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Multi-objective inventory optimization tool development

Case EKE-Electronics Oy

Implementing MOPSO and TOPSIS algorithms

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Abstract

EKE-Electronics Oy has been significantly growing over the past years and, therefore, it required an improved inventory management system. The main goal of this system was to shorten the production lead times, and thus, enhance the customer service. To support this mission, a change from a Make to Order to a Make to Stock production system was proposed.

Furthermore, the optimization possibilities of the inventory management system proposed were analyzed. The developed system incorporated the company's characteristics, management requirements, and room for the future development of the company.

The objectives included the selection and setup of the inventory management system and a selection and development of the system's optimization methods. Therefore, an extensive literature review of existing inventory management and optimization systems was performed.

The objectives were met by implementing an Order Point, Order Quantity (s, Q) inventory management system and coding the MOPSO and TOPSIS algorithms in the Python programming language. The implementation stage included a comprehensive statistical data analysis of historic data for the past 2.5 years and an analysis of the backlogged orders for the following 1.5 years.

The main results consisted of creating a General Bill of Materials (G-BOM) for several selected systems and analyzing data for sixty modules from the portfolio of the company. The G-BOM included an array of stockout probabilities on a 0% to 100% scale, expected total costs of the listed service levels, and the required safety stocks for each module. The outcomes required knowledge of inventory management, coding, statistics, and mathematical optimization to achieve the desired results.

Keywords/tags (subjects). Inventory management, mathematical optimization, MOPSO, TOPSIS, case study, multiobjective optimization, Python, the (s, Q) model, mathematical modeling

Miscellaneous (Confidential information). ***Appendix 1 is confidential, and it has been removed from the public thesis. Grounds for secrecy: Act on the Openness of Government Activities 621/1999, Section 24, 17: business or professional secret. The period of secrecy is twenty-five years and it ends on 18.5.2045.***

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1 Introduction

How to keep customers satisfied with product availability at the lowest cost? This is the question that puzzles any inventory manager at some point. If there is too little inventory, then stockouts are unavoidable and the disappointed customer group is growing. The longer the customers need to wait for their order, the more chances there are that they will not order next time from the same supplier or that they will even cancel the existing orders. (Chopra & Meindl 2015, 1-2.)

On the other hand, high inventory stock leads to high invested capital, higher storing and inventory management costs, and increased risks in case of the goods becoming obsolete, especially in case of electronics. This leads to increased overall expenditure and, thus, to higher prices for customers, which leads again to unsatisfied clients. (ibid., 2-3.)

Many warehousing and purchasing managers, and often companies in general, rely on experience, intuition, and almost no calculations when setting up inventory policies and determining ordering sizes and turnover rates. While this behavior allows satisfying the overall needs of the customers, it often does not bring the most optimal results. Moreover, when competition switches from features to prices, the company with the best operational structure usually emerges victorious. (Senior & Swailes 2016, 143-145.)

In order to help companies optimize their operations, an array of mathematical and more general solutions has been developed. It ranges from whole philosophies of reducing excessive use of resources throughout the company to sophisticated and detailed simulation models for trying out new ideas. (Silver, Pyke & Thomas 2017, 46-47.)

In this study, several optimization methods found in literature were analyzed. Next, their practical applications were evaluated for the case company EKE-Electronics Oy.

1.1 Company description

EKE-Electronics Oy (later in the text - EKE) is a Finnish company based in Espoo that provides cutting-edge technologies for train automation. The main customers of the company are train manufacturers, operators, and train integrators. EKE focuses on developing customized solutions for train automation, communications, diagnostics, and safety.

Being established in 1986, EKE has acquired invaluable experience in manufacturing a range of train management and automation products, and it has installed tens of thousands of systems around the globe. Nowadays, more than 90% of EKE's projects are international in its portfolio with the Trainnet® systems being used in more than 30 countries.

The solutions provided to the customers range from supplying solitary modules to developing turnkey solutions, including design, manufacturing, software development, and testing. The systems developed by EKE are the backbone of the data management network of the train, and they are central to managing train functions.

Products developed by EKE are highly reliable as proven by several certificates. All solutions comply with the railway industry standard EN 50155, and EKE-Electronics is IRIS (International Railway Industry Standard) certified. Moreover, many systems are developed to meet the Safety Integrity Level (SIL) standards. This allows guaranteeing systematic safety management on trains, which is essential to the mobility products offered by train operators.

The company is a part of a family-owned EKE-Group that has two other international divisions. One specializes in the construction of residential and business areas while the second focuses on leasing business premises. EKE-Electronics is a leading family-owned company in the field of train automation. This allows EKE to enjoy certain flexibility and a very stable organization, which leads to high motivation and retention rates among the employees.

1.2 Production processes at EKE

EKE develops modular units that are installed into racks to produce an operational system. Modularity allows for highly customizable and flexible systems to be created from an optimal number of different modules. The systems are, furthermore, combined into solutions – clusters of systems to execute certain functions. Figure 1 below represents the product hierarchy at EKE.

Figure 1. Product hierarchy at EKE-Electronics

The far-left unit in Figure 1 is a module. Modules developed by EKE are designed for specific applications. For example, EKE's portfolio includes Input/output modules, Memory modules, Power Supply modules, and other categories of modules. Currently, EKE produces approximately 60 module types, with each module being a building part of a system.

In the middle in Figure 1 is a system – an assembly of modules. Systems are designed for the individual requirements of customers and trains, and overall, they carry out a variety of applications. A development of a system also includes customer- and train-specific software design and management. An example of a system is a train computer or an event recorder.

Finally, on the right in Figure 1 there is a solution that combines several systems on a train. The solutions can carry out specific functions on a train and they can include speed monitoring, door control based on the location of the train, safety monitoring, and other vital functions.

Modules and systems are mainly produced by a subcontractor with certain activities being delegated to EKE's own production premises. The subcontractor manufactures modules on request and maintains its own production lots with a small stock of systems or modules.

The current production model is make-to-order (Arnold, Chapman, & Clive 2012, 4-5), in other words, the modules' and systems' orders are sent to production mainly after the order has been internally received. This leads to optimized production costs and standard lead times of 16 weeks.

1.3 Background of the research

To support the sustainable growth of the enterprise, EKE would like to offer its customers better lead times along with reliable and functional products. To achieve that, one possibility is to move away from the make-to-order approach (Arnold et al. 2012, 4-5). The production systems that facilitate such a change are described in the literature review chapter.

In order to improve the lead times, an imminent measure would be to increase the inventory held by EKE, excluding the optimization of the production processes. Production processes were not included in this thesis due to the ownership of production being at a subcontractor, while EKE could exercise direct control over inventory management.

Due to the high modularity of the produced systems, it would be possible to acquire a stock of modules that would allow assembling a greater variety of customer-specific systems. In this case, the question presented at the beginning of the introduction chapter becomes viable: “How to keep customers satisfied with product availability at the lowest cost?” This work tries to offer a solution for optimizing potential inventory levels while offering improved customer service.

2 Research methods

In order to solve the problem stated above, it is advisable to carry out research. Scientific research is required to generalize empirical findings and provide a base for theory building. In conducting research, it is important to define the research methods that aid in the work. For their definition, researchers should start with the nature of their research question and then continue with the methods of data collection. (Newman & Benz 1998, 14-16.)

The main way to classify research methods relies on the polarized description of qualitative and quantitative methods. The continuum between these is called a mixed research method. The selection of the most suitable research method depends on the goal of the research, available data, and means of its analysis. (ibid.) Therefore, this section starts by defining the research question, continues with the data collection methods and ends by describing and choosing the most appropriate research method.

2.1 Research questions

As mentioned in the introduction, the goal of this paper was to develop an inventory optimization tool for EKE. The tool should calculate an optimized module inventory in order to decrease costs and increase customer service at the same time. Since these optimization objectives may conflict with each other, some preference for finding the optimization balance should be given by the user of the tool. Moreover, the tool should be flexible and simple to use.

Put in one sentence, the main research question is: "*What could be a suitable inventory optimization system for EKE-Electronics?"*

This general topic can be broken down into two subtopics. One general topic highlighted in the research question considers inventory management, while the other relates to optimization. Therefore, it is sensible to pay attention to both fields of research. The first sub-question can be: "*What is a suitable inventory management model for EKE?"*

The second part of the research focused on the optimization side and built upon the inventory management model selected. Therefore, "*How could the selected inventory management model be optimized?"* was the second sub-question. The answer to this question would also have to consider potential changes in the production and business strategies of the company and should account for the precision of the sales forecasts.

2.2 Methods of collecting data

Every research process relies on data in order to produce conclusions and to be able to generalize and prove theories based on the findings. The data collected for research can be classified as primary and secondary. Both types of data have different methods of collection and define the nature of the research. (Westat 2002, 43-44.)

Primary data is collected from the actual interaction with the environment in contrast to secondary data that is collected through research of already published works. Primary data is more reliable, authentic, and objective because it has not yet been altered by human beings. Therefore, this data is genuine and unbiased. (Kabir 2016, 204.)

Secondary data has been collected and published before the research in question is carried out. Therefore, any literature study relies on secondary data obtained from written sources. This type of data is easier to obtain, but it may often represent only partial or modified data. Researchers who have collected the primary data in the first place may have omitted or altered certain pieces of that data to satisfy the needs of their research. (ibid.) To improve the reliability of secondary data, peer-reviewed journals rely on their members to validate the published materials. Moreover, certain checks are carried out to maintain the validity of the published research over time. (Newman & Benz 1998.)

Although being more preferred in research for its validity, primary data is often harder and more expensive to obtain. Furthermore, researchers often need to rely on secondary data for the generalizations made and proved by others. The sources of each data type vary as well.

Primary data is collected through surveys, interviews, focus groups, observations, tests, case studies, and experiments. Each of these methods differs in terms of the cost of using them and the quality and detail of the data provided. For example, surveys are easy to carry out on a big population of users, but they produce fewer insights than interviews. (Westat 2002, 49-62; Kabir 2016, 204-205.)

Secondary data can be obtained from course books, research articles, records, biographies, newspapers, published statistical data, data archives, internet articles, and private document studies. The quality of secondary data often depends on the costs of used methods. Some data types, such as newspapers and internet articles are faster and cheaper to obtain and analyze, but they provide usually less reliable data than course books and research articles. (ibid.)

The table below summarizes the advantages and disadvantages of both types of data collection.

Table 1. Advantages and disadvantages of primary and secondary data

	Primary data	Secondary data
Advantages	The data is collected specifically for the study carried out The quality of the data is certain for the researcher Additional data can be obtained if required	A cheap and fast way of data collection Opportunity to study trends over time and historical data Possibility to use the conclusions of previous research
Disadvantages	Additional costs concerning the organization of data collection Costs of ensuring the validity of the data Required skills of the researcher	Less reliability Less focus on and customization to the topic of the research Potential obsoleteness of data and dependence on location and population of previous research

The data used for this research came from both sources. To answer theory-based sub-questions, it was necessary to analyze the already developed inventory and optimization models. These models had already been developed, and they could be obtained from course books and research articles.

The main research question required an analysis of the primary data collected from the production and inventory systems of EKE. Moreover, the primary data received from the company’s management and thesis supervisor proved to be invaluable.

After this data was analyzed, it was possible to select and develop a tailor-made inventory optimization system as stated in the research question.

2.3 Qualitative research method

After the collected data is defined and research questions are established, it is possible to select the suitable research method out of three methods discussed earlier: qualitative, quantitative, and mixed. Qualitative research aims to describe and analyze the collected data in words. This allows us to better discuss the meaning of the collected information, and it also benefits problem-solving (Eyisi 2016.)

Often information collected during research is unorganized, and it is present in the form of experience, models, opinions, frameworks and other data presented in the form of text and figures. Such data requires careful interpretation and can provide valuable insights. Overall, the attitude of the researcher plays a noticeable role during qualitative analysis, and it may lead to biases if the researcher wants to prove a certain theory. (Newman & Benz 1998, 16-17.)

The methods of collecting data for qualitative research usually include in-depth interviews, case studies, and observations. Thus, qualitative research methods can provide detailed and unique data for the research. Furthermore, often in case of developing a theory, qualitative research is needed to interpret the collected primary and secondary data. Finally, this approach is essential for analyzing human thought and behavior if such an analysis should take place. (Eyisi 2016.)

The disadvantages of this approach are as great as the mentioned advantages. As with many sources of primary data, qualitative research may be more expensive to carry out or it can include only a limited variety of data for analyzing. Especially if secondary data is analyzed under this research method, the validity of the conclusions can be severely affected by the inexperience of the researcher. It requires an experienced person to correctly conduct an interview (Westat 2002, 49-51) and even a more experienced person to correctly study it. Finally, the observations may be overcomplicated compared to a quantitative approach. (Eyisi 2016.)

2.4 Quantitative research method

Quantitative methods are often used in hypothesis testing. In this case, a hypothesis is stated based on certain data obtained by the researcher, and it is then proved to be correct or wrong using statistical evaluations of the collected primary and secondary data. Thus, statistical methods and data presented in numbers are central to the quantitative research methods. (Newman & Benz 1998, 18-19.)

Implementing more scientific methods in research brings its advantages. Often quantitative research is faster to execute and describe thanks to the clarity of numbers and ease of use of modern statistical software. Generalization is easier to obtain through this approach due to a greater array of data analyzed and the application of statistical methods. Furthermore, the research is easier to replicate and validate by other researchers. This can bring more value for a paper and increase its chances of being published in a journal. (Eyisi 2016.)

On the other hand, this approach less often produces deep insights into the phenomena being analyzed, mostly due to the detachment between the researcher and the environment being analyzed. The results of the research are heavily dependent on the overall quality and uniformity of the data analyzed, and the research process requires noticeably greater volumes of collected data. Finally, the work with statistical instruments and a predefined form of quantitative research reduces the flexibility and creativity of the researcher and may impact the quality of the work produced. (ibid.)

2.5 Mixed research method

As mentioned earlier, the mixed research method is a combination of both qualitative and quantitative methods. Such a combination allows combining the strength of both approaches at the same time desiring to reduce their weaknesses.

As presented by Newman and Benz (1998, 20-21), mixed research could start with a qualitative approach and then validate the developed theories with a quantitative analysis. First, data can be collected and analyzed using a qualitative study to produce a hypothesis or a theory. Next, this hypothesis is validated by introducing statistical methods and data analysis techniques from the quantitative field of research.

Such a combination allows the research process to incorporate insights received from an in-depth analysis of the collected data, flexibility, and theory creation by analyzing a wide variety of data and implementing scientific methods to prove the created theory. Therefore, often such methods can be seen in the creation of new and disrupting theories. The main disadvantage of mixed research methods comes from the cost of collecting different sources of data and analyzing them. (ibid.)

As for the present study, it was necessary to employ both qualitative and quantitative study of the collected data. It was necessary to analyze verbally the inventory management and optimization models and systems presented in the literature. After the suitable models were chosen for application to the case of EKE-Electronics, it was necessary to conduct a quantitative analysis of the product data, such as forecasted demand and the historical sales of individual modules. Therefore, this work proceeded with a mixed research method.

2.6 Overview of the research structure

As outlined in the previous sections, this work employed both primary and secondary data and qualitative and quantitative research methods. The obtained secondary data is presented in the next section of the literature review. This section is divided into two parts: the first is dedicated to inventory management in general, and it aims at defining the potential systems that could be implemented at EKE.

The second section is committed to optimization. It discusses the available means of optimizing single- and multi-objective problems, and it also discusses the selection of the approach used for the application stage.

As the main goal of the work, the selected systems were developed in the real world and applied to the primary data received from the company. The process of the model development and the results are documented in section 4. Finally, the last part of this work is devoted to a discussion about the progress of this thesis, a list of the limitations of the research, and future ways to develop the work done.

3 Literature review

3.1 Inventory management

Seeing the rapid growth of the service industry and the less noticeable development of production, it is easy to suggest that manufacturing is losing the previous importance that it had during the industrial age. Researchers believe that production may be on its path to extinction (Silver, Pyke & Thomas 2017, 3). When considering new developments in 3D printing and the declining rate of consumerism, it might even be true.

However, it is clear why nations care about manufacturing. Two pillars of industrial production are food and machine tools industries. The latter produces tools for manufacturing companies, such as metal cutting machines. The sustainable food industry is essential in providing the population in case of conflict with means to live. The machine industry, in its turn, is essential for supporting this and other industrial groups in the nation. (ibid.)

In contrast to the service industry, which focuses on the knowledge and expertise of the employees, manufacturing relies on an intertangled network of various industries, employed capital, low- and high-skilled personnel, and methods. One of the essential parts of production is inventory that is included both in the capital and in the methods sections presented above. (ibid., 4.)

Concurrent with Silver and colleagues (2017) Hopp and Spearman (2011) emphasize the fact that the procurement of raw materials and components comprises an average of 70% of the total expenditure in the production industry. Therefore, procurement and inventory management can be of equal importance for a factory as production management. (48.)

Despite its importance, inventory management is often being underdeveloped by many companies involved in the production. This leaves room for improvement and connecting theoretical frameworks with real-life operations. As an example of this Silver and colleagues (2017) provide data on the results of their students applying theoretical knowledge. Over several years it was recorded that the students consulting local manufacturing firms were able to improve their inventory systems in 90% of cases with an average of 20% cost savings without sacrificing the customer service level. It means that professional advice could bring greater benefits. (4.)

3.1.1 Definitions

In this work, the term “inventory” is used often, and therefore, it must be defined in the first place. There are several definitions by various authors in the literature works (Hopp & Spearman 2011, 188; Silver, Pyke & Thomas 2017, 26). To select one, this work uses the definition of Kenton (2019) as the reference point: “Inventory is the term for the goods available for sale and raw materials used to produce goods available for sale”. Inventory is an interchangeable concept with stock (Hopp & Spearman 2011, 188).

Inventory can include a magnitude of varied items (Vermorel 2013). Therefore, in order to improve the handling and bookkeeping of stock, a basic unit of inventory is usually created. All items that are considered by management to be the same, for example, a "pen" can be a basic unit of inventory at a certain retailer who sells only one type of pens. Another might need to classify items, such as "black pen" and "orange pen", while a third will add information about the producer, such as "black pen Parker". This basic unit of inventory is called the Stock Keeping Unit (SKU). (Silver et al. 2017, 28.)

Often, especially in a production environment, some Stock Keeping Units consist of other SKUs, which, in turn, can consist of other items. This creates a picture of a tree, in which one item, such as a ready-for-sale pen, requires several other items for its production, and these are held in stock and produced or externally purchased. This tree is called the Bill of Material (BOM) and an example of it is Figure 2 below. (Giard & Sali 2012.)

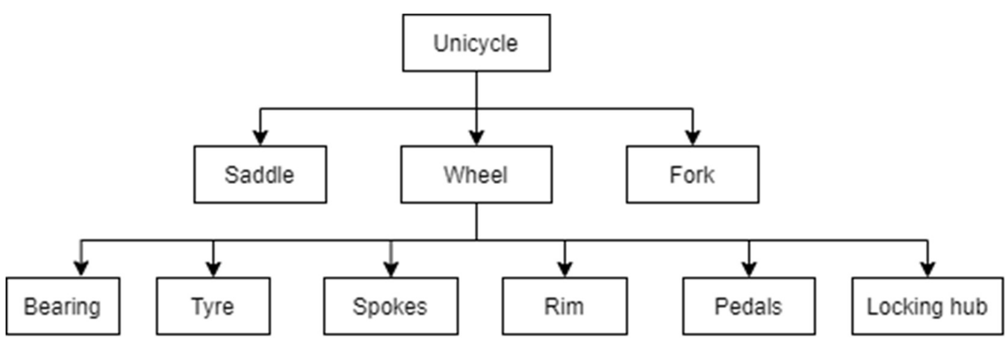


Figure 2. Example of a Bill of Material (Sipilä 2017)

Inventory is further classified into several types. Five types in the inventory classification by Silver and colleagues (2017) are cycle inventory, congestion stock, safety stock, work in progress (or pipeline) inventory, and finished goods inventory. A cycle inventory is the number of goods accumulated at hand, which results from the desire to run the production of every item in batches instead of single-item production. Batch production has at least three proven benefits: economies of scale due to the economy of setup costs, batch discounts for purchasing and freight costs, and the necessity of production restrictions, such as a required minimum volume of a processing tank at a chemical plant. (26.)

A congestion stock results from several stock-keeping units competing for a piece of production machinery. If the setup times are long or the production time of a certain piece of equipment takes a longer time than in the rest of the line, the inventory upwards from the production line tends to accumulate before being processed at that stage of the line. This accumulated inventory is the congestion stock. (ibid.)

An important element of inventory is the safety stock (SS). Silver and colleagues (2017) define safety stock as the number of goods kept for demand and supply uncertainty. Therefore, the safety stock is useless if demand and supply are known with certainty. (26.) There are many ways to determine the optimal safety stock and they are presented in the respective chapter later in this work.

Finally, the pipeline stock is the type of inventory that is used at some stage of the production or distribution process. For example, goods that are being transported between warehouses, production facilities, and production equipment on any mode of transport from a pallet truck to a cargo vessel are pipeline stock. Moreover, goods that are being processed on machinery at any stage are also called work in progress (or pipeline stock). (Silver et al. 2017, 26.)

Once inventory as a term is defined, the next logical step is to describe the inventory management systems used. One of the oldest and still featured systems is the ABC classification. While it does not offer exact guidance on actions like most other inventory management systems, it is simple to use, and it offers some insights into the company. (Hopp & Spearman 2011, 587.) More sophisticated inventory control systems are discussed in detail later in this chapter.

The ABC classification relies on the Pareto rule, according to which about 20% of all inventory items constitute about 80% of the total turnover or stock value. Therefore, it is possible to classify every inventory item according to its annual turnover or value to be either an A, B or C category item. Usually, A items are goods that contribute 80% of the total turnover or value, B items – between 80% and 95% and C items take the rest 5% of the total turnover or value. Thus, A items are seen as the most important, and they earn the most tailored attention from management, sales, and production. (ibid., 588.)

Inventory management systems can be compared and controlled by several indicators. Some of them are inventory turnover, service level, and cost. Inventory turnover is the ratio of $\frac{annual\ sales\ (in\ units\ or\ €)}{average\ inventory\ (in\ units\ or\ €)}$. If currency values are used, then the sales should be valued in the same way as inventory – as goods value without a sales margin. In other words, inventory turnover measures how often the inventory rotates throughout a year, and it can be measured per single item or for a whole warehouse or supply chain. The less this value is, the smaller is the average inventory compared to the annual sales and the greater is the challenge to maintain service level. (Sipilä 2017.)

Service level defines the percentage of customer orders that are satisfied from the shelf. In other words, it is a ratio of units sold to the units wanted by customers. The same idea can be applied to internal customers, such as production, as well as to the external paying customers. While this value can hardly be correctly measured in the hectic environment of fast-moving consumer goods (FMCG), for example, in business to business (B2B) sales it often becomes more apparent thanks to separate requests regarding the availability of products. In contrast to the real world, many inventory models rely on this parameter for a rough estimation of stockout likelihood. (Hopp & Spearman 2011, 70.)

According to Silver and colleagues (2017), quite often managers strive for high service levels, ranging from 90% to 99%. To allow for a good service level in case of a hardly predictable demand, a company needs to carry an extra inventory. Moreover, keeping an inventory always costs a company money. (246.)

This expenditure arises mainly from two different sources: tied-in capital and warehouse costs. Tied-in capital can be measured as a total value of the inventory multiplied by an opportunity cost factor defined by the company. The total value of inventory is merely a sum of the value of each stocked item. Moreover, the value of each item should be defined by examining the resources required to produce the item. This step is not easy to execute accurately, but most production companies have some estimation of the unit's costs for each produced item. Finally, the opportunity costs factor is theoretically a percentage that the company could earn on its money being used elsewhere. While the actual interest earned on different opportunities may fluctuate over time, many companies define a fixed level of opportunity costs to help in decision making. (ibid., 41-43.)

One way to determine the opportunity costs factor is to use the weighted average cost of capital (WACC) formula. The idea behind this approach takes root in the fact that any investment needs to be supplied with capital obtained from private investments in the company or from taking a bank loan. Since most companies, even private ones, have a mix of both in different proportions, the WACC formula allows estimating a weighted cost of each euro obtained by the company. (Kumar, Colombage, & Rao 2017.)

The formula consists of two parts: the cost of debt and cost of equity (private investment). The bank loan interest rate is equal to the pre-tax cost of debt. The after-tax cost of debt considers also corporate tax rate making the following equation: *cost of debt = interest rate * (1 – corporate tax rate)*. (ibid.) Therefore, the greater the tax rate is, the better the loan is "shielding" the company from it.

The cost of equity can be determined for public and less often private companies according to the capital asset pricing model (CAPM). It depends on three factors: risk-free rate, companies comparative risk assessment and expected market return. The risk-free rate is usually compared to guaranteed financial instruments, such as the US

Treasury Bonds. Risk measurement depends on the company's situation compared to the stock market. If a company is less stable than the market, such as in case of a start-up, the risk measurement is greater than one. If the company is more stable, then this value is less than one. Expected market returns can be compared to a market index, such as the S&P 500. The formula is then: *Cost of equity = risk-free rate + risk measure * (expected market return – risk-free rate).* (Thibblin & Numminen 2019.)

Often companies may use their measurement as the cost of equity, such as Return on Investment. In this case, the company will compare its profits to the amount of invested capital. (Zamfir, Manea, & Ionescu 2016.) Overall, both methods aim at determining the expectation of investors regarding the potential profits on their capital.

Based on the equations for the cost of equity and cost of capital, the weighted average cost of capital for a company is:

$$\begin{aligned} WACC \; = \; & percentage\ of\ equity\ in\ the\ financial\ structure * cost\ of\ equity \\ & + percentage\ of\ debt * cost\ of\ debt \end{aligned}$$

The warehouse costs of an item can depend on the item's bulkiness and the amount of space occupied by the item. If warehousing at the current capacity costs a company x euro and offers y m^3 of storage space, then the costs of storing an item taking space z m^3 are *x * z/y* euro.

Besides merely keeping stock in place, the company also bears the costs of replenishing the stock. The replenishment costs are the result of salary and other costs related to the purchasing department and processes as well as possible freight and shipment-related costs incurred with every order. In practice, however, a precise value of the ordering costs is hard to obtain as it is hard to exactly measure the average time that employees spend on ordering and following up on a single order. Sometimes, companies estimate their annual overhead purchasing expenditure and divide it into the annual number of purchasing orders. (Silver et al. 2017, 42-45.)

The final reviewed cost component is the backlog cost and lost sales cost. It essentially results from a desire to optimize the costs mentioned above – i.e. having less inventory and ordering it less often. This behavior leads to an increased chance of demand exceeding the stock held at the moment, and thus, it leads to lost sales or in the best case to delayed customer orders. (Hopp & Spearman, 462.)

Although the backlog cost is not an obvious expenditure of a company, it is still an important one. If a company cannot satisfy demand, it may lose customers, immediate and future sales as well as goodwill. In addition to these nebulous costs, the company may need to bear costs of overtime, expediting, split lots, and other such costs. (ibid.) These costs can sum up to 65% of the total inventory costs, according to Vermorel (2013).

If a company faces a stockout occasion and manages to convince a potential customer to wait for their order – i.e. to turn the lost sales into a backlog – the customer will usually be promised that they will receive the order in a certain time. This time is called lead time and applies to every process in supply chain management. In essence, the lead time is a period from the start of an activity to its end. Lead times are measured for purchasing and sales – the time from order placement to its receipt as well as for production – from the start of processing an item to receiving the ready for sales item and transportation – from loading to unloading. The same logic is applied to other areas of supply chain management and production. (Silver et al. 2017, 49.)

Stockouts usually happen due to an unexpected demand rise that was not predicted or expected during previous replenishment. Such a demand that can happen with a known probability is called thus probabilistic or stochastic. It is contrasted to deterministic demand that is known beforehand – i.e. determined before it occurs in the long term. There is also fuzzy demand – demand, which probability is impossible to predict, and that, thus, is happening completely randomly. The most widely used probability density function for probabilistic demand is the normal distribution. It requires just two parameters – mean and standard deviation. (ibid.) Its graph is presented in Figure 3 below.

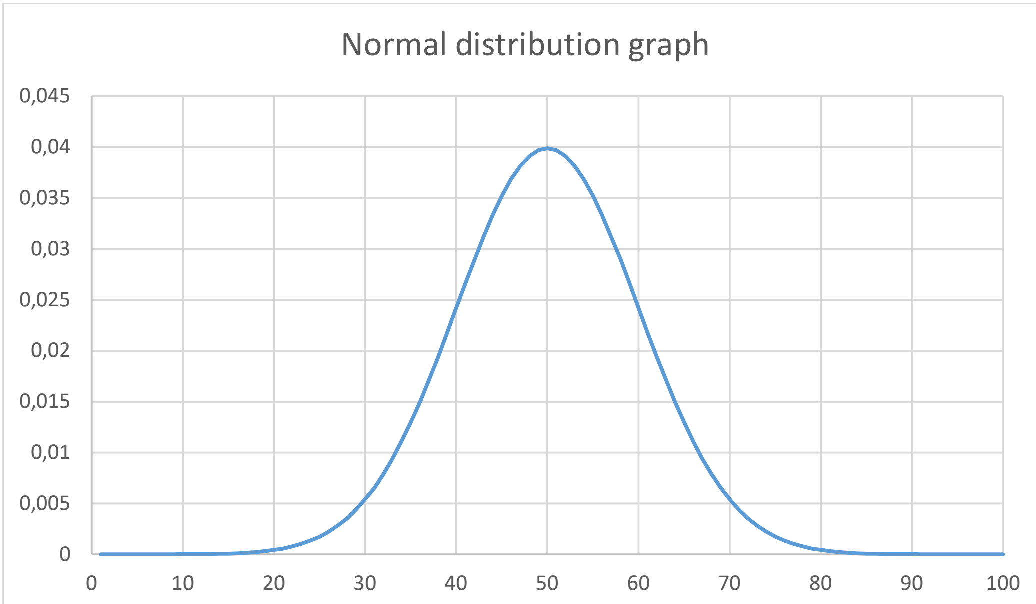


Figure 3. Normal distribution graph

Long procurement lead times may create situations, in which several purchasing orders are outstanding to a single supplier for the same item. For example, if goods are reordered when the inventory level reaches 100 units, and the ordering quantity is 50 units with a long lead time, it often may happen that another purchasing order should be placed to avoid potential stock out. However, it is difficult to know when to place the second order.

In order to track the inventory level including outstanding orders, the inventory position is used instead of a net-stock. Net stock is the on-hand inventory reduced by the number of backorders, and the on-hand inventory is the actual number of goods on the shelf. While the on-hand inventory cannot be negative, net stock can be.

(Silver et al. 2017, 238.) For example, if there were 100 items back-ordered at the moment when replenishment of the size of 80 items arrived, then on-hand inventory would become 80 once the goods are received. However, the net stock would still be -20 items, since 20 items would be left back-ordered after the arrived 80 units are cross-docked to customers.

The inventory position, furthermore, adds outstanding orders into consideration. In other words, it is *On-hand inventory + outstanding orders – backorders – committed units*. The committed units are the units sold to a customer or reserved by production from the awaited shipments. In other words, the inventory position indicates the total amount of available for sale or use items that have been ordered but not yet used. (ibid.) In the example above the inventory position would be the same just before the arrival of 80 ordered units and after the goods were received at the warehouse.

Table 2 summarizes the definitions used in this chapter in one place. It includes the essential terms that are used later in the study and can be used as a reference.

Table 2. List of inventory management definitions

Term	Definition
Inventory	Goods available for sale and raw materials used to produce the goods
SKU	The basic unit of inventory as defined by management
BOM	A list of SKUs required for the production of an item
Safety stock	Part of inventory that is held to allow for demand uncertainty and avoid stockouts
ABC classification	Classification of stock-keeping units according to contribution to the annual turnover
Inventory turnover	$\frac{annual\ sales\ (in\ units\ or\ €)}{average\ inventory\ (in\ units\ or\ €)}$
Service level	$\frac{annual\ sales\ (in\ units)}{annual\ potential\ sales\ (in\ units)} = \frac{cases\ of\ sale}{cases\ of\ sale + cases\ of\ stockout}$
Tied-in capital	Capital invested in any kind of inventory
Ordering cost	Costs incurred with placing an order – usually measured on an annual basis
Lost sales cost	Nebulous cost of lost goodwill and potential profit incurred in case customer cannot buy a product and cancels the order
Backorder cost	Cost of lost goodwill and the potential cost of overtime and expediting in case customer cannot buy a product and decides to wait until the product is delivered
Lead time	Time from the start of a process to its end
Deterministic demand	Demand that is exactly predicted in the long term in the future
Probabilistic demand	Demand that may occur in the future with known probability
Inventory position	Amount of available for sale or use items that have been ordered but not yet used

3.1.2 Production systems

As was mentioned in the background of this research, one option to improve production lead time for EKE is to update its production system. Arnold and colleagues (2012, 4-5) identify four types of production systems: engineer-to-order, make-to-order, assemble-to-order, and make-to-stock.

Engineer-to-order (ETO) approach is applied when a customer requires highly customized products and new engineering or design. Often the customer is also deeply involved in the design and sometimes production process. Due to the added design phase, the delivery times are often long. Moreover, most of the material is procured only after the order or even design stage, because before an order, it can be difficult to estimate the number of inventories required. (ibid.) The examples of engineer-to-order items are luxury yachts, villas, and unique constructions, such as nuclear plants.

Make-to-order (MTO) production system is used when customer specifications still vary but can be satisfied by the production of standardized components. In contrast to the engineer-to-order approach, the design stage is omitted in this system and a certain inventory of raw materials is usually held. (ibid.) This is the case of EKE, where only the raw material stock is held and owned by the subcontractor.

Assemble-to-order (ATO) arrangement is implemented to allow customers to enjoy the possibility of modifying a system according to their needs while keeping shorter lead times compared to the previous two models. In this case, the manufacturer keeps an inventory of standardized modules that are purchased externally or produced internally. Then according to a customer request, the final product is assembled from the items held in the inventory with little to no manufacturing involved in this step. (ibid.) Examples vary from pizza baking to luxury cars (BMW), tractors (Valtra), and computers (Dell).

ATO strategy relies on detailed Bills of Material to maintain optimized inventory levels. Each final product will also have a different BOM that consists of the same standardized components as in BOMs of other products with some components changed. In case of a car, these can be different models of the motor, assorted colors of the frame, several types of leather used in furnishing, and other components. If BOMs are implemented correctly, then demand forecasts for each Stock Keeping Unit can be calculated as a sum of forecasts for final products. (ibid.)

Make-to-stock (MTS) is the most widely used method of production in retail. Here, items are produced ready-to-be-sold and are held in the inventory until an order is received. In this case, the lead times are the shortest compared to other systems presented in this chapter, however, the variety is the most expensive to maintain. (ibid.).

Figure 4 below, as adapted from Arnold and colleagues (2012, 4), compares four production systems presented above. It is noticeable that the main difference is in the movement of inventory closer to the final customer and, thus, the reduction of lead times between the order and shipping phases.

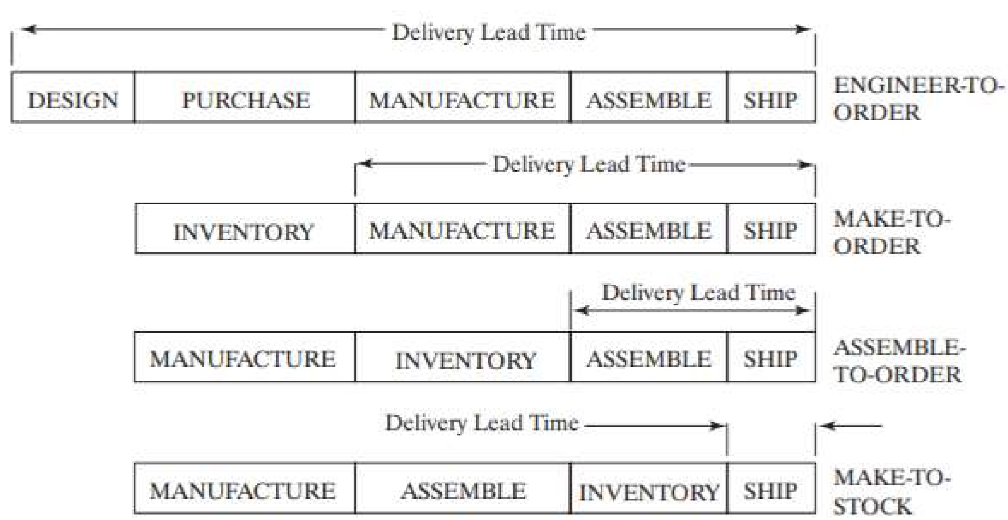


Figure 4. Comparison of production systems (Arnold et al. 2012, 4)

3.1.3 Safety stock

One way to calculate safety stock in case of uncertain demand is presented in the work of Hopp and Spearman (2011): *customer service level (CSL) * standard deviation of demand during lead time.* Customer service level is discussed later in this section and the standard deviation is a value calculated from the sales records. Often letter *k* or *z* is used to mark CSL and σ is commonly used as a standard deviation mark. (74.) In other words, $SS = k^* \sigma$. This system assumes that the supply lead time is constant enough to ignore the possible time fluctuation.

For the case when the supply lead times also noticeably vary, Talluri, Cetin, and Gardner (2004) have used an updated formula for the calculation of the standard deviation of demand *and supply* during lead time. Here is the formula presented in their work:

$$\sigma = \sqrt{\sigma_R^2 * L + R^2 * s_L^2}$$

In this formula, σ_R stands for a standard deviation of demand during a period (for example a month). *L* is the lead time in the same unit of time. *R* is average demand during the period and s_L is the standard deviation of the lead time. The calculation for the safety stock, however, remains the same: $SS = k^* \sigma$.

Silver and colleagues (2017) present three other ways to establish safety stock. The first and most widely used one is a simple approach to keeping safety stock equal to a certain time of demand. For example, if demand was estimated at 100 units in a month and the company adopted the policy of having two weeks of demand as safety stock, then the stock would equal 50 units. This system, while easy to understand, does not account for a variation in demand and relies only on the mean level of demand. If there is a high likelihood of noticeable variations in demand, then stockouts still can occur and if demand is very predictable, the company will invest in unnecessarily high stock levels. (249.)

The second method is based on minimizing the cost of a potential stockout and unavoidable stock keeping. The costs considered are cost per stockout occasion, cost per unit short, cost per unit short per time, and cost per order line short. All methods except the third (cost per unit short per time) can be applied to backorder and lost sales concept. (ibid.) The cost minimization method is as easy to understand as the one presented in the previous chapter. However, the actual costs required for this approach are extremely difficult to obtain correctly and they have a profound effect on the inventory system developed. Therefore, this approach can be applied with difficulty in situations when the cost of failing to deliver goods to customers on time cannot be established.

The third approach is to evaluate the required safety stock based on the service level indicators. One of these was presented at the beginning of this chapter from Hopp and Spearman (2011). Other indicators can be the fraction of demand to be satisfied from stock, a fraction of time when a net stock (stock on the shelf) is positive, and a defined average time between stockouts (Silver et al. 2017, 249-250). Each of these indicators has a different impact on the complexity of the modeling system used, however, they all take root in the defined by management service rate the company wishes to pay for. Due to versatility and connection to the company’s strategy, these methods are often used.

Finally, Silver and colleagues (2017) present a possibility to allocate predefined safety stock among the inventory. In case there is a determined by management investment possible to make into the safety stock, the goal is to optimally allocate it to each given SKU. The two methods to do this are to minimize expected total stockout occasions per year and expected total value short per year (in euro). Both methods include aggregated considerations of the total investment made and service level provided. They are usually built upon the stockout price methods presented earlier. (250.)

Safety stock is an essential part of inventory management in the world of unstable demand. The choice of the suitable model depends on the strategy the company pursues and the modeling technique applied to the inventory system. The next chapter takes a deeper look at the modeling part of inventory management.

3.1.4 Modeling

As was mentioned in the introduction, inventory is an essential concept for manufacturing. Therefore, the human mind has created a variety of techniques and models to deal with inventory optimization, demand uncertainty, and production management. Regarding inventory optimization, Silver and colleagues (2017) distinguish three modeling strategies:

1. precise modeling,
2. broad-scope modeling, and
3. optimization with a little modeling

Precise modeling is built upon a selected limited number of decision variables and a defined set of values describing a system. (Silver et al., 47.) In other words, the modeler selects variables that they can change according to the outputs of the model, such as purchasing order size, safety stock or the reordering point. Then, they gather data regarding the environment being analyzed. This data can comprise of ordering cost, annual demand, stock carrying cost, and other indicators. Finally, an algorithm or a model is created and upon execution, they provide the decision-maker (DM) with a relevant result for the variables. A classic example is the Economic Order Quantity, which is discussed in the next chapter.

Typically, an inventory optimization mathematical model allows three types of solutions: a deductive, iterative, and trial-and-error solution. The deductive solution is also called a closed-form approach that aims at defining the output - usually the best value of a variable to optimize the objective function – with a given set of parameters. These solutions are often fast and easy to use, although may often require a deep understanding of mathematics. An example here is again the Economic Order Quantity. (Silver et al. 2017, 46.)

Iterative solutions focus on arriving at the best value of a variable or variables by iteratively optimizing these values and following the behavior of the objective function or functions (ibid.). For example, the Multi-Objective Particle Swarm Optimization (MOPSO) method, that is presented later, simulates an array of particles that iteratively scan available values of the variables and search for the point of potentially best solutions to the given objective functions (Padhye 2016).

Trial-and-error solutions strive for developing an approximate model of the environment and they allow the user to iteratively change variables to arrive at the desired or suitable result. This category includes all forms of simulation modeling, such as Flexim. (Silver et al. 2017, 46.)

Broad-scope modeling is more of a philosophy rather than a strategy. It aims at developing a system that is close to real-life and is free from limitations and assumptions of the precise mathematical modeling. Less stiff modeling, however, leads to vaguer or non-existent objective functions. In other words, it is harder to optimize the broad-scope model with mathematical means and the decision-maker must take independent actions based on own experience and data provided by the modeling system. (Silver et al. 2017, 47.) A good example is Material Requirements Planning. The system aggregates enterprise-wide data to allow the user to make decisions regarding production schedules and quantities without providing potential solutions. Since this philosophy has a little to do with mathematical modeling, it remained out of the scope of this thesis.

The final strategy defined by Silver departs yet further from mathematical modeling. According to this philosophy, decision-makers may attempt to minimize inventories without the help of mathematical methods. This category includes widely featured philosophies of Just in Time (JIT) and Optimized Production Technology (OPT). These philosophies strive for eliminating (or at least reducing) wastes. (ibid.) Just in Time, for example, distinguishes eight categories of waste: inventory, waiting, overproduction, over-processing, goods moving, employee moving, defects, and unused talent. Just in Time philosophy is an integral part of the Toyota Production System (TPS). (Monden 1993, 173.) Again, this type of modeling was not included in this thesis.

3.1.5 Lot sizing and Economic Order Quantity

As mentioned in the previous chapter, one of the most featured inventory optimization systems is Economic Order Quantity or EOQ. It is the oldest model in the inventory optimization literature, according to the research of Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014). It was discovered by Ford Whitman Harris in 1913, but it then passed into oblivion and was rediscovered again only in 1988. According to the research of Andriolo and colleagues (2014), in the years from 1996 to 2009 there were 352 papers published concerning EOQ in relevant peer-reviewed journals.

The Economic Order Quantity model, as the name suggests, determines the most economically profitable purchasing or production order quantity. Essentially, it is a formula that returns the number of items required to order every time the stock depletes to minimize the sum of ordering and stock keeping costs. The following assumptions were used in the work of Harris (1913, 135-136):

1. the demand is constant and deterministic,
2. the order quantity can be any real number, i.e. it can be fractional,
3. there are no limits on the minimum or maximum purchasing order quantity, and
4. there are no discounts on the unit's costs related to the order size. For example, purchasing and transportation costs are independent of purchasing lot size.
5. Cost factors remain constant over time,
6. each item is treated individually, and the benefits of joint review and ordering are negligible,
7. replenishment is instantaneous,

8. no shortages are allowed,
9. the entire order quantity is delivered at the same time regardless of the order size, and
10. all values are expected to remain constant over time.

The model works under strict assumptions that are difficult to achieve in the real world. However, the penalty of using Economic Order Quantity instead of a more complex and more optimal system is less than 11.8%. Therefore, under certain circumstances, it may be more efficient to implement a model based on Economic Order Quantity rather than investing in more precise framework development. (Silver et al. 2017, 148.)

Moreover, the model is simple to understand and requires just six variables:

- Q = the replenishment order size in units,
- D = annual demand in units,
- T = order lead time,
- A = fixed order cost per each order placed,
- v = the cost (or value) of one unit of the item analyzed. Contrary to the sales price, this value consists of the purchasing price and potentially additional processing costs. It is measured in €.
- r = the carrying charge – the cost of capital and stock-keeping as was discussed in section 3.1.1. It is used as a percentage of the capital invested.

Since the given parameters are always constant (assumption 10), it is logical to have the same order quantity every time an order is placed. Moreover, the replenishment lead time is 0 (assumption 7), no shortages are allowed (assumption 8), and demand is deterministic (assumption 1). Therefore, it is advisable to place the order once the inventory reaches the 0-level. The inventory level pattern under these assumptions is presented in Figure 5 below.

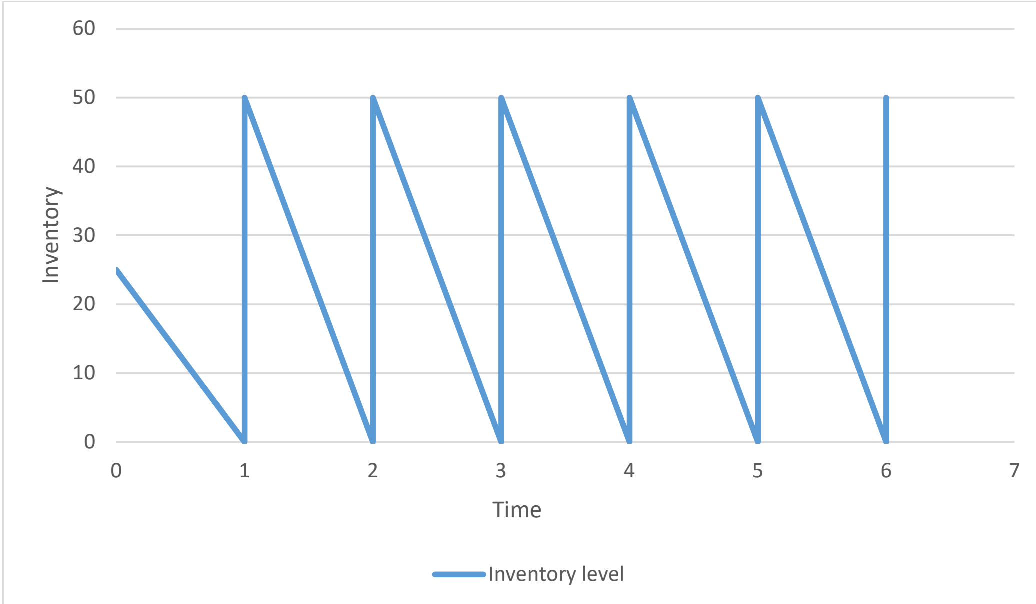


Figure 5. Inventory level under the EOQ model

It can be noted that within this pattern, there are exactly *D/Q* orders in a year (or another unit of time if chosen). Therefore, if a cost A is incurred with every order, then annually the ordering cost would amount to *D/Q*A*.

According to the definition of the inventory costs presented in chapter 3.1.1, a common method of calculating inventory carrying costs is through the multiplication of the average inventory level to its price multiplied by the carrying charge: *I*v*r*, where *I* is the average inventory.

In case of the sawtooth diagram presented in Figure 5 above, the average inventory level is *Q/2*. Thus, inventory carrying costs are:

$$\frac{Q * v * r}{2}$$

The total annual costs would be then:

$$\frac{AD}{Q} + \frac{Q * v * r}{2} \tag{3.1}$$

A graph of these two functions is illustrated by Figure 6 below, which is adapted from Silver and colleagues (2017, 150).

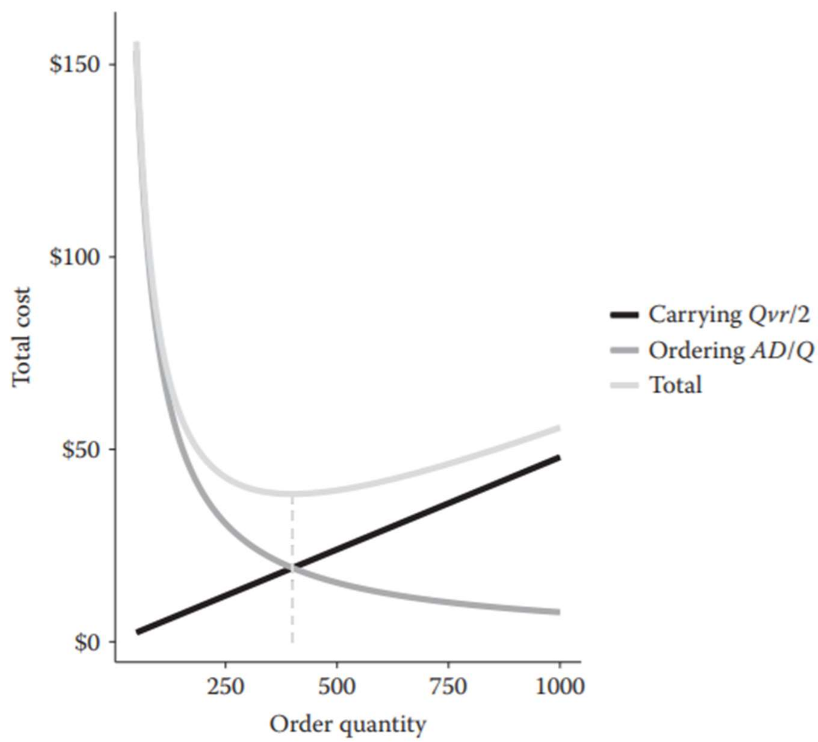


Figure 6. Total costs in EOQ (Silver et al. 2017, 150)

The derivative is used to find the minimum of the total costs function:

$$\frac{vr}{2} - \frac{AD}{Q^2} = 0$$

The solution to this equation is the value of the order quantity that minimizes the sum of both cost functions. It is:

$$EOQ = \sqrt{\frac{2AD}{vr}} \tag{3.2}$$

Finally, by substituting the value from equation 3.2 into 3.1, the minimum point of the total cost is $\sqrt{2ADvr}$.

Summing up, this model is easy and fast to use, and it provides relatively reliable results. At least, it can rapidly evaluate if the current lot sizes used in the company are logical from the cost perspective or if they require a deeper analysis. However, an array of assumptions has a significant impact on the quality of the output produced and reduces the significance of the Economic Ordering Quantity concept in the modern environment.

Due to the attention the approach has attracted and the simplicity of the basic principles, in the last 100 years, many improvements have reduced or eliminated every assumption of Harris. Andriolo and others (2014) illustrate, that extensions to formula 3.2 have incorporated input data with a probability density function for all parameters including added lead time and uncertain information about input parameters. Moreover, these extensions also include potential volume discounts, trade credits, varying quality of purchased items, inflation effects, shortages, and finite production rate. (22.) These extensions remained out of the scope of this thesis because they do not relate to the multi-objective optimization, focusing instead on a single objective of minimizing costs.

In addition to the Economic Order Quantity model that is the basis of a variety of more sophisticated models, it would be useful for this work to see one extension to this model. All assumptions regarding the EOQ model can be approximated or solved as shown in the works of Andriolo and colleagues (2014), Silver and coworkers (2017, 145-179), and Holbom and Segerstedt (2014). The lead time assumption is the most critical one concerning the real-life environment.

To account for a fixed lead time with deterministic demand, it is enough to place the order early enough for the inventory to last until the replenishment arrives. This time is defined by the inventory level, at which the order needs to be placed, and which is called the reordering point. (Silver et al. 2017, 168.)

Reordering point (ROP) is calculated as the demand received during the lead time. In other words, it is the annual demand multiplied by the ratio of the duration of the replenishment time to the total length of the year. The time units used should be the same.

3.1.6 The dynamic version of Economic Order Quantity

In case the demand is predefined for N periods but is not levelled in each period, the Economic Order Quantity model will not always produce the optimized result, according to Wagner and Whitin (1958). It will happen more often if the inventory carrying costs also vary. To overcome these complications of the real world, Wagner and Whitin (1958) have developed a dynamic version of the Economic Order Quantity algorithm.

The following assumptions remain the same as in the original model:

1. there are no limits on minimum or maximum purchasing order quantity and
2. there are no discounts on the unit’s costs related to the order size. For example, purchasing and transportation costs are independent of purchasing lot size.
3. Each item is treated individually, and benefits of joint review and ordering are negligible,

4. replenishment is instantaneous,
5. no shortages are allowed,
6. the entire order quantity is delivered at the same time regardless of the order size,
7. all values are expected to remain constant over time, and
8. the inventory position in the first period analyzed is less than demand in the same period.

The variables used in the model are:

- d_t = amount demanded in period t,
- r_t = inventory carrying costs to the period $t + 1$. In this model this value includes the cost of goods sold, in other words, no separate value for the goods' price is needed.
- A_t = order costs in period t, and
- q_t = amount ordered/produced in period t.

The most basic approach to solve the problem of identifying the best q_t for periods $t = 1,2, .. , N$, would be to enumerate all possible 2^{N-1} combinations and determine the case with minimal costs. The proposed model, however, reduces the maximum needed number of combinations to be checked to $N * \frac{N+1}{2}$. Furthermore, usually, it requires fewer calculations to achieve the optimal result. (ibid.)

The model defines five theorems. These theorems refer to periods as t, t_1, and t_2. The numbering is always used in a way that $t_2 \leq t_1 \leq t$, i.e. period t_2 occurs before period t.

Theorem 1. At period t there is either starting inventory left in the system or there is a new order placed and goods received. Since the system is deterministic and delivery times are instantaneous, this theorem is true because there is no point in paying for keeping inventory in stock if it can be ordered and delivered in the period t. (ibid.)

Theorem 2. The optimal solution produces variants of either 0 purchasing amounts or procurement quantities big enough to satisfy demand in the months, following the order as per the formula below. Thus, there is either demand satisfied from stock or there is a new purchasing order placed to satisfy one or several future months' demand. The proofs for this and the following theorems are presented in the work of Wagner and Whitin (1958).

$$q_t = 0 \; or \; \sum_{j=t}^{k} d_j \;\; for \; some \; k, t \leq k \leq N$$

Theorem 3. If the demand in period *t* is satisfied by a purchasing order placed in period t_1 (such that t_1 is less than *t)*, then the demand in all periods from t_1 to *t* is satisfied by that order.

Theorem 4. If the demand in period *t* is 0, then the periods 1 to *t - 1* can be considered separately from the rest of the solutions. In other words, the decisions made after a purchasing order is placed do not affect any previous decision.

According to the four presented theorems, the minimal cost function for periods 1, 2, .. , t is:

$$F(t) = \min\{\min_{1\leq j\leq t}[A_j + \sum_{h=j}^{t-1}\sum_{k=h+1}^{t} r_h * d_k + F(j-1)]; A_t + F(t-1)\} \qquad (3.3)$$

F(1) = A_1 and *F(0) = 0.* This recurring formula or a program means that the optimized cost for the first *t* periods would comprise the setup costs for period *j* and the inventory carrying expenditure for filling demand d_k from period *j + 1* to *t*. Therefore, the last replenishment ordered before period *t* is made in period *j*. Furthermore, the formula includes the cost of adopting the optimal policy through periods 1 to *j - 1* as signified by *F (j - 1*). The minimum cost received through this formula is not necessarily unique, as several optimized solutions can be found for a case of predetermined demand. (ibid.)

Theorem 5. *The Planning Horizon Theorem*. The final theorem introduced in the Dynamic Economic Order Quantity model aims at further reducing the number of required iterations to arrive at the desired result. According to the theorem, if for a period t_1 the minimum of equation (3.3) occurs for $j = t_2 \leq t_1$, then in the following t periods after t_1 $(t > t_1)$ it is enough to study only values of j between t_2 and t: $t_2 \leq j \leq t$. In fact, if $t_1 = t_2$, it is enough to analyze formulas with $q_{t1} > 0$. In other words, an order must be placed in the period t_1. The proof of this theorem is also presented in the work of Wagner and Whitin (1958).

Finally, all requirements to deduct the algorithm are defined. The algorithm is described in the following way: at any period t_1 from 1 to N the following actions are to be carried out:

1. Analyze the possibility to place an order in period t_2 between 1 and t_1 to fill demand between the order placement and the current period t_1.
2. Calculate the total cost of t_1 different policies by summing the ordering cost of the order placed in period t_2 and the costs of acting according to the algorithm in the periods 1 to $t_2 - 1$. Additionally, the holding costs of inventory from the procurement in period t_2 are added.
3. From the t_1 alternatives, choose the most cost-optimized solution through 1 to t_1.
4. Proceed to period $t_1 + 1$ *if* t_1 *is not yet equal* N. Stop if $t_1 = N$. (ibid.)

The dynamic concept of the lot-sizing model is noticeably more complicated in both theory and application. It indeed allows varied planning under deterministic demand over a long horizon. A good example of such a case would be production planning that has received sales estimation or actual orders in advance and can proceed with a defined production for a long time.

With modern programmable software, the application of the algorithm of Wagner and Whitin (1958) is reduced to writing a program in any of the modern programming languages and executing it with given data. Thanks to the increase in mathematical capacity of modern tools and wide use of programming, new and more complicated methods and models are developed. These models reduce assumptions (and thus limitations) of the model described in the previous two chapters. For example, the case of non-deterministic demand, which is explored in the next section.

3.1.7 Probabilistic demand

According to Silver and colleagues (2017, 239), a replenishment control system should aim at answering three fundamental questions:

1. How often the inventory status should be determined?
2. When a replenishment order should be placed?
3. How large a replenishment should be?

Under deterministic demand discussed in the previous part, these questions can be answered at ease. Economic Order Quantity framework allows calculating inventory level at any point in time given input values and current inventory level. The second question was discussed as well, and the reordering point can be either at the depletion of stock in case of zero lead time or early enough for the order to arrive before a potential stockout. The last question is answered with the help of equation 3.2 that defines the size of each procurement or production order.

In case of uncertain demand, however, the same questions become noticeably harder to answer. To help decision-makers in their job Silver and coworkers (2017, 241-245) defined a combination of inventory replenishment policies. To select the appropriate one, four questions need to be answered (ibid., 240):

1. How valuable is the item?
2. Is inventory reviewed periodically or constantly?
3. What stock policy form to choose?
4. What objectives need to be met?

The first question is often answered by implementing the ABC classification described in section 3.1.1. The second one depends on the production and inventory management systems used and the company’s situation. The selection between the two systems depends on the review interval (*R*), which signifies the time between each consecutive inventory status update. (ibid., 240-241)

In the periodical system, according to the name, the inventory is checked in periods every time interval *R*. A common example is a conventional vending machine. For instance, every week a driver arrives and checks the remaining stock levels after which he/she decides if some items need refilling. In this case, *R* is one week. If the stock out happens between the checks, no action is taken until the review period. Thus, the longer the review period is, the more uncertain the probabilistic demand is, which happens between the order placement (once the reordering point is reached) and the inventory replenishment. (ibid.)

The extreme case is a continuous review policy – a situation when the inventory level is updated at every point of time. In case of Enterprise Resource Planning (ERP) systems and Point of Sale (POS) data collection systems, such monitoring becomes trivial and usual. These systems update corporate-wide inventory levels after each transaction, and at any point in time (given the transactions are carried out correctly) any user can know the exact inventory level. Even with these systems, the inventory is usually checked intervally, such as every day. (ibid.)

Based on the four possible combinations of answers to the first two questions, four stock policy forms can be formed (ibid.):

1. order point, order quantity *(s, Q)* system,
2. order point, order-up-to-level *(s, S)* system,
3. periodic review, order-up-to-level *(R, S)* system, and
4. periodic review, order quantity, order-up-to-level (*R, s, S*) system.

The following table connects these four systems to the potential answers to the first two questions of the analysis:

Table 3. Guide to select the stock policy form

	Continuous review	Periodic review
A-class items	(*s, S*) system	(*R, s, S*) system
B- and C-class items	(*s, Q*) system	(*R, S*) system

In more detail, these four systems work in the following way. The first, order point, order quantity *(s, Q)* system has two operating parameters as signified by the name. Every time the inventory position (not on-hand or net stock) surpasses ordering point *s*, an order of size *Q* is placed. Inventory position, contrary to the net stock, is used to avoid situations, in which an order is placed due to low stock despite a replenishment arriving the next day. It is a robust and easy understand system that is supported by most ERP systems. (ibid., 242.)

Order point, order-up-to-level *(s, S)* system works in a similar style. Once the inventory level reaches the ordering point *s*, an order is placed of the amount, which would refill the inventory position to the number *S* units. (ibid., 242-243.) This system accounts for potential transactions that surpass the reordering point and strives to keep the inventory level at a more optimized point. However, it also requires data on the probability of a customer order surpassing the reordering point to determine the optimized up-to-level quantity of goods. It adds complexity to the data gathering and analysis stages. Therefore, it is suggested to be used only for A-class items. For B- and C-class items this system may often be overcomplex and consume more resources than adding value.

In companies without sophisticated computer control, the Periodic review, order-up-to-level *(R, S)* system is often being implemented. According to this form of inventory management, every period *R* an order is placed to the supplier to increase the inventory position to the level *S*. (ibid.) This system can produce significant savings in situations when several items are ordered from a single provider. Especially in cases of overseas shipments with a shipment unit being a Full Container Load (FCL), it is advisable to order items in bulk instead of delivering a single item once the reordering point is reached.

The idea of the fourth form, Periodic review, order quantity, order-up-to-level *(R, s, S)* system, is in reviewing inventory position each *R* period. If the stock position is lower than reordering point *s*, then an order is placed to increase the inventory position to the level *S*. This system has shown to produce the lowest total costs than any other system under quite general assumptions. On the other hand, the required data gathering and analysis can prove an obstacle in applying this system to every class of items. (ibid., 244-245.)

3.1.8 Decision rules under probabilistic demand

Previous parts of the thesis outlined different possibilities for establishing safety stock and four inventory models. Consequently, there is a variety of decision principles on how to combine the rules for safety stock with the inventory management systems in question. As this work could not cover the whole array of decision rules presented by Silver and others (2017) and Hopp and Spearman (2011), it restrained to the most used inventory management system – continuous monitoring, reorder point, order quantity *(s, Q)* model. This system, as discussed earlier, is chosen because of its good results compared to manageable computation difficulty.

The *(s, Q)* model relies on the use of the optimal order quantity that can be derived, for example, from the systems presented above, such as the Economical Order Quantity or the Dynamic EOQ. In the following equations d_L stands for expected demand during the replenishment time, *k* is a safety factor that can be determined as described later and σ_L is the standard deviation of the demand forecasts during the lead time. The basic equations are thus:

$$\text{Reorder point } s = d_L + Safety\ Stock$$

$$\text{Safety stock } SS = k * \sigma_L$$

Demand during the lead time has a probability density function $f_d(d_0)$. The function is defined so that $f_d(d_0)\partial d_0$ is the probability that the expected during the replenishment time demand lies between *d₀* and ∂d_0. In this case, the safety stock (which is the stock expected to be on the shelf when the next replenishment arrives) is:

$$SS = \int_0^\infty (s - d_0) * f_d(d_0) * \partial d_0 = s - \ d_L$$

Probability of a stockout during a replenishment cycle is $\int_s^\infty f_d(d_0) * \partial d_0$ – in other words, this formula represents the chance that demand will exceed the reordering point *s* (which includes both mean expected demand and safety stock). The models above are applied for the backorder model that allows the net stock to be negative, i.e. that the amount of sales can exceed the stock on the shelf.

In case the demand during the lead time is assumed to be of the normal distribution, then the probability of the stockout is simplified to $p_{u\geq}(k)$ where this value represents the probability that a standard normal will assume a value of *k* and larger. The unit normal is a normal distribution with mean 0 and standard deviation 1. The graph of the unit normal is presented in Figure 7 below. The marked area starting from 1 to the infinity represents the probability of a stockout when service factor *k* is 1.

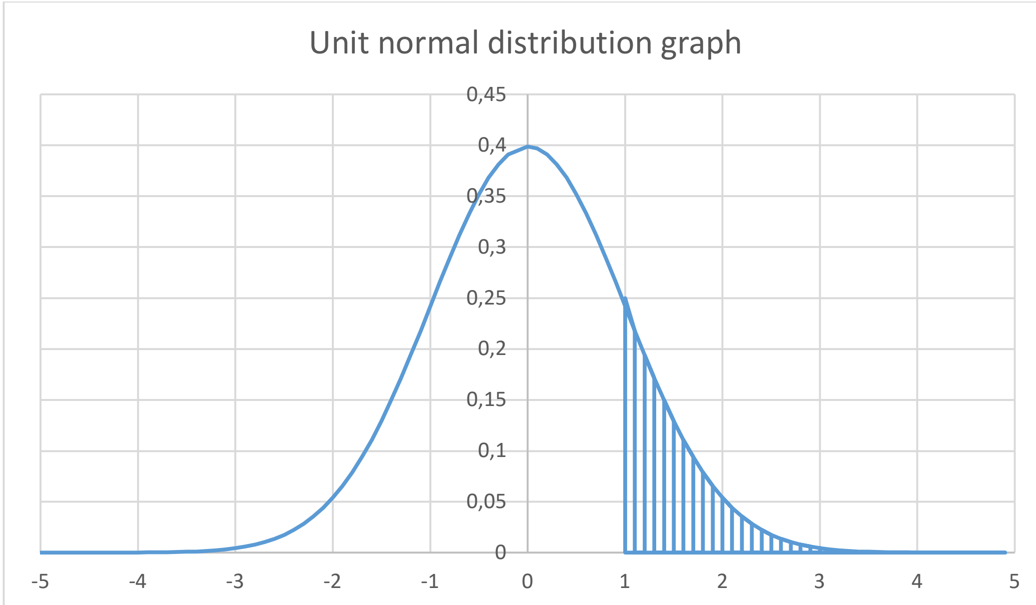


Figure 7. Unit normal graph

A possible graph of the inventory position under *(s, Q)* system is presented in Figure 8 below.

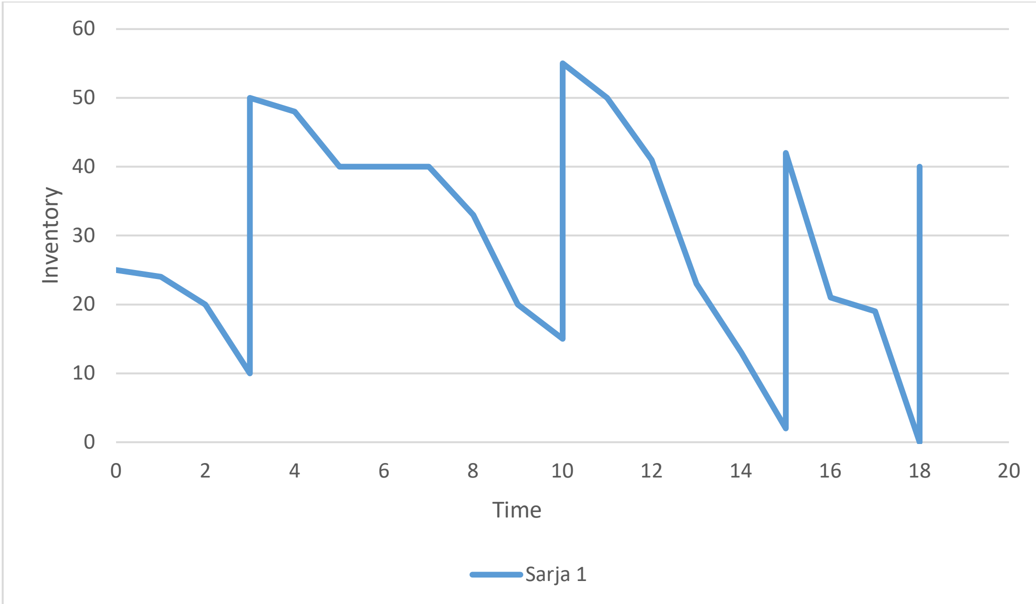


Figure 8. Inventory level in *(s, Q)* model

In this model, an order of 40 units is placed at the reordering point, which equals 20 units. The lead time is one period. The safety stock is kept at 10 units.

The decision rule for a specified safety factor k would be, as discussed earlier, simple: $Safety\ Stock\ = k * \sigma_L$ and reorder point $s = d_L + SS$. In case a known service level *P* is set up, then *k* should be chosen to satisfy $p_{u\geq}(k) = 1 - P$ (Silver et al. 2017, 265-266.)

In this system, total costs are $TC = ordering\ costs + inventory\ holding\ costs = \frac{A*D}{Q} + (\frac{Q}{2} + SS) * r * v$, where *A* is the ordering cost, *D* is the annual expected demand, *Q* is a purchasing lot size, *r* is the inventory holding cost (in % of value) and *v* is the total inventory (or item) valuation. This formula approximates the average throughput inventory level at $\frac{Q}{2}$. (ibid.)

The case of a specified cost per stockout occasion requires more calculation. It also adds the cost of stockout that is $\frac{D}{Q} * p_{u\geq}(k) * B_1$ to the total cost calculation where B_1 is the cost of a stockout occasion. Under the assumption of a normal demand distribution during the lead time, this formula evaluates the expected number of stockout occasions per year and multiplies it by the cost of a single stockout. (ibid.)

The following decision rule is established to determine the optimal *k* value. First, evaluate if $\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r} \geq 1$. If so, then $k = \sqrt{2\ln\left(\frac{DB_1}{\sqrt{2\pi}Qv\sigma_L r}\right)}$, otherwise, it is the lowest possible service level decided by the management (ibid., 266-267). The first evaluation is necessary because a natural logarithm of 1 is 0 and it is impossible under real numbers to evaluate a square root of a negative number.

In case there is an established value for a unit short B_2, it would be necessary to evaluate if $\frac{Qr}{DB_2} \leq 1$. If so, then *k* should be chosen to satisfy $p_{u\geq}(k) = \frac{Qr}{DB_2}$, otherwise, it should be an assigned from the management value due to impossibility of calculation - $p_{u\geq}(k)$. As a likelihood of an event, it can assume values of 0 to 1 only. (ibid.)

In the event of the safety stock is established based on the number of customer lines short (i.e. number of goods not delivered from a single order), the value *z* is introduced as an average number of units ordered per customer order. The cost of an item short is measured as *B₄.* Then *k* should be chosen to satisfy $p_{u\geq}(k) = \frac{Qrvz}{DB_4}$ or the lowest value set by the management. (ibid., 267-268).

If management desires to limit stockouts to a certain frequency, time between stockouts (TBS) can be established. Then, the demand during this timeframe *D(TBS)* is also measured. If $\frac{Q}{D(TBS)} \leq 1$, then *k* is chosen according to $p_{u\geq}(k) = \frac{Q}{D(TBS)}$. Otherwise, the minimum allowed service level *k* is used.

3.2 Optimization

Naturally, people strive to succeed. When it comes to decision making, humanity has been working long to learn to select the best possible choice out of offered possibilities. (Harari 2015, 79-80.) Nevertheless, individuals are not perfect in decision-making, neither they like this process, as can be proven by experiments of Kahneman and Tversky (2000). Therefore, a way to find the best solution to a given problem has been a lucrative promise for a long time. Now, this is the goal of optimization field of science.

Mathematical optimization or mathematical programming is a process of selecting the most suitable element from an array of potential solutions. Usually, the most suitable item is defined by mathematical functions that are called objectives. Potential solutions are evaluated by using these objective functions as well as restricted by additional constraints that also take the form of mathematical functions. (Dantzig 1963, 6.)

The previous sections have already presented optimization methods that are discussed in more detail in this chapter. For example, the Dynamic Economic Order Quantity algorithm presented in chapter 3.1.5 perfectly illustrated the definition given by Dantzig (1963, 6). It established the objective function (3.3) that described the best solution – periods at which an order should be placed. The possible solutions were constrained, as an example, by the absence of stockouts and by the nonnegativity of the order sizes.

Mathematical optimization is divided into different fields based on the form and number of the objective and constraint functions as well as the number of solutions to be identified. Optimization with one objective function is the most common type and, according to Bazaraa, Sherali, and Shetty (2013, 67-68), it includes 22 sub-fields, among which are linear and nonlinear programming, stochastic optimization, and heuristics. Optimization problems with several objective functions are solved by applying multi-objective optimization algorithms. Finally, if the goal of the algorithm is to find all suitable solutions, it is considered to be a part of global optimization. Global optimization algorithms are contrasted to local optimization ones, which identify optimal solutions on a part of the feasible region. (Amorim, Antunes, and Almada-Lobo 2011.)

3.2.1 Single-objective optimization

As mentioned in the previous paragraph, there are a few sub-fields of single-objective programming available. To give some overview of the techniques, this work describes linear, nonlinear, and stochastic optimizations as well as heuristics approach due to their use in the optimization of logistics problems.

Linear programming. Dantzig (1963), the father of linear programming, defines a linear problem as a problem that satisfies four assumptions: proportionality, nonnegativity, additivity, and it has a linear objective function. The first, proportionality, requires that the results of objective and constraint functions are proportional to the activity level. For example, the costs of investing in inventory are proportional to the value of inventory *v* a company has. If the company was to double its inventory, the costs would also be doubled in case they are defined as $v * r$, where *r* is the inventory holding costs in % of the value. (32-36.)

The second assumption is nonnegativity. According to the name, it requires that all decision variables used in the process are equal or above 0, although they still may be real numbers. (ibid.) The requirement is obvious for a real-world situation. For example, a factory cannot produce a negative number of products. However, mathematical models do not always account for that and an optimization algorithm may suggest procuring a negative number of units to optimize costs. This issue is usually handled by specifying appropriate constraints so that the optimizer would not analyze negative solutions.

As for the last two assumptions, additivity requires that the problem involves a system, where item flows are regarded as complete (ibid.). In other words, a warehouse can receive 10 items only if the same 10 items were sent there from another place.

A good way of solving linear problems is introduced in Excel spreadsheets with one of the algorithms presented by Dantzig (1963). MS Excel Solver requires a complete model coded into a spreadsheet to operate correctly. The inputs into the program are the variables' cells, constraint cells, and the objective cell. The variables' cells do not have to contain any information because the algorithm sets the optimized values into these cells. Constraint cells should be a result of one or several formulas applied to the variables' cells that produce a number or a Boolean as a result. In this case, the algorithm compares the values in these cells to the constraints entered and ensures the results satisfy these constraints. Finally, the algorithm adjusts the variables cells in a way that the objective cell is either maximized, minimized, or is as close to the set value as possible. (Saleh & Latif 2009.)

For linear problems, MS Excel Solver uses the Simplex LP algorithm that can be viewed in detail in the work of Dantzig (1963). This algorithm is universal and can be applied to any number of constraints and variables and, therefore, can be implemented to a variety of linear problems.

To describe the concept of this algorithm, several definitions need to be explained. First, a vector is an object that has both direction and magnitude. It is represented as $\vec{x}$ or $\overrightarrow{AB}$, where *A* is the originating point and *B* is the ending point in a two-dimensional space. Every vector $\vec{x}$ that satisfies all the constraints is called a feasible solution and the collection of all such solutions is called the feasible region. Feasible solutions region is the collection of all values of the objective function for the values of variables from the feasible region. Finally, the optimal solution to a single-objective optimization problem is defined as the point in the feasible region at which the objective function is no more than its value at any other point in the feasible region. This definition is appropriate for a minimization problem, however, in case of maximization problem, the objective function can be multiplied by *(-1)* to satisfy the minimization objective. In case there are several optimal solutions, they are referred to as alternative optimal solutions. Another definition useful to mention is of an extremum or extreme point. A local extremum is a value of the objective function that is greater or smaller than the values next to it. If the value is greater than its neighbor, it is called a maximum, otherwise it is a minimum. Global extremum is the value of the function that is greater or smaller than all other values of the function. (ibid.)

The feasible space formed by linear constraint functions has vertexes at points where constraints intersect. The main idea of Simplex is to iteratively move using a vector from one vertex to another in a way of improving the objective function. Since a combination of linear functions is also linear, the objective function has no extrema. Therefore, when an optimal solution is found, it becomes the solution to the problem. This solution lies on the boundary space defined by the constraints. A more formal definition is presented in the works of Dantzig (1963, 94-100) and Bazaraa and colleagues (2013, 75-80).

In addition to algebraic solutions, such as the Simplex algorithm, linear optimization problems can be solved graphically, if they have two variables. Moreover, graphical solutions offer a better understanding of the concepts of feasible region (or set), constraints and objective functions on a set of two variables. A typical linear optimization problem for two variables can be put as follows (Schulze 1998):

Maxime: $x + y$

Constraints:

$$2x + y \leq 14$$

$$2y - x \leq 8$$

$$2x - y \leq 10$$

$$x \geq 0, y \geq 0$$

The graphical representation of the constraints and maximized objective functions are presented in Figure 9 below. The feasible region is marked in blue and it defines the set of *(x, y)* pairs that can satisfy all constraints. The objective function is maximized at the point *(4, 6)* at value 10. In case of minimization, the optimal solution would be 0 at the point *(0, 0)*, since no negative values are allowed.

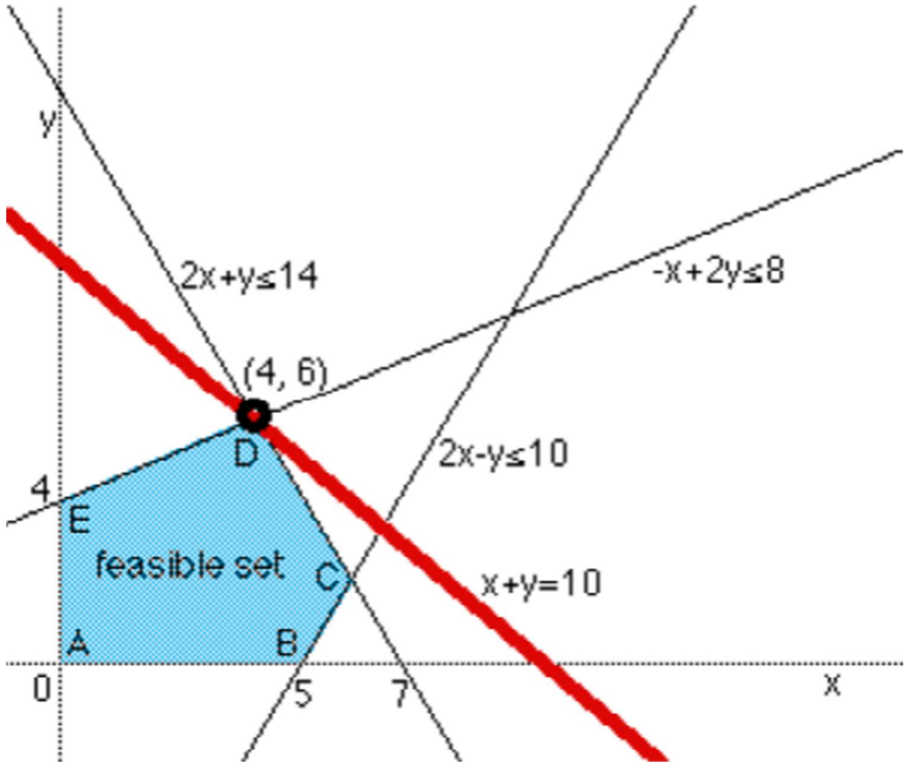


Figure 9. Graphical solution of a linear optimization problem

Nonlinear programming. In addition to linear problems, MS Excel Solver can solve nonlinear optimization problems using the Generalized Reduced Gradient (GRG Nonlinear) algorithm. Thus, the Solver Add-in allows solving the most common programming problems in the form of a spreadsheet. (Saleh & Latif 2009.) Nonlinear programming is discussed in the next paragraph.

A nonlinear optimization problem is formulated in the same way, however, the assumptions stated by Dantzig (1963) do not hold. Obvious from the name, objective and constraints function can be both linear and nonlinear. Although, same as in the linear optimization, the functions can take negative values, the real-world optimization often limits the value to nonnegative sets. (Bazaraa et al. 2013, 2-3.)

Nonlinear functions, in the first place, do not need to be proportional. An example of a nonlinear function can be $x^2 + \frac{1}{y}$. Some problems, as illustrated by Bazaraa and colleagues (2013, 3), can still be solved by the graphical method presented earlier. Figure 10 below represents a solution to the following problem:

$$Minimize: (x-3)^2 + (y-2)^2$$
$$Constraints: x^2 - y \leq 3$$
$$y \leq 1$$
$$x \geq 0$$

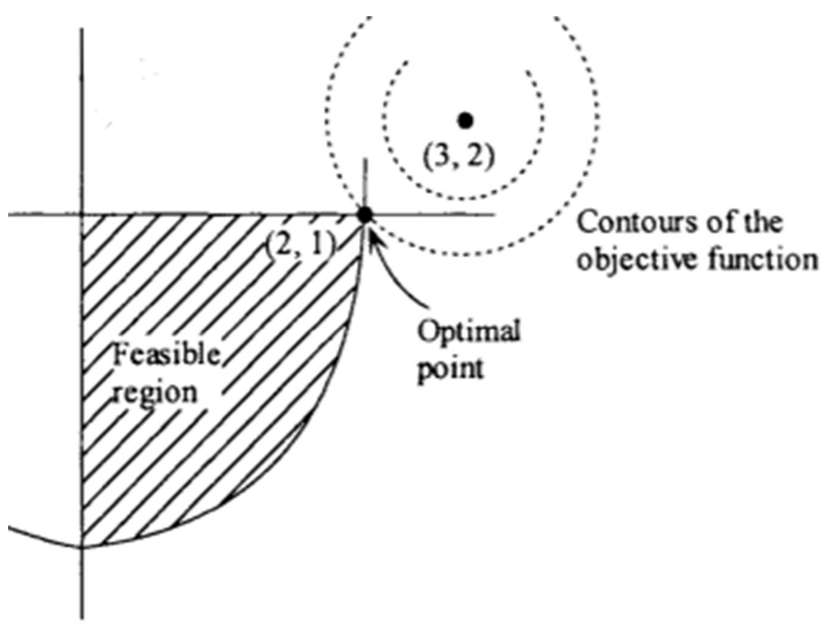


Figure 10. Graphical solution of a nonlinear optimization problem (Bazaraa et al. 2013, 3)

The feasible region, as restricted by three constraints, is marked in the first and second quadrants of the graph. The objective function is a circle with its center in *(3, 2)* and radius equal to the value of the function. Since the objective is to find such a pair of *x* and *y* that the solution is at the same time feasible and minimizes the objective function, the solution will be to increase the radius of the circle until it intersects with a solution from the feasible set. It happens at the point *(2, 1)* with the radius being 2. It is also the answer to the problem stated above. This is the most basic solution – to incrementally increase the objective function and analyze its value. In case of minimization problem, the solution would be to move towards decreasing the value and vice versa in case of maximization until reaching a point when any change to the variables would cause an undesirable change to the objective function's value. (ibid.)

However, this approach is not always applicable to nonlinear functions because they might have several local extremums. A global extremum is either the largest or the smallest value of a function over the feasible range of its variables. A local extremum is the smallest or largest point of a function on any of its parts such that a change to the variable in any direction would cause the function to change in the same direction (an increase or decrease). (ibid.) An example of a function with several local minima and maxima is in Figure 11 below. The function is $2 * \sin(x) + x$ and it has no global extremums due to both being infinities. Local extrema are marked in red.

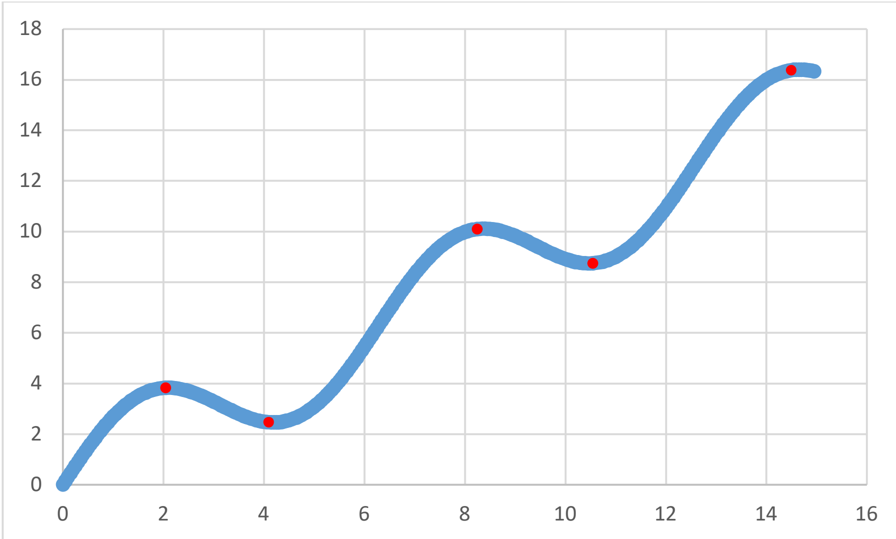


Figure 11. Graph of a function with several local minima and maxima

The idea of the algorithm used in MS Excel Solver for optimizing nonlinear problems is similar to the one described above. The algorithm is called Generalized Reduced Gradient (DRD) and, as the name suggests, relies on the gradient of the function to arrive at the optimal solution. The algorithm computes the gradient at a feasible solution (i.e. a set of variables from the feasible set) and determines the direction from the gradient, along which the objective function improves. Then the program adjusts to a new feasible solution following the direction calculated and establishes the gradient again. The process continues until the program arrives at an extremum of the objective function. A graphical example of the execution of the program is in Figure 12 below. A gradient is marked with red and it is always perpendicular to the value curves of the function.

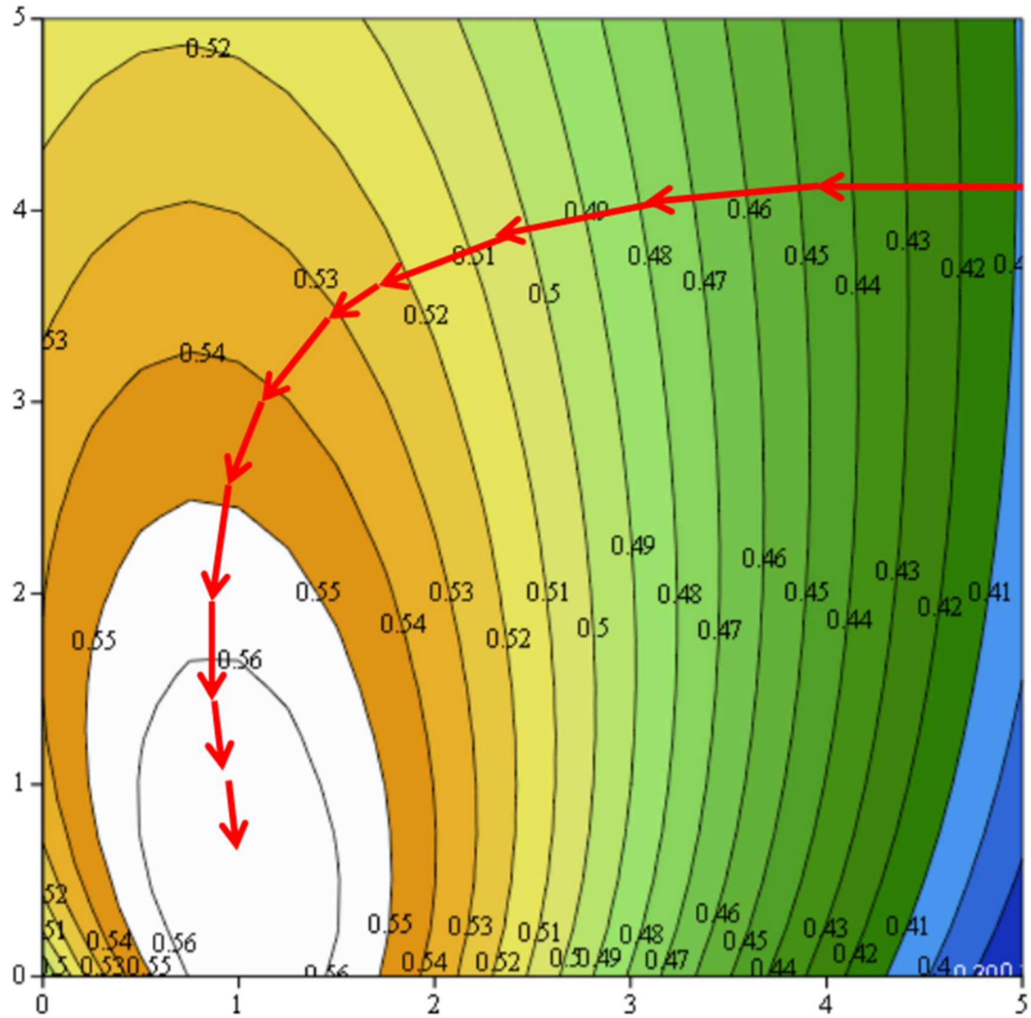


Figure 12. Graphical representation of the gradient method (Lehtola 2017)

3.2.2 Multi-objective optimization

The previous section detailed just a few of a variety of developed single objective optimization algorithms. Another algorithm of this field is mentioned in this section – Particle Swarm Optimization (PSO). This amount of work done to solve optimization problems proves the importance of this topic for decision-makers all around the globe, and it gives a robust base for future developments. However, modern challenges often require consideration of several objectives that are usually contradicting each other.

To address this need, a field of Multiple Criteria Decision-Making (MCDM) was developed. It allows to address problems with several objectives scientifically rather than allowing decisions to be made according to the intuition of a decision-maker. Multiple Criteria Decision-Making is further divided into two sub-fields: multi-attribute decision analysis and multi-objective optimization. The former helps in identifying optimal solutions on a discrete set of feasible solutions. An example can be identifying the best place for a factory out of 11 available sites optimizing production costs, delivery times, and workforce availability. Multi-objective optimization solves MCDM problems that include an undefined and usually indefinite set of feasible solutions. (Miettinen 1999, 3-4.) An example here is inventory optimization of costs and service levels, considering order size and safety factors as variables. Both variables can take any value restricted only by constraint function, such as that both should be positive.

Multi-objective optimization is further developed into linear and nonlinear optimization, which were discussed in the previous section. In multi-objective linear programming, all objective and constraint functions need to be linear. These problems are often solved with mathematically easier methods that exploit characteristics of a linear function and that do not apply to nonlinear multi-objective tasks. However, nonlinear methods can be applied to linear problems, although it could cause inefficiency. (ibid., 4-5.)

The main fundamental difference of multi-objective optimization comes from the idea of the optimal solution. In single-objective optimization, there is only one value of the objective function that is considered optimal (although there may be several or even infinite number of variables that produce this result). It is usually possible to distinguish if the objective function’s value 10 is better than 1 in case of maximization. (ibid., 10-13.)

In multi-objective optimization, it is also possible to conclude that a solution with the values for two objectives (6, 10) is better than (5, 9) in case of maximization. However, it is not so easy to compare results (6, 10) and (9, 5). To define an optimal solution in multi-objective optimization problems, the Pareto optimality concept is used. (Lalwani, Singhal, Kumar, and Gupta 2013.)

According to the Pareto optimality concept (or sometimes Edgeworth-Pareto), a solution can be taken into consideration if none of its components can be improved without deteriorating others. Formally for maximization problems, a decision vector x^1 is considered Pareto optimal if there exists no vector *x* such that $f_i(x) \leq f_i(x^1)$ for all *i = 1, .. , k* and $f_j(x) < f_j(x^1)$ for at least one index *j.* Where f_j are the objective functions and *k* is the number of objective functions in the problem analyzed. (ibid.)

Another way to define the set of optimal solutions can be carried out through a concept of domination. A decision vector *x* is said to dominate a vector x^1 if it is better in at least one objective function *f(x)* while not being worse in all objective functions. A vector is called nondominated if no feasible solution vector, which dominates it, exists. According to the two definitions presented above, any point on the Pareto optimal front is a nondominated decision vector and a set of all nondominated vectors form Pareto optimal solutions. (Miettinen 1999, 23-25.)

Figure 13 below illustrates the Pareto Optimality concept. The space marked in black is the feasible solution region – any of the dots in this zone can be achieved at certain values of the two (in this case) variables for two objective functions. The red line is the Pareto optimal front of solutions, which answers the task of optimization of the given functions. There are no other solutions to the problem that could improve at least one function without sacrificing progress of another.

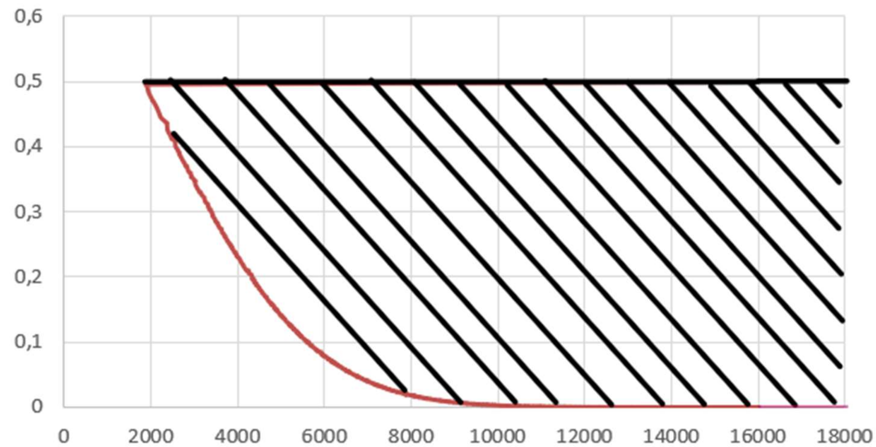


Figure 13. Example of the Pareto Optimality concept

Since there can be an infinite (or nearly infinite) number of solutions to a given problem, a new element is required to produce the final solution. It is the decision-maker who can analyze the Pareto optimal solutions produced by an algorithm and choose the best one according to his or her own experience, values, instructions, and other guiding principles. (Miettinen 1999, 14-15.) There are several mathematical ways to rank produced solutions, one of which is the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS). This technique evaluates all Pareto optimal solutions and searches for the closest to the ideal solution. (Lalwani et al. 2013.) It is examined in more detail later in this work.

Miettinen (1999) outlines four big groups of methods developed for multi-objective nonlinear optimization:

- no-preference methods,
- a posteriori methods,
- a priori methods, and
- interactive methods.

Several methods presented in the work of Miettinen (1999), such as weighted sum and ε-constraint, simplify the multi-objective problem by converting it to a single-objective and then solving using existing and proven methods. (67-211.)

3.2.3 The MOPSO

This work does not aim at giving a comprehensive overview of optimization methods that are presented, for example, in Miettinen (1999), Bazaraa and coworkers (2013), and Srivastav and Agrawal (2017). Since the goal is to illustrate a practical application of one of the methods presented in literature, later in this work only the methods chosen for development were studied in detail. Therefore, after analyzing potential approaches to solving the problem in question, it was decided to apply Multi-Objective Particle Swarm Optimization (MOPSO).

This algorithm is robust, allows to easily update objective functions and constraints, and effectively identifies Pareto optimal solutions as proven in Srivastav and Agrawal (2017), Padhye, Branke, and Mostaghim (2016), Lalwani and colleagues (2013), Tsou (2007), Tsou, Cheng, Lee, Huang, Song, and Teng (2013). The MOPSO is an extension to the Particle Swarm Optimization (PSO) algorithm that aims at solving single-objective optimization problems by relying on a population of particles searching the feasible solutions region for nondominated vectors.

PSO. The particle swarm optimization algorithm was first presented by Kennedy and Eberhart (1995) and soon gained popularity due to simplicity and versatility of its use. The algorithm was inspired by the aesthetics of bird flocking and slightly simulates it. It generates an array of particles that are conscious of their position, speed, and best-found value during movement. All particles are aware of the most suitable solution found by the whole array. With every iteration, particles move by adjusting their speed to get closer to their personal best point and the global best point. (ibid.)

A more formal description requires the use of formulas. Number of variables *d* produces a d-dimensional feasible space (such as in 2D, 3D, etc.) Let a particle's *i* position be $\vec{x}_i$= (x_{i1}, x_{i2}, .. , x_{id}), velocity $\vec{V}_i$= (V_{i1}, V_{i2}, .. , V_{id}), and personal best-achieved position $\vec{P}_i = (p_{i1}, p_{i2}, .., p_{id})$, as well as its location $\vec{x}_{pbest,i} = (x_{pbest,i1}, x_{pbest,i2}, .., x_{pbest,id})$. Index *i* can be a value from 1 to *n,* where *n* is the number of particles. (Lalwani et al. 2013.)

With every iteration t, every particle *i* updates its speed for the next iteration:

$$V_i^{t+1} = \omega * V_i^t + c_1 r_1 (x_{pbest,i} - x_i^t) + c_2 r_2 (x_{gbest} - x_i^t) \tag{3.4}$$

where ω is the inertia weight, which is usually between 0.8 and 1.2, c_1 is the cognitive acceleration coefficient, usually between 1 and 4, c_2 is the social acceleration coefficient, r_1 and r_2 are random weights from 0 to 1 and x_{gbest} is the location of the best-found value for the whole array of particles.

Once the new speed is found, the particle moves to a new position:

$$x_i^{t+1} = x_i^t + V_i^{t+1} \tag{3.5}$$

An optimization problem is stated as usual: minimize/maximize function $f(\vec{x})$, subject to *j* constraints $g(\vec{x})$. Chart 14 below represents the flow of the PSO algorithm:

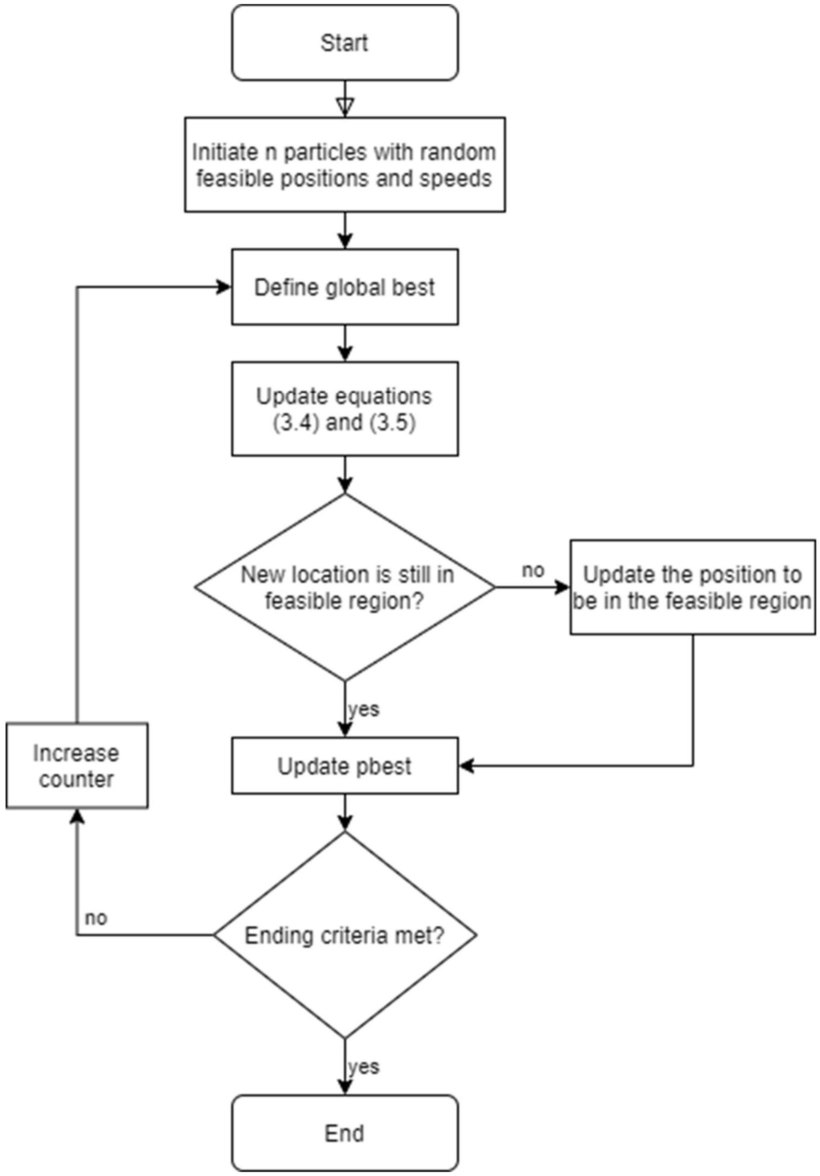


Figure 14. PSO algorithm's execution chart

The MOPSO. Multi-Objective Particle Swarm Optimization algorithm is based on the same execution pattern as described above for the PSO algorithm, except, for multiple objective functions. This creates issues with identifying the global best solution since there can be several nondominated solutions.

To solve this, the MOPSO algorithms usually maintain an external array of nondominated solutions that is updated with each new solution found by particles. With every iteration a global best solution is selected from this archive and is used for an iteration over all particles. To increase the diversity of solutions, some authors (such as Padhye and colleagues (2016), Srivastav and Agrawal (2017)) propose keeping an archive of personal nondominated solutions. There are several ways to select that global and personal best solution, as outlined in Padhye and coworkers (2016):

1. Random selection. It is the least computationally expensive and is easy to implement. It can be implemented for both personal and global solutions.
2. Sigma method. In this approach, a nondominated solution is selected for each particle to be the closest to the direct line between the particle and its origin. This method requires more computational power and sometimes generates a quite small variety of solutions. It is originally developed for the global solutions archive but can be used for personal best as well.
3. WSum (weighted sum) method. In this approach, the preference is given to solutions that enhance the function where the particle is already good relative to other particles. It leads to an increase in the divergence of particles and a variety of solutions. WSum approach was developed for personal solutions but can be applied for the global archive too.
4. The Pareto dominance guide selection methods. This set of methods suggests that the most suitable guide for a particle is a solution that dominates it (i.e. not worse in any function and better at least in one). A few methods from this set are Rounds, Random, and Prob. The rounds methods is the most mathematically complex, because it aims at assigning each particle a global solution that dominates over the least number of particles, thus promoting diversity. In Prob, each solution is assigned a weight according to the number of particles it dominates and then solutions are randomly assigned the given weight as a probability.

Srivastav and Agrawal (2017) compare the performance of the MOPSO algorithm with the extensions presented above to conventional evolutionary algorithms: Vector Evaluated Genetic Algorithm (VEGA), Multi-Objectives Genetic Algorithm (MOGA), Non-dominated Sorting Genetic Algorithm (NSGA), NSGA-II, and Multi-Objective Cuckoo Search (MOCS). As supported by the results of Padhye and colleagues (2016), the Random MOPSO algorithm performs well in error ratio, hypervolume, running time, and maximum spread metric compared to other algorithms and versions of the MOPSO.

The Multi-Objective Particle Swarm Optimization algorithm is presented in Chart 15 below. It includes several starting steps that are omitted in the previous PSO algorithm's illustration: initialization of parameters and generation of empty arrays. These steps are included here to represent the programming code written for the algorithm more accurately. The selection of the next and the first particles, which is also present in the Particle Swarm Optimization algorithm's code, is outlined in Figure 15 below for the same reasons.

The main noticeable difference between both algorithms is in dealing with several objective functions. The description of the control of nondominated solutions in the global best archive as well as the comparison of the new position of each particle to the archive were omitted from this part and are described in the implementation stage as they were developed in the course of this work and not borrowed from literature.

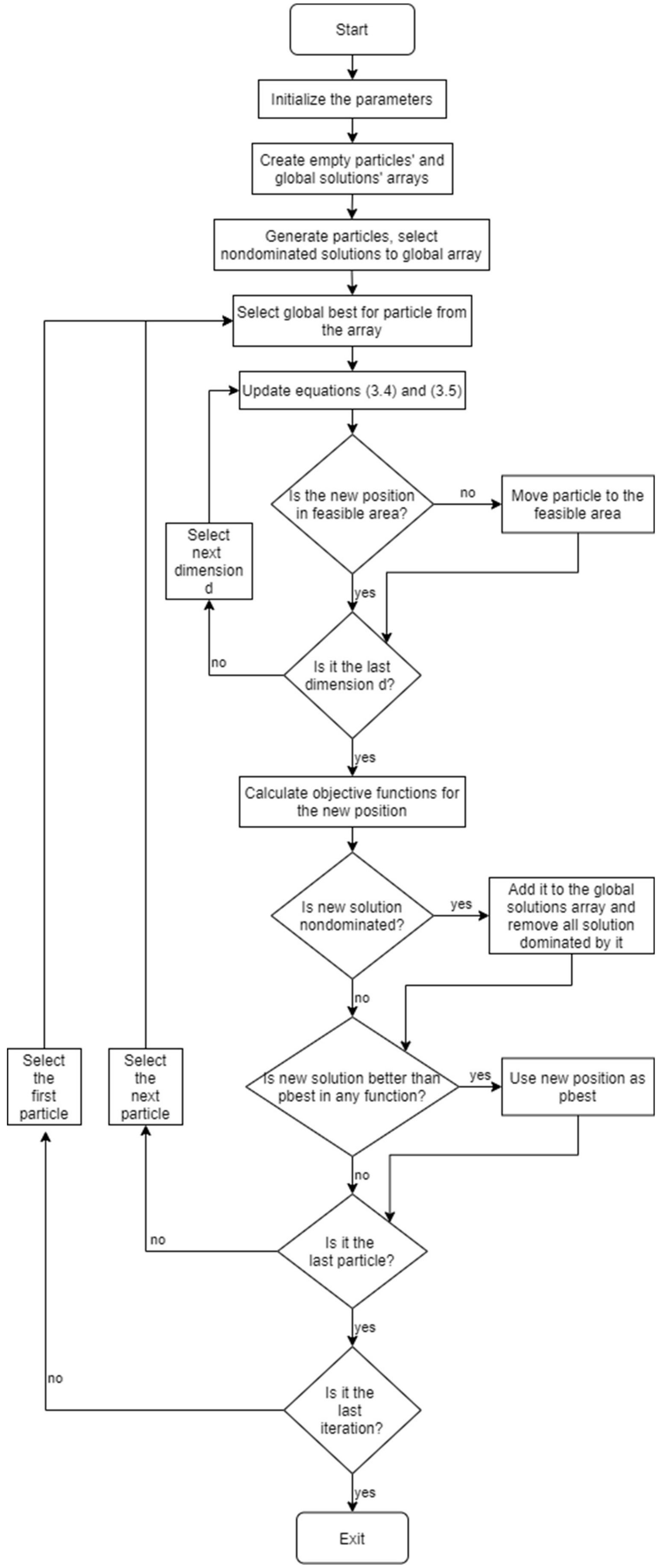

Start
Initialize the parameters
Create empty particles' and global solutions' arrays
Generate particles, select nondominated solutions to global array
Select global best for particle from the array
Update equations (3.4) and (3.5)
Is the new position in feasible area?
no
Move particle to the feasible area
yes
Select next dimension d
no
Is it the last dimension d?
yes
Calculate objective functions for the new position
Is new solution nondominated?
yes
Add it to the global solutions array and remove all solution dominated by it
no
Is new solution better than pbest in any function?
yes
Use new position as pbest
no
Select the first particle
Select the next particle
no
Is it the last particle?
yes
no
Is it the last iteration?
yes
Exit

Figure 15. The MOPSO algorithm

There are plenty of decisions to make, while setting the algorithm and its sub-algorithm. Some of the possibilities for particle guide selection are presented at the beginning of the MOPSO section, while a non-dominance validation sub-algorithm is presented in the implementation stage. Another decision considers the step of keeping particles within the feasible area boundaries.

There are several options to handle the particles that leave the feasible area. One is to return them on the border point of the feasible area, which they have crossed, with zero speed. The particle will start moving in the next iteration toward the global and local best positions. Another way is to “bounce” the particle from the border. It means that the particle, which travels towards the border, reaches the border, then changes its direction and travels back the remaining distance. The speed is changed to the opposite. A third method is used to return the particles to their starting positions or a chosen position from the personal best archive. Finally, the particles can be randomly set inside the feasible area. (Fu, Wang, Zhang, Zhao & Wang 2019.)

Each of the methods presented can offer satisfactory results since particles will move towards selected guides in any case of the particle placement. The research indicates that over various problems, these boundary handling techniques perform well compared to each other. Therefore, the final algorithm’s solution may be chosen by a researcher at their consideration. (Padhye et al. 2016.)

Noteworthy, there was not much said regarding the objective and constraint functions during the description of the algorithm. It is due to the versatility of the MOPSO execution programming code. If written in a general manner, the MOPSO algorithm can easily handle the change of functions, feasible area, and feasible solutions area as well as settings changes.

Due to the versatility of the presented approach, it is used in different engineering and programming areas. Lalwani and colleagues (2013) identify 196 research works that have applied the MOPSO algorithm from the year 2006 to 2012. The areas of application range from aerospace, civil, industrial, and software engineering to biological sciences, image processing, and neural networks.

The main restraints of this algorithm arise from the calculation demands from the execution computer. The results are the most exact if the global solutions' array is unlimited, as well as particles' speeds and even their personal non-dominated solutions' archive. Moreover, more particles and iterations usually bring more detailed results. (Padhye et al. 2016)

However, since such enhancements usually lead to longer execution times, it is not always feasible to have unrestricted solutions arrays and high numbers of iterations and particles. Constraints are often determined by comparing the quality of solutions at different algorithm's settings to decide on the most appropriate balance between quality and speed. (Tsou 2007). It is a multi-objective problem of its own.

3.2.4 The TOPSIS

As was discussed in the previous sections, the MOPSO algorithm produces an array of nondominated solutions that need to be classified to help in the selection of the potentially best value. One of the ways, as proposed by Tsou (2007) and Srivastav and Agrawal (2017), is to combine the MOPSO algorithm with Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS).

Although the name may sound vague, the algorithm is straightforward. First presented by Hwang and Yoon (1981), the algorithm was reproduced in the work of Gwo-Hshiung and Jih-Jeng (2011, 69-71), from where it is adapted to this study.

Let a set of alternatives $A = \{A_k | k = 1, .., n\}$, i.e. there are *n* alternative solutions. Set of criteria $C = \{C_j | j = 1, .., m\}$, so that there are *m* criteria for each alternative. A value of each alternative *k* for a given criteria *j* is denoted as performance rating

$X = \{x_{jk} \,|k = 1,..,n; j = 1,..,m\}$. Finally, $w = \{w_j | j = 1,..,m\}$ is the set of weights for each criterion *j,* as defined by decision-maker. (ibid.)

The idea of the algorithm is to identify the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for the data given. The positive ideal solution in case of minimization is the smallest value in each criterion and vice versa in case of maximization of each criterion. Then the algorithm compares each alternative to its Euclidian distance to the PIS and the NIS. Finally, the algorithm awards ratings to each of the alternatives in a way that the highest rating is given to the solution that is the closest to the PIS and the farthest from the NIS. (ibid.)

First, the TOPSIS calculates normalized ratings for each performance rating x_{jk} to have the same scale of units across all criteria. A normalized rating r_{jk} is calculated as each performance rating divided by the square root of the sum of all performance ratings for the same criterion *j*:

$$r_{jk}(x) = \frac{x_{jk}}{\sqrt{\sum_{k=1}^{n} x_{jk}^2}}$$

Weighted normalized ratings are $v_{jk}(x) = w_j * r_{jk}(x)$.

In case of minimization objective, PIS and NIS are defined as:

$$PIS = \{v_1^+, v_2^+, .., v_m^+\} = \{\min_k v_{jk} \,|k = 1,..,n\}$$

$$NIS = \{v_1^-, v_2^-, .., v_m^-\} = \{\max_k v_{jk} \,|k = 1,..,n\}$$

where v_j^+ is the smallest value of all *n* alternatives in the criterion *j* and v_j^- is the largest value. It is important to note that usually the Positive Ideal Solution is not in the feasible region. Otherwise, it would be the dominant solution and the data would have only one nondominant solution, i.e. the objective functions would not conflict.

Then for each alternative *k (k = 1, ... , n)* the distances to the PIS and NIS solutions are calculated:

$$D_k^+ = \sqrt{\sum_{j=1}^{m} \left(v_{jk} - v_j^+\right)^2}$$

$$D_k^- = \sqrt{\sum_{j=1}^{m} \left(v_{jk} - v_j^-\right)^2}$$

The similarities to the Positive Ideal Solution are:

$$C_k^+ = \frac{D_k^-}{D_k^+ + D_k^-} \qquad (3.6)$$

As can be noticed, the shorter the distance to the Negative Idea Solution is, the smaller the rating of the TOPSIS for the same value of the distance to PIS is. Furthermore, the closer the alternative to the PIS while being at the same distance from the NIS is, the higher the rating is. The rating can take values from 0 to 1 (in case of the NIS and PIS solutions respectively). Figure 16 below illustrates this algorithm.

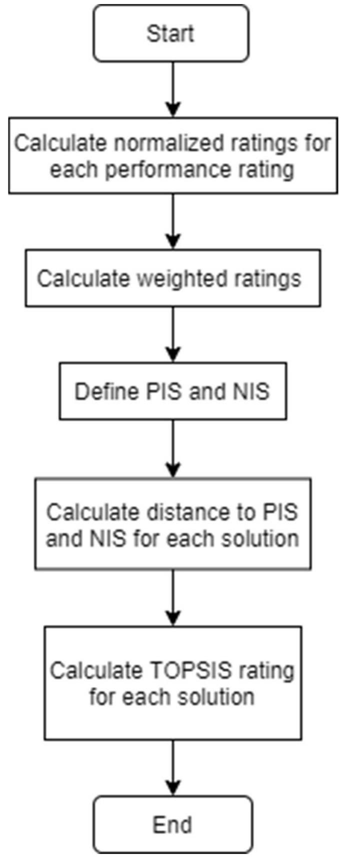


Figure 16. The TOPSIS algorithm

3.2.5 Coding languages

The algorithms of the MOPSO and the TOPSIS presented above can be calculated by spreadsheets or even by hand, although, they were designed to be programmed and executed on a computer. During programming evolution, humanity has developed several programming languages to satisfy various needs.

A programming (or coding) language is a formal language that defines a set of instructions to execute in computer programming. Sheikh and Islam (2016) identify eight major programming languages and conduct a qualitative study. The chosen languages were C, C++, Python, Java, Pascal, GW-Basic, and JavaScript.

The analysis of these programming languages is done according to ten criteria: simplicity, writability, reliability, appropriate data structures, availability, market demand, community support, machine limitations, libraries, and coverage. Out of the eight chosen languages, three are selected as the best general-purpose programming languages: Java, Python, and C++. (ibid.)

As it is not connected to the core of this work, the analysis did not go deeper into the differences of the defined programming languages. The main idea of these four paragraphs is that it is advisable to use computer programming languages with respective software installed on the execution computer to implement many optimization models. Programming languages differ in details of use and user-friendliness, however, most of them can perform the defined algorithms if they are coded correctly, according to the requirements of each language. (ibid.)

4 Research implementation

A significant amount of literature is written on the principles of research implementation. In case of this work, the goal is to develop an optimization system that acts as a tool for operations of EKE-Electronics Oy. To develop a system, it is advisable, as suggested by Little (1970, 466– 485) and Shiflet (2006), to carry out the following steps:

1. Problem analysis. Identify fundamental questions, such as research questions stated at the beginning of this work.

2. Data gathering. Collect primary and secondary data as discussed in the research methods.

3. Simplification of the model to the most reasonable level. As advised by Silver and coworkers (2017, 47-48), the model should be adjusted to balance between exact optimization and complexity. Overcomplex models may

consider every possible detail of the environment but prove to be difficult in use. Thus, user error may overweigh the potential benefits of exact modeling.

4. Variables and units. Establish variables required for analysis, their units, objectives, and potential constraints.

5. Connection of the variables and objectives by formulas.

6. Model implementation and solution.

7. Solution validation.

8. Modeling process documentation.

9. Maintenance of the model.

In the section "Research methods", the definition of the first two steps was already put. The main research question was defined as "*What could be a suitable inventory optimization system for EKE-Electronics?".* The selected methods of data collection are both primary and secondary data. Secondary data was already analyzed in the previous section "Literature review" and this section focuses on the primary data analysis.

The "Literature review" section presented several potential solutions to be implemented at EKE-Electronics. For example, there were four types of replenishment policies under probabilistic demand presented, such as order point, order quantity *(s, Q)* and order point, order-up-to-level *(s, S)* systems. Moreover, classical and time-proved systems for deterministic demand were studied, as Economic Order Quantity with a Dynamic extension. Besides, decision-making and simplification were already implied in the process of literature selection, as was also mentioned in the Optimization section.

4.1 Inventory replenishment policy

The major step in designing the inventory optimization system in this work was to identify the most appropriate inventory system to use. First, the characteristics of the problem are summarized in the table below. This data was received from the discussion with EKE’s management and data analysis of the product and module inventories in ERP software. The table is followed by a more detailed explanation of each point.

Table 4. Model's characteristics

Demand type (deterministic, probabilistic or fuzzy)	Probabilistic
Importance of items optimized	A, B, and C classes
Continuous or periodic review	Continuous review
Inventory policy form	Order point, order quantity *(s, Q)* system
Objectives	Order quantity, safety stock quantity per service level, turnover level

Deterministic demand is a case when sales forecasts are exact, and it is possible to plan shipments of the exact amount of goods on exact days over a long horizon. If the quantities shipped are moreover the same, then the EOQ model can be applied directly. If quantities differ, then the Dynamic EOQ model is suitable.

Probabilistic demand is an uncertain scenario, where expected demand is estimated to be a certain number with a given probability. Different probability distributions can be used, such as normal, Poisson, lognormal, and even discrete distribution. This is the case of EKE-Electronics, where future demand can be estimated based on forecasts and historical data. Probability demand inventory systems can be applied in this case.

Fuzzy demand pattern is impossible to predict and therefore, is hard to plan. Since this is not the case of EKE, such demand systems were not presented in the literature review. More can be learned from Silver and colleagues (2017) and Hopp and Spearman (2011), as an example.

To select an appropriate inventory policy, it is useful to analyze the type of items to be optimized under that system. More computationally complicated systems usually may pay off only by application to the most important, A-class, items. While to more general items, it may be useful to apply a more heuristic approach. (Silver et al. 2017, 47-49.)

A useful and clear approach for item classification is the ABC system. This system groups inventory items into three clusters based on the annual turnover (in value) of each item. Thus, items that bring the most revenue to the company, receive the highest attention. It does not explicitly mean that these items are the most expensive or the most sold, though. (Hopp & Spearman 2011, 587.)

In case of EKE-Electronics, the optimization system includes all three types of items because it should be designed now for the whole inventory selection. As a development opportunity, a more sophisticated approach may be applied to A-class items in the future. Thus, a general inventory policy is selected.

The last classification point for inventory systems presented in the literature review chapter is the regularity and frequency of the inventory review. If it is done regularly within a short interval of time (such as daily or every few days), then the system can be approximately considered as continuous. In other words, a manager may at any moment know the exact inventory position and can notice when there are fewer items than the set reorder point.

The opposite of the continuous review is a periodic review system. In this system, the inventory is checked still regularly but at noticeably longer periods – such as a week. In this case, the inventory policy needs to consider the possible demand during the review period. This review period can be included in the lead time as one of the solutions.

EKE-Electronics has implemented a modern ERP system, which employees use daily. Therefore, inventory position may be checked continuously, and orders placed when required to replenish inventory.

Considering the three decision points presented above and guidelines from Silver and others (2017), it may be concluded that the order point, order quantity *(s, Q)* system can be implemented in this case. Later, when A-class items are separated from B- and C-class modules, another system may be implemented.

Under *(s, Q)* system, the main goal is to determine the reorder point s, including objectives set by management and personnel. A possible list of objectives is presented in the literature review part, from which EKE has selected the expected stockout frequency objective, according to the strategy of focusing on customer satisfaction. Another major objective of the inventory system is to minimize the total costs of the inventory management process. A supporting objective, that can be derived from the *(s, Q)* system is the turnover level that is calculated as the ratio $\frac{annual\ sales\ (in\ units\ or\ €)}{average\ inventory\ (in\ units\ or\ €)}$. Thus, EKE wants to find a balance between the main two objectives.

The two optimization objectives are: total costs function and stockout probability. Total costs are calculated as a sum of ordering costs, inventory holding costs, and warehouse costs. The only real payable price in this formula is the warehouse space, other components account for the work of the purchasing department, and opportunity costs of invested capital. Although these costs are not being directly paid for each inventory item, they are necessary for the optimization process not to set too small ordering sizes that would be hard to replenish continuously.

During the data collection process, it was found out that the main production subcontractor does not charge warehousing prices now and has space to store modules until assembly into components. Therefore, the warehousing element is omitted from the formula for the moment. If the subcontractor, however, notifies that an extra price for the increased stock of material is to be charged, then it would be easy to add this cost element to the formula and would not require major modifications to the model.

Finally, the cost formula used in this work is the following:

$$Total\ Costs\ (TC) = A * D \,/\, Q + h * c * (Q \,/\, 2 + k * \sigma)$$

where *A* is the costs of single purchasing order (measured in time), *D* is the annual demand of an item, *Q* is a size of a single replenishment order, *h* is an annual carrying cost of a single inventory unit, measured as a percentage of the item's value, *c* is the cost of a single inventory unit, *k* represents the chosen service level, and σ is the standard deviation of demand during lead time.

As for the inventory holding costs, the Weighted Average Cost of Capital (WACC) value is calculated and used. The WACC formula and description are presented in the first part of the Literature Review. The calculation values are presented in Appendix 1 with the final value for WACC. This value is used as *h* in the optimization model.

Customer satisfaction with the quality of the delivery process is measured according to the frequency of stockouts. It is calculated as $\int_s^\infty f_d(d_0) * \partial d_0$, where $f_d(d_0)$ is the statistical distribution of demand during lead time. In this case, it is considered as normal distribution. The reorder point *s* is calculated as $s = D_L + k * \sigma$. D_L is the demand during the lead time variable that is calculated for each SKU individually.

Finally, there are two constraints included in the model. The first one considers the optimization variable of the order quantity. The order quantity should not be less than 0 and not more than the annual demand (to limit the search space to reasonable bounds). The second one concerns the customer service objective – *k.* It should not be less than 0 as well and should not lead to safety stock exceeding the annual demand. Both formulas are presented in the table below.

A table below summarizes the objective functions used in the model as well as variables and constants that are required from the user. The explanation for each variable and formula can be found in this chapter.

Table 5. Summary of optimization data

Decision variables	
Order quantity, units	*Q*
Service level, %	*k*
Variables entered by a user	
Ordering cost, €	*A*
Lead time, months	*L*
Annual demand, units	*D*
Inventory carrying costs, %	*h*
Item's value, €	*c*
The standard deviation of lead time demand	*σ*
Objective and other functions	
Cost function, to minimize	$A * \frac{D}{Q} + h * c * (\frac{Q}{2} + k * \sigma)$
Stockout likelihood, to minimize	$\int_{s}^{\infty} f_d(d_0) * \partial d_0$
Inventory turnover	$\frac{2 * D}{Q}$
Reordering point	$D_L + k * \sigma$
The table continues on the next page	

The table continues from the previous page	
Constraints	
Order quantity, units	$0 \leq Q \leq D$
Service level	$0 \leq k \leq \frac{D}{\sigma}$
Calculated values	
Expected demand during lead time, units	$D_L = L * \frac{D}{12}$
Reordering point, units	$s = D_L + k * \sigma$

Previous sections covered the steps 1-5 from the model developing system, as identified by Little (1970, 466– 485) and Shiflet (2006). This part discussed the potential for simplification of the model while satisfying the requirements of EKE. Such a simplification is to use *(s, Q)* inventory management system. Then variables are defined in the last part of this chapter together with formulas connecting each variable and constant. Finally, the table above summarizes the progress.

4.2 The MOPSO model

The next step of the Little's (1970, 466– 485) and Shiflet's (2006) system is to implement the model. To satisfy the requirement of optimizing both costs and service level, it is necessary to use multi-objective optimization techniques. Both cost and stockout likelihood functions are nonlinear. The cost function is nonlinear due to the ordering cost $A * \frac{D}{Q}$, since division by the variable turns the function into nonlinearly dependent on the order quantity. The stockout likelihood is nonlinear because of at least integrating. Constraints and reordering point calculation are linear (only addition and multiplication of the variables with constants), while the inventory turnover is again nonlinear due to division by *Q*.

Since at least one function in the optimization model is nonlinear, the problem at hand is a case of nonlinear multi-objective optimization. To solve it, the MOPSO model is chosen and designed as described in the Literature Review section. The MOPSO is preferred due to its versatility and relative ease of implementation. Moreover, theoretical results and comparisons prove the algorithm to be as good as other multi-objective nonlinear algorithms or even outperform them. Finally, it supports potential for the initial inventory model modification and development well.

The algorithm was implemented in the Python programming language. Two of the programming languages discussed at the end of the Literature Review section were available for use for this work due to previous experience of studying and using – Java and Python. Python is faster to install on new computers and debatably easier to use. Since there was no programming code presented in the literature, the program was created from scratch according to the outlined algorithm in Figure 15.

A custom enhancement to the algorithm was made in the step of new nondominated solution inclusion. If a new nondominated solution was identified during the update of a position of a particle, then the global array of nondominated solutions should be updated. All dominated solutions should be deleted at this step and the new particle's position should be added.

Figure 17 below represents the form of the Pareto-optimal front for this problem. The vertical axis is the chance of stockout during a single replenishment cycle (with the maximum being 50%) and the horizontal axis is the total cost of the solution. It is logical to see that better service levels cost more. The worst service level of 50% is justified by the basis of the normal distribution. As it is symmetrical (as in Figure 3) the chance that value is more than the mean of the distribution is the same as the chance of being lower than mean – exactly 50%.

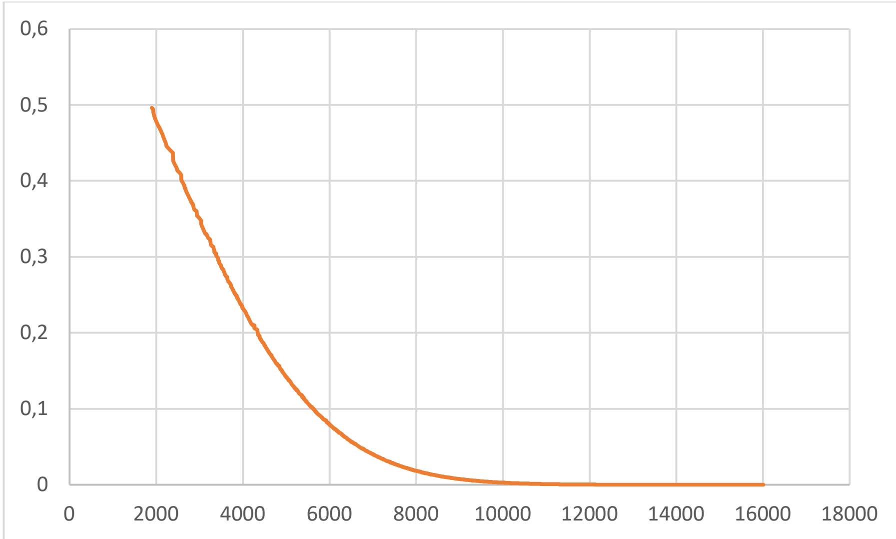


Figure 17. Example of the Pareto-optimal front for the model of EKE

From Figure 17, it is clear that the cost factor conflicts with the service level objective. In other words, in the Pareto-optimal front, the lower the chance of stock-out is, the higher the overall cost is. To optimize this process, the global best solutions' array is sorted from the beginning according to the cost factor. Therefore, when the sort order is descending for cost, it is ascending for stock-out probability and vice versa.

To check if a new solution is non-dominated, the fact that there are just two objective functions is used. It is enough to find the first solution saved in the global array that has the cost function's value lower than the new solution if the sorting order is descending for costs. If this solution has a higher chance of a stockout, then the solution found is not dominated by the examined solution because it has a higher cost and lowers stockout probability. It is not dominated by any other solution that has lower costs – because the checked solution is not dominated by these, i.e. they have a higher stockout chance. Furthermore, it is not dominated by solutions that have higher costs because they are worse in at least one objective function.

However, the solutions that have higher costs may be dominated by the newly found solution. To check that, the solution before the added one is checked. By definition of a non-dominance and sorted array, this solution has a higher cost than the new one. If the stockout probability in this solution is also higher than in the new solution, then this solution is worse than the added solution in two objectives. Therefore, it is dominated and is deleted from the archive.

This process continues until the first solution that has a higher cost but lower stockout likelihood is found. This solution is still non-dominated because while it is worse in the cost function, it is better in customer service objective. Moreover, all the solutions that have even higher costs are not dominated by the newly added solution because they are not dominated by the lastly checked one. In other words, they have higher costs and lower stockout percentage. The algorithm is presented in Figure 18 below.

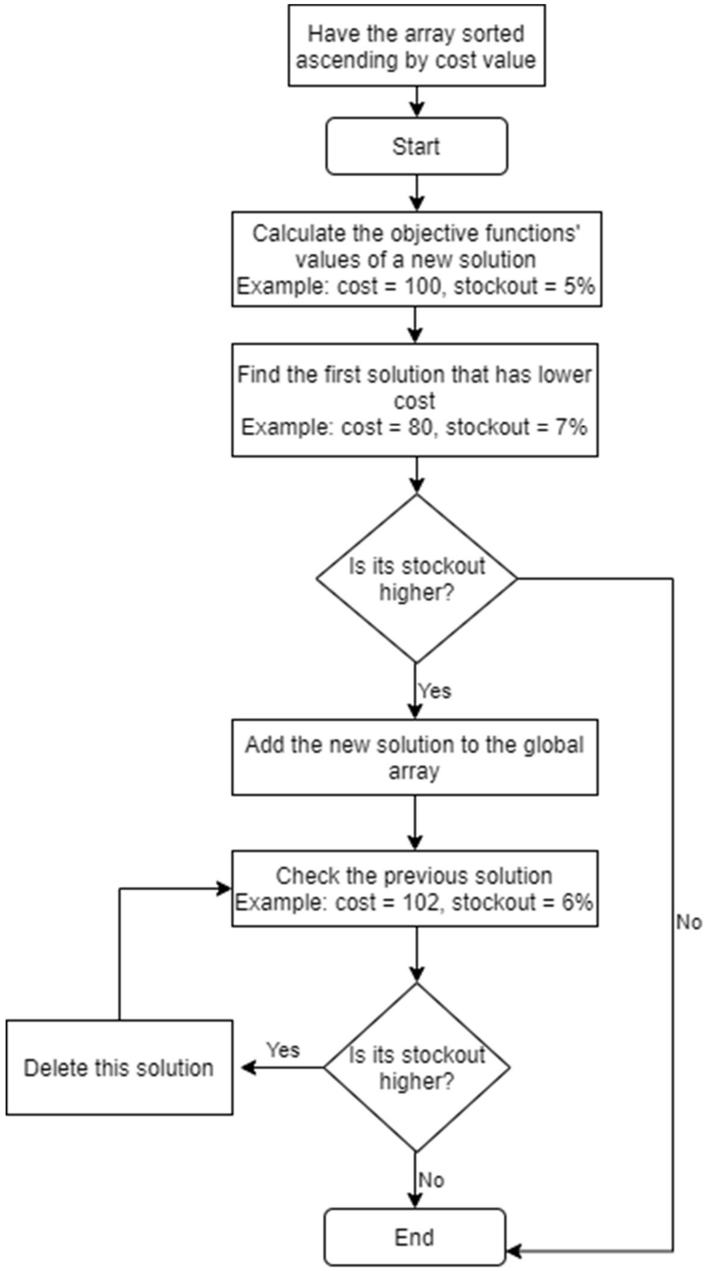


Figure 18. Optimality validation sub-algorithm

This relatively straightforward algorithm allows to keep the global solution array sorted and non-dominated. Moreover, it saves time, not validating a new solution against every solution for optimality previously added to the array. Furthermore, it allows to keep unlimited global solutions array along with reasonable execution time. Most likely this sub-algorithm was not present in the reviewed literature due to its specificity to the analyzed problem of two objective functions.

The literature review outlined several tested variations of the MOPSO algorithm. They are Random selection, Sigma method, WSum, and the Pareto dominance guide selection. The Random selection algorithm without the personal best archive version was selected for several reasons.

First, and essential, the Random selection worked well in the tests of Padhye and colleagues (2016) and was noticed for the balance between execution times and precision. Second, in order to produce the exact Pareto-optimal front of solutions, there was no limit induced on the number of optimal global solutions. Thus, every optimal global position found is saved and could be used in the guide selection for each particle. In this scenario, mathematically demanding algorithms of Sigma, WSum, and the Pareto dominance would significantly increase the execution time.

Finally, the algorithm was tested with the Random selection method and was able to produce almost the precise optimal front of solutions. In order to achieve that, experimenting was used with the settings of the algorithm to reach balance. Moreover, the analysis of different algorithm settings from Tsou (2007) was tested. The table below summarizes the settings used in the final version of the MOPSO algorithm implemented in Python coding language.

Another decision to be made concerns the boundary handling technique. As outlined in the literature review, this decision is not as important since it does not significantly impact the results of the algorithm. For this algorithm, the random return of the particle to the feasible area is chosen for several reasons.

First, the feasible area is defined in four simple inequalities for two variables. These constraints define a clear feasible region, to which the particle can be returned to randomly. Second, it improves the diversity of the solution because the particles that cross the feasible area may go in the opposite direction following another global guide. This is important for algorithms that have several local minima, which is not the case of this solution, however. Nevertheless, it does not bring drawbacks either. Finally, mathematically it is the fastest way, given that there is no personal best solutions' archive implemented to the algorithm.

Table 6. The MOPSO algorithm settings

Variable	**Definition**	**Value used**
N	Number of particles	100
T	Number of iterations	100
D	Number of dimensions	2
ω	Speed inertia	1
c_1	Cognitive acceleration coefficient	2.5
c_2	Social acceleration coefficient	2.5
$x_{0,Q}$	The starting position of a particle in order size variable	Random between $\frac{D}{6} \pm \frac{D}{24}$
$x_{0,k}$	The starting position of a particle in customer service variable	Random between $k_{max} * 0.1 \; and \; k_{max} * 0.75$
$v_{0,Q}$	Starting velocity of a particle in order size variable	Random between -20 and 20
$v_{0,k}$	Starting velocity of a particle in customer service variable	Random between -0.1 and 0.1

4.3 The TOPSIS model

The Multi-Objective Particle Swarm Optimization algorithm produces a set of solutions ready to print them into an Excel file after execution. The final set consists of values of the global solutions’ array used in the algorithm during calculations. The output graph looks similar to Figure 17.

To help a decision-maker to make an optimal decision, some ranking of the results is needed. The model algorithm chosen for this work is the TOPSIS due to its straightforward application and ease of logical understanding. The system is implemented according to the algorithm presented in the Literature review section.

The only parameter used in the model is the weight distribution between the two objective functions: total costs and stockout probability. This parameter is entered by the user of the tool. After fast execution (under 1 sec), the algorithm prints the global solutions’ archive of the MOPSO with the TOPSIS ranking from 0 to 1 into an Excel file. The best 10 solutions can be selected and highlighted for a user.

4.4 Results

Before the optimization step, raw data should have been collected and analyzed. The historical sales data and sales forecast data were available from EKE’s ERP system on both module and system levels. The historical data was analyzed for the years 2018, 2019, and 2020 and forecasts were taken for years 2020 and 2021.

Due to the fast development and growth of EKE, the data from previous years could not be used reliably in the forecasts to determine current demand distribution. Since the demand has dramatically increased, new products introduced, and new customers attracted, the historical demand from the years 2019 and 2020 was eventually used in the model.

Therefore, more attention was paid to the backlogged orders for the years 2020 and 2021 as more reliable data. Since backlogged orders for the current year are almost the same as confirmed orders, this data is considered as the most reliable. However, it is essential to add that unexpected orders during the year can occur and the expected annual demand is greater than the sum of the already received orders.

To account for this potential increase, the expected sales growth value from the sales department is used. Given that most of the data was analyzed in the early beginning of the second quarter of 2020, the expected growth as of the beginning of 2020 was reduced by a quarter, assuming that 25% of unexpected orders were already received and 75% are yet to come.

For example, the sales department would expect a 40% increase in backlogged orders during the year 2020 as of 1.1.2020. The backlogged orders analyzed on 1.4.2020 are expected to grow by 30% during the rest of the year accounting for orders received from 1.1.2020 to 1.4.2020. Thus, the expected mean demand for the year 2020 was multiplied by this parameter.

The data was analyzed on a monthly basis and two parameters were calculated. The first parameter is the expected annual demand in the year 2020 calculated as described in the previous two paragraphs. The second one is the standard deviation parameter.

For the standard deviation, it was not enough to use only the backlogged data for the following year because the shipping dates for such forecasted orders are not always exact. The data for the previous year, however, is always confirmed and may be used to improve the validity of the forecasted data. To combine the standard deviation values for the year 2020 and 2019, the average of two was used. Another possible approach would be to use a weighted average with more weight given to the forecasted data or to use only the forecasted data without relying on the historical analysis.

Analysis of each module's optimal order quantities and service levels required other parameters as well. The data collection included the procurement values of each module, which included the raw material cost and work required to produce the module. Furthermore, the ordering costs, lead time, and the weight of the cost objective against service level objective were required. The ordering and material costs together with lead time were necessary for the cost function as presented earlier in this section. The lead time was required also for the stockout probability calculation. Finally, the TOPSIS algorithm relied on the provided ratio of service level significance.

The scope of analyzed data included 60 modules from EKE's portfolio and aggregation of data on a system level to facilitate decision making. A new tool in the form of an Excel spreadsheet was created to store nondominant solutions from each module and link them to imported Bills of Material for analyzed systems.

Since each module included more than a hundred of non-dominant solutions, a combination of values of just two modules would create more than ten thousand solutions – if all stockout probabilities of one module were analyzed for every stockout probability of the second module. An example would be a decision to allow for a 10% stockout likelihood of a module while requiring a second module to be available in 99% of orders. Such an arrangement would require a separate study executed to optimize the overall stockout likelihood on the system level while maintaining different service levels on the module level.

Therefore, aggregation of module data included the assumption that each module installed to a system would have the same stockout probability. Then the total stockout likelihood of a system is a probability of a single module not being available for sale when required. Suppose a system consists of *n* modules, which are *x* likely to face a stockout. Then the system will have a total stockout likelihood of $1-(1-x)^n$

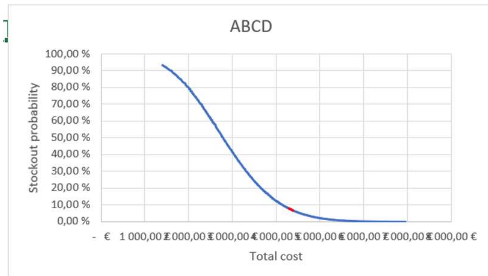
(Jaynes 2003, 26-32). Since $1 - x$ is always less than 1, the greater the number of modules in a system is, the more likely it will be out of stock.

The stockout scale was selected from one of the modules to be the baseline for the dataset. Optimized order sizes, total costs, the TOPSIS values, and proposed safety stock amounts were collected from non-dominated solutions of each module that had equal or the closest stockout probability to the one in the baseline. With this method, all data was aggregated to 400 data points on the stockout scale.

The total cost, the TOPSIS ranking, and total safety stock were summed up for each module to aggregate data on the system level. Thus, the TOPSIS ranking would be the highest for a solution, which had the best cumulative rating for each analyzed module. However, it might happen that the best solution on the system level would not be considered by the TOPSIS as the best for each individual module.

Figure 19 below is an example of the user interface of the aggregated data on the system level. Material and total costs, as well as safety stocks presented in Figure 19, were multiplied by a certain multiplicator and module names were changed to respect the sensitive information. In the table on the left, OKA means the material cost from Finnish, Safety Stock from TOPSIS is the number of stored modules according to the highest rating from TOPSIS and the total investment is a product of the material cost and proposed safety stock. The table on the right shows the aggregated stockout probabilities, sums of total costs and TOPSIS ratings of each module used in BOM, and the safety stock required to achieve the given stockout level.

BOM	Module	OKA	Safety stock from TOPSIS	Total investment
ABCD	XXX	82,47 €	131	10 773,03 €
	YYY	83,96 €	671	56 317,24 €
	ZZZ	71,91 €	57	4 128,27 €
	AAA	143,05 €	16	2 287,13 €



Stockout				Safety stock				
Module	System	Cost	TOPSIS	Total	XXX	YYY	ZZZ	AAA
0,00 %	0,00 %	6 948,08 €	3,39	642	494	42	11	94
0,01 %	0,04 %	6 551,30 €	3,43	596	459	39	10	87
0,02 %	0,08 %	6 351,08 €	3,44	573	441	38	10	84
0,03 %	0,12 %	6 226,50 €	3,46	559	431	37	10	82
0,04 %	0,16 %	6 117,45 €	3,47	547	421	36	10	80
0,05 %	0,20 %	6 045,76 €	3,47	538	414	35	9	79
0,06 %	0,24 %	5 979,77 €	3,48	530	408	35	9	78
0,07 %	0,28 %	5 913,32 €	3,49	523	403	34	9	77
0,08 %	0,32 %	5 854,75 €	3,49	517	398	34	9	76
0,09 %	0,36 %	5 813,75 €	3,50	512	394	34	9	75
0,10 %	0,40 %	5 769,13 €	3,50	507	390	33	9	75
0,11 %	0,44 %	5 728,63 €	3,51	502	387	33	9	74
0,12 %	0,48 %	5 700,63 €	3,51	499	384	33	9	73
0,13 %	0,52 %	5 660,92 €	3,51	495	381	32	9	73
0,14 %	0,56 %	5 630,26 €	3,52	491	378	32	9	72
0,15 %	0,60 %	5 600,38 €	3,52	487	375	32	9	72
0,16 %	0,64 %	5 566,95 €	3,52	484	372	32	9	71
0,17 %	0,68 %	5 548,96 €	3,52	481	370	32	8	71
0,18 %	0,72 %	5 535,50 €	3,53	478	368	31	8	70
0,19 %	0,76 %	5 500,42 €	3,53	476	367	31	8	70
0,20 %	0,80 %	5 473,18 €	3,53	473	364	31	8	69
0,21 %	0,84 %	5 455,46 €	3,53	471	363	31	8	69

Figure 19. Example of the user interface

The chart in the left corner is a scatter chart, whose horizontal axis is the total cost array and the vertical axis is the stockout probability on the system level. It is easy to notice the difference between this chart and Figure 17, which was made for the module level. Figure 20 below illustrates an example of the aggregation of modules' nondominated solutions for a given system with 15 modules. Although being similar to Figure 17 in the lower part of the chart, the plot in Figure 20 extends symmetrically to 100% stockout probability.

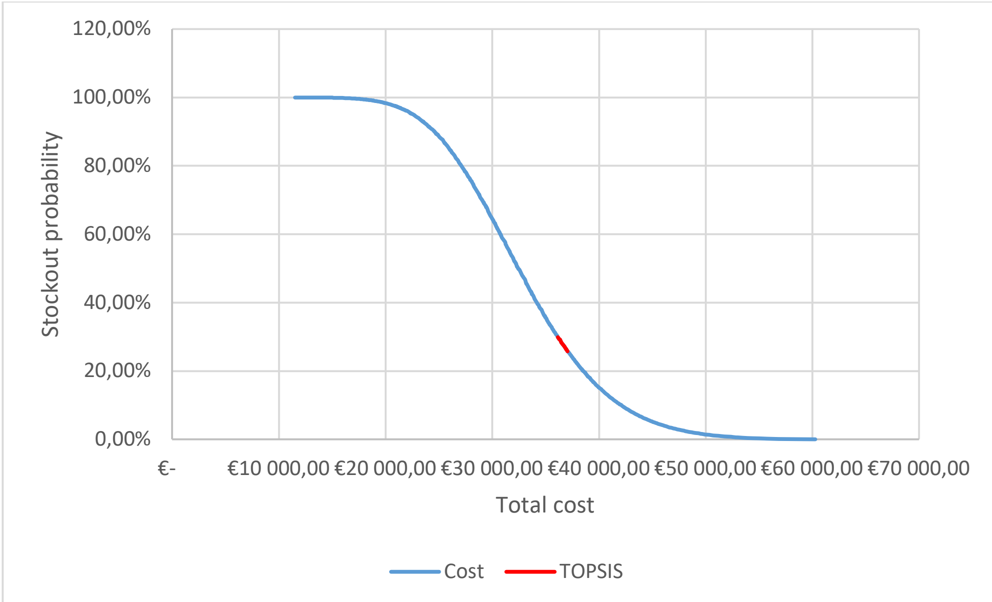


Figure 20. The results on system level

The next section presents the results of the MOPSO and TOPSIS algorithms for a module's data given. Then the results are compared to the methods presented earlier in the literature review section and works of other researchers.

4.5 Validation

To check the validity of the results of the MOPSO algorithm, a test dataset was selected. All used parameters are presented in Table 7 below. To avoid repetition, the options outlined in Table 6 are valid for this test run.

Table 7. Test data

Variable	Name	Test 1 values	Test 2 values
A	Order cost	50€	100€
L	Lead time	2 months	4 months
D	Annual expected demand	1200 units	1000 units
h	Inventory carrying costs	90%	10%
c	Item's value	0.5€	1€
σ	The standard deviation of demand per month	100 units	10 units
-	Importance of cost factor against service level for TOPSIS	70%	30%

The results of both tests are presented in Appendix 2 supported by charts. These numbers can be validated in two ways. Since the current model is built upon the theory presented in the Literature Review section, the results for the order quantity can be validated by comparing to the Economic Order Quantity Formula 3.2. In other words, the assumptions used in the EOQ sections are also implied for the current model.

For the first test set, the Economic Order Quantity is equal

$$EOQ = \sqrt{\frac{2AD}{hc}} = \sqrt{\frac{2 * 50 * 1200}{0.9 * 0.5}} = 516\ units$$

The proposed order quantity from the MOPSO algorithm varies according to the points checked and left unchecked by the particles. However, the average of all 743 results is 517 units for the *Q* parameter. Thus, the algorithm finds the most optimal order quantity and arranges the results to it. Ultimately, it can be considered valid, since the actual EOQ formula is not used anywhere and the algorithm arrives at the same solution by the particle swarm method.

The second data set is included to illustrate the handling of the boundaries by the algorithm. EOQ in the second set is equal

$$EOQ = \sqrt{\frac{2AD}{hc}} = \sqrt{\frac{2 * 100 * 1000}{0.1 * 1}} = 1414\, units$$

However, the model incorporated a boundary for the order quantity to be between *0* and the annual expected demand *D*. In other words, the algorithm is discouraged from ordering more than a year's worth amount of goods at once despite potential economical advantages. One of the reasons may be the constantly changing environment that may change the input data in a year and, thus, have an impact on the results of, for example, order quantities. Ultimately, the algorithm has a set boundary for *Q* of *1000 units.*

Therefore, obeying the rules, no values of *Q* exceeds *1000 units* in the second table of results in Appendix 2. The average value is *991 units* out of the found 56 solutions. The second set of results contained noticeably fewer solutions because the particles constantly tried to escape the feasible region and were returned to starting positions not reaching all possible non-dominated solutions. The formulas for the cost, stockout probability, TOPSIS rating, safety stock, average stock, and inventory turnover were validated in Excel to be correct.

5 Discussion

5.1 Research purpose

The main research question of this work was, as presented in chapter 2.1: “What could be a suitable inventory optimization system for EKE-Electronics?” Section 4 “Research implementation” tried to answer this question implementing theory presented in Section 3. First, a potentially suitable inventory system is selected from the literature based on the characteristics of EKE’s business. Then this system is implemented into an optimization algorithm coded in Python. Finally, the results of the optimization are aggregated on the system level and they are presented to the management for decision making. Ultimately, the main research question is answered with one proposal out of a variety of acceptable systems.

In a broader scope, the goal of this work was to develop a customizable inventory optimization platform for EKE-Electronics. This platform could illustrate the investments required to achieve a selected customer service level and it should be flexible to account for future developments of the company and constantly changing business environment. Therefore, the optimization through customized coding of the MOPSO algorithm was selected because it allows for flexibility and future enhancement, as discussed in the “Topics for further development” part.

5.2 Research results

The system developed allows EKE to overview its inventory requirements and question the existing Make to Order approach. The change proposed goes beyond only inventory optimization field because it requires cooperation and development with suppliers, different sales approaches, and updates to current production and material planning systems. This work facilitates a scientifically supported decision-making process in the transition from MTO to Assembly to Order production system.

A major part of the work included a comprehensive data analysis. The results of the combination of the historical statistics of module sales together with backlogged orders may be useful in other decision-making fields, such as sales and strategic purchasing.

The methods and algorithms used in this study are possible to reproduce for implementation in other companies and other fields, where inventory management might be required or may be useful. The MOPSO algorithm guarantees flexibility necessary for application in unconnected business areas and it assures company-specific application of objectives. The inventory management systems used are proved by time and comprehensive practical application studies, such as works of Hopp and Spearman (2011) and Silver and colleagues (2017).

Knowledge from different fields was implemented to achieve the desired results. Most of the literature review was based on inventory management systems on one side and optimization methods on the other side. Additionally, the statistical analysis was needed in the primary data analysis and module data aggregation on the system level. Financial management methods were employed for the calculation of the Weighted Average Cost of Capital (WACC). Coding skills were essential for the optimization model development and validation. Finally, mathematical skills were necessary for the analysis of both inventory management and optimization studies as well as for the model construction and verification.

Finally, the user experience was enhanced by allowing for simplified input of data – the user needs to enter only seven numbers in a row and then receive an Excel table with all global non-dominated solutions found and a graph of costs against service level. This method also ensures the robustness of the model, since the user cannot accidentally corrupt the calculations while viewing them. Moreover, the user is not required to possess any sophisticated mathematical knowledge to use the results of the developed tool.

Once the management selects the desired service level individually for each module or as a general level for all SKUs, the results from the developed tool can be implemented to practice. A possible way for that would be to set up the order point, order quantity *(s, Q)* system in the existing ERP software. Then the order quantity and order point values would be imported to the Enterprise Resource Planning software from the module-individual results. The values would be selected according to the defined customer service level. The ERP software would constantly monitor the stock levels of individual modules and once the reorder point would be reached, a notification would be sent to the procurement staff. Then a purchasing order could be placed to the supplier.

5.3 Limitations of the research

Limitations of this study are as various as its applications. First, the work applied models existing in the literature to the specific case assigned by EKE-Electronics Oy. Therefore, the literature review section focused on the specifics of the practical application and could not include the whole overview of inventory management and optimization systems presented in the literature. For example, only scientific inventory management systems were included in the analysis, thus, leaving other ways of cost optimization, such as Just in Time philosophy, mostly untouched.

Moreover, the results of the system developed are as valid as the input data. The fast growth of the enterprise hardly allowed to consider historical data as valid and neither backlogged orders were a fully precise source of information. Finally, sometimes small population sizes resulted in noticeable confidence intervals. Since these limitations are company-specific, they require a more careful interpretation of the results by the management. Moreover, the results should be considered as a recommendation for action but not as a guarantee of the selected service levels or costs.

This research, moreover, does not include potentially unexpected changes in sales volumes of the analyzed products, emergency cases, such as pandemic situations, or errors in the source data. Production optimization, purchasing development, product consolidation, and standardization remained out of the scope of this work, despite their ability to decrease costs and increase service levels. Finally, the introduction of new products would require a potential demand estimation based on the experience of the employees.

Another limitation of this study is considering a production system as either MTO or ATO (or any other production system). However, as criticized by Giard and Sali (2012), modern supply chains often rely on a combination of MTO and, for example, Make to Stock production systems. The integration of several systems requires a different approach and could influence optimal purchasing lots and expected costs of service levels.

5.4 Topics for further development

In addition to the examples of the limitations of this work listed above, there are several options to develop the achieved results. As already mentioned in the "Research implementation" section, there are several suitable inventory management systems for EKE. For example, an order point, order-up-to-level *(s, S)* system theoretically could provide more precise results and offer additional savings. These benefits would come from planning for situations when the inventory position would be noticeably lower than the reorder point after a sale of a module. For example, the set reorder point is 50 units and the order quantity is 200 units. At inventory position of 57 units and an order of 25 units is placed. Then the inventory position reduces to 35 units and, after a purchasing order is placed, it becomes 235. If the procurement order was sent at 50 units, the expected inventory position would become 250 and this value is the basis to the inventory management cost calculation under (s, Q) system. (Silver et al. 2017, 327-331.)

A further study could conduct required analysis of ordering quantities of individual modules for EKE and update the MOPSO algorithm with cost and service calculations of *(s, S)* inventory management system. The results then could be compared in practice to the *(s, Q)* system and the savings could be analyzed against the data analysis costs.

Another way to continue optimization of the model would be to use more precise demand functions for individual modules. The current choice is normal distribution that is advised in the literature (Silver et al. 2017, 275-277). However, more of available historical data would empower an analysis of demand distribution and selection of another function for analysis for certain modules.

Paying attention to the advice of Giard and Sali (2012), further research can analyze a combination of ATO and MTO production systems. Moreover, an analysis of Assembly to Order system can be enhanced by studying saving possibilities at different assembly levels. Current selection is to keep a stock of all required modules and assemble systems on request. It would be possible, however, to keep some stock of essential modules and raw materials for assembly of other modules that are used less often.

Furthermore, automation of the data aggregation process can be attempted. Currently the data from individual module results is linked to the general BOM Excel file as described in the "Research Implementation" section. The Python code can be adjusted to print the modules' results directly to the general BOM file instead of generating individual files for each module.

Finally, the developed tool can be used to compare potential savings if the manufacturer's lead time are reduced. This knowledge could be a starting point for future development of the production process and enhanced cooperation.

5.5 Reflection on research

This study was my longest and possibly the richest academical project. Besides developing my skills in supply chain management, optimization, coding, corporate finance, and statistics, I was able to apply the gained knowledge to a practical case. It was incredibly satisfying to create a tool that could be beneficial for EKE and to analyze the currently used business procedures.

This work, furthermore, opens future opportunities for development both on personal and professional level for me. As an example, I understood that I enjoy the optimization of logistics problems and would love to continue developing in this field. Moreover, now I can develop more complicated inventory management systems based on this study.

This research was possible only due to invaluable help of many involved people. I would like to express my warm gratitude to my thesis supervisors, Juha Sipilä and Tommi Franssila. The case study was made possible thanks to EKE-Electronics and I especially thank all my colleagues and my supervisor Juha Paldanius for support, guidance, and comments. Vital support came from my family and friends, whom I express my appreciation. Finally, a great part of basic knowledge required for this work came from all JAMK staff and the design of the International Logistics programme.

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Appendices

Appendix 2. Results of the algorithm tests

Test 1.

				Final optimized values			
Q	k	Cost	Stockout	TOPSIS grade	Safety stock	Average stock	Inventory turnover
540	4,80	664,34 $	0 %	61,92 %	959	1229	0,98
487	4,78	663,00 $	0 %	61,99 %	956	1199	1,00
584	4,76	662,80 $	0 %	62,00 %	953	1245	0,96
459	4,75	661,75 $	0 %	62,06 %	951	1180	1,02
533	4,75	659,64 $	0 %	62,18 %	949	1216	0,99
529	4,74	659,07 $	0 %	62,22 %	948	1213	0,99
566	4,72	658,04 $	0 %	62,28 %	944	1227	0,98
565	4,71	657,62 $	0 %	62,30 %	943	1226	0,98
541	4,71	656,81 $	0 %	62,35 %	943	1213	0,99
490	4,69	654,77 $	0 %	62,47 %	938	1183	1,01
505	4,69	654,16 $	0 %	62,50 %	937	1190	1,01
521	4,67	652,68 $	0 %	62,59 %	934	1195	1,00
477	4,66	652,46 $	0 %	62,60 %	932	1170	1,03
473	4,66	652,43 $	0 %	62,60 %	931	1168	1,03
585	4,65	652,28 $	0 %	62,61 %	929	1221	0,98
595	4,63	651,83 $	0 %	62,64 %	927	1224	0,98
459	4,62	650,08 $	0 %	62,74 %	925	1154	1,04
576	4,62	649,33 $	0 %	62,78 %	924	1211	0,99
492	4,62	648,06 $	0 %	62,86 %	923	1169	1,03
495	4,61	647,38 $	0 %	62,90 %	922	1169	1,03
533	4,61	647,07 $	0 %	62,92 %	921	1188	1,01
476	4,58	645,33 $	0 %	63,02 %	916	1154	1,04
522	4,58	644,46 $	0 %	63,07 %	916	1177	1,02
514	4,57	644,11 $	0 %	63,09 %	915	1172	1,02
619	4,53	643,93 $	0 %	63,10 %	906	1216	0,99
587	4,52	640,91 $	0 %	63,28 %	904	1197	1,00
529	4,51	638,48 $	0 %	63,43 %	902	1167	1,03
521	4,50	637,68 $	0 %	63,48 %	901	1161	1,03
508	4,50	637,55 $	0 %	63,49 %	900	1154	1,04
453	4,46	636,22 $	0 %	63,57 %	893	1119	1,07
464	4,45	634,64 $	0 %	63,66 %	891	1123	1,07
582	4,45	634,30 $	0 %	63,68 %	889	1180	1,02
494	4,44	631,81 $	0 %	63,84 %	887	1134	1,06
535	4,43	630,97 $	0 %	63,89 %	885	1153	1,04
502	4,42	630,13 $	0 %	63,94 %	884	1134	1,06
524	4,40	628,84 $	0 %	64,02 %	881	1143	1,05
508	4,40	628,68 $	0 %	64,03 %	881	1135	1,06
494	4,38	626,99 $	0 %	64,13 %	876	1123	1,07
487	4,38	626,88 $	0 %	64,14 %	876	1119	1,07
483	4,37	626,51 $	0 %	64,16 %	875	1116	1,07

471	4,35	624,54 $	0 %	64,29 %	869	1105	1,09
495	4,34	623,22 $	0 %	64,37 %	868	1116	1,08
495	4,33	622,70 $	0 %	64,40 %	867	1115	1,08
506	4,33	622,34 $	0 %	64,42 %	866	1120	1,07
500	4,33	621,82 $	0 %	64,46 %	865	1115	1,08
516	4,32	621,07 $	0 %	64,50 %	864	1122	1,07
505	4,30	619,84 $	0 %	64,58 %	861	1114	1,08
502	4,29	618,71 $	0 %	64,65 %	858	1109	1,08
510	4,29	618,45 $	0 %	64,67 %	858	1113	1,08
607	4,25	617,97 $	0 %	64,70 %	850	1154	1,04
537	4,25	614,72 $	0 %	64,91 %	849	1118	1,07
541	4,23	613,17 $	0 %	65,01 %	846	1116	1,08
495	4,22	611,95 $	0 %	65,08 %	843	1091	1,10
567	4,19	610,31 $	0 %	65,19 %	838	1121	1,07
503	4,17	607,31 $	0 %	65,38 %	833	1085	1,11
527	4,16	607,04 $	0 %	65,40 %	832	1096	1,09
517	4,15	606,18 $	0 %	65,46 %	831	1089	1,10
529	4,15	605,99 $	0 %	65,47 %	830	1094	1,10
504	4,14	605,19 $	0 %	65,52 %	828	1080	1,11
493	4,13	603,89 $	0 %	65,61 %	825	1072	1,12
498	4,12	603,24 $	0 %	65,65 %	824	1073	1,12
559	4,10	602,46 $	0 %	65,70 %	821	1100	1,09
530	4,09	600,59 $	0 %	65,82 %	818	1083	1,11
501	4,08	599,92 $	0 %	65,87 %	817	1067	1,12
490	4,07	599,43 $	0 %	65,90 %	815	1060	1,13
528	4,07	598,55 $	0 %	65,96 %	814	1078	1,11
518	4,06	597,35 $	0 %	66,04 %	811	1070	1,12
555	4,04	596,69 $	0 %	66,08 %	808	1086	1,11
477	4,04	596,53 $	0 %	66,09 %	808	1046	1,15
537	4,04	595,89 $	0 %	66,14 %	807	1076	1,12
533	4,02	594,63 $	0 %	66,22 %	805	1071	1,12
509	4,01	593,42 $	0 %	66,30 %	802	1057	1,14
495	4,00	592,66 $	0 %	66,35 %	800	1048	1,15
545	3,99	591,90 $	0 %	66,40 %	798	1071	1,12
521	3,99	591,50 $	0 %	66,43 %	798	1058	1,13
542	3,99	591,42 $	0 %	66,43 %	797	1068	1,12
512	3,98	590,89 $	0 %	66,47 %	797	1053	1,14
517	3,97	590,02 $	0 %	66,53 %	795	1053	1,14
535	3,97	589,51 $	0 %	66,56 %	793	1061	1,13
521	3,96	588,92 $	0 %	66,60 %	792	1053	1,14
524	3,95	587,53 $	0 %	66,70 %	789	1051	1,14
516	3,94	587,27 $	0 %	66,72 %	789	1047	1,15
524	3,94	586,65 $	0 %	66,76 %	787	1049	1,14
547	3,93	586,64 $	0 %	66,76 %	786	1060	1,13
517	3,93	585,68 $	0 %	66,82 %	785	1044	1,15
489	3,91	584,95 $	0 %	66,87 %	783	1027	1,17
534	3,91	584,31 $	0 %	66,92 %	782	1049	1,14
532	3,90	583,77 $	0 %	66,95 %	781	1046	1,15
544	3,90	583,76 $	0 %	66,95 %	780	1052	1,14
514	3,89	582,67 $	0 %	67,03 %	778	1035	1,16

506	3,89	582,35 $	0 %	67,05 %	778	1031	1,16
536	3,89	582,31 $	0 %	67,05 %	777	1045	1,15
469	3,88	582,23 $	0 %	67,06 %	775	1010	1,19
539	3,87	581,33 $	0 %	67,12 %	775	1045	1,15
508	3,86	580,19 $	0 %	67,20 %	773	1027	1,17
471	3,85	579,61 $	0 %	67,24 %	769	1005	1,19
494	3,84	578,58 $	0 %	67,31 %	769	1016	1,18
562	3,83	578,12 $	0 %	67,34 %	766	1048	1,15
555	3,82	576,34 $	0 %	67,47 %	763	1040	1,15
542	3,81	575,88 $	0 %	67,50 %	763	1034	1,16
486	3,80	574,69 $	0 %	67,58 %	760	1003	1,20
579	3,78	574,49 $	0 %	67,59 %	757	1046	1,15
562	3,78	573,34 $	0 %	67,67 %	756	1037	1,16
504	3,78	572,52 $	0 %	67,73 %	756	1008	1,19
488	3,77	572,40 $	0 %	67,74 %	755	999	1,20
495	3,77	571,70 $	0 %	67,79 %	754	1001	1,20
500	3,76	571,28 $	0 %	67,82 %	753	1003	1,20
543	3,76	571,10 $	0 %	67,83 %	752	1023	1,17
513	3,75	569,88 $	0 %	67,92 %	750	1006	1,19
540	3,74	569,61 $	0 %	67,94 %	749	1019	1,18
557	3,73	568,64 $	0 %	68,00 %	746	1024	1,17
465	3,72	568,50 $	0 %	68,01 %	744	977	1,23
525	3,72	566,81 $	0 %	68,13 %	743	1006	1,19
531	3,71	566,50 $	0 %	68,16 %	742	1008	1,19
491	3,70	566,06 $	0 %	68,19 %	741	986	1,22
485	3,70	565,72 $	0 %	68,21 %	740	982	1,22
538	3,70	565,40 $	0 %	68,23 %	740	1008	1,19
530	3,70	565,08 $	0 %	68,26 %	739	1004	1,19
556	3,68	563,96 $	0 %	68,34 %	735	1013	1,18
480	3,67	563,73 $	0 %	68,35 %	735	975	1,23
506	3,67	563,14 $	0 %	68,40 %	735	988	1,21
515	3,66	562,04 $	0 %	68,47 %	733	990	1,21
492	3,65	561,15 $	0 %	68,54 %	730	976	1,23
484	3,64	560,70 $	0 %	68,57 %	729	970	1,24
480	3,63	559,93 $	0 %	68,63 %	726	966	1,24
549	3,63	559,44 $	0 %	68,66 %	726	1001	1,20
468	3,61	557,97 $	0 %	68,77 %	721	955	1,26
475	3,60	557,53 $	0 %	68,80 %	721	959	1,25
489	3,60	556,77 $	0 %	68,85 %	720	964	1,24
559	3,60	556,72 $	0 %	68,86 %	719	999	1,20
482	3,59	555,69 $	0 %	68,93 %	717	958	1,25
494	3,58	555,05 $	0 %	68,98 %	717	963	1,25
485	3,58	554,63 $	0 %	69,01 %	715	958	1,25
499	3,57	553,97 $	0 %	69,06 %	714	964	1,25
522	3,57	553,84 $	0 %	69,07 %	714	975	1,23
531	3,57	553,70 $	0 %	69,08 %	714	979	1,23
514	3,56	552,68 $	0 %	69,15 %	712	969	1,24
528	3,55	552,32 $	0 %	69,18 %	711	975	1,23
516	3,55	552,07 $	0 %	69,19 %	710	969	1,24
487	3,55	552,04 $	0 %	69,20 %	709	953	1,26

519	3,55	551,57 $	0 %	69,23 %	709	969	1,24
525	3,53	549,98 $	0 %	69,35 %	706	968	1,24
517	3,52	549,57 $	0 %	69,38 %	705	963	1,25
503	3,52	549,45 $	0 %	69,39 %	704	956	1,26
544	3,52	549,06 $	0 %	69,41 %	703	975	1,23
492	3,50	547,76 $	0 %	69,51 %	700	946	1,27
530	3,50	547,42 $	0 %	69,54 %	700	965	1,24
528	3,49	546,55 $	0 %	69,60 %	698	962	1,25
518	3,49	546,23 $	0 %	69,62 %	697	957	1,25
560	3,48	546,07 $	0 %	69,63 %	695	975	1,23
482	3,47	545,63 $	0 %	69,67 %	695	936	1,28
520	3,46	544,24 $	0 %	69,77 %	693	953	1,26
572	3,45	544,19 $	0 %	69,77 %	690	976	1,23
492	3,45	542,85 $	0 %	69,87 %	689	935	1,28
538	3,44	542,41 $	0 %	69,91 %	689	958	1,25
537	3,44	542,31 $	0 %	69,91 %	688	957	1,25
512	3,44	542,06 $	0 %	69,93 %	688	944	1,27
502	3,44	542,00 $	0 %	69,94 %	688	939	1,28
524	3,44	541,88 $	0 %	69,94 %	688	950	1,26
516	3,44	541,74 $	0 %	69,96 %	687	946	1,27
549	3,43	541,46 $	0 %	69,98 %	686	960	1,25
501	3,43	540,88 $	0 %	70,02 %	685	936	1,28
515	3,42	540,16 $	0 %	70,07 %	684	941	1,27
525	3,42	540,15 $	0 %	70,07 %	684	946	1,27
507	3,41	539,71 $	0 %	70,11 %	683	936	1,28
493	3,41	539,38 $	0 %	70,13 %	682	928	1,29
502	3,41	539,20 $	0 %	70,14 %	682	933	1,29
477	3,40	538,98 $	0 %	70,16 %	680	918	1,31
559	3,40	538,73 $	0 %	70,18 %	679	959	1,25
505	3,40	538,02 $	0 %	70,23 %	679	931	1,29
523	3,39	537,86 $	0 %	70,25 %	679	940	1,28
503	3,39	537,68 $	0 %	70,26 %	678	930	1,29
499	3,39	537,50 $	0 %	70,27 %	678	927	1,29
524	3,39	537,18 $	0 %	70,30 %	677	939	1,28
570	3,37	536,97 $	0 %	70,31 %	674	959	1,25
550	3,36	535,65 $	0 %	70,41 %	673	948	1,27
509	3,36	535,10 $	0 %	70,45 %	673	927	1,29
492	3,36	534,68 $	0 %	70,48 %	671	917	1,31
544	3,35	534,45 $	0 %	70,50 %	671	943	1,27
552	3,34	533,92 $	0 %	70,54 %	669	945	1,27
502	3,34	533,25 $	0 %	70,59 %	668	919	1,31
519	3,34	533,12 $	0 %	70,60 %	668	928	1,29
541	3,34	533,11 $	0 %	70,60 %	668	938	1,28
511	3,34	532,61 $	0 %	70,64 %	667	923	1,30
542	3,32	531,36 $	0 %	70,73 %	664	935	1,28
504	3,32	531,08 $	0 %	70,76 %	664	915	1,31
542	3,32	531,03 $	0 %	70,76 %	663	934	1,28
541	3,31	530,42 $	0 %	70,81 %	662	932	1,29
500	3,30	529,76 $	0 %	70,86 %	661	910	1,32
565	3,28	528,59 $	0 %	70,94 %	656	939	1,28

545	3,28	527,61 $	0 %	71,02 %	655	928	1,29
511	3,27	526,93 $	0 %	71,07 %	655	910	1,32
520	3,26	526,08 $	0 %	71,14 %	653	913	1,31
495	3,26	525,86 $	0 %	71,15 %	652	899	1,33
500	3,26	525,45 $	0 %	71,18 %	651	901	1,33
520	3,25	525,15 $	0 %	71,21 %	651	911	1,32
566	3,24	524,99 $	0 %	71,22 %	648	931	1,29
483	3,24	524,41 $	0 %	71,26 %	648	890	1,35
541	3,24	524,11 $	0 %	71,29 %	648	918	1,31
513	3,23	523,00 $	0 %	71,37 %	646	902	1,33
506	3,23	522,88 $	0 %	71,38 %	645	898	1,34
513	3,22	522,40 $	0 %	71,42 %	644	901	1,33
503	3,22	521,95 $	0 %	71,45 %	643	895	1,34
538	3,21	521,73 $	0 %	71,47 %	643	912	1,32
485	3,21	521,47 $	0 %	71,49 %	641	884	1,36
487	3,20	520,49 $	0 %	71,56 %	639	883	1,36
530	3,19	519,56 $	0 %	71,64 %	638	903	1,33
528	3,19	519,33 $	0 %	71,65 %	638	902	1,33
491	3,18	519,06 $	0 %	71,67 %	636	882	1,36
575	3,17	519,04 $	0 %	71,68 %	634	922	1,30
463	3,17	518,98 $	0 %	71,68 %	634	865	1,39
486	3,17	517,95 $	0 %	71,76 %	634	877	1,37
511	3,17	517,37 $	0 %	71,81 %	633	889	1,35
543	3,16	517,13 $	0 %	71,82 %	632	903	1,33
492	3,15	515,79 $	0 %	71,93 %	629	875	1,37
489	3,14	515,23 $	0 %	71,97 %	628	872	1,38
543	3,14	514,87 $	0 %	72,00 %	627	899	1,34
497	3,13	514,37 $	0 %	72,04 %	626	875	1,37
516	3,13	514,18 $	0 %	72,05 %	626	884	1,36
518	3,13	514,01 $	0 %	72,07 %	626	885	1,36
527	3,13	513,71 $	0 %	72,09 %	625	889	1,35
545	3,12	513,13 $	0 %	72,13 %	623	896	1,34
495	3,11	512,79 $	0 %	72,16 %	623	870	1,38
541	3,10	511,67 $	0 %	72,25 %	620	891	1,35
529	3,10	511,13 $	0 %	72,29 %	619	884	1,36
529	3,09	510,69 $	0 %	72,33 %	618	883	1,36
491	3,09	510,45 $	0 %	72,34 %	617	863	1,39
517	3,08	509,99 $	0 %	72,38 %	617	876	1,37
494	3,08	509,66 $	0 %	72,41 %	616	863	1,39
501	3,07	508,45 $	0 %	72,50 %	613	864	1,39
532	3,06	508,29 $	0 %	72,51 %	613	879	1,37
480	3,05	507,50 $	0 %	72,57 %	610	850	1,41
560	3,05	507,33 $	0 %	72,59 %	609	889	1,35
494	3,05	506,73 $	0 %	72,64 %	609	856	1,40
521	3,04	506,15 $	0 %	72,68 %	608	869	1,38
498	3,03	505,48 $	0 %	72,73 %	607	856	1,40
535	3,03	505,37 $	0 %	72,74 %	606	874	1,37
484	3,03	505,37 $	0 %	72,74 %	606	848	1,42
559	3,03	505,37 $	0 %	72,74 %	605	884	1,36
538	3,02	504,29 $	0 %	72,83 %	604	873	1,38

508	3,02	504,13 $	0 %	72,84 %	604	858	1,40
572	3,00	504,03 $	0 %	72,85 %	601	887	1,35
558	3,00	503,40 $	0 %	72,90 %	601	880	1,36
546	3,00	502,99 $	0 %	72,93 %	601	873	1,37
560	3,00	502,99 $	0 %	72,93 %	600	880	1,36
492	3,00	502,48 $	0 %	72,97 %	600	845	1,42
536	2,99	502,09 $	0 %	73,00 %	599	867	1,38
557	2,98	501,50 $	0 %	73,05 %	597	875	1,37
552	2,98	501,23 $	0 %	73,07 %	596	872	1,38
569	2,97	501,01 $	0 %	73,09 %	595	879	1,36
469	2,97	500,73 $	0 %	73,11 %	594	828	1,45
574	2,97	500,57 $	0 %	73,12 %	593	880	1,36
476	2,97	500,06 $	0 %	73,16 %	593	831	1,44
539	2,97	499,45 $	0 %	73,21 %	593	863	1,39
490	2,95	498,46 $	0 %	73,29 %	591	836	1,44
496	2,95	498,26 $	0 %	73,31 %	590	838	1,43
517	2,95	497,99 $	0 %	73,33 %	590	849	1,41
540	2,94	497,20 $	0 %	73,39 %	588	858	1,40
464	2,92	496,34 $	0 %	73,46 %	584	816	1,47
505	2,92	494,94 $	0 %	73,57 %	583	836	1,44
536	2,91	494,34 $	0 %	73,62 %	582	850	1,41
516	2,91	493,99 $	0 %	73,65 %	581	839	1,43
488	2,90	493,82 $	0 %	73,66 %	580	824	1,46
558	2,89	493,39 $	0 %	73,69 %	578	858	1,40
537	2,89	492,46 $	0 %	73,77 %	578	846	1,42
498	2,88	491,82 $	0 %	73,82 %	576	825	1,45
491	2,88	491,52 $	0 %	73,84 %	575	821	1,46
477	2,87	491,37 $	0 %	73,85 %	574	812	1,48
490	2,87	490,56 $	0 %	73,92 %	573	818	1,47
518	2,86	489,94 $	0 %	73,97 %	572	831	1,44
482	2,85	489,82 $	0 %	73,98 %	571	812	1,48
496	2,85	489,36 $	0 %	74,02 %	571	819	1,47
563	2,84	488,59 $	0 %	74,08 %	567	849	1,41
550	2,83	487,88 $	0 %	74,13 %	567	842	1,43
532	2,83	487,35 $	0 %	74,18 %	566	833	1,44
514	2,83	486,98 $	0 %	74,21 %	566	823	1,46
574	2,80	486,00 $	0 %	74,28 %	561	848	1,42
556	2,80	485,26 $	0 %	74,34 %	561	839	1,43
471	2,80	485,19 $	0 %	74,35 %	560	795	1,51
516	2,80	483,96 $	0 %	74,45 %	559	817	1,47
532	2,79	483,68 $	0 %	74,47 %	558	824	1,46
502	2,78	482,93 $	0 %	74,53 %	557	808	1,49
499	2,78	482,43 $	0 %	74,57 %	555	805	1,49
526	2,77	481,28 $	0 %	74,67 %	553	816	1,47
537	2,76	480,91 $	0 %	74,70 %	552	820	1,46
531	2,76	480,60 $	0 %	74,72 %	551	817	1,47
546	2,75	479,89 $	0 %	74,78 %	549	822	1,46
504	2,74	478,84 $	0 %	74,86 %	548	800	1,50
506	2,74	478,81 $	0 %	74,87 %	548	801	1,50
545	2,73	478,75 $	0 %	74,87 %	547	819	1,46

539	2,73	478,62 $	0 %	74,88 %	547	816	1,47
514	2,72	477,51 $	0 %	74,97 %	545	802	1,50
507	2,72	477,26 $	0 %	74,99 %	544	797	1,50
507	2,71	476,47 $	0 %	75,05 %	542	796	1,51
530	2,71	476,26 $	0 %	75,07 %	542	807	1,49
532	2,71	476,26 $	0 %	75,07 %	542	808	1,49
502	2,71	475,98 $	0 %	75,09 %	541	792	1,51
543	2,70	475,78 $	0 %	75,11 %	540	812	1,48
543	2,70	475,58 $	0 %	75,13 %	540	811	1,48
531	2,68	473,49 $	0 %	75,30 %	536	801	1,50
523	2,67	472,82 $	0 %	75,35 %	534	796	1,51
526	2,66	472,20 $	0 %	75,40 %	533	796	1,51
512	2,66	472,11 $	0 %	75,41 %	533	789	1,52
505	2,66	471,77 $	0 %	75,44 %	532	784	1,53
556	2,65	471,14 $	0 %	75,48 %	529	807	1,49
530	2,64	470,39 $	0 %	75,55 %	529	794	1,51
526	2,64	469,69 $	0 %	75,60 %	527	790	1,52
502	2,63	469,58 $	0 %	75,61 %	527	778	1,54
542	2,63	469,53 $	0 %	75,62 %	526	798	1,50
512	2,62	468,06 $	0 %	75,74 %	524	780	1,54
509	2,62	467,98 $	0 %	75,74 %	523	778	1,54
503	2,61	467,73 $	0 %	75,76 %	523	775	1,55
503	2,61	467,50 $	0 %	75,78 %	522	774	1,55
564	2,59	466,72 $	0 %	75,84 %	519	801	1,50
561	2,59	465,93 $	0 %	75,91 %	517	798	1,50
553	2,59	465,63 $	0 %	75,93 %	517	794	1,51
500	2,58	464,95 $	0 %	75,99 %	517	767	1,57
553	2,57	464,42 $	1 %	76,03 %	514	791	1,52
557	2,57	464,05 $	1 %	76,06 %	513	792	1,52
538	2,57	463,47 $	1 %	76,11 %	513	782	1,53
507	2,56	463,24 $	1 %	76,13 %	513	766	1,57
526	2,56	462,86 $	1 %	76,16 %	512	775	1,55
514	2,56	462,82 $	1 %	76,16 %	512	769	1,56
553	2,55	462,52 $	1 %	76,18 %	510	787	1,53
525	2,55	461,97 $	1 %	76,23 %	510	772	1,55
509	2,54	461,44 $	1 %	76,27 %	509	764	1,57
482	2,53	461,03 $	1 %	76,30 %	507	748	1,60
508	2,53	460,52 $	1 %	76,35 %	507	761	1,58
507	2,53	460,38 $	1 %	76,36 %	507	760	1,58
512	2,53	459,98 $	1 %	76,39 %	506	762	1,58
501	2,52	459,37 $	1 %	76,44 %	504	755	1,59
569	2,51	459,04 $	1 %	76,46 %	501	786	1,53
507	2,51	457,91 $	1 %	76,56 %	501	755	1,59
559	2,49	457,49 $	1 %	76,59 %	499	778	1,54
478	2,49	457,44 $	1 %	76,59 %	499	737	1,63
483	2,49	456,92 $	1 %	76,64 %	498	739	1,62
514	2,48	455,69 $	1 %	76,74 %	496	753	1,59
502	2,48	455,60 $	1 %	76,75 %	496	747	1,61
521	2,47	454,94 $	1 %	76,80 %	495	755	1,59
527	2,46	453,43 $	1 %	76,92 %	491	755	1,59

545	2,45	452,82 $	1 %	76,97 %	489	762	1,58
497	2,44	452,53 $	1 %	76,99 %	489	737	1,63
522	2,44	451,72 $	1 %	77,06 %	487	748	1,60
493	2,43	451,38 $	1 %	77,09 %	486	733	1,64
509	2,42	450,35 $	1 %	77,17 %	484	739	1,62
529	2,42	450,22 $	1 %	77,18 %	484	748	1,60
541	2,41	449,51 $	1 %	77,24 %	482	753	1,59
484	2,41	449,51 $	1 %	77,24 %	481	723	1,66
513	2,41	448,92 $	1 %	77,29 %	481	738	1,63
518	2,40	448,42 $	1 %	77,33 %	480	739	1,62
565	2,39	448,24 $	1 %	77,33 %	478	760	1,58
501	2,39	447,37 $	1 %	77,41 %	478	728	1,65
498	2,38	447,17 $	1 %	77,43 %	477	726	1,65
514	2,37	445,82 $	1 %	77,54 %	474	731	1,64
523	2,37	445,59 $	1 %	77,55 %	474	735	1,63
504	2,37	445,31 $	1 %	77,58 %	473	725	1,66
546	2,35	444,69 $	1 %	77,62 %	471	744	1,61
537	2,35	443,86 $	1 %	77,69 %	470	738	1,63
520	2,35	443,50 $	1 %	77,72 %	469	729	1,65
521	2,34	443,06 $	1 %	77,76 %	468	728	1,65
516	2,34	442,62 $	1 %	77,79 %	467	725	1,65
542	2,33	442,36 $	1 %	77,81 %	466	737	1,63
498	2,33	442,22 $	1 %	77,82 %	466	715	1,68
514	2,33	441,97 $	1 %	77,84 %	466	723	1,66
542	2,33	441,95 $	1 %	77,84 %	465	736	1,63
556	2,32	441,56 $	1 %	77,87 %	463	741	1,62
499	2,32	440,96 $	1 %	77,92 %	463	713	1,68
556	2,31	440,81 $	1 %	77,93 %	462	740	1,62
464	2,29	439,65 $	1 %	78,01 %	458	690	1,74
477	2,28	438,74 $	1 %	78,09 %	457	695	1,73
510	2,28	437,54 $	1 %	78,19 %	456	711	1,69
462	2,26	437,38 $	1 %	78,19 %	452	683	1,76
496	2,26	436,10 $	1 %	78,30 %	452	700	1,71
543	2,26	435,84 $	1 %	78,32 %	451	723	1,66
531	2,26	435,55 $	1 %	78,35 %	451	717	1,67
494	2,25	435,28 $	1 %	78,37 %	450	698	1,72
489	2,25	435,14 $	1 %	78,38 %	450	694	1,73
545	2,25	435,00 $	1 %	78,39 %	450	722	1,66
481	2,24	434,67 $	1 %	78,41 %	448	689	1,74
513	2,24	433,93 $	1 %	78,47 %	448	704	1,70
498	2,24	433,90 $	1 %	78,47 %	447	697	1,72
519	2,24	433,75 $	1 %	78,49 %	447	707	1,70
510	2,22	432,43 $	1 %	78,59 %	445	699	1,72
500	2,22	431,94 $	1 %	78,63 %	443	693	1,73
488	2,21	431,62 $	1 %	78,65 %	442	686	1,75
529	2,20	430,64 $	1 %	78,73 %	440	705	1,70
520	2,20	430,18 $	1 %	78,76 %	440	699	1,72
524	2,19	429,70 $	1 %	78,80 %	438	701	1,71
536	2,19	429,40 $	1 %	78,82 %	437	705	1,70
517	2,19	429,06 $	1 %	78,85 %	437	696	1,73

497	2,17	428,23 $	1 %	78,91 %	435	683	1,76
501	2,16	426,89 $	2 %	79,01 %	432	682	1,76
501	2,16	426,80 $	2 %	79,02 %	432	682	1,76
527	2,15	426,02 $	2 %	79,08 %	430	694	1,73
471	2,14	425,84 $	2 %	79,07 %	428	663	1,81
477	2,14	425,37 $	2 %	79,11 %	427	666	1,80
511	2,13	424,38 $	2 %	79,20 %	427	682	1,76
532	2,13	424,35 $	2 %	79,20 %	426	692	1,73
528	2,13	423,94 $	2 %	79,23 %	426	689	1,74
515	2,12	423,08 $	2 %	79,29 %	424	681	1,76
516	2,10	421,64 $	2 %	79,40 %	421	679	1,77
502	2,10	421,12 $	2 %	79,43 %	419	670	1,79
486	2,09	420,70 $	2 %	79,46 %	418	660	1,82
524	2,09	420,26 $	2 %	79,50 %	417	679	1,77
512	2,08	419,76 $	2 %	79,53 %	416	672	1,78
537	2,08	419,31 $	2 %	79,56 %	415	684	1,76
497	2,07	418,99 $	2 %	79,58 %	414	663	1,81
529	2,07	418,76 $	2 %	79,60 %	414	678	1,77
546	2,07	418,58 $	2 %	79,61 %	413	686	1,75
528	2,06	417,70 $	2 %	79,68 %	412	676	1,78
501	2,06	417,48 $	2 %	79,69 %	411	661	1,81
537	2,05	417,31 $	2 %	79,70 %	411	679	1,77
513	2,05	416,93 $	2 %	79,73 %	410	666	1,80
556	2,04	416,84 $	2 %	79,72 %	409	687	1,75
510	2,04	416,06 $	2 %	79,79 %	408	663	1,81
525	2,03	415,55 $	2 %	79,83 %	407	669	1,79
499	2,03	415,33 $	2 %	79,84 %	406	656	1,83
570	2,01	414,41 $	2 %	79,87 %	402	687	1,75
554	2,00	413,08 $	2 %	79,98 %	400	677	1,77
522	2,00	412,33 $	2 %	80,04 %	400	661	1,82
503	2,00	412,02 $	2 %	80,06 %	399	650	1,85
523	1,99	411,93 $	2 %	80,07 %	399	660	1,82
517	1,99	411,57 $	2 %	80,09 %	398	657	1,83
532	1,99	411,29 $	2 %	80,11 %	397	663	1,81
484	1,98	411,09 $	2 %	80,11 %	396	638	1,88
502	1,98	410,69 $	2 %	80,15 %	396	647	1,85
521	1,97	409,87 $	2 %	80,20 %	394	655	1,83
506	1,97	409,61 $	2 %	80,21 %	394	646	1,86
501	1,97	409,58 $	2 %	80,21 %	394	644	1,86
508	1,97	409,28 $	2 %	80,24 %	393	647	1,85
548	1,96	408,93 $	3 %	80,24 %	391	665	1,80
556	1,95	408,36 $	3 %	80,27 %	390	668	1,80
486	1,95	408,06 $	3 %	80,30 %	389	632	1,90
496	1,93	406,62 $	3 %	80,39 %	387	635	1,89
520	1,93	406,13 $	3 %	80,43 %	386	646	1,86
548	1,92	405,88 $	3 %	80,43 %	385	659	1,82
532	1,92	405,54 $	3 %	80,46 %	385	651	1,84
559	1,91	405,14 $	3 %	80,46 %	382	662	1,81
468	1,90	404,81 $	3 %	80,46 %	381	615	1,95
543	1,90	403,74 $	3 %	80,55 %	380	652	1,84

532	1,89	402,96 $	3 %	80,60 %	379	645	1,86
550	1,89	402,63 $	3 %	80,60 %	377	653	1,84
564	1,88	402,25 $	3 %	80,61 %	375	658	1,82
516	1,88	401,30 $	3 %	80,69 %	375	633	1,89
495	1,87	401,03 $	3 %	80,70 %	374	622	1,93
532	1,87	400,84 $	3 %	80,71 %	374	640	1,87
501	1,87	400,51 $	3 %	80,73 %	373	624	1,92
518	1,87	400,34 $	3 %	80,74 %	373	632	1,90
531	1,86	399,75 $	3 %	80,77 %	372	637	1,88
523	1,86	399,58 $	3 %	80,78 %	372	633	1,90
533	1,85	399,12 $	3 %	80,80 %	370	637	1,88
545	1,85	399,05 $	3 %	80,79 %	370	642	1,87
521	1,85	398,57 $	3 %	80,83 %	369	630	1,91
517	1,84	397,95 $	3 %	80,86 %	368	626	1,92
517	1,84	397,85 $	3 %	80,86 %	368	626	1,92
529	1,83	397,53 $	3 %	80,87 %	367	631	1,90
511	1,83	397,00 $	3 %	80,90 %	366	621	1,93
510	1,82	396,44 $	3 %	80,93 %	365	620	1,94
490	1,82	396,30 $	3 %	80,91 %	364	608	1,97
553	1,81	395,86 $	4 %	80,92 %	362	638	1,88
486	1,81	395,52 $	4 %	80,94 %	362	605	1,98
564	1,79	394,38 $	4 %	80,96 %	358	640	1,88
486	1,79	393,88 $	4 %	81,00 %	358	601	2,00
492	1,79	393,70 $	4 %	81,02 %	358	604	1,99
529	1,79	393,10 $	4 %	81,06 %	357	621	1,93
540	1,78	392,93 $	4 %	81,05 %	356	626	1,92
511	1,78	392,47 $	4 %	81,08 %	356	611	1,96
506	1,77	391,69 $	4 %	81,11 %	354	607	1,98
485	1,76	391,34 $	4 %	81,09 %	352	594	2,02
545	1,76	391,00 $	4 %	81,11 %	352	624	1,92
516	1,75	389,58 $	4 %	81,17 %	349	607	1,98
495	1,73	388,48 $	4 %	81,19 %	346	594	2,02
486	1,73	388,08 $	4 %	81,18 %	345	588	2,04
498	1,72	387,60 $	4 %	81,21 %	345	594	2,02
489	1,72	387,36 $	4 %	81,20 %	344	588	2,04
530	1,71	386,15 $	4 %	81,24 %	342	606	1,98
533	1,70	385,58 $	4 %	81,25 %	340	607	1,98
503	1,70	385,18 $	4 %	81,26 %	339	591	2,03
476	1,69	385,08 $	5 %	81,20 %	338	576	2,09
519	1,69	384,10 $	5 %	81,28 %	337	596	2,01
504	1,68	384,03 $	5 %	81,28 %	337	589	2,04
510	1,68	383,68 $	5 %	81,28 %	336	591	2,03
506	1,67	383,09 $	5 %	81,29 %	335	588	2,04
567	1,66	382,80 $	5 %	81,20 %	332	615	1,95
544	1,65	381,60 $	5 %	81,27 %	331	603	1,99
466	1,64	381,55 $	5 %	81,19 %	329	562	2,14
495	1,64	380,15 $	5 %	81,28 %	328	575	2,09
518	1,63	379,12 $	5 %	81,30 %	326	585	2,05
513	1,62	378,56 $	5 %	81,29 %	325	581	2,06
529	1,62	378,28 $	5 %	81,28 %	324	589	2,04

508	1,62	377,93 $	5 %	81,28 %	323	577	2,08
488	1,61	377,75 $	5 %	81,24 %	322	566	2,12
499	1,60	376,92 $	5 %	81,26 %	321	570	2,10
552	1,59	376,24 $	6 %	81,20 %	319	595	2,02
551	1,58	374,91 $	6 %	81,18 %	316	591	2,03
532	1,58	374,32 $	6 %	81,21 %	315	581	2,06
519	1,57	373,54 $	6 %	81,20 %	314	573	2,09
512	1,57	373,33 $	6 %	81,19 %	313	569	2,11
503	1,55	372,21 $	6 %	81,14 %	311	562	2,14
527	1,55	372,16 $	6 %	81,15 %	311	574	2,09
501	1,55	372,01 $	6 %	81,13 %	310	561	2,14
499	1,55	371,85 $	6 %	81,12 %	310	559	2,15
514	1,55	371,55 $	6 %	81,13 %	309	566	2,12
504	1,55	371,51 $	6 %	81,12 %	309	561	2,14
529	1,54	371,43 $	6 %	81,12 %	309	573	2,09
518	1,54	371,02 $	6 %	81,11 %	308	567	2,12
527	1,52	369,24 $	6 %	81,02 %	304	567	2,11
482	1,51	368,85 $	7 %	80,93 %	302	543	2,21
530	1,51	368,00 $	7 %	80,95 %	301	566	2,12
513	1,50	367,61 $	7 %	80,94 %	301	557	2,15
520	1,50	367,19 $	7 %	80,91 %	300	560	2,14
524	1,49	366,54 $	7 %	80,87 %	298	560	2,14
487	1,47	365,43 $	7 %	80,73 %	295	538	2,23
539	1,47	364,98 $	7 %	80,73 %	294	564	2,13
496	1,47	364,91 $	7 %	80,73 %	294	542	2,21
517	1,47	364,50 $	7 %	80,73 %	294	552	2,17
506	1,46	363,52 $	7 %	80,64 %	291	545	2,20
483	1,45	363,47 $	7 %	80,56 %	290	532	2,26
508	1,45	362,75 $	7 %	80,58 %	290	543	2,21
552	1,44	362,22 $	8 %	80,45 %	287	563	2,13
538	1,44	361,79 $	8 %	80,46 %	287	556	2,16
521	1,43	360,91 $	8 %	80,41 %	286	546	2,20
504	1,42	360,45 $	8 %	80,36 %	284	536	2,24
498	1,42	360,36 $	8 %	80,33 %	284	533	2,25
536	1,42	360,35 $	8 %	80,33 %	284	552	2,17
522	1,41	359,57 $	8 %	80,28 %	283	544	2,21
481	1,40	359,03 $	8 %	80,11 %	280	521	2,30
546	1,40	358,71 $	8 %	80,12 %	280	553	2,17
563	1,39	358,07 $	8 %	79,94 %	277	559	2,15
497	1,38	357,03 $	8 %	79,96 %	277	525	2,28
536	1,38	356,83 $	8 %	79,93 %	276	544	2,21
479	1,37	356,33 $	9 %	79,77 %	274	513	2,34
489	1,36	355,03 $	9 %	79,66 %	272	516	2,32
543	1,35	354,61 $	9 %	79,61 %	271	543	2,21
552	1,34	353,90 $	9 %	79,46 %	269	545	2,20
544	1,34	353,63 $	9 %	79,47 %	269	541	2,22
481	1,34	353,49 $	9 %	79,39 %	268	508	2,36
506	1,34	352,96 $	9 %	79,43 %	268	521	2,30
545	1,33	352,78 $	9 %	79,34 %	267	539	2,22
509	1,33	352,22 $	9 %	79,33 %	266	521	2,30

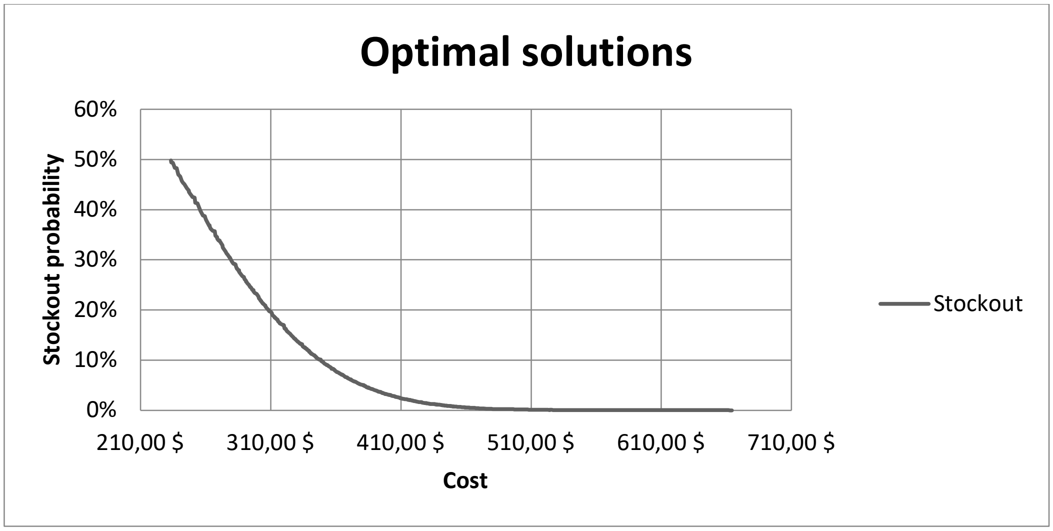
508	1,32	351,45 $	9 %	79,21 %	265	519	2,31
524	1,32	351,35 $	9 %	79,19 %	264	526	2,28
492	1,31	350,70 $	9 %	79,02 %	262	508	2,36
481	1,30	350,34 $	10 %	78,89 %	261	501	2,39
538	1,30	349,68 $	10 %	78,87 %	260	529	2,27
557	1,29	349,55 $	10 %	78,74 %	259	537	2,23
478	1,29	349,51 $	10 %	78,72 %	259	498	2,41
543	1,29	349,01 $	10 %	78,73 %	259	530	2,26
573	1,27	348,35 $	10 %	78,37 %	255	541	2,22
537	1,27	347,26 $	10 %	78,44 %	255	523	2,29
519	1,27	346,68 $	10 %	78,38 %	254	513	2,34
510	1,27	346,67 $	10 %	78,37 %	254	509	2,36
508	1,27	346,42 $	10 %	78,32 %	253	507	2,37
498	1,25	345,37 $	10 %	78,08 %	251	500	2,40
536	1,25	345,12 $	11 %	78,03 %	250	518	2,32
544	1,24	344,66 $	11 %	77,90 %	249	521	2,30
486	1,24	344,48 $	11 %	77,83 %	248	491	2,44
542	1,24	344,24 $	11 %	77,82 %	248	519	2,31
490	1,24	343,96 $	11 %	77,75 %	247	492	2,44
480	1,23	343,70 $	11 %	77,61 %	246	486	2,47
493	1,23	343,28 $	11 %	77,62 %	246	492	2,44
542	1,23	343,03 $	11 %	77,57 %	245	516	2,33
536	1,22	342,39 $	11 %	77,46 %	244	512	2,34
540	1,21	341,77 $	11 %	77,30 %	243	513	2,34
540	1,21	341,41 $	11 %	77,22 %	242	512	2,34
525	1,21	340,98 $	11 %	77,18 %	241	504	2,38
504	1,21	340,97 $	11 %	77,16 %	241	493	2,43
547	1,19	340,07 $	12 %	76,86 %	238	512	2,34
515	1,19	339,57 $	12 %	76,86 %	238	496	2,42
565	1,18	339,40 $	12 %	76,53 %	236	518	2,32
496	1,18	338,62 $	12 %	76,57 %	236	483	2,48
492	1,18	338,62 $	12 %	76,54 %	235	482	2,49
536	1,17	338,14 $	12 %	76,46 %	235	502	2,39
472	1,16	338,01 $	12 %	76,19 %	233	469	2,56
480	1,16	337,67 $	12 %	76,20 %	233	473	2,54
561	1,16	337,37 $	12 %	76,07 %	232	512	2,34
545	1,15	336,65 $	12 %	76,03 %	231	503	2,38
488	1,15	336,62 $	12 %	76,01 %	231	475	2,53
480	1,15	336,39 $	13 %	75,86 %	230	470	2,55
502	1,15	335,63 $	13 %	75,84 %	229	480	2,50
523	1,14	335,35 $	13 %	75,79 %	229	490	2,45
532	1,14	335,06 $	13 %	75,68 %	228	494	2,43
522	1,14	334,88 $	13 %	75,66 %	228	489	2,45
510	1,14	334,77 $	13 %	75,63 %	227	482	2,49
478	1,13	334,38 $	13 %	75,30 %	225	464	2,59
572	1,12	334,33 $	13 %	75,11 %	224	510	2,35
530	1,12	333,08 $	13 %	75,15 %	224	489	2,46
478	1,11	332,97 $	13 %	74,91 %	222	461	2,60
500	1,11	332,32 $	13 %	74,91 %	222	472	2,54
501	1,10	331,53 $	14 %	74,70 %	220	471	2,55

537	1,09	331,05 $	14 %	74,53 %	219	488	2,46
527	1,09	330,91 $	14 %	74,53 %	219	482	2,49
515	1,08	329,27 $	14 %	74,06 %	215	473	2,54
491	1,07	328,96 $	14 %	73,87 %	214	460	2,61
506	1,07	328,38 $	14 %	73,78 %	213	466	2,57
514	1,06	328,14 $	14 %	73,72 %	213	470	2,55
493	1,06	327,59 $	15 %	73,45 %	211	457	2,62
509	1,05	327,10 $	15 %	73,38 %	210	465	2,58
504	1,04	325,96 $	15 %	73,00 %	208	460	2,61
489	1,02	324,81 $	15 %	72,52 %	205	449	2,67
506	1,02	324,11 $	15 %	72,41 %	204	457	2,63
537	1,01	323,05 $	16 %	72,00 %	201	469	2,56
512	1,00	322,74 $	16 %	71,96 %	201	457	2,63
504	0,99	321,61 $	16 %	71,55 %	198	450	2,67
559	0,98	321,55 $	16 %	71,26 %	197	476	2,52
508	0,98	320,64 $	16 %	71,23 %	196	450	2,66
459	0,96	320,16 $	17 %	70,43 %	192	421	2,85
580	0,96	320,06 $	17 %	70,41 %	191	481	2,49
581	0,95	319,91 $	17 %	70,32 %	191	482	2,49
563	0,95	318,98 $	17 %	70,29 %	190	472	2,54
520	0,94	316,98 $	17 %	69,93 %	188	448	2,68
493	0,94	316,90 $	17 %	69,80 %	187	434	2,77
500	0,93	315,85 $	18 %	69,47 %	185	435	2,76
504	0,93	315,79 $	18 %	69,46 %	185	437	2,74
485	0,92	315,54 $	18 %	69,20 %	184	426	2,82
548	0,92	315,33 $	18 %	69,15 %	183	457	2,62
475	0,91	315,30 $	18 %	68,97 %	182	420	2,86
484	0,91	314,47 $	18 %	68,78 %	181	423	2,84
539	0,90	313,76 $	18 %	68,64 %	180	450	2,67
483	0,90	313,71 $	18 %	68,48 %	180	421	2,85
506	0,90	313,06 $	19 %	68,44 %	179	432	2,78
546	0,89	312,97 $	19 %	68,27 %	178	451	2,66
518	0,88	311,39 $	19 %	67,82 %	176	434	2,76
470	0,86	310,93 $	19 %	67,19 %	172	407	2,95
478	0,85	309,99 $	20 %	66,97 %	171	410	2,93
511	0,85	308,91 $	20 %	66,84 %	170	426	2,82
504	0,85	308,87 $	20 %	66,80 %	170	422	2,85
549	0,84	308,82 $	20 %	66,62 %	169	443	2,71
523	0,84	308,34 $	20 %	66,61 %	169	430	2,79
502	0,83	307,45 $	20 %	66,22 %	167	418	2,87
534	0,83	307,45 $	20 %	66,21 %	167	433	2,77
535	0,83	307,19 $	20 %	66,09 %	166	434	2,77
521	0,82	306,10 $	21 %	65,71 %	164	424	2,83
556	0,81	305,90 $	21 %	65,35 %	162	440	2,73
490	0,81	305,37 $	21 %	65,28 %	162	407	2,95
496	0,80	304,78 $	21 %	65,10 %	160	409	2,94
538	0,80	304,74 $	21 %	65,07 %	160	430	2,79
499	0,80	304,16 $	21 %	64,86 %	159	409	2,94
496	0,79	304,12 $	21 %	64,82 %	159	407	2,95
533	0,79	303,92 $	21 %	64,78 %	159	425	2,82

493	0,79	303,82 $	21 %	64,67 %	158	404	2,97
532	0,79	303,47 $	22 %	64,60 %	158	424	2,83
531	0,78	302,76 $	22 %	64,31 %	156	422	2,85
501	0,78	302,61 $	22 %	64,24 %	156	406	2,96
499	0,78	302,43 $	22 %	64,15 %	155	405	2,96
516	0,77	301,93 $	22 %	64,01 %	155	413	2,91
507	0,77	301,91 $	22 %	63,98 %	154	408	2,94
492	0,77	301,77 $	22 %	63,81 %	154	399	3,00
522	0,76	300,94 $	22 %	63,59 %	152	413	2,90
477	0,75	300,67 $	23 %	63,15 %	150	389	3,09
470	0,73	299,40 $	23 %	62,47 %	147	382	3,14
512	0,73	297,73 $	23 %	62,25 %	145	401	2,99
524	0,73	297,69 $	23 %	62,22 %	145	407	2,95
500	0,72	296,93 $	24 %	61,85 %	143	393	3,05
475	0,71	296,81 $	24 %	61,48 %	141	379	3,17
514	0,70	295,81 $	24 %	61,44 %	141	398	3,02
558	0,69	295,31 $	24 %	60,90 %	138	417	2,88
535	0,69	294,43 $	25 %	60,79 %	138	405	2,96
528	0,69	294,33 $	25 %	60,78 %	138	402	2,99
503	0,68	293,94 $	25 %	60,61 %	137	388	3,09
493	0,67	293,35 $	25 %	60,27 %	135	381	3,15
534	0,67	292,99 $	25 %	60,18 %	134	401	2,99
529	0,67	292,89 $	25 %	60,17 %	134	399	3,01
519	0,66	292,15 $	25 %	59,88 %	133	392	3,06
546	0,65	291,61 $	26 %	59,49 %	131	404	2,97
526	0,65	291,29 $	26 %	59,50 %	131	394	3,05
496	0,65	290,85 $	26 %	59,24 %	130	377	3,18
528	0,64	290,11 $	26 %	58,99 %	128	392	3,06
512	0,64	289,89 $	26 %	58,92 %	128	384	3,13
474	0,63	289,56 $	27 %	58,37 %	125	362	3,32
549	0,62	288,86 $	27 %	58,28 %	125	399	3,01
500	0,62	288,13 $	27 %	58,12 %	124	374	3,21
505	0,61	287,34 $	27 %	57,81 %	122	375	3,20
505	0,60	286,52 $	27 %	57,46 %	120	373	3,22
512	0,59	285,80 $	28 %	57,18 %	119	375	3,20
482	0,59	285,67 $	28 %	56,87 %	117	358	3,35
521	0,59	285,05 $	28 %	56,87 %	117	377	3,18
502	0,58	284,85 $	28 %	56,74 %	116	367	3,27
527	0,58	284,29 $	28 %	56,53 %	115	379	3,17
516	0,58	284,24 $	28 %	56,53 %	115	373	3,22
528	0,57	284,19 $	28 %	56,48 %	115	379	3,16
478	0,56	283,52 $	29 %	55,90 %	112	351	3,42
566	0,55	282,97 $	29 %	55,55 %	110	393	3,05
554	0,55	282,33 $	29 %	55,46 %	110	387	3,10
546	0,55	282,10 $	29 %	55,47 %	110	383	3,14
529	0,55	281,71 $	29 %	55,43 %	109	374	3,21
522	0,54	281,21 $	29 %	55,26 %	108	369	3,25
527	0,53	280,33 $	30 %	54,87 %	106	370	3,24
497	0,52	279,16 $	30 %	54,33 %	104	352	3,41
513	0,51	278,64 $	30 %	54,19 %	103	359	3,34

491	0,51	278,63 $	30 %	54,06 %	102	348	3,45
519	0,51	277,88 $	31 %	53,89 %	101	360	3,33
538	0,49	277,09 $	31 %	53,47 %	99	368	3,26
521	0,49	276,56 $	31 %	53,34 %	98	359	3,34
508	0,48	275,98 $	31 %	53,10 %	97	351	3,42
495	0,48	275,66 $	32 %	52,89 %	96	343	3,50
505	0,48	275,39 $	32 %	52,85 %	95	348	3,45
527	0,45	273,37 $	32 %	52,05 %	91	354	3,39
561	0,44	272,99 $	33 %	51,57 %	88	369	3,25
471	0,43	272,14 $	33 %	51,16 %	86	322	3,73
565	0,43	271,89 $	33 %	51,07 %	86	368	3,26
480	0,42	270,89 $	34 %	50,83 %	84	324	3,70
565	0,42	270,80 $	34 %	50,66 %	83	366	3,28
493	0,42	270,07 $	34 %	50,68 %	83	330	3,64
530	0,41	269,19 $	34 %	50,42 %	82	347	3,46
563	0,40	268,94 $	35 %	49,99 %	79	361	3,33
488	0,40	268,35 $	35 %	49,98 %	79	323	3,72
497	0,39	267,62 $	35 %	49,79 %	78	326	3,68
456	0,37	267,05 $	36 %	48,91 %	73	301	3,98
550	0,36	265,62 $	36 %	48,94 %	73	348	3,45
508	0,35	264,03 $	36 %	48,54 %	70	324	3,70
510	0,34	263,05 $	37 %	48,20 %	68	323	3,71
548	0,34	263,03 $	37 %	48,04 %	67	341	3,52
494	0,34	262,83 $	37 %	48,04 %	67	314	3,82
520	0,34	262,55 $	37 %	48,03 %	67	327	3,67
556	0,31	260,66 $	38 %	47,14 %	61	339	3,54
477	0,30	260,10 $	38 %	46,91 %	60	298	4,02
465	0,29	259,48 $	39 %	46,51 %	57	290	4,14
517	0,28	257,95 $	39 %	46,49 %	57	315	3,80
552	0,26	256,22 $	40 %	45,75 %	52	328	3,66
458	0,22	253,89 $	41 %	44,65 %	44	273	4,39
537	0,22	252,10 $	41 %	44,64 %	43	312	3,85
535	0,22	251,96 $	41 %	44,61 %	43	311	3,86
467	0,20	251,78 $	42 %	44,21 %	41	274	4,38
453	0,19	251,55 $	42 %	43,88 %	38	265	4,53
480	0,19	249,93 $	43 %	43,88 %	38	277	4,33
560	0,19	249,80 $	43 %	43,81 %	37	317	3,79
532	0,17	248,02 $	43 %	43,55 %	35	300	3,99
493	0,16	247,25 $	44 %	43,30 %	32	279	4,30
560	0,16	247,14 $	44 %	43,11 %	31	311	3,86
546	0,15	245,86 $	44 %	42,92 %	29	302	3,97
497	0,12	243,63 $	45 %	42,44 %	25	273	4,39
501	0,12	242,87 $	45 %	42,29 %	23	274	4,38
523	0,11	241,97 $	46 %	42,11 %	21	283	4,24
482	0,08	240,56 $	47 %	41,66 %	17	258	4,65
519	0,08	239,14 $	47 %	41,51 %	15	275	4,37
541	0,07	238,90 $	47 %	41,40 %	14	284	4,22
583	0,04	238,02 $	48 %	40,88 %	9	300	4,00
539	0,04	236,23 $	48 %	40,90 %	8	278	4,32
482	0,04	236,13 $	49 %	40,80 %	7	248	4,84

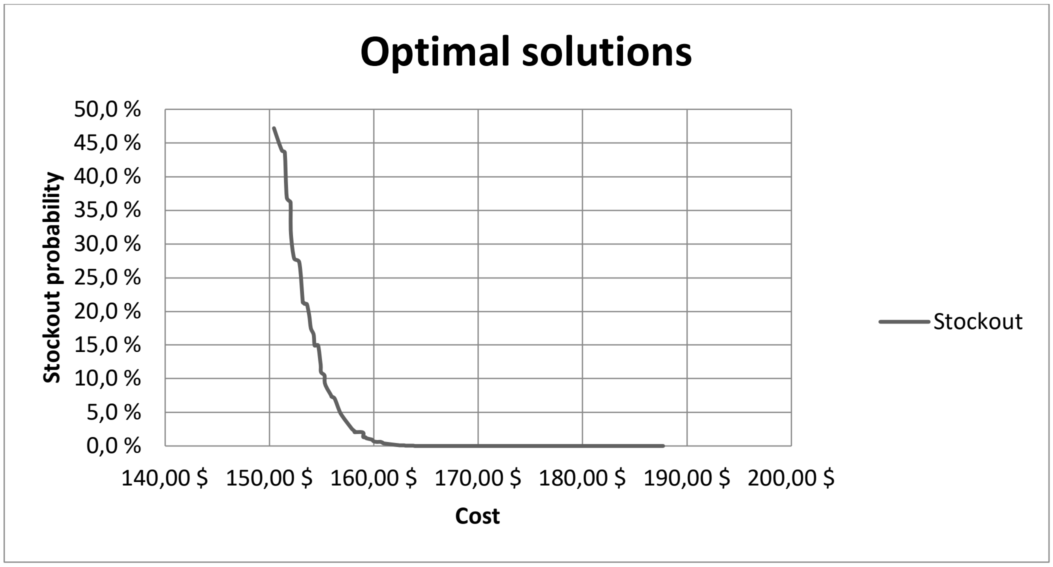
571	0,01	234,89 $	49 %	40,44 %	3	288	4,16
531	0,01	233,71 $	49 %	40,48 %	3	268	4,48
480	0,00	233,36 $	50 %	40,30 %	1	241	4,99



Test 2.

				Final optimized values			
Q	k	Cost	Stockout	TOPSIS grade	Safety stock	Average stock	Inventory turnover
769	4,80	187,72 $	0,0 %	97 %	192	576	1,74
870	4,79	177,63 $	0,0 %	98 %	192	626	1,60
952	4,76	171,65 $	0,0 %	98 %	190	667	1,50
997	4,60	168,53 $	0,0 %	99 %	184	683	1,46
973	4,21	168,28 $	0,0 %	99 %	168	655	1,53
999	4,00	166,09 $	0,0 %	99 %	160	659	1,52
996	3,89	165,78 $	0,0 %	99 %	156	653	1,53
987	3,60	165,08 $	0,0 %	99 %	144	637	1,57
986	3,51	164,73 $	0,0 %	99 %	140	634	1,58
996	3,43	163,92 $	0,0 %	99 %	137	635	1,57
990	3,30	163,70 $	0,0 %	99 %	132	627	1,59
995	3,29	163,38 $	0,1 %	99 %	131	629	1,59
999	3,26	163,06 $	0,1 %	99 %	130	630	1,59
991	3,10	162,87 $	0,1 %	99 %	124	620	1,61
998	3,09	162,44 $	0,1 %	99 %	124	623	1,61
991	2,93	162,22 $	0,2 %	99 %	117	613	1,63
995	2,67	160,94 $	0,4 %	99 %	107	604	1,66
990	2,58	160,85 $	0,5 %	99 %	103	598	1,67
988	2,54	160,76 $	0,5 %	99 %	102	596	1,68
989	2,51	160,61 $	0,6 %	99 %	100	594	1,68
996	2,50	160,20 $	0,6 %	99 %	100	598	1,67
996	2,44	159,95 $	0,7 %	98 %	98	596	1,68

992	2,35	159,81 $	0,9 %	98 %	94	590	1,70
996	2,27	159,28 $	1,2 %	98 %	91	589	1,70
994	2,23	159,21 $	1,3 %	97 %	89	586	1,71
998	2,21	158,98 $	1,3 %	97 %	89	587	1,70
985	2,05	158,96 $	2,0 %	96 %	82	575	1,74
1000	2,04	158,17 $	2,1 %	96 %	82	581	1,72
998	2,02	158,17 $	2,2 %	96 %	81	580	1,72
999	1,97	157,93 $	2,5 %	95 %	79	578	1,73
998	1,92	157,79 $	2,7 %	94 %	77	576	1,74
996	1,66	156,82 $	4,9 %	90 %	66	564	1,77
993	1,47	156,23 $	7,1 %	86 %	59	555	1,80
997	1,45	155,97 $	7,3 %	86 %	58	557	1,80
997	1,45	155,96 $	7,4 %	85 %	58	556	1,80
1000	1,33	155,32 $	9,2 %	82 %	53	553	1,81
995	1,26	155,28 $	10,5 %	79 %	50	548	1,83
1000	1,23	154,93 $	11,0 %	78 %	49	549	1,82
996	1,17	154,90 $	12,1 %	76 %	47	545	1,84
990	1,04	154,67 $	14,9 %	71 %	42	537	1,86
997	1,04	154,33 $	15,0 %	71 %	42	540	1,85
993	0,97	154,22 $	16,5 %	68 %	39	536	1,87
995	0,93	153,97 $	17,6 %	66 %	37	535	1,87
993	0,86	153,81 $	19,4 %	62 %	34	531	1,88
993	0,80	153,59 $	21,1 %	59 %	32	528	1,89
997	0,80	153,36 $	21,1 %	59 %	32	531	1,88
997	0,80	153,32 $	21,2 %	59 %	32	531	1,88
1000	0,79	153,20 $	21,4 %	59 %	32	532	1,88
991	0,60	152,86 $	27,3 %	49 %	24	520	1,92
1000	0,59	152,37 $	27,8 %	48 %	24	523	1,91
997	0,48	152,05 $	31,5 %	42 %	19	518	1,93
988	0,35	152,02 $	36,2 %	35 %	14	508	1,97
991	0,35	151,86 $	36,4 %	35 %	14	509	1,96
994	0,33	151,65 $	36,9 %	35 %	13	510	1,96
984	0,16	151,47 $	43,6 %	28 %	6	498	2,01
989	0,15	151,20 $	43,9 %	28 %	6	500	2,00
997	0,07	150,44 $	47,2 %	26 %	3	501	1,99



Optimal solutions
Stockout probability
50,0 %
45,0 %
40,0 %
35,0 %
30,0 %
25,0 %
20,0 %
15,0 %
10,0 %
5,0 %
0,0 %
140,00 $
150,00 $
160,00 $
170,00 $
180,00 $
190,00 $
200,00 $
Cost
Stockout