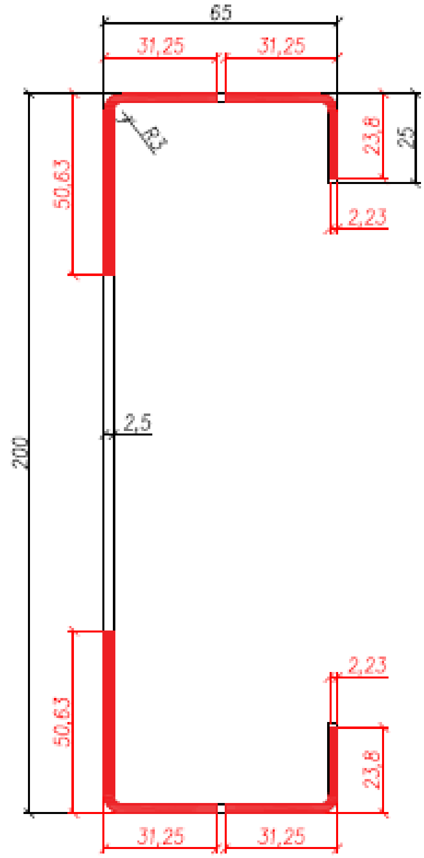
Lets research what it will be if we change the thickness of the cross-section. We take a step equal to 0.5 mm. But as it was written earlier with the given width we can not use a profile with the thickness equal to 1 mm or less.

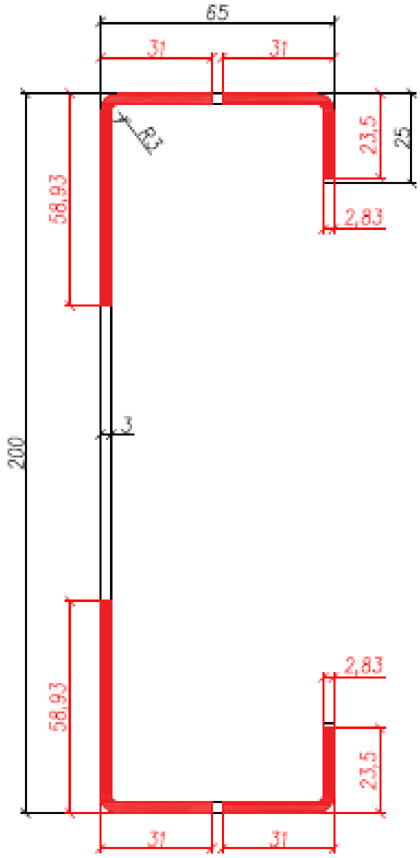
Calculation is done in program MathCad and the results are summarized in a tabular form. Examples of calculations from Mathcad are in Appendix.

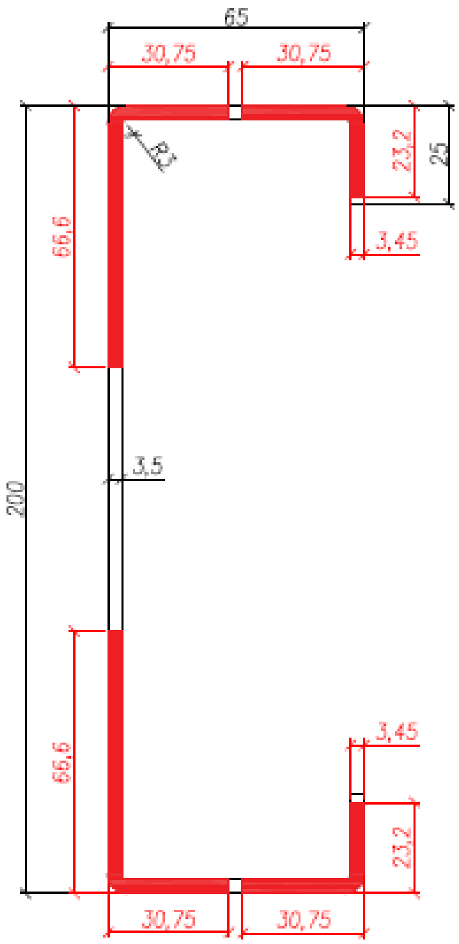
Table 4. Changes of thickness. Results

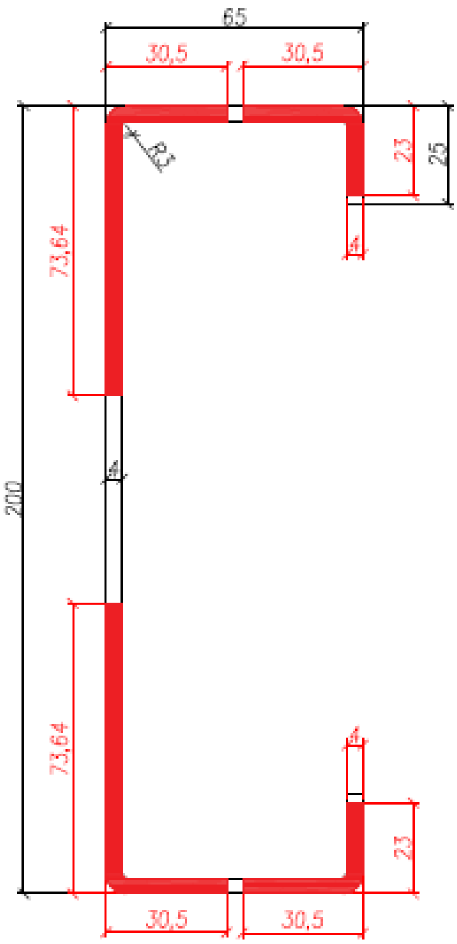
| Feature | Original dimensions | Effective dimensions with different thickness | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | t=1.5 | t=2 | t=2.5 | t=3 | t=3.5 | t=4 | t=4.5 | t=5 |
| Height | 200 | 64.40 | 83.46 | 101.26 | 117.86 | 133.20 | 147.28 | 160.08 | 171.62 |
| Width | 65 | 52.92 | 62.82 | 62.50 | 62.00 | 61.50 | 61.0 | 60.5 | 60.0 |
| Length of edge stiffener | 25 | 22.0 | 24.0 | 23.8 | 23.5 | 23.2 | 23.0 | 22.8 | 22.5 |
| Thickness | ~ | 1.05 | 1.65 | 2.23 | 2.83 | 3.45 | 4.0 | 4.5 | 5 |




65
31,25
31,25
R3
23,8
25
2,23
50,63
2,5
200
50,63
2,23
23,8
31,25
31,25


65
31
31
R3
23,5
25
2,83
58,93
3
200
58,93
2,83
23,5
31
31


65
30,75
30,75
R3
23,2
25
3,45
66,6
3,5
200
66,6
3,45
23,2
30,75
30,75


65
30,5
30,5
R3
23
25
4
73,64
4
200
73,64
4
23
30,5
30,5

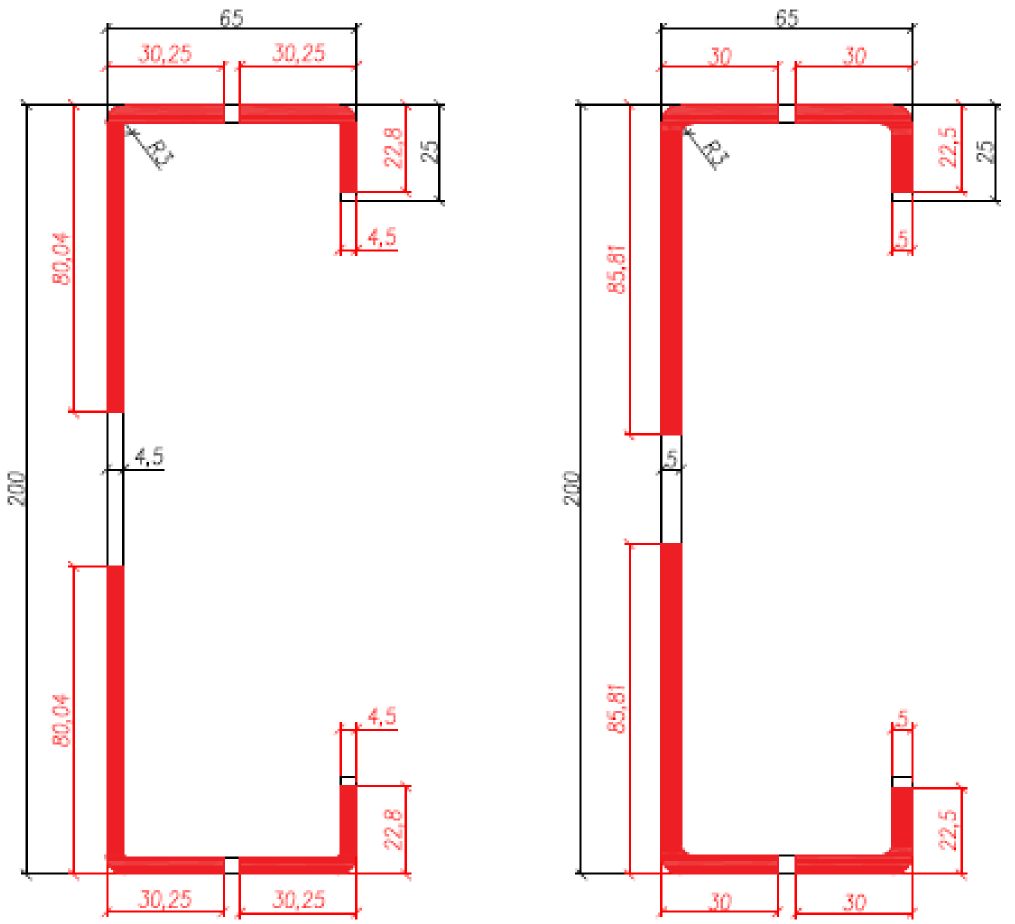



Figure 24. Changes of thickness. Results

From the results of this calculation we can see the following things:

1. The effective height of the cross-section steadily increases with rising of thickness. But the dependence is not linear. It a bit go down further.
   On Figure 25 we can see dependence between the effective height and the thickness of cross-section.

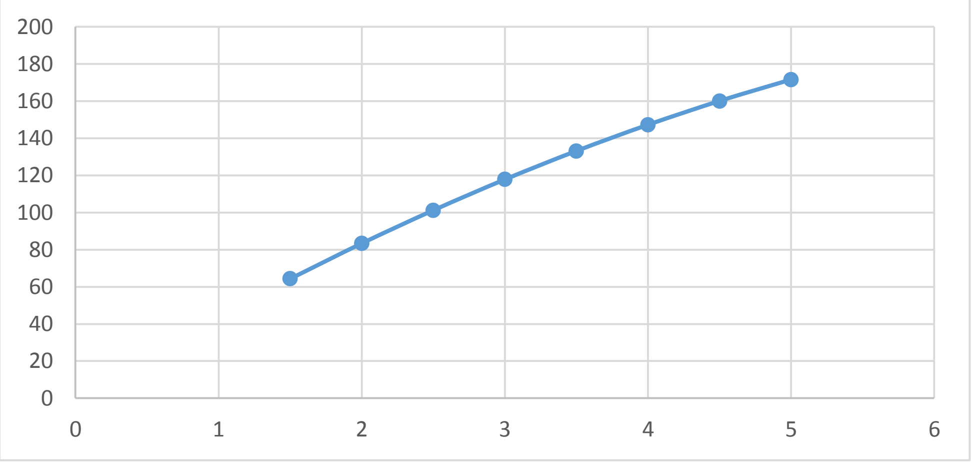



Figure 25. Dependence between effective height and thickness

2. Up to thickness equal to two, the width of cross-section growsw rapidly. Then it slowly decrease on half of one millimeter.

   On the Figure 26 we can see the dependence between the effective width and the thickness of cross-section.

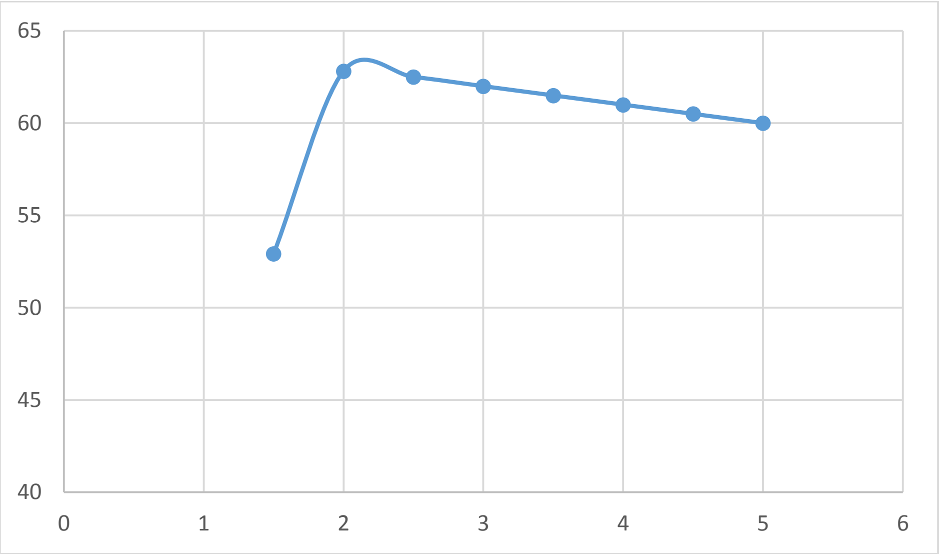



Figure 26. Dependence between effective width and thickness

3. There is quite a common situation with the dependence between the effective length of the edge stiffener and the thickness of the cross-section.

   Up to thickness equal to two the width of cross-section grows and then it very slowly decreases. It practically remains unchanged.

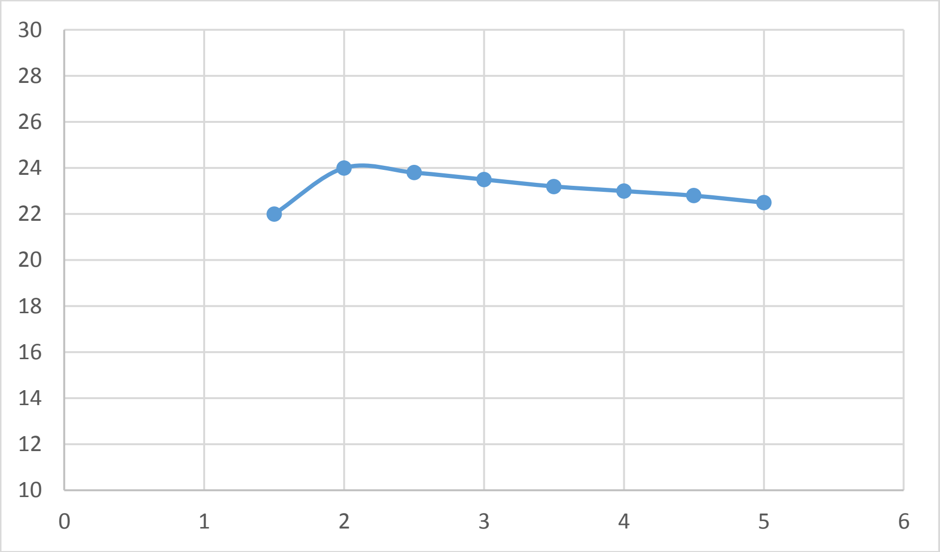



Figure 27. Dependence between effective length of edge stiffener and thickness

4. Up to thickness of cross-section equal four, the effective thickness increases and approaches directly to the width value itself. After this values of thickness and effective thickness of cross-section are equal and remain without changing.

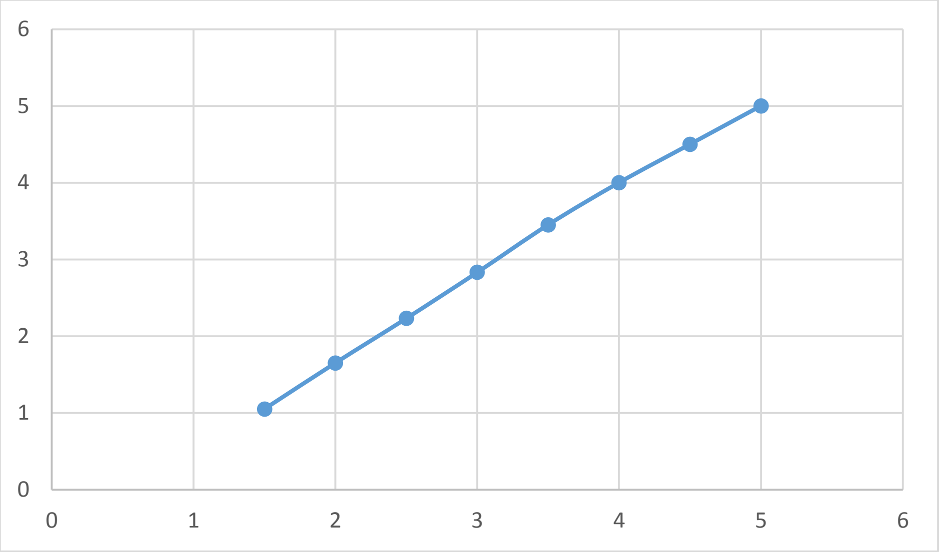


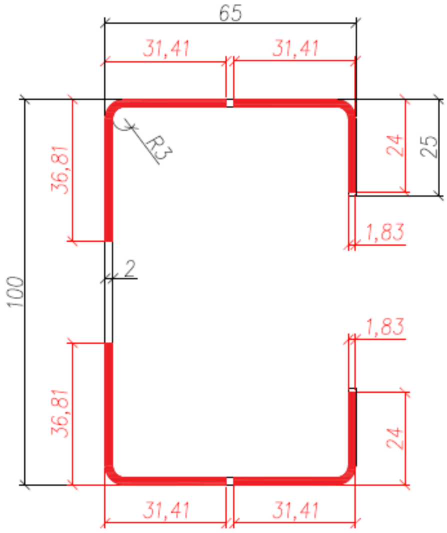

Figure 28. Dependence between effective thickness and thickness of cross-section

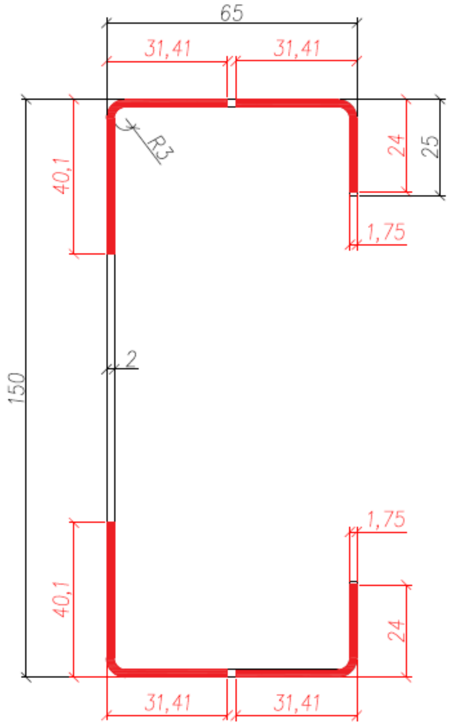
## 4.4. Changes of height

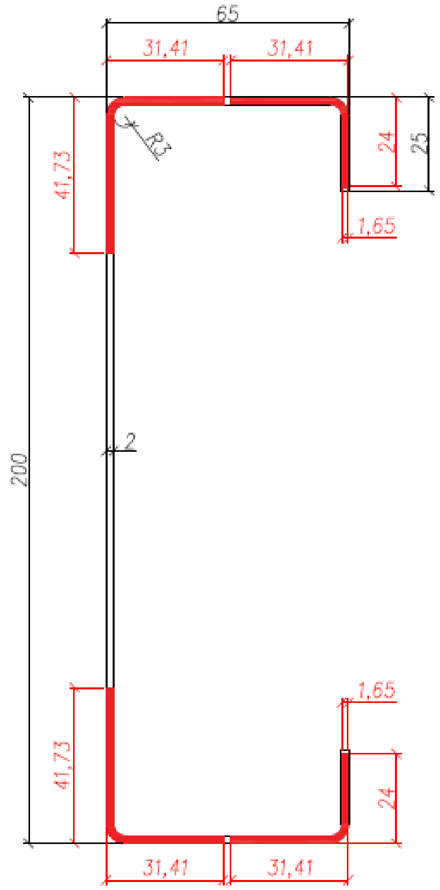
Now we research what it will be if we change the height of the cross-section. We take a step equal to 50 mm.

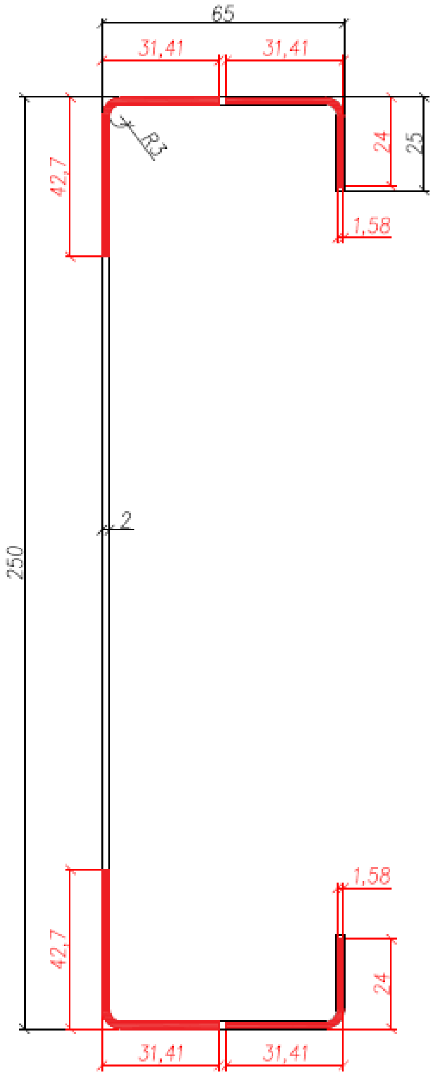
Table 5. Changes of height. Results

| Feature | Original dimensions | Effective dimensions with different thickness | | | | | |
|---|---|---|---|---|---|---|---|
| | | h=100 | h=150 | h=200 | h=250 | h=300 | h=350 |
| Height | ~ | 73.62 | 80.2 | 83.46 | 85.4 | 86.70 | 87.60 |
| Width | 65 | 62.82 | 62.82 | 62.82 | 62.82 | 62.82 | 62.82 |
| Length of edge stiffener | 25 | 24.0 | 24.0 | 24.0 | 24.0 | 24.0 | 24.0 |
| Thickness | 2 | 1.82 | 1.75 | 1.65 | 1.58 | 1.53 | 1.48 |


65
31,41
31,41
R3
24
25
36,81
1,83
2
100
1,83
36,81
24
31,41
31,41


65
31,41
31,41
R3
24
25
40,1
1,75
2
150
1,75
40,1
24
31,41
31,41


65
31,41
31,41
R3
24
25
41,73
1,65
2
200
1,65
41,73
24
31,41
31,41


65
31,41
31,41
R3
24
25
42,7
1,58
2
250
1,58
42,7
24
31,41
31,41

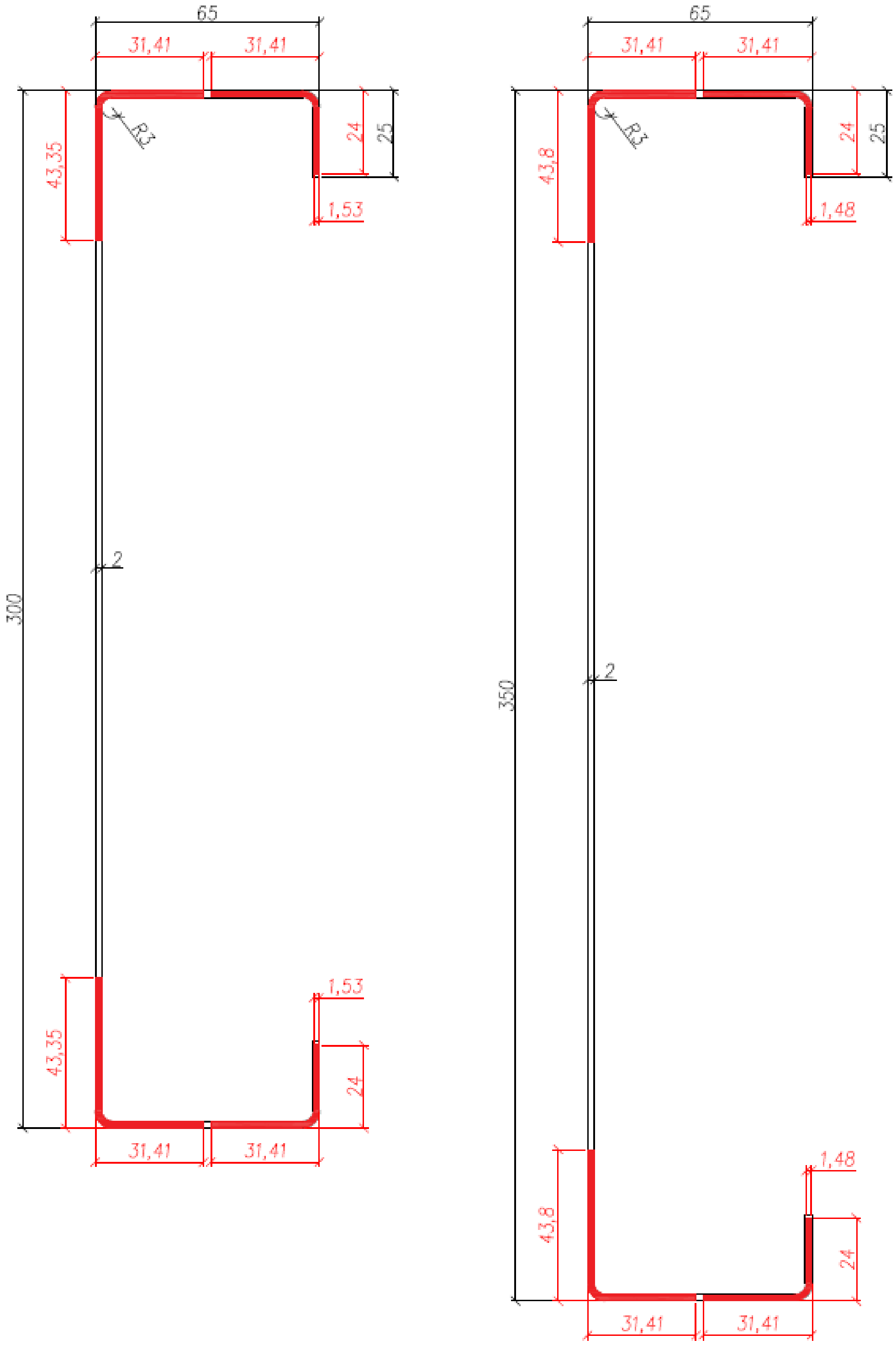



Figure 29. Changes of height. Results

As it is seen from the results only the effective height and effective thickness depend on the height of cross-section. The ffective width and the length of edge stiffener do not change at all.

1. The effective height grows a little bit with increasing of the height of cross-section. However, the difference between the height of cross-section and

effective height also grow up. So that the common effective square of cross-section decreases and the efficiency of using this profile reduces.

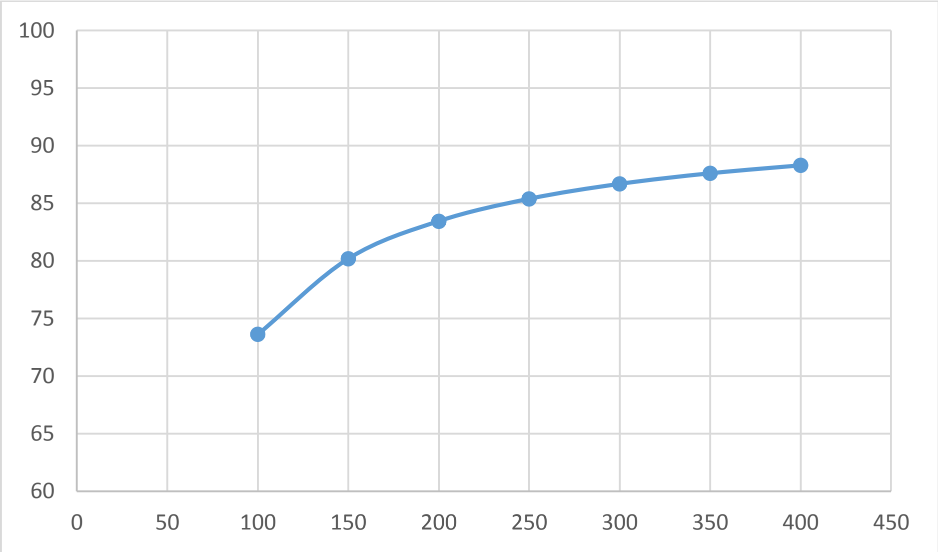



Figure 30. Dependence between height and effective height

2. Effective thickness decreases with increasing of height of cross-section.

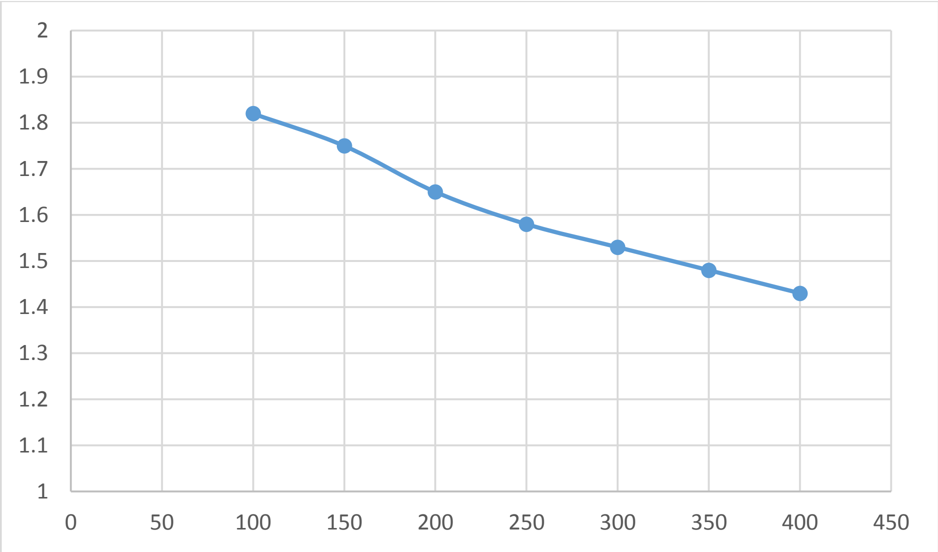


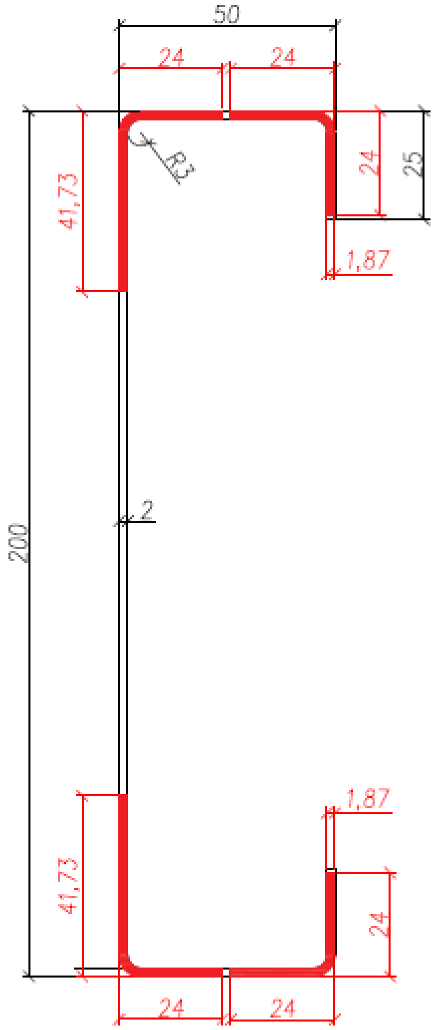

Figure 31. Dependence between height and effective height

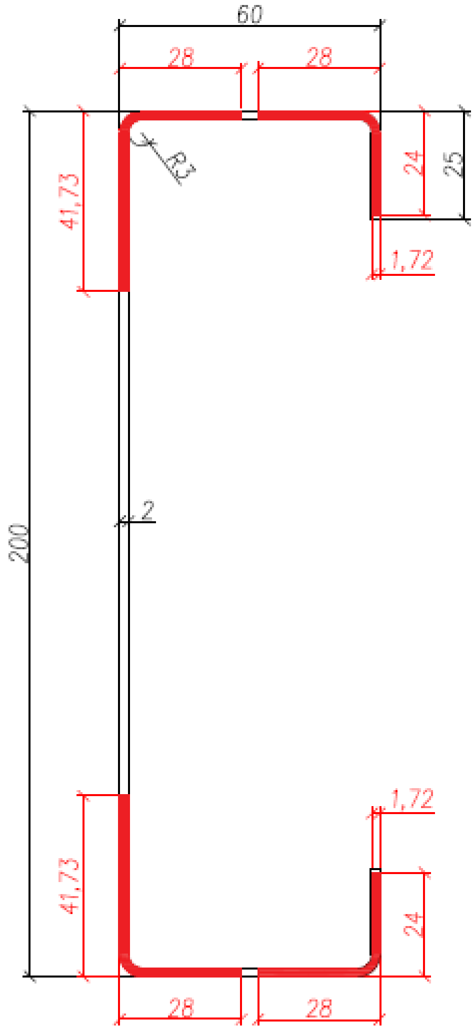
## 4.5 Changes of width

Then we research what it will be if we change the width of the cross-section. We take a step equal to 10 mm.

Table 6. Changes of width. Results

| Feature | Original dimensions | Effective dimensions with different thickness | | | | | |
|---|---|---|---|---|---|---|---|
| | | b=50 | b=60 | b=70 | b=80 | b=90 | b=100 |
| Height | 200 | 83.46 | 83.46 | 83.46 | 83.46 | 83.46 | 83.46 |
| Width | ~ | 48.0 | 58.0 | 65.04 | 68.64 | 71.42 | 73.64 |
| Length of edge stiffener | 25 | 24.0 | 24.0 | 24.0 | 18.5 | 14.53 | 11.72 |
| Thickness | 2 | 1.87 | 1.72 | 1.58 | 1.26 | 0.93 | 0.76 |



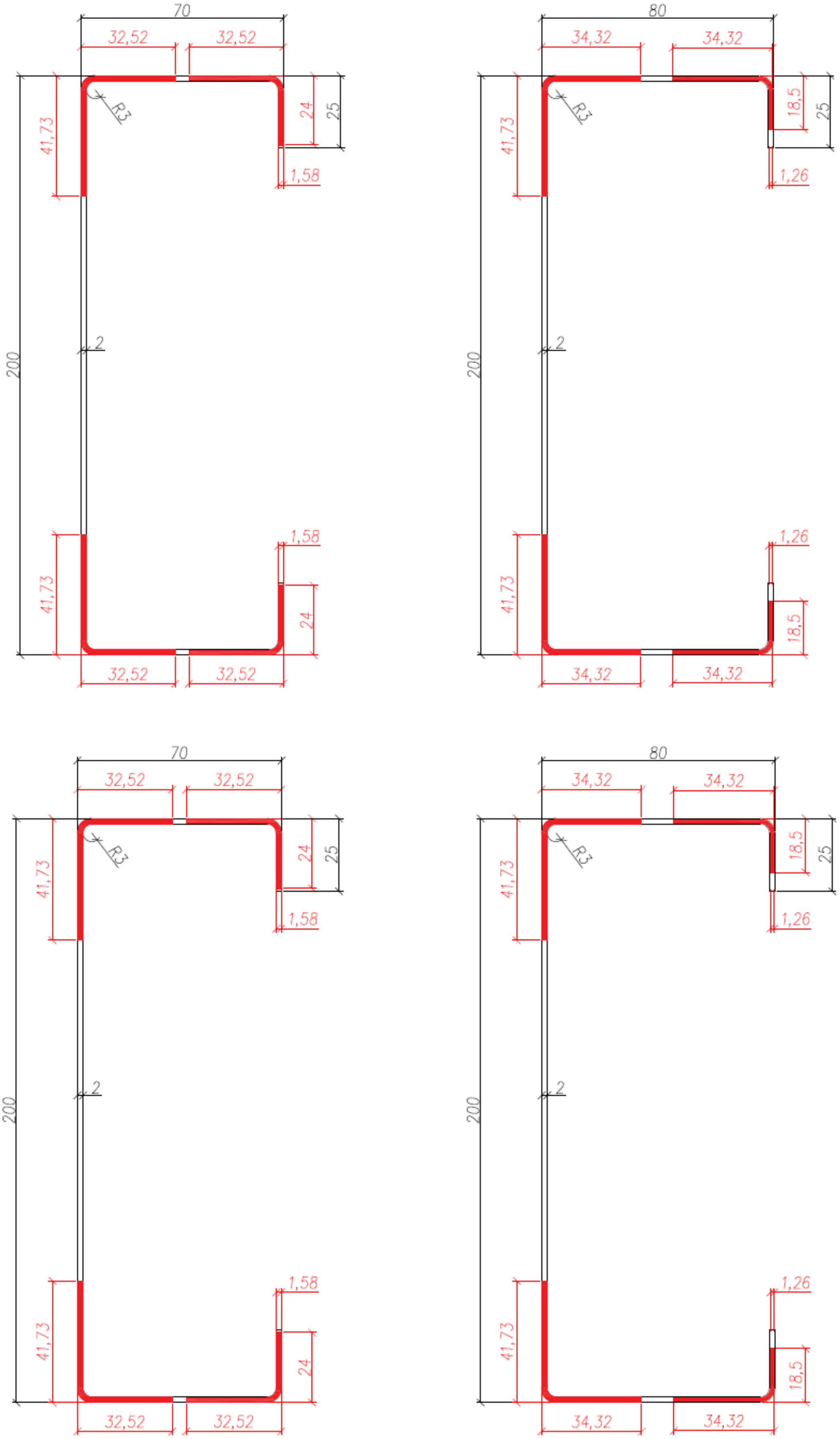



Figure 32. Changes of width. Results

As it is seen from the results only the effective width and effective thickness depend on the width of cross-section. The effective height and the length of the edge stiffener do not change at all.

1. The effective width grows a little bit with increasing of the width of cross-section. However, the difference between the width of cross-section and the effective width also increase. So that the common effective square of cross-section decreases and the efficiency of using this profile reduces.

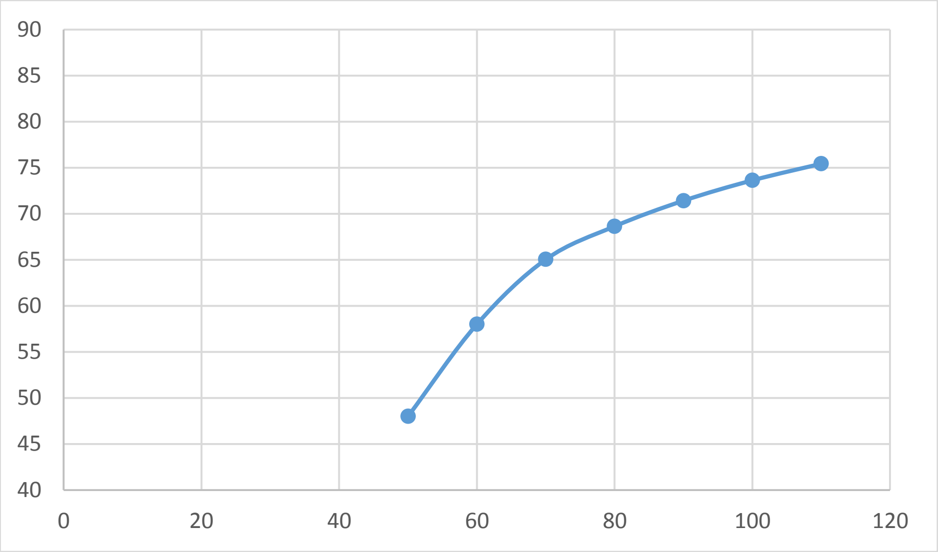



Figure 33. Dependence between height and effective height

2. Effective thickness decreases with increasing of height of cross-section.

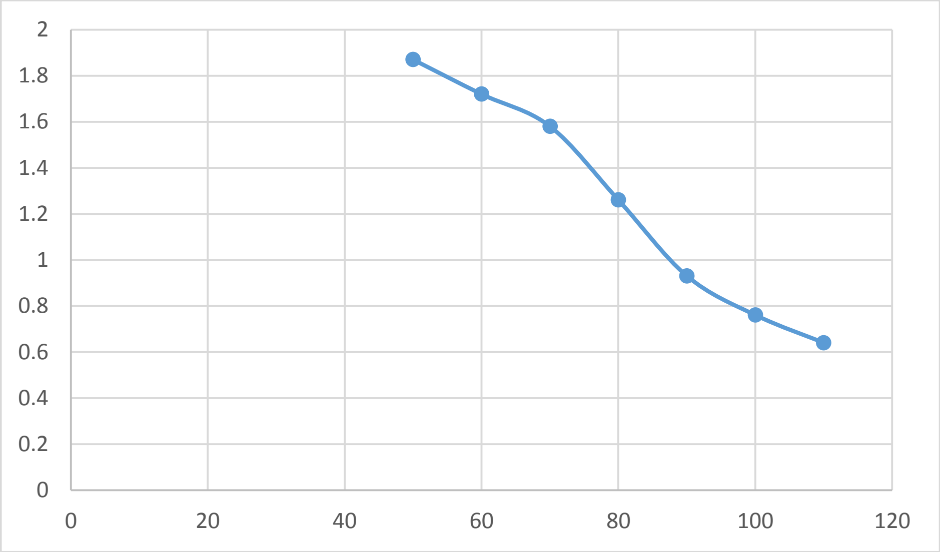


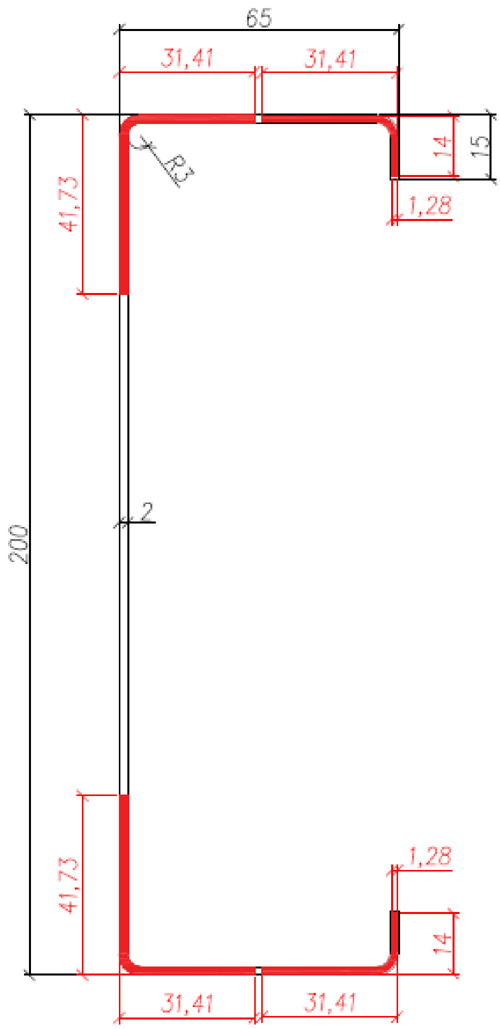

Figure 34. Dependence between height and effective height

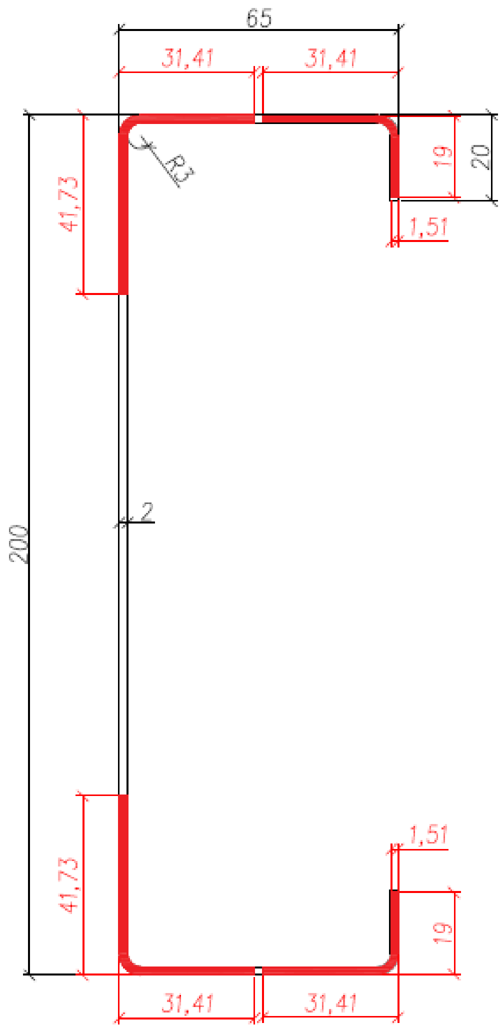
## 4.6 Changes of width length of edge stiffener

Then we research what it will be if we change the length of edge stiffener. We take a step equal to 5 mm.

Table 7. Changes of length of edge stiffener

| Feature | Original dimensions | Effective dimensions with different thickness | | | | |
|---|---|---|---|---|---|---|
| | | c=15 | c=20 | c=25 | c=30 | c=35 |
| Height | 200 | 83.46 | 83.46 | 83.46 | 83.46 | 83.46 |
| Width | 65 | 62.82 | 62.82 | 62.82 | 62.82 | 62.82 |
| Length of edge stiffener | ~ | 14.0 | 19.0 | 24.0 | 29.0 | 34.0 |
| Thickness | 2 | 1.28 | 1.51 | 1.65 | 1.74 | 1.82 |



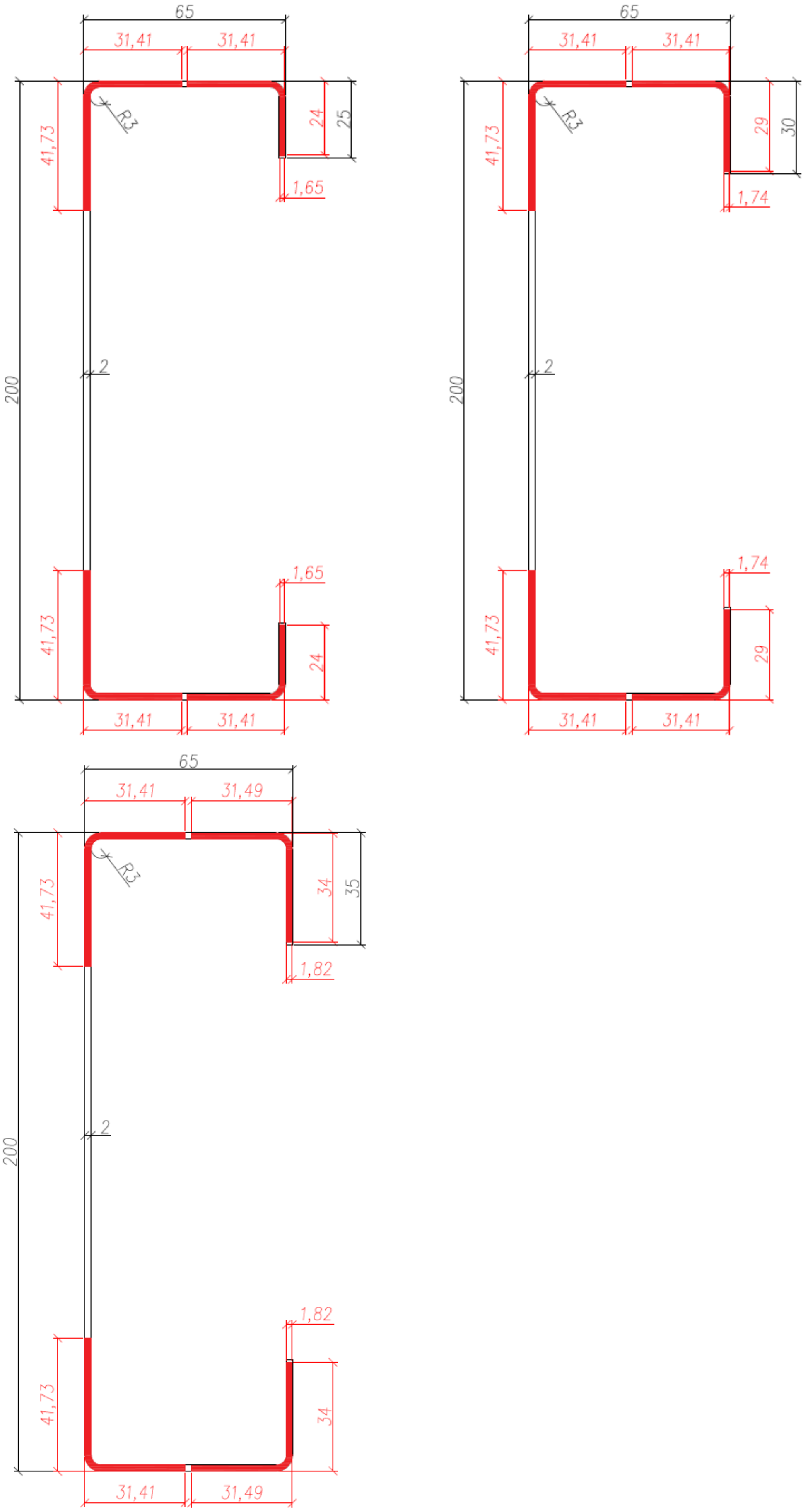



Figure 35. Changes of length of edge stiffener. Results

As it is seen from the results only the effective length of the edge stiffener and the effective thickness depend on edge length of cross-section. The effective height and the width do not change at all.

1. The effective length of the edge stiffener grows regularly with the same step as on what increase changing of edge length.

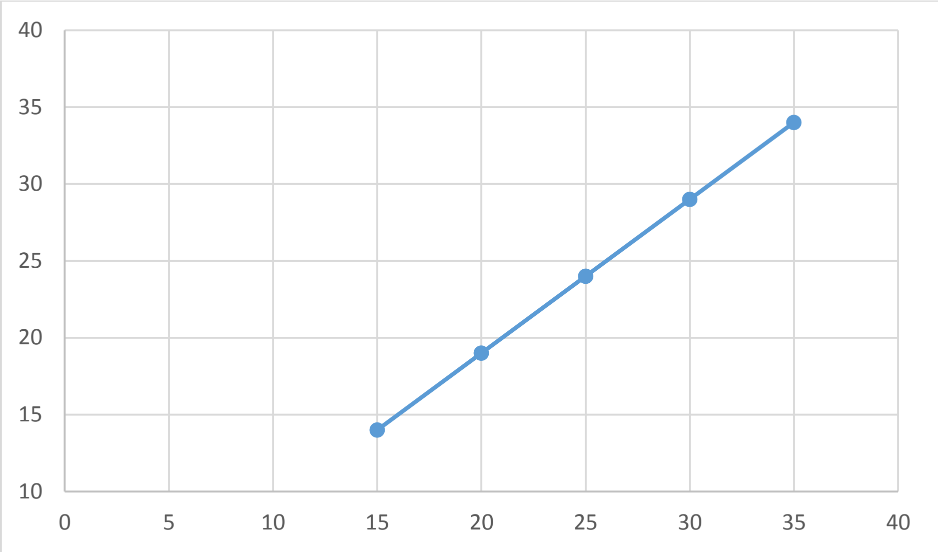



Figure 36. Dependence between length of edge stiffener and effective length

2. The effective thickness increases with growing up of edge length of cross-section. But in time increasing gets lower.

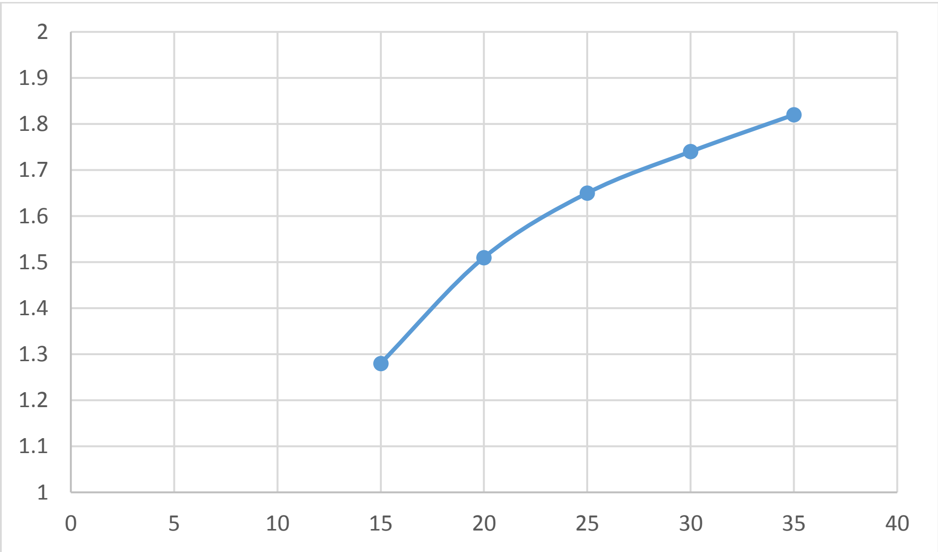



Figure 37. Dependence between length of edge stiffener and effective thickness

We consider how the changes of dimensions of cross-section effect on effective characteristics. This help us to understand what part of cross-section of the profile will be working under distortional buckling.

Eurocode 3 uses the effective width method to consider the reduction of resistant capacity of the member in a post-buckling status. The process of getting the reduced (effective) area of the cross-section is sometimes difficult to understand and has little correspondence with the reality of the problem. Therefore, it is easy to make mistakes and hard to detect them. Another limitation of Eurocode 3 concerns the necessity of having a cross-section with a series of limitations to its geometry. Any complex cross-section is difficult to analyze using EC3 and sometimes even impossible. This discourages the possibility of cross-section optimization. [4]

# 5 Conclusion

This thesis has described an investigation into the behaviour of cold-formed steel members subjected to distortional buckling. We have a little bit improved our knowledge and understanding of the distortional buckling behavior and analyzed how changes of dimensions of cross-section effect on the effective characteristics of the profile.

Local buckling is the most common failure mode for the majority of existing structures especially profiles with C cross-section. This is due to the fact that these members have slender webs, and as a result local buckling is a more common limit state than distortional buckling. Rack sections and other members with h/b ratios around 1, members with intermediate stiffeners in the web, and members with particularly small edge stiffeners, are all examples of members that are prone to distortional failures. For these members obvious checks on distortional buckling are required for successful design.

In conclusion, one can say that there is a need to do more investigations into the behaviour of distortional buckling. And also this phenomenon should be taken into account in Russian norms where there is no mention of them at all.

## Figures

## References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# Example 1. Calculation with t=1.5 mm

**Calculasion of thin-walled cold-form section on distortional buckling**

***Determination of section properties of the initial section:*** ORIGIN := 1

$h := 200mm$

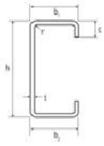
$b := 65mm$

$c := 25mm$

$t := 1.5mm$

$r := 3mm$

$\varphi := 90°$



Checking element size ratios to determine whether this profile can be applied to the calculation of EN 1993-1-3 (in accordance with par. 5.2 (1) and table 5.1 [5]):

$$\begin{vmatrix} \text{"Can calculate"} & \text{if } \left(\frac{b}{t} \le 60\right) \wedge \left(\frac{c}{t} \le 50\right) \wedge \left(\frac{h}{t} \le 500 \cdot \sin(\varphi)\right) \\ \text{"Can't calculat"} & \text{otherwise} \end{vmatrix} = \text{"Can calculate"}$$

In order to provide sufficient stiffness and to avoid primary buckling of the stiffness element itself, the sizes of stiffeners shoud be within the following ranges (in accordance with par. 5.2 (2) [5]):

$$\begin{vmatrix} \text{"Stiffness is provided"} & \text{if } 0.2 \le \frac{c}{b} \le 0.6 \\ \text{"Stiffener does't take into account in the calculation"} & \text{if } \frac{c}{b} < 0.2 \\ \text{"Stiffness is not provided"} & \text{otherwise} \end{vmatrix} = \text{"Stiffness is provided"}$$

Theoretical dimensions of flat areas to medlines of elements (in accordance with par. 1.5.3 (2) and 1.6.5 (1) [5]):

$hp := h - t = 198.5 \cdot mm$

$bp := b - t = 63.5 \cdot mm$

$cp := c - 0.5 \cdot t = 24.25 \cdot mm$

Theoretical widths of flat areas of cross-section (in accordance with par. 5.1 (1) and fig. 5.1 [5]):

$rm := r + 0.5 \cdot t = 3.75 \cdot mm$

$gr := rm \cdot (\tan(0.5 \cdot \varphi) - \sin(0.5 \cdot \varphi)) = 1.1 \cdot mm$

$bph := hp - 2 \cdot gr = 196.3 \cdot mm$

$bpb := bp - 2 \cdot gr = 61.3 \cdot mm$

$bpc := cp - gr = 23.15 \cdot mm$

The need to take into account the influence of rounded corners on cross-section resistance (in accordance with par. 5.1 (3) [5]):

$$\begin{vmatrix} \text{"Shouldn't take into account"} & \text{if } (r \le 5 \cdot t) \wedge (r \le 0.1 \cdot bph) \wedge (r \le 0.1 \cdot bpb) \wedge (r \le 0.1 \cdot bpc) \\ \text{"Should take into account"} & \text{otherwise} \end{vmatrix} = \text{"Should take into account"}$$

Notation: For cross-section stiffness properties the influence of rounded corners should always be taken into account.

The influence of rounded corners on section properties may be taken into account by reducing the propeliies calculated for an otherwise similar cross-section with sharp corners (in accordance with par. 5.1 (4) [5]):

$$\delta := \frac{4 \cdot 0.43 \cdot r}{bph + 2 \cdot bpb + bpc} \cdot \frac{\varphi}{1.571} = 0.015$$

Section properties with sharp corners:

$Agsh := t \cdot (hp + 2 \cdot bp + 2 \cdot cp) = 561 \cdot mm^2$

$$Igsh := \frac{t \cdot hp^3}{12} + 2 \cdot \left[\frac{bp \cdot t^3}{12} + \left(\frac{hp}{2}\right)^2 \cdot bp \cdot t\right] + 2 \cdot \left[\frac{t \cdot cp^3}{12} + \left(\frac{hp - cp}{2}\right)^2 \cdot cp \cdot t\right] = 341 \cdot cm^4$$

Section properties with the influence of rounded corners by approximate formulas (in accordance with par. 5.1 (4) [5]):

$Ag := Agsh \cdot (1 - \delta) = 552.54 \cdot mm^2$

$Ig := Igsh \cdot (1 - 2 \cdot \delta) = 330.72 \cdot cm^4$

***Determination of section properties of the effective section on central compression:***

**Step 1a. Finding the effective width of the shelves**

Setting the nominal value of the basic yield strength (in accordance with table 3.1a [5]):

$fyb := 350MPa$

Then the basic yield strength (in accordance with par. 3.2.2 (1) [5]):

$fy := fyb = 350 \cdot MPa$

Factor ε (in accordance with par. 4.4 (2) [7]):

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$\psi := 1$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$k\sigma := 4.0$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pb := \frac{bp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.91$$

Reduction factor for loss of stability of compressed plate plate with double-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{vmatrix} \frac{\lambda pb - 0.055 \cdot (3 + \psi)}{\lambda pb^2} & \text{if } \lambda pb > 0.673 \\ 1 & \text{otherwise} \end{vmatrix} = 0.83$$

Effective width of the shelf (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$beff := \rho \cdot bp = 52.9275 \cdot mm$

$be1 := 0.5 \cdot beff = 26.4637 \cdot mm$

$be2 := 0.5 \cdot beff = 26.4637 \cdot mm$

**Step 1b. Finding the effective width of the edge stiffener**

Stability loss factor for single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$k\sigma := \begin{vmatrix} 0.5 & \text{if } \frac{bpc}{bp} \le 0.35 \\ 0.5 + 0.83 \cdot \sqrt[3]{\left(\frac{bpc}{bp} - 0.35\right)^2} & \text{if } 0.35 < \frac{bpc}{bp} < 0.60 \end{vmatrix} = 0.55$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.94$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{vmatrix} \frac{\lambda pc - 0.188}{\lambda pb^2} & \text{if } \lambda pb > 0.748 \\ 1 & \text{otherwise} \end{vmatrix} = 0.91$$

Effective width of single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$ceff := \rho \cdot cp = 21.96 \cdot mm$

The effective cross-sectional area of the edge stiffener (in accordance with par. 5.5.3.2 (6) [5]):

$As := t \cdot (be2 + ceff) = 7.26 \times 10^{-5} m^2$

$As1 := As = 7.26 \times 10^{-5} m^2$

$As2 := As = 7.26 \times 10^{-5} m^2$

**Step 2. Determination of bearing capacity reduction rate due to distortional buckling**

Elastic modulus and Poisson's ratio in the elastic stage (in accordance with par. 3.2.6 [8]):

$E := 210000MPa$

$\nu := 0.3$

The distance from the intersection of the wall and shelves to the centers of gravity of the effective area of the edge stiffener:

$$b1 := bp - \frac{0.5 \cdot be2 \cdot t \cdot be2}{t \cdot (be2 + ceff)} = 0.06 \, m$$

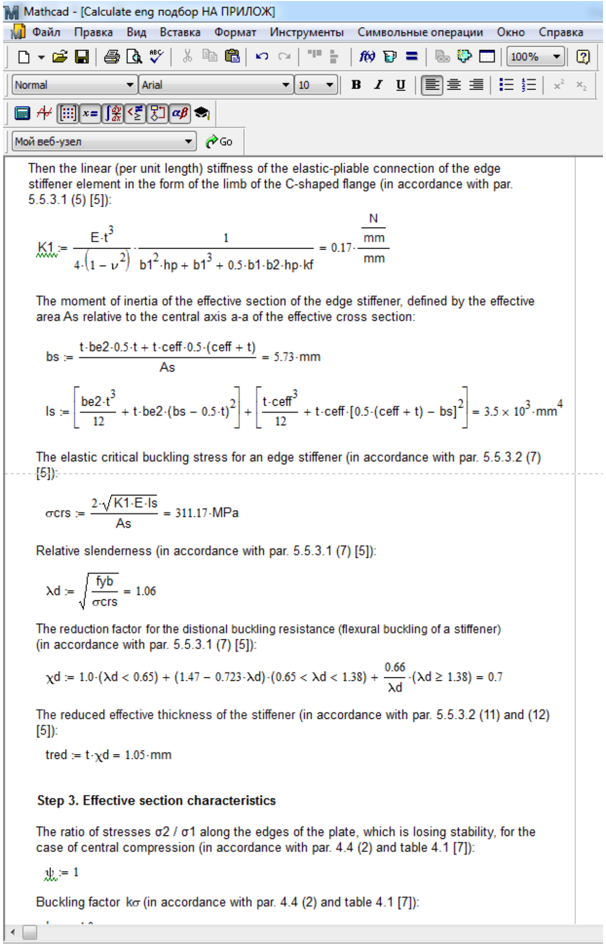
$b2 := b1 = 0.06 \, m$

Stress factor kf (in accordance with par. 5.5.3.1 (5) [5]):

$kf := 1$ - for a symmetric section in compression

Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par.

Example 1. Calculation with t=1.5 mm. Continuance



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Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

$$K1 := \frac{E \cdot t^3}{4 \cdot (1 - \nu^2)} \cdot \frac{1}{b1^2 \cdot hp + b1^3 + 0.5 \cdot b1 \cdot b2 \cdot hp \cdot kf} = 0.17 \cdot \frac{\frac{N}{mm}}{mm}$$

The moment of inertia of the effective section of the edge stiffener, defined by the effective area As relative to the central axis a-a of the effective cross section:

$$bs := \frac{t \cdot be2 \cdot 0.5 \cdot t + t \cdot ceff \cdot 0.5 \cdot (ceff + t)}{As} = 5.73 \cdot mm$$

$$Is := \left[\frac{be2 \cdot t^3}{12} + t \cdot be2 \cdot (bs - 0.5 \cdot t)^2\right] + \left[\frac{t \cdot ceff^3}{12} + t \cdot ceff \cdot [0.5 \cdot (ceff + t) - bs]^2\right] = 3.5 \times 10^3 \cdot mm^4$$

The elastic critical buckling stress for an edge stiffener (in accordance with par. 5.5.3.2 (7) [5]):

$$\sigma crs := \frac{2 \cdot \sqrt{K1 \cdot E \cdot Is}}{As} = 311.17 \cdot MPa$$

Relative slenderness (in accordance with par. 5.5.3.1 (7) [5]):

$$\lambda d := \sqrt{\frac{fyb}{\sigma crs}} = 1.06$$

The reduction factor for the distional buckling resistance (flexural buckling of a stiffener) (in accordance with par. 5.5.3.1 (7) [5]):

$$\chi d := 1.0 \cdot (\lambda d < 0.65) + (1.47 - 0.723 \cdot \lambda d) \cdot (0.65 < \lambda d < 1.38) + \frac{0.66}{\lambda d} \cdot (\lambda d \ge 1.38) = 0.7$$

The reduced effective thickness of the stiffener (in accordance with par. 5.5.3.2 (11) and (12) [5]):

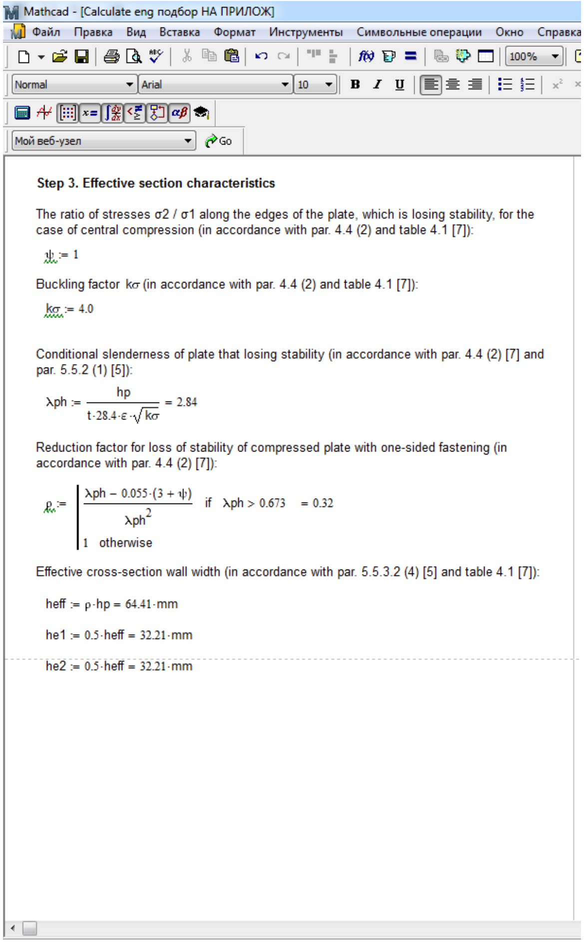
$$tred := t \cdot \chi d = 1.05 \cdot mm$$

**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):



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**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda ph := \frac{hp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 2.84$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda ph - 0.055 \cdot (3 + \psi)}{\lambda ph^2} & \text{if } \lambda ph > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.32$$

Effective cross-section wall width (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$heff := \rho \cdot hp = 64.41 \cdot mm$$

$$he1 := 0.5 \cdot heff = 32.21 \cdot mm$$

$$he2 := 0.5 \cdot heff = 32.21 \cdot mm$$

# Example 2. Calculation with h=100 mm

**Calculasion of thin-walled cold-form section on distortional buckling**

***Determination of section properties of the initial section:***

ORIGIN := 1

h := 100mm

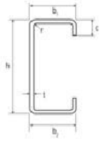
b := 65mm

c := 25mm

t := 2mm

r := 3mm

φ := 90°



Checking element size ratios to determine whether this profile can be applied to the calculation of EN 1993-1-3 (in accordance with par. 5.2 (1) and table 5.1 [5]):

$$\begin{cases} \text{"Can calculate"} & \text{if } \left(\frac{b}{t} \le 60\right) \wedge \left(\frac{c}{t} \le 50\right) \wedge \left(\frac{h}{t} \le 500 \cdot \sin(\varphi)\right) \\ \text{"Can't calculat"} & \text{otherwise} \end{cases} = \text{"Can calculate"}$$

In order to provide sufficient stiffness and to avoid primary buckling of the stiffness element itself, the sizes of stiffeners shoud be within the following ranges (in accordance with par. 5.2 (2) [5]):

$$\begin{cases} \text{"Stiffness is provided"} & \text{if } 0.2 \le \frac{c}{b} \le 0.6 \\ \text{"Stiffener does't take into account in the calculation"} & \text{if } \frac{c}{b} < 0.2 \\ \text{"Stiffness is not provided"} & \text{otherwise} \end{cases} = \text{"Stiffness is provided"}$$

Theoretical dimensions of flat areas to medlines of elements (in accordance with par. 1.5.3 (2) and 1.6.5 (1) [5]):

$hp := h - t = 98 \cdot mm$

$bp := b - t = 63 \cdot mm$

$cp := c - 0.5 \cdot t = 24 \cdot mm$

Theoretical widths of flat areas of cross-section
(in accordance with par. 5.1 (1) and fig. 5.1 [5]):

$rm := r + 0.5 \cdot t = 4 \cdot mm$

$gr := rm \cdot (\tan(0.5 \cdot \varphi) - \sin(0.5 \cdot \varphi)) = 1.17 \cdot mm$

$bph := hp - 2 \cdot gr = 95.66 \cdot mm$

$bpb := bp - 2 \cdot gr = 60.66 \cdot mm$

$bpb := bp - 2 \cdot gr = 60.66 \cdot mm$

$bpc := cp - gr = 22.83 \cdot mm$

The need to take into account the influence of rounded corners on cross-section resistance (in accordance with par. 5.1 (3) [5]):

$$\begin{cases} \text{"Shouldn't take into account"} & \text{if } (r \le 5 \cdot t) \wedge (r \le 0.1 \cdot bph) \wedge (r \le 0.1 \cdot bpb) \wedge (r \le 0.1 \cdot bpc) \\ \text{"Should take into account"} & \text{otherwise} \end{cases} = \text{"Should take into account"}$$

Notation: For cross-section stiffness properties the influence of rounded corners should always be taken into account.

The influence of rounded corners on section properties may be taken into account by reducing the propeliies calculated for an otherwise similar cross-section with sharp corners (in accordance with par. 5.1 (4) [5]):

$$\delta := \frac{4 \cdot 0.43 \cdot r}{bph + 2 \cdot bpb + bpc} \cdot \frac{\varphi}{1.571} = 0.022$$

Section properties with sharp corners:

$Agsh := t \cdot (hp + 2 \cdot bp + 2 \cdot cp) = 544 \cdot mm^2$

$$Igsh := \frac{t \cdot hp^3}{12} + 2 \cdot \left[\frac{bp \cdot t^3}{12} + \left(\frac{hp}{2}\right)^2 \cdot bp \cdot t\right] + 2 \cdot \left[\frac{t \cdot cp^3}{12} + \left(\frac{hp - cp}{2}\right)^2 \cdot cp \cdot t\right] = 89.8 \cdot cm^4$$

Section properties with the influence of rounded corners by approximate formulas (in accordance with par. 5.1 (4) [5]):

$Ag := Agsh \cdot (1 - \delta) = 532.3 \cdot mm^2$

$Ig := Igsh \cdot (1 - 2 \cdot \delta) = 85.94 \cdot cm^4$

***Determination of section properties of the effective section on central compression:***

**Step 1a. Finding the effective width of the shelves**

Setting the nominal value of the basic yield strength (in accordance with table 3.1a [5]):

$fyb := 350MPa$

Then the basic yield strength (in accordance with par. 3.2.2 (1) [5]):

$fy := fyb = 350 \cdot MPa$

Factor ε (in accordance with par. 4.4 (2) [7]):

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$\psi := 1$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$k\sigma := 4.0$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pb := \frac{bp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.68$$

Reduction factor for loss of stability of compressed plate plate with double-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pb - 0.055 \cdot (3 + \psi)}{\lambda pb^2} & \text{if } \lambda pb > 0.673 \\ 1 & \text{otherwise} \end{cases} = 1$$

Effective width of the shelf (in
accordance with par. 5.5.3.2 (4) [5]
and table 4.1 [7]):

$beff := \rho \cdot bp = 62.8268 \cdot mm$

$be1 := 0.5 \cdot beff = 31.4134 \cdot mm$

$be2 := 0.5 \cdot beff = 31.4134 \cdot mm$

**Step 1b. Finding the effective width of the edge stiffener**

Stability loss factor for single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$k\sigma := \begin{cases} 0.5 & \text{if } \frac{bpc}{bp} \le 0.35 \\ 0.5 + 0.83 \cdot \sqrt[3]{\left(\frac{bpc}{bp} - 0.35\right)^2} & \text{if } 0.35 < \frac{bpc}{bp} < 0.60 \end{cases} = 0.54$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.7$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.7$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pc - 0.188}{\lambda pb^2} & \text{if } \lambda pb > 0.748 \\ 1 & \text{otherwise} \end{cases} = 1$$

Effective width of single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$ceff := \rho \cdot cp = 24 \cdot mm$

The effective cross-sectional area of the edge stiffener (in accordance with par. 5.5.3.2 (6) [5]):

$As := t \cdot (be2 + ceff) = 1.11 \times 10^{-4} m^2$

$As1 := As = 1.11 \times 10^{-4} m^2$

$As2 := As = 1.11 \times 10^{-4} m^2$

**Step 2. Determination of bearing capacity reduction rate due to distortional buckling**

Elastic modulus and Poisson's ratio in the elastic stage (in accordance with par. 3.2.6 [8]):

$E := 210000MPa$

$\nu := 0.3$

The distance from the intersection of the wall and shelves to the centers of gravity of the effective area of the edge stiffener:

$$b1 := bp - \frac{0.5 \cdot be2 \cdot t \cdot be2}{t \cdot (be2 + ceff)} = 0.05 m$$

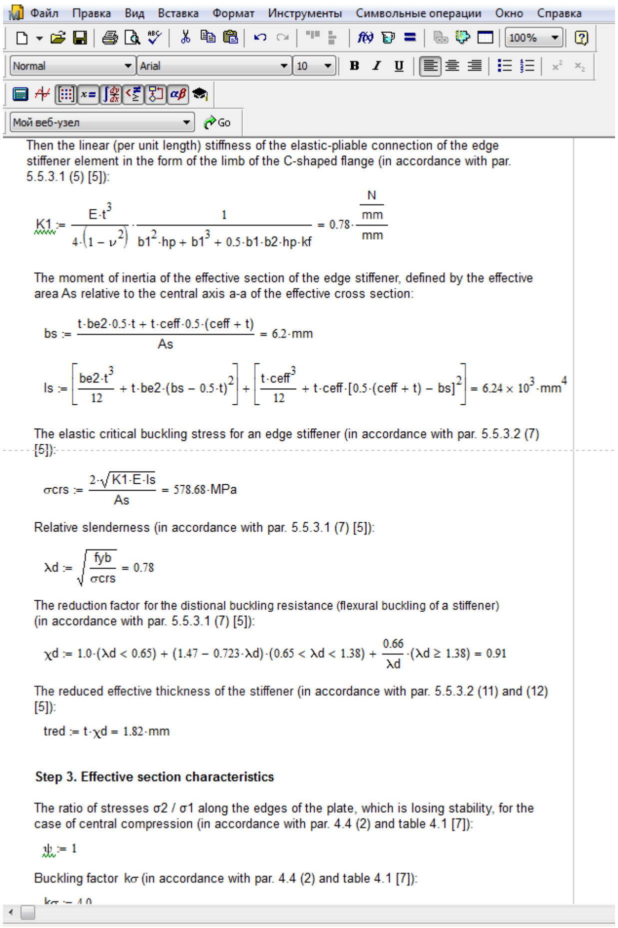
$b2 := b1 = 0.05 m$

Stress factor kf (in accordance with par. 5.5.3.1 (5) [5]):

$kf := 1$ - for a symmetric section in compression

Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

# Example 2. Calculation with h=100 mm. Continuance



Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

$$K1 := \frac{E \cdot t^3}{4 \cdot (1 - \nu^2)} \cdot \frac{1}{b1^2 \cdot hp + b1^3 + 0.5 \cdot b1 \cdot b2 \cdot hp \cdot kf} = 0.78 \cdot \frac{\frac{N}{mm}}{mm}$$

The moment of inertia of the effective section of the edge stiffener, defined by the effective area As relative to the central axis a-a of the effective cross section:

$$bs := \frac{t \cdot be2 \cdot 0.5 \cdot t + t \cdot ceff \cdot 0.5 \cdot (ceff + t)}{As} = 6.2 \cdot mm$$

$$Is := \left[\frac{be2 \cdot t^3}{12} + t \cdot be2 \cdot (bs - 0.5 \cdot t)^2\right] + \left[\frac{t \cdot ceff^3}{12} + t \cdot ceff \cdot [0.5 \cdot (ceff + t) - bs]^2\right] = 6.24 \times 10^3 \cdot mm^4$$

The elastic critical buckling stress for an edge stiffener (in accordance with par. 5.5.3.2 (7) [5]):

$$\sigma crs := \frac{2 \cdot \sqrt{K1 \cdot E \cdot Is}}{As} = 578.68 \cdot MPa$$

Relative slenderness (in accordance with par. 5.5.3.1 (7) [5]):

$$\lambda d := \sqrt{\frac{fyb}{\sigma crs}} = 0.78$$

The reduction factor for the distional buckling resistance (flexural buckling of a stiffener) (in accordance with par. 5.5.3.1 (7) [5]):

$$\chi d := 1.0 \cdot (\lambda d < 0.65) + (1.47 - 0.723 \cdot \lambda d) \cdot (0.65 < \lambda d < 1.38) + \frac{0.66}{\lambda d} \cdot (\lambda d \ge 1.38) = 0.91$$

The reduced effective thickness of the stiffener (in accordance with par. 5.5.3.2 (11) and (12) [5]):

$$tred := t \cdot \chi d = 1.82 \cdot mm$$

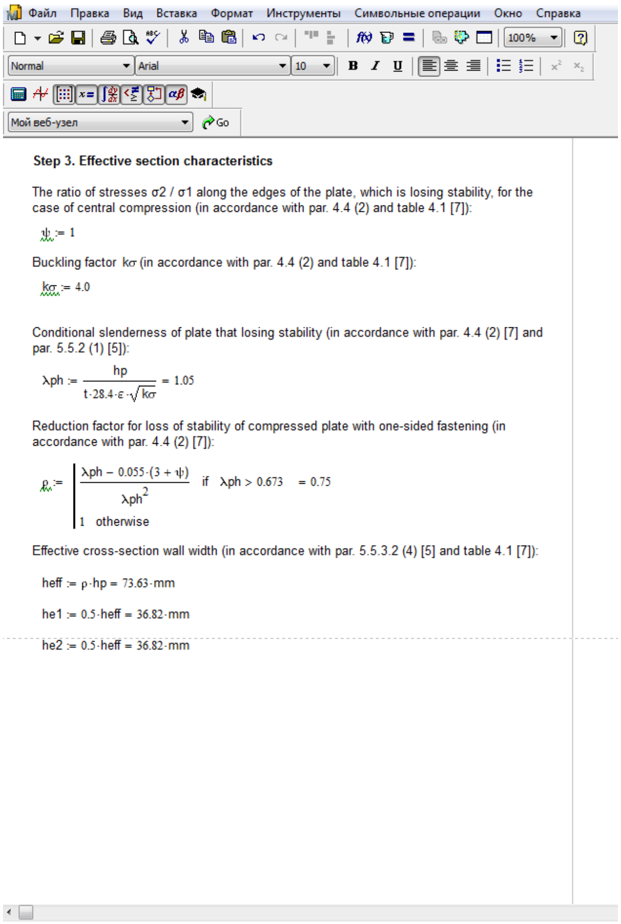
**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$



**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda ph := \frac{hp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 1.05$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda ph - 0.055 \cdot (3 + \psi)}{\lambda ph^2} & \text{if } \lambda ph > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.75$$

Effective cross-section wall width (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$heff := \rho \cdot hp = 73.63 \cdot mm$$

$$he1 := 0.5 \cdot heff = 36.82 \cdot mm$$

$$he2 := 0.5 \cdot heff = 36.82 \cdot mm$$

# Example 3. Calculation with b=80 mm

**Calculasion of thin-walled cold-form section on distortional buckling**

***Determination of section properties of the initial section:*** $ORIGIN := 1$

$h := 200mm$

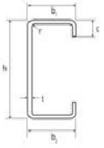
$b := 80mm$

$c := 25mm$

$t := 2mm$

$r := 3mm$

$\varphi := 90°$




Checking element size ratios to determine whether this profile can be applied to the calculation of EN 1993-1-3 (in accordance with par. 5.2 (1) and table 5.1 [5]):

$$\begin{cases} \text{"Can calculate"} & \text{if } \left(\frac{b}{t} \le 60\right) \wedge \left(\frac{c}{t} \le 50\right) \wedge \left(\frac{h}{t} \le 500 \cdot \sin(\varphi)\right) \\ \text{"Can't calculat"} & \text{otherwise} \end{cases} = \text{"Can calculate"}$$

In order to provide sufficient stiffness and to avoid primary buckling of the stiffness element itself, the sizes of stiffeners shoud be within the following ranges (in accordance with par. 5.2 (2) [5]):

$$\begin{cases} \text{"Stiffness is provided"} & \text{if } 0.2 \le \frac{c}{b} \le 0.6 \\ \text{"Stiffener does't take into account in the calculation"} & \text{if } \frac{c}{b} < 0.2 \\ \text{"Stiffness is not provided"} & \text{otherwise} \end{cases} = \text{"Stiffness is provided"}$$

Theoretical dimensions of flat areas to medlines of elements (in accordance with par. 1.5.3 (2) and 1.6.5 (1) [5]):

$hp := h - t = 198 \cdot mm$

$bp := b - t = 78 \cdot mm$

$cp := c - 0.5 \cdot t = 24 \cdot mm$

Theoretical widths of flat areas of cross-section
(in accordance with par. 5.1 (1) and fig. 5.1 [5]):

$rm := r + 0.5 \cdot t = 4 \cdot mm$

$gr := rm \cdot (\tan(0.5 \cdot \varphi) - \sin(0.5 \cdot \varphi)) = 1.17 \cdot mm$

$bph := hp - 2 \cdot gr = 195.66 \cdot mm$

$bpb := bp - 2 \cdot gr = 75.66 \cdot mm$

$bpc := cp - gr = 22.83 \cdot mm$

The need to take into account the influence of rounded corners on cross-section resistance (in accordance with par. 5.1 (3) [5]):

$$\begin{cases} \text{"Shouldn't take into account"} & \text{if } (r \le 5 \cdot t) \wedge (r \le 0.1 \cdot bph) \wedge (r \le 0.1 \cdot bpb) \wedge (r \le 0.1 \cdot bpc) \\ \text{"Should take into account"} & \text{otherwise} \end{cases} = \text{"Should take into account"}$$

Notation: For cross-section stiffness properties the influence of rounded corners should always be taken into account.

The influence of rounded corners on section properties may be taken into account by reducing the propeliies calculated for an otherwise similar cross-section with sharp corners (in accordance with par. 5.1 (4) [5]):

$$\delta := \frac{4 \cdot 0.43 \cdot r}{bph + 2 \cdot bpb + bpc} \cdot \frac{\varphi}{1.571} = 0.014$$

Section properties with sharp corners:

$Agsh := t \cdot (hp + 2 \cdot bp + 2 \cdot cp) = 804 \cdot mm^2$

$$Igsh := \frac{t \cdot hp^3}{12} + 2 \cdot \left[\frac{bp \cdot t^3}{12} + \left(\frac{hp}{2}\right)^2 \cdot bp \cdot t\right] + 2 \cdot \left[\frac{t \cdot cp^3}{12} + \left(\frac{hp - cp}{2}\right)^2 \cdot cp \cdot t\right] = 508.3 \cdot cm^4$$

Section properties with the influence of rounded corners by approximate formulas (in accordance with par. 5.1 (4) [5]):

$Ag := Agsh \cdot (1 - \delta) = 792.78 \cdot mm^2$

$Ig := Igsh \cdot (1 - 2 \cdot \delta) = 494.11 \cdot cm^4$

***Determination of section properties of the effective section on central compression:***

**Step 1a. Finding the effective width of the shelves**

Setting the nominal value of the basic yield strength (in accordance with table 3.1a [5]):

$fyb := 350MPa$

Then the basic yield strength (in accordance with par. 3.2.2 (1) [5]):

$fy := fyb = 350 \cdot MPa$

Factor ε (in accordance with par. 4.4 (2) [7]):

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$\psi := 1$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$k\sigma := 4.0$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pb := \frac{bp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.84$$

Reduction factor for loss of stability of compressed plate plate with double-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pb - 0.055 \cdot (3 + \psi)}{\lambda pb^2} & \text{if } \lambda pb > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.88$$

Effective width of the shelf (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$beff := \rho \cdot bp = 68.6456 \cdot mm$

$be1 := 0.5 \cdot beff = 34.3228 \cdot mm$

$be2 := 0.5 \cdot beff = 34.3228 \cdot mm$

**Step 1b. Finding the effective width of the edge stiffener**

Stability loss factor for single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$k\sigma := \begin{cases} 0.5 & \text{if } \frac{bpc}{bp} \le 0.35 \\ 0.5 + 0.83 \cdot \sqrt[3]{\left(\frac{bpc}{bp} - 0.35\right)^2} & \text{if } 0.35 < \frac{bpc}{bp} < 0.60 \end{cases} = 0.5$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.73$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pc - 0.188}{\lambda pb^2} & \text{if } \lambda pb > 0.748 \\ 1 & \text{otherwise} \end{cases} = 0.77$$

Effective width of single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$ceff := \rho \cdot cp = 18.5 \cdot mm$

The effective cross-sectional area of the edge stiffener (in accordance with par. 5.5.3.2 (6) [5]):

$As := t \cdot (be2 + ceff) = 1.06 \times 10^{-4} m^2$

$As1 := As = 1.06 \times 10^{-4} m^2$

$As2 := As = 1.06 \times 10^{-4} m^2$

**Step 2. Determination of bearing capacity reduction rate due to distortional buckling**

Elastic modulus and Poisson's ratio in the elastic stage (in accordance with par. 3.2.6 [8]):

$E := 210000MPa$

$\nu := 0.3$

The distance from the intersection of the wall and shelves to the centers of gravity of the effective area of the edge stiffener:

$$b1 := bp - \frac{0.5 \cdot be2 \cdot t \cdot be2}{t \cdot (be2 + ceff)} = 0.07\,m$$

$b2 := b1 = 0.07\,m$

Stress factor kf (in accordance with par. 5.5.3.1 (5) [5]):

$kf := 1$ - for a symmetric section in compression

Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

# Example 3. Calculation with b=80 mm. Continuance

$$K1 := \frac{E\cdot t^3}{4\cdot(1-\nu^2)}\cdot\frac{1}{b1^2\cdot hp + b1^3 + 0.5\cdot b1\cdot b2\cdot hp\cdot kf} = 0.28\cdot\frac{\frac{N}{mm}}{mm}$$

The moment of inertia of the effective section of the edge stiffener, defined by the effective area As relative to the central axis a-a of the effective cross section:

$$bs := \frac{t\cdot be2\cdot 0.5\cdot t + t\cdot ceff\cdot 0.5\cdot(ceff+t)}{As} = 4.24\cdot mm$$

$$Is := \left[\frac{be2\cdot t^3}{12} + t\cdot be2\cdot(bs-0.5\cdot t)^2\right] + \left[\frac{t\cdot ceff^3}{12} + t\cdot ceff\cdot[0.5\cdot(ceff+t)-bs]^2\right] = 3.14\times 10^3\cdot mm^4$$

The elastic critical buckling stress for an edge stiffener (in accordance with par. 5.5.3.2 (7) [5]):

$$\sigma crs := \frac{2\cdot\sqrt{K1\cdot E\cdot Is}}{As} = 258.81\cdot MPa$$

Relative slenderness (in accordance with par. 5.5.3.1 (7) [5]):

$$\lambda d := \sqrt{\frac{fyb}{\sigma crs}} = 1.16$$

The reduction factor for the distional buckling resistance (flexural buckling of a stiffener) (in accordance with par. 5.5.3.1 (7) [5]):

$$\chi d := 1.0\cdot(\lambda d < 0.65) + (1.47 - 0.723\cdot\lambda d)\cdot(0.65 < \lambda d < 1.38) + \frac{0.66}{\lambda d}\cdot(\lambda d \ge 1.38) = 0.63$$

The reduced effective thickness of the stiffener (in accordance with par. 5.5.3.2 (11) and (12) [5]):

$$tred := t\cdot\chi d = 1.26\cdot mm$$

**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and

**Step 3. Effective section characteristics**

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda ph := \frac{hp}{t\cdot 28.4\cdot\varepsilon\cdot\sqrt{k\sigma}} = 2.13$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases}\frac{\lambda ph - 0.055\cdot(3+\psi)}{\lambda ph^2} & \text{if } \lambda ph > 0.673 \\ 1 & \text{otherwise}\end{cases} = 0.42$$

Effective cross-section wall width (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$heff := \rho\cdot hp = 83.46\cdot mm$$

$$he1 := 0.5\cdot heff = 41.73\cdot mm$$

$$he2 := 0.5\cdot heff = 41.73\cdot mm$$

# Example 4. Calculation with c=15 mm

**Calculasion of thin-walled cold-form section on distortional buckling**

***Determination of section properties of the initial section:*** ORIGIN := 1

h := 200mm

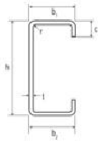
b := 65mm

c := 15mm

t := 2mm

r := 3mm

φ := 90°



Checking element size ratios to determine whether this profile can be applied to the calculation of EN 1993-1-3 (in accordance with par. 5.2 (1) and table 5.1 [5]):

$$\begin{cases} \text{"Can calculate"} & \text{if } \left(\frac{b}{t} \le 60\right) \wedge \left(\frac{c}{t} \le 50\right) \wedge \left(\frac{h}{t} \le 500 \cdot \sin(\varphi)\right) \\ \text{"Can't calculat"} & \text{otherwise} \end{cases} = \text{"Can calculate"}$$

In order to provide sufficient stiffness and to avoid primary buckling of the stiffness element itself, the sizes of stiffeners shoud be within the following ranges (in accordance with par. 5.2 (2) [5]):

$$\begin{cases} \text{"Stiffness is provided"} & \text{if } 0.2 \le \frac{c}{b} \le 0.6 \\ \text{"Stiffener does't take into account in the calculation"} & \text{if } \frac{c}{b} < 0.2 \\ \text{"Stiffness is not provided"} & \text{otherwise} \end{cases} = \text{"Stiffness is provided"}$$

Theoretical dimensions of flat areas to medlines of elements (in accordance with par. 1.5.3 (2) and 1.6.5 (1) [5]):

$hp := h - t = 198 \cdot mm$

$bp := b - t = 63 \cdot mm$

$cp := c - 0.5 \cdot t = 14 \cdot mm$

Theoretical widths of flat areas of cross-section (in accordance with par. 5.1 (1) and fig. 5.1 [5]):

$rm := r + 0.5 \cdot t = 4 \cdot mm$

$gr := rm \cdot (\tan(0.5 \cdot \varphi) - \sin(0.5 \cdot \varphi)) = 1.17 \cdot mm$

$bph := hp - 2 \cdot gr = 195.66 \cdot mm$

$bpb := bp - 2 \cdot gr = 60.66 \cdot mm$

$bpc := cp - gr = 12.83 \cdot mm$

The need to take into account the influence of rounded corners on cross-section resistance (in accordance with par. 5.1 (3) [5]):

$$\begin{cases} \text{"Shouldn't take into account"} & \text{if } (r \le 5 \cdot t) \wedge (r \le 0.1 \cdot bph) \wedge (r \le 0.1 \cdot bpb) \wedge (r \le 0.1 \cdot bpc) \\ \text{"Should take into account"} & \text{otherwise} \end{cases} = \text{"Should take into account"}$$

Notation: For cross-section stiffness properties the influence of rounded corners should always be taken into account.

The influence of rounded corners on section properties may be taken into account by reducing the propeliies calculated for an otherwise similar cross-section with sharp corners (in accordance with par. 5.1 (4) [5]):

$$\delta := \frac{4 \cdot 0.43 \cdot r}{bph + 2 \cdot bpb + bpc} \cdot \frac{\varphi}{1.571} = 0.016$$

Section properties with sharp corners:

$Agsh := t \cdot (hp + 2 \cdot bp + 2 \cdot cp) = 704 \cdot mm^2$

$$Igsh := \frac{t \cdot hp^3}{12} + 2 \cdot \left[\frac{bp \cdot t^3}{12} + \left(\frac{hp}{2}\right)^2 \cdot bp \cdot t\right] + 2 \cdot \left[\frac{t \cdot cp^3}{12} + \left(\frac{hp - cp}{2}\right)^2 \cdot cp \cdot t\right] = 423.86 \cdot cm^4$$

Section properties with the influence of rounded corners by approximate formulas (in accordance with par. 5.1 (4) [5]):

$Ag := Agsh \cdot (1 - \delta) = 692.99 \cdot mm^2$

$Ig := Igsh \cdot (1 - 2 \cdot \delta) = 410.6 \cdot cm^4$

***Determination of section properties of the effective section on central compression:***

**Step 1a. Finding the effective width of the shelves**

Setting the nominal value of the basic yield strength (in accordance with table 3.1a [5]):

$fyb := 350MPa$

Then the basic yield strength (in accordance with par. 3.2.2 (1) [5]):

$fy := fyb = 350 \cdot MPa$

Factor ε (in accordance with par. 4.4 (2) [7]):

$$\varepsilon := \sqrt{\frac{235MPa}{fy}} = 0.82$$

The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$\psi := 1$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$k\sigma := 4.0$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pb := \frac{bp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.68$$

Reduction factor for loss of stability of compressed plate plate with double-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pb - 0.055 \cdot (3 + \psi)}{\lambda pb^2} & \text{if } \lambda pb > 0.673 \\ 1 & \text{otherwise} \end{cases} = 1$$

Effective width of the shelf (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$beff := \rho \cdot bp = 62.8268 \cdot mm$

$be1 := 0.5 \cdot beff = 31.4134 \cdot mm$

$be2 := 0.5 \cdot beff = 31.4134 \cdot mm$

**Step 1b. Finding the effective width of the edge stiffener**

Stability loss factor for single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$$k\sigma := \begin{cases} 0.5 & \text{if } \frac{bpc}{bp} \le 0.35 \\ 0.5 + 0.83 \cdot \sqrt[3]{\left(\frac{bpc}{bp} - 0.35\right)^2} & \text{if } 0.35 < \frac{bpc}{bp} < 0.60 \end{cases} = 0.5$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda pc := \frac{cp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 0.43$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda pc - 0.188}{\lambda pb^2} & \text{if } \lambda pb > 0.748 \\ 1 & \text{otherwise} \end{cases} = 1$$

Effective width of single edge stiffener (in accordance with par. 5.5.3.2 (5a) [5]):

$ceff := \rho \cdot cp = 14 \cdot mm$

The effective cross-sectional area of the edge stiffener (in accordance with par. 5.5.3.2 (6) [5]):

$As := t \cdot (be2 + ceff) = 9.08 \times 10^{-5} m^2$

$As1 := As = 9.08 \times 10^{-5} m^2$

$As2 := As = 9.08 \times 10^{-5} m^2$

**Step 2. Determination of bearing capacity reduction rate due to distortional buckling**

Elastic modulus and Poisson's ratio in the elastic stage (in accordance with par. 3.2.6 [8]):

$E := 210000MPa$

$\nu := 0.3$

The distance from the intersection of the wall and shelves to the centers of gravity of the effective area of the edge stiffener:

$$b1 := bp - \frac{0.5 \cdot be2 \cdot t \cdot be2}{t \cdot (be2 + ceff)} = 0.05 m$$

$b2 := b1 = 0.05 m$

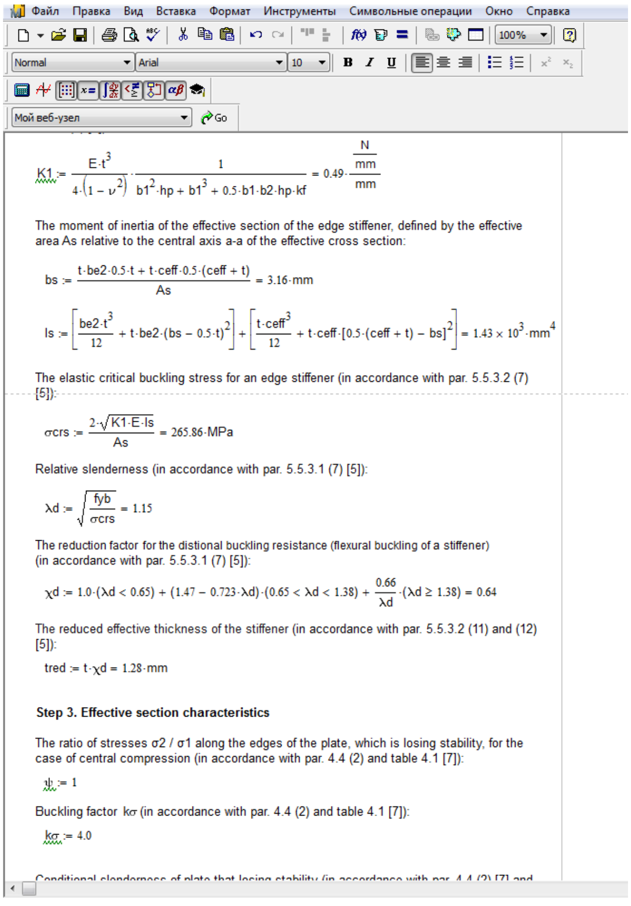
Stress factor kf (in accordance with par. 5.5.3.1 (5) [5]):

$kf := 1$ - for a symmetric section in compression

Then the linear (per unit length) stiffness of the elastic-pliable connection of the edge stiffener element in the form of the limb of the C-shaped flange (in accordance with par. 5.5.3.1 (5) [5]):

$$K1 := \frac{E \cdot t^3}{4 \cdot (1 - \nu^2)} \cdot \frac{1}{b1^2 \cdot hp + b1^3 + 0.5 \cdot b1 \cdot b2 \cdot hp \cdot kf} = 0.49 \cdot \frac{\frac{N}{mm}}{mm}$$

# Example 4. Calculation with c=15 mm. Continuance



$$K1 := \frac{E \cdot t^3}{4 \cdot (1 - \nu^2)} \cdot \frac{1}{b1^2 \cdot hp + b1^3 + 0.5 \cdot b1 \cdot b2 \cdot hp \cdot kf} = 0.49 \cdot \frac{\frac{N}{mm}}{mm}$$

The moment of inertia of the effective section of the edge stiffener, defined by the effective area As relative to the central axis a-a of the effective cross section:

$$bs := \frac{t \cdot be2 \cdot 0.5 \cdot t + t \cdot ceff \cdot 0.5 \cdot (ceff + t)}{As} = 3.16 \cdot mm$$

$$Is := \left[\frac{be2 \cdot t^3}{12} + t \cdot be2 \cdot (bs - 0.5 \cdot t)^2\right] + \left[\frac{t \cdot ceff^3}{12} + t \cdot ceff \cdot [0.5 \cdot (ceff + t) - bs]^2\right] = 1.43 \times 10^3 \cdot mm^4$$

The elastic critical buckling stress for an edge stiffener (in accordance with par. 5.5.3.2 (7) [5]):

$$\sigma crs := \frac{2 \cdot \sqrt{K1 \cdot E \cdot Is}}{As} = 265.86 \cdot MPa$$

Relative slenderness (in accordance with par. 5.5.3.1 (7) [5]):

$$\lambda d := \sqrt{\frac{fyb}{\sigma crs}} = 1.15$$

The reduction factor for the distional buckling resistance (flexural buckling of a stiffener) (in accordance with par. 5.5.3.1 (7) [5]):

$$\chi d := 1.0 \cdot (\lambda d < 0.65) + (1.47 - 0.723 \cdot \lambda d) \cdot (0.65 < \lambda d < 1.38) + \frac{0.66}{\lambda d} \cdot (\lambda d \ge 1.38) = 0.64$$

The reduced effective thickness of the stiffener (in accordance with par. 5.5.3.2 (11) and (12) [5]):

$$tred := t \cdot \chi d = 1.28 \cdot mm$$

**Step 3. Effective section characteristics**

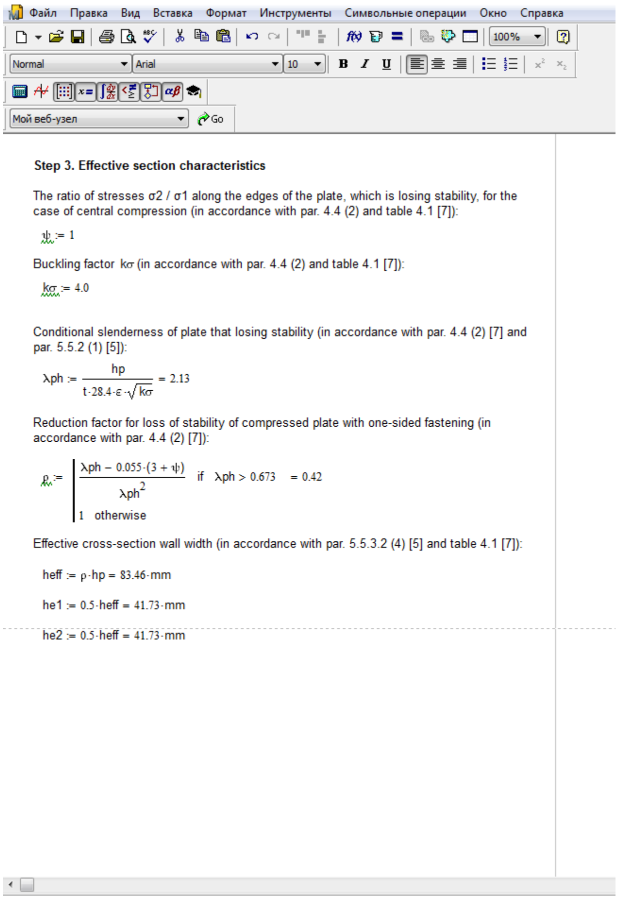
The ratio of stresses σ2 / σ1 along the edges of the plate, which is losing stability, for the case of central compression (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$\psi := 1$$

Buckling factor kσ (in accordance with par. 4.4 (2) and table 4.1 [7]):

$$k\sigma := 4.0$$

Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and



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Conditional slenderness of plate that losing stability (in accordance with par. 4.4 (2) [7] and par. 5.5.2 (1) [5]):

$$\lambda ph := \frac{hp}{t \cdot 28.4 \cdot \varepsilon \cdot \sqrt{k\sigma}} = 2.13$$

Reduction factor for loss of stability of compressed plate with one-sided fastening (in accordance with par. 4.4 (2) [7]):

$$\rho := \begin{cases} \frac{\lambda ph - 0.055 \cdot (3 + \psi)}{\lambda ph^2} & \text{if } \lambda ph > 0.673 \\ 1 & \text{otherwise} \end{cases} = 0.42$$

Effective cross-section wall width (in accordance with par. 5.5.3.2 (4) [5] and table 4.1 [7]):

$$heff := \rho \cdot hp = 83.46 \cdot mm$$

$$he1 := 0.5 \cdot heff = 41.73 \cdot mm$$

$$he2 := 0.5 \cdot heff = 41.73 \cdot mm$$