Buckling Prevention In Lightweight Stiffened Structures.

Charles Nutakor

Degree Thesis

Plastics Technology

2012

DEGREE THESIS	
Arcada	
Degree Programme:	Plastics Technology
Identification number:	10292
Author:	Charles Nutakor
Title:	Buckling Prevention In Lightweight Stiffened Structures
Supervisor (Arcada):	Rene Herrmann
Commissioned by:	

Abstract:

The focus of this thesis is to investigate the buckling prevention in lightweight stiffened structures using a finite element model. The emphasis of the investigation presented in this thesis is to find the critical length at which buckling preventers can be attached along the side of a stiffening beam so it can effectively prevent the stiffened structure from buckling.

The investigation was conducted using a finite element model of a lightweight structure and the analysis and simulations were carried out using the NX NASTRAN finite element analysis software. The analysis method is based on a general form of argument known as the reductio ad absurdum. During the investigation the critical length *Lc* was studied as a function of slenderness ratio S_r of a slender beam, where the width *t* is constant and the height *h* is variable.

The critical length *Lc* at which the buckling preventer becomes irrelevant was determined for five different slenderness of the beam. The imperial finding is

$$Lc = 222{,}3S_r + 981{,}7 \quad \text{[mm]}$$

$$R^2 = 0{,}9984 \quad 10 < S_r < 200 \,;\, t = 10 \,,\, h = \text{variable}$$

where R^2 is the coefficient of determination

Keywords:	Lightweight, stiffened structures, buckling, critical length, finite element analysis, slenderness ratio, NX NASTRAN
Number of pages:	48
Language:	ENGLISH
Date of acceptance:	24.04.2012

Contents

Figures

Tables

FOREWORD

First and foremost I would like to thank God for his abundant love in my life. Without his love I could not have done anything and this thesis would not have been a success.

I am sincerely grateful to my thesis supervisor Rene Hermann. For his enthusiasm, encouragement and resolute dedication to my thesis. He has in diverse ways consistently helped me improve and supported me gain my accomplishments. Under his guidance the thesis objectives were achieved in a timely and orderly manner.

I would also like to thank my parents Rita Nutakor and Godwin Nutakor as well as my siblings for their unconditional love and constant support throughout my studies.

I gratefully acknowledge the contributions by Jukka Airaksinen and Pasi Marttila at IDEAL PLM and COMSOL OY Finland respectively for tutoring me on the finite element analysis and helping with my NX NASTRAN queries.

I would like to thank all my friends especially Emilia Österman who stood by me and was a great source of encouragement.

March 4th 2012

1 INTRODUCTION

The presented Bachelor's Thesis is about how to prevent buckling in lightweight stiffened structures, with respect to the spacing of buckling preventers along the stiffener of light weight structures. In today's world structural efficiency is the primary concern in the ship, aerospace and aircraft industries. That is why there is a high need for thin, strong and lightweight materials which are capable of withstanding high stresses. Thin plates of various shapes used in naval and aeronautical structures are often subjected to normal compressive and shearing loads acting in the middle plane of the plate (in-plane loads). Under certain conditions such loads can result in a plate buckling. Buckling or elastic instability of plates is of great practical importance. The buckling load depends on the plate thickness: the thinner the plate, the lower is the buckling load. In many cases, a failure of thin plate elements may be attributed to an elastic instability and not to the lack of their strength. Therefore, plate buckling analysis presents an integral part of the general analysis of a structure. This is why it is important for engineers in the above mentioned industries to know how far apart to space buckling preventers along the sides of structural stiffeners.

1.1 Background

Product weight and durability are very vital to the engineers in the aerospace, boat and aircraft industries. The common way to ensure that buckling resistance in a structural member is sufficient, may be either to increase the web thickness of the member or by using stiffeners. The choice is in most cases based on total economy that is the cost for increasing the web thickness of the member and hence result in a heavy weight product and the cost involved in reinforcing thin walled members with light weight stiffeners. This thesis is focused on thin walled structural members hence, the buckling prevention in lightweight stiffened structures. Engineers of today are most concerned about light weight structural performance and efficiency. This means high performance products need to be lightweight, yet strong enough to take harsh loading conditions. For the product to be light in weight, that means the skin has to be thin and if the skin is thin it is obvious that the product under goes lateral twisting, that is why the product member needs to be reinforced with light weight stiffeners, to help provide the required buckling resistance in the member system.

To illustrate this, consider the example of the simply supported thin walled member (slender column) shown in figure 1. If a force F is applied centrally over a stiffener of length *l*, height *h*, and thickness *t*. With any small force *F* the stiffened panel will re-main straight and support it.

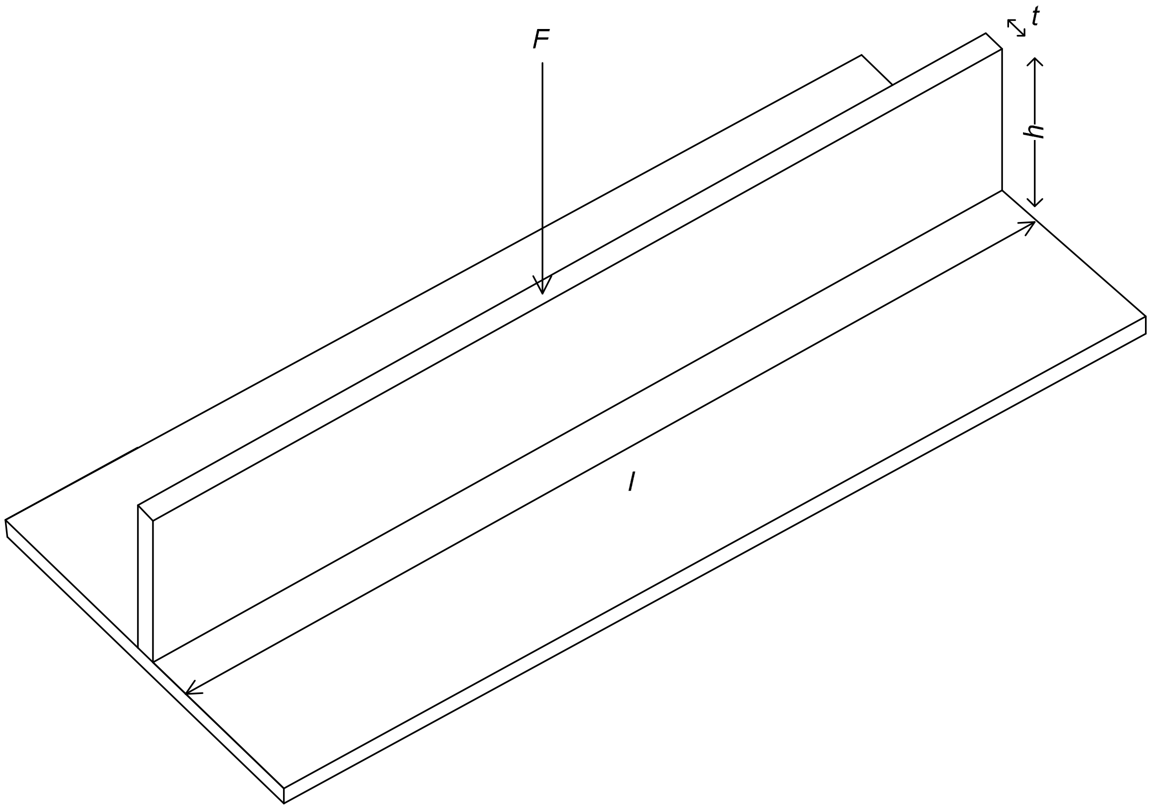

Figure 1: Slender column attached to the surface of a plate.

When a slender column is subjected to small compressive stresses or loads, as shown in figure 2, where the actual compressive stress at this point is less than the ultimate compressive stresses that the material is capable of withstanding, the structure will not undergo any visibly large displacement.

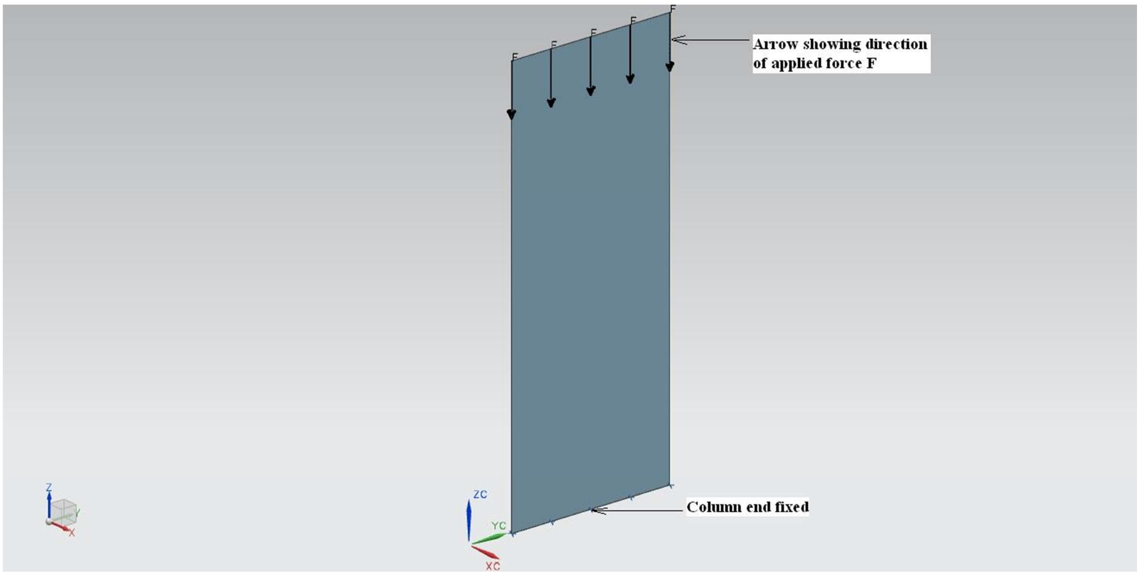

Figure 2: Compressive stresses applied to column

This means that, the only deformation that takes place is the axial shortening of the column as shown in figure 3. For low values of the applied force, if the column were to be deflected laterally by a force perpendicular to the column, and the lateral force were thereafter removed, the column will return to its straight position, even with the force F remaining in place. This means the structure remains in stable equilibrium.

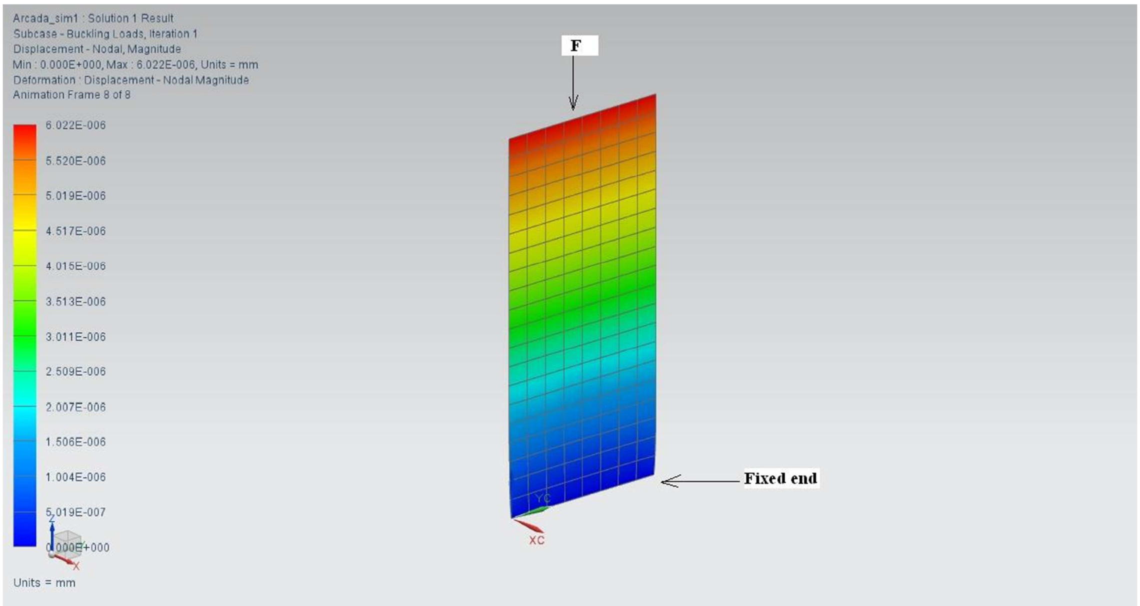

Figure 3: Shortening of the column

As compressive loads increase, a slender column can buckle, as shown in figure 4 if the applied load (or stress) reaches a critical value. This form of buckling usually manifests itself in the form of excessive distortion of thin plate elements. When very thin plates are specified, in the desire to achieve minimum weight and supposedly minimum cost, distortion may induce initial out-of-plane deformations that then develop into local buckling when the member is loaded. The use of transverse or longitudinal stiffeners, while maintaining recommended width-thickness limitations on plates and stiffeners minimize the probability of local buckling.

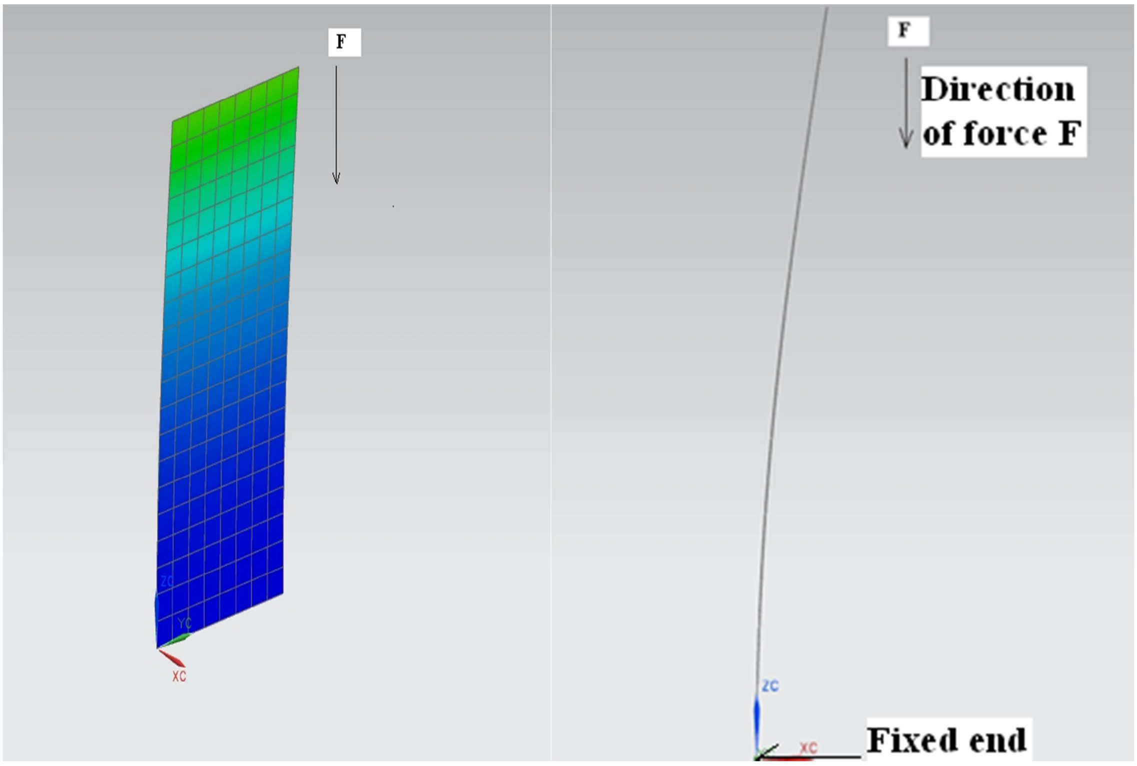

Figure 4: Showing buckling of column

The figure 4 above shows a front and side view of a buckled thin column (example of Euler buckling)

1.2 Problem Definition

In the boat, aerospace and car industries, structural performance and efficiency is of most importance to the engineer. And as stated earlier that is when high performance products need to be lightweight, yet strong enough to take harsh loading conditions. Whenever a member is designed, it is very important that it satisfy specific strength, stability and deflection requirements. Since most members are subjected to loads in its plane. Most commonly, uniaxial or biaxial compressive loads are considered; however, buckling may occur with biaxial loads, which are compressive in one direction and tensile in the other, or with shearing loads, or with the combination of any of these loadings. Indeed, it may even occur with uniaxial tensile loading if the load is not uniform [1], and if these members are long and slender the loading may be large enough to cause the member to suddenly bow out sideways. These sideways deformations are normally too large to be acceptable; consequently, the member is considered to have failed. The sideways deformation that occurs is called buckling. Buckling of a member can often lead to a sudden and catastrophic failure of a structure or mechanism, for this reason, extra attention must be given to the design of members so that they can safely support their intended loads without buckling. [2]

This is where the use of lightweight stiffeners comes into play. Having stiffeners significantly increases the load resistance capacity of structural members without much increase in weight.

The reader should note that just applying stiffeners to structural elements, does not necessarily take away all issues of buckling, yes they do but to just limited extents, that is why this thesis aims at investigating the use of a nose shape buckling preventer, along the sides of the stiffening beam and more necessarily finding the critical length at which these buckling preventers becomes useless and no more prevents the buckling of structural members.

1.3 Aims and objectives of Thesis

This thesis has elaborated on the stability problems of structures. Particularly it has investigated the problem of buckling prevention in stiffeners which is still insufficiently explored. Obviously the fact that the stiffeners are made to be light in weight will aid to reduce material usage and production costs. Therefore buckling of these light weight stiffeners is inevitable. The objective of this thesis is:

- To investigate the critical length at which buckling preventers becomes useless and no more prevents buckling when attached to the sides of stiffeners.

1.4 Method

In order to achieve the aim of this thesis and get theoretical knowledge of the problem at hand, literature studies have been undertaken and a study on analyses of buckling problems for different shell elements have been carried out. A comparison with a more basic model (Euler Column model), which can be verified by literature, has been made.

However the buckling analysis of a composite material shell element has been studied, in undertaking these studies the Finite Element Method was used. There are several Finite Element Analyses software available for commercial use but Nastran NX originally developed for NASA, was used to execute the buckling analysis. Results obtained from the validated Finite Element Analyses have been presented and discussed in this thesis.

1.5 Scope and Limitations

The scope of this thesis has been focused on the stability problem of lightweight stiffened structures and more importantly it has investigated at which critical length buckling preventers placed on the sides of these lightweight stiffened structures becomes useless and no more prevents buckling.

The entire thesis consisted of five chapters. Introductory section in first chapter. For undertaking the investigation an intense literature review on buckling and the use of stiffeners for the stability problems which exist in structural elements and the application of finite element method for the study of these stability problems is presented in the second chapter. Also in the second chapter is a brief review of buckling and the elements which plays a role in its existence and a review on the finite element analysis. The third chapter details the methodology of the thesis and the use of the nose shape buckling preventer in acquiring the critical length at which it will be absurd to place buckling preventers since they will no more be preventing buckling in structural members. The fourth chapter presents the results obtained after investigating a slender column with five slenderness parameters.

This is followed by conclusion and recommendations in the fifth chapter and then the references used for the entire achievement.

2 LITERATURE REVIEW

The catastrophic failure in structures has been generally dealt with in two ways and results and conclusions have been drawn. From the analytical and numerical point of view the problem of buckling of stiffened plates enormously has been studied with pioneer work of Bryan (1891) [3] who applied energy criteria to the study of the stability of plates under uniform compression, while Timoshenko in (1936) [4] and Timoshenko and Gere in (1951) [5] presented numerical tables for buckling loads of rectangular plates stiffened by longitudinal and transverse ribs. The effect of eccentricity of the stiffener was introduced as the effective moment of inertia of the stiffener by Siede (1953) [6]. While Troitsky (1976) [7] discussed the earlier developments in this field. Nonetheless, because of the complexity of the problem the existing solutions are limited to simple loads, and boundary conditions and more importantly simple geometry stiffened plates. Among these methods majority of researchers have used the finite element method (FEM). The first attempt to apply the finite element method to the stability analysis of unstiffened plates is due to Kapur and Hartz (1966) [8] and to stiffened plates is due to Dawe (1969) [9]. Later, several finite element solutions (Shastry, Venkateswara, Rao, and Reddy, 1976 [10]; Shen, Huang and Wang, 1987 [11]; Madhujit and Abhijit, 1990 [12]; Meiwen and Issam, 1992 [13]; Sabir and Djoudi, 1995 [14]; Grondin, Elwi and Cheng, 1999 [15]; Sheikh, Elwi, and Grondin, 2003 [16]; Vörös, 2007 [17]; Vörös, 2007 [18]) have been developed for stability problems of slab-and-beam structures. It should be noted that the finite element method is a good tool for the solution of the aforementioned problem. Besides this method other researchers also employed the boundary element method (BEM), among these are (Sapountzakis and Mokos, 2009 [19]; Tan et. al., 2009 [20]; Liu, 2007 [21]; Sapountzakis and Tsiatas, 2007 [22]; Dziatkiewicz and Fedelinski, 2007 [23]; Wang et. al., 2006 [24];Sanz et. al., 2006 [25]; Zhou et. al. (2006) [26]; Fernandes and Venturini, 2005 [27]; Botta and Venturini, 2005 [28]; Divo and Kassab, 2005 [29];Miers and Telles, 2004 [30]; Rashed, 2004 [31]; Zhang and Savaidis, 2003 [32]; Hatzigeorgiou and Beskos, 2002 [33]; Ochiai, 2001 [34]; Providakis, 2000 [35]; Shiah, and Tan, 2000 [36]; de Paiva, 1996 [37]; Katsikadelis and Sapountzakis, 1991 [38];Katsikadelis and Sapountzakis, 1985 [39],even though the boundary element method has been used successfully on unstiffened plates ,to my

knowledge there is no viable evidence as to if it has been used in the solution of stiffened plates yet.
In the period 1902 to 1914. Boobnov [40] made a great contribution, by applying a stress analysis to steel plates stiffened by a system of interconnected longitudinal and transverse beams.

He made further important contributions to the theory of stiffened plates. The first to apply the theory of the bending of plates in the structural design of ships, he showed that deflections of plates under hydrostatic pressure are not usually small, so that not only bending but also stretching of the middle plane of the plates must be considered. He provided the general solution to the problem and also prepared numerical tables to simplify its application.

The theory of interconnected longitudinal and transverse beams is of great importance in the design of ships, and Boobnov contributed much to this theory. Considering a system of parallel equidistant longitudinal beams supported by a crossbeam, Boobnov showed that this support can be treated as if it were a beam on an elastic foundation and he prepared tables simplifying the analysis of this beam. Later Boobnov extended his method to the case of several crossbeams.

Timoshenko [41] in the capacity of a consulting engineer participated in the analysis of the elastic stability of stiffened plates under various kinds of loading and edge conditions, performed for the first time in the design of the Russian dreadnoughts. To solve the stability problem of plates reinforced by stiffeners, Timoshenko proposed a method based on the consideration of the energy of a system and successfully applied it to practical problems.

2.1 Buckling

Buckling is a catastrophic mode of failure characterized by a sudden failure of a structural member subjected to high compressive stresses, where the actual compressive stress at the point of failure is less than the ultimate compressive stresses that the material is capable of withstanding. As shown in figure 4 above.

In 1757 Leonhard Euler derived a formula that shows a critical load for buckling of a column. The critical load is the maximum load that a structure can support prior to structural instability or collapse. A column of course, is simply a common case of a compression member. The critical load causes the column to be in a state of unstable equilibrium; that is the column deforms with hardly noticing the change in the geometry. At the point of critical load value, the introduction of the slightest lateral force will cause the column to fail by buckling, which is characterized by the column bending sideways with an indefinitely large displacement. The formula derived by Euler for columns with no consideration for lateral forces is given below. However, it should be noted that if lateral forces are considered the value of the critical load remains approximately the same. [42]

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \qquad \text{Eq. 1.0}$$

Where

P_{cr} = critical force (vertical load on column),

E = Young's modulus,

I = area moment of inertia,

L = length of column,

K = effective length factor, whose value depends on how the ends of the column are fixed.

2.2 Effective Length Coefficients and End Support Conditions

Theoretically, end supports are either pinned or fixed. In reality they can be designed to be pinned or rigid and may actually fall somewhere in between truly pinned or fixed.

The support conditions will have an impact on the effective length (Le), of the column and can be different in each plane.

The effective length (Le), of a column is the distance between points on the column where the moment is zero (inflection points). [43]

The effective length can be expressed as:

$$Le = KL \qquad \text{Eq. 1.1}$$

Where L is the actual length of the column in the plane of buckling and K is an effective length coefficient factor, whose value depends on the condition of end support of the column, as follows

For both ends pinned (hinged, free to rotate), $K = 1.0$.

For both ends fixed, $K = 0.50$.

For one end fixed and the other end pinned, $K = 0.699$....

For one end fixed and the other end free to move laterally, $K = 2.0$.

2.3 Slenderness Ratio

The relationship between the length of the column, its lateral dimensions and the end conditions will strongly affect the resistance of the column to buckling. Obviously, the greater the height of a column and the tendency towards buckling, the more critical becomes the relationship between the thickness and height. This relationship is known as the slenderness ratio; as this ratio increases, so the loadbearing capacity of the column increases.

Slenderness ratio of a column is defined as the ratio of the effective height ℓ of the column to the least radius of gyration r of the column section. It is used in determining the strength of a column. As per the slenderness ratio the columns are categorised as:

a) *Short Columns*- having slenderness ratio, $S_r < 60$
b) *Intermediate Columns*- having S_r in the range of $60 < S_r < 100$
c) *Long Columns-having* S_r in the range of $S_r > 100$

Where r is the least radius of gyration calculated on the basis of the minor principal moment of inertia I. [44]

$$S_r = \frac{\ell}{r} \qquad \text{Eq.2.0}$$

S_r= slenderness ratio

r = radius of gyration

ℓ = effective height

2.4 Radius of Gyration

The radius of gyration introduces the effects of cross-sectional size and shape to slenderness. It is one measure of effectiveness to resisting buckling. The radius of gyration r is given by the following formula

$$r = \sqrt{\frac{I}{A}} \qquad \text{Eq. 3.0}$$

Where I is the second moment of area and A is the total cross-sectional area. It should be noted that the radius of gyration is very useful in estimating the stiffness of a column. [45]

2.5 Stiffening System

The web of a girder may buckle locally under pure shear due to diagonal compression, or under flexure due to bending compressive stress, or under concentrated loads due to bearing compressive stress. Providing stiffeners prevents this local buckling of the web. [46]

There are three different arrangements of stiffeners commonly used to reinforce plate. The longitudinal stiffeners placed parallel to the in-plane load carry a portion of the applied load. The transverse stiffeners are used merely to subdivide the plate into smaller panels, since the portion of the load carried by them is relatively small. A combination of longitudinal and transverse stiffeners results in orthogonally stiffened plates. Conventional structural shapes used for stiffeners are flat plates; angles, channels, T-, and inverted T- sections. In aerospace structures Z, U and Y type's stiffeners are also common.

Stiffeners are used to give the required design bending and buckling resistance at less weight than columns of uniform thickness. They act as struts and thereby transmit compressive forces. A stiffener also provides additional support to plates at beam connection locations and is added when the strength of the plate is exceeded but full moment strength of the beam section is desired. It is obvious that reinforcing the plate by transverse stiffeners will have little effect upon the buckling strength of the plate unless these are spaced very closely. The critical compressive stress of the plate will be increased to any considerable extent only if the distance between transverse stiffeners is far smaller than those of the unstiffened plate.

Introducing one or more longitudinal stiffeners as show in figure 5 could yield a more economical construction, these stiffeners not only carry a portion of the compressive load but also subdivide the column (plate) into smaller panels, hence increasing the critical stress at which the plate will buckle. [47]

Consider a rectangular plate of length a, width b, and thickness t, which is reinforced by two longitudinal stiffeners. The plate is loaded by a uniformly distributed load σt .The stiffeners are assumed to be attached to the plate and having the same compressive stress σ as the plate.

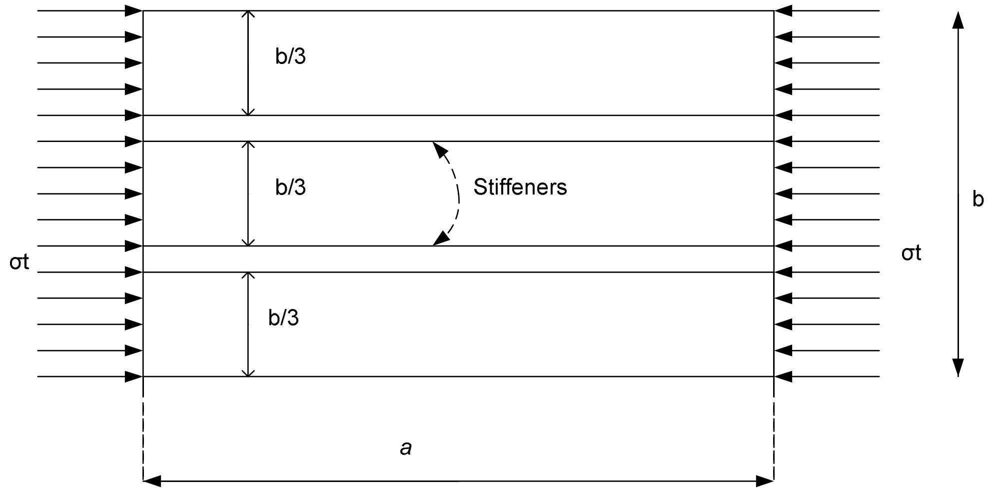

Figure 5: Simply supported plates having two longitudinal stiffeners (Bleich Friedrich 1952:361)

2.6 Lateral –Torsional Buckling of Stiffeners

The lateral-torsional buckling (or tripping) of stiffeners is a phenomenon in which the failure of a stiffened panel occurs after the stiffener bends sideways about the edge of the stiffener web attached to the platting. When torsional rigidity of the stiffener is small this phenomenon is more likely to take place.

Example of such phenomenon is the elastic local buckling of stiffener webs; this is a possibility that must be put into serious consideration when dealing with built-up sections. Such an occurrence of local buckling in the stiffener cross-section can sometimes be a sudden phenomenon resulting in subsequent unloading of the stiffener panel, particularly with the use of flat-bar stiffeners. In a situation like this, once the stiffener web buckling occurs, the buckled or collapsed plating is left with no stiffening and hence overall stiffened panel collapse may follow with little increase in the loading.

A plate – stiffener combination with the attached effective platting under combined axial compression and lateral line loads is typically considered to collapse if tripping occurs after the platting between stiffeners collapses.

In a continuous steel stiffened panel, tripping may generally involve a coupling of sideways and vertical deflection and rotation of the stiffener web together with the local buckling of the attached platting. Unlike an ordinary beam – column in steel framed structures, the attached platting of a plate- stiffener combination in steel plated structures is restricted from deflecting sideways, while the stiffener flange is relatively free to deflect sideways and vertically.

For unsymmetrical section profiles, vertical bending, sideways bending and torsion are typically coupled, while for symmetric section profiles, only sideways bending and torsion are normally coupled. This means the overall flexural Euler buckling and lateral–torsional buckling can sometimes be closely coupled for plate–stiffener combinations. [48]

2.7 Finite Element Method and NX NASTRAN

The finite element method (FEM) is a numerical procedure, which can be applied to obtain solutions of partial differential equations (PDE) as well as integral equations in engineering. Steady, transient, linear, or nonlinear problems in stress analysis, heat transfer, fluid flow, and electromagnetism problems may be analysed with finite element methods. The solution approach is based either on eliminating the differential equation completely or rendering the PDE into an approximating system of ordinary differential equations, which are then numerically integrated .In this method all the complexities of the problems like varying shape, boundary conditions and loads are maintained as they are but the solutions obtained are approximate. Because of its diversity and flexibility as an analysis tool, it is receiving much attention in engineering. [49]

Finite Element Analysis (FEA) is a computer simulation technique used in engineering analysis. The pace at which computer technology is improving has boosted this method, since the computer is the basic need for the application of this method.

One of the commercially widely used Finite Element Analysis (FEA) package is NX NASTRAN. NX NASTRAN is primarily a solver for finite element analysis, and NASTRAN NX 8 to be precise allows for graphically building a model or meshing. It offers a wide range of analysis from concept simulation to advanced analysis. In addition to pre- and post-processing capabilities it is integrated with linear and nonlinear capabilities.

2.8 Stability Problem of Thin Plates

A structural element is unstable if any disturbance of the system results in a sudden change in deformation mode or displacement value after which the system does not return to its original equilibrium state. [50]

Due to high strength to weight ratio, most structural elements are made of relatively thin plates. When a thin plate is under compression, local buckling may occur if the width to thickness ratio is too high. Most plates found in ships and aircrafts are relatively thin so as to make structural parts more lighter in weight, which in turn makes it more easy for them to be applied in various structures, for example they may be homogeneous and isotropic, they may be stiffened or have a composite construction. Depending on the mode of application the plate may be subjected to various loads and if the load increases, the least disturbance will cause the plate to bend sideways hence the use of stiffeners to reinforce these thin plates to prevent them from failing catastrophically.

3 METHOD

You like to reinforce thin walled surfaces using a stiffening beam (system) say a longitudinal stiffener attached to this surface. As the beams stiffness is related to the third power of its height the engineer will be tempted to put more material in height (h) rather than width (b) as shown in Eq.5.0. Let S_r be the slenderness ratio of a beam and suppose that at some slenderness ratio the stiffener may theoretically function, but later the stiffener turns to be unstable due to buckling.

The engineering challenge is then to place a buckling preventing mechanism on the side of the stiffener to:

a) Give the required design bending and buckling resistance at less weight than columns of uniform thickness.
b) Act as struts and thereby transmit compressive forces.
c) Provide additional support to plates at beam connection locations and is added when the strength of the plate is exceeded but full moment strength of the beam section is desired

The hypothesis of this thesis is that a buckling preventer can efficiently prevent buckling while attached to the side of a stiffener, if sufficiently placed along the beams length. As shown in figure 11.

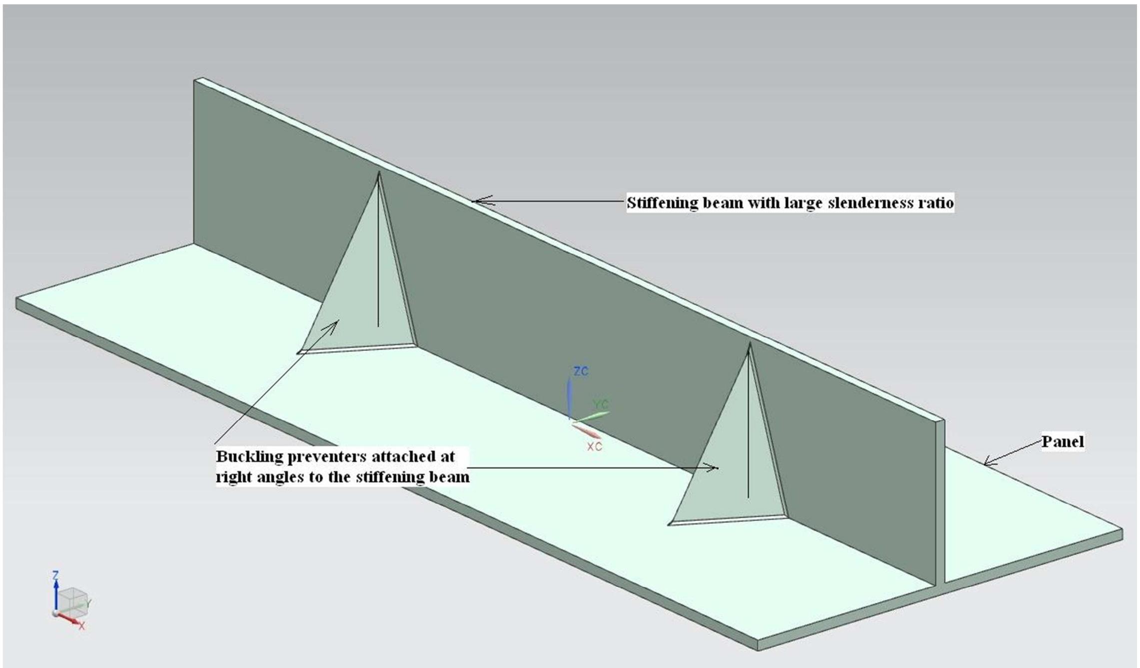

Figure 6: Thin walled stiffened panel with buckling preventers

The study of the problem is done using a proofing method called Proof by contradiction (also known as reductio ad absurdum, Latin for "by reduction to the absurd"). It is shown that if some statement were true, a logical contradiction occurs, hence the statement must be false. Example if one wants to disprove proposition p. The procedure is to show that assuming p leads to a logical contradiction. Thus, according to the law of non-contradiction, p must be false.

On the other hand if one wish to prove proposition p. One can proceed by assuming "not p" (i.e. that p is false), and show that it leads to a logical contradiction. Thus, according to the law of non-contradiction, "not p" must be false, and so, according to the law of the excluded middle, p is true. [51]

This method is used in my thesis to show at which critical length along a beam a buckling preventer becomes useless due to the fact that it no longer prevents buckling.

3.1 Case Study of Slender Column

The problem which is being analysed is a simple Euler buckling case of a slender column subjected to compressive forces. Axially loaded slender columns in compression experience a mode of instability when such compressive forces increase, they continually deforms while the load keeps increasing until reaching the critical load. At this point failure occurs and the column deforms into an irreversibly different pattern. This is where the use of stiffeners comes into play as they act as struts and thereby transmit these compressive forces. They also provide the web effective in withstanding buckling due to shear. It should be noted that reinforcing a structural member with stiffeners will have little effect upon its buckling strength unless these are spaced very closely.

3.1.1 NX NASTRAN Simulation

This work involves buckling analysis of a thin plate stiffening beam (slender column).

The beam remains stable even as deformations increases but it becomes unstable only when the applied load exceeds the beams load carrying capacity.

a. Geometry:

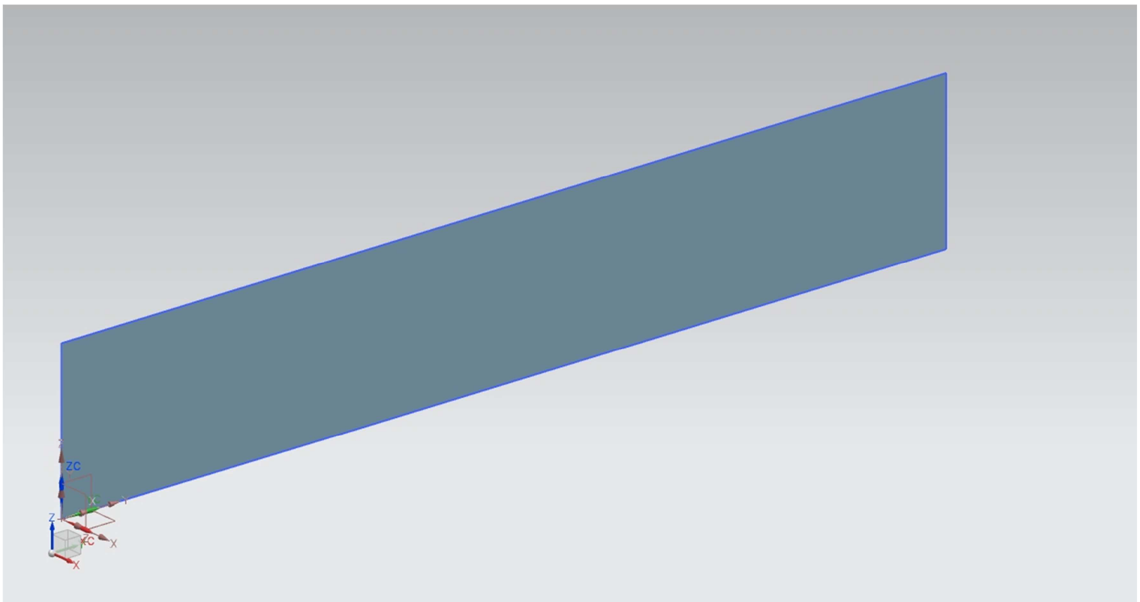

Figure 7: The stiffening beam (slender column)

The dimensions of the slender column are as follows

Table 1: Dimensions of slender column

Height(h)	500 mm
Width (b)	10 mm
Length (l)	1200 mm

b. Material Properties

NX NASTRAN offers a wide range of material properties to be used for different type of analysis; the material property used for this thesis is isotropic.

Table 2: Material Properties for slender column

Young's modulus	1000 (MPa)
Poisson ratio	0.3

c. Boundary Conditions

Fixed Constraint

For the slender column, a 2d mesh is formed by using CQUAD4 mesh. The elements in the mesh have three degrees of freedom at each node: translations in the element X and Y directions and rotation about the element Z-axis. The beam is analysed and after several trials the element size 50, was found to be satisfactory and implemented.

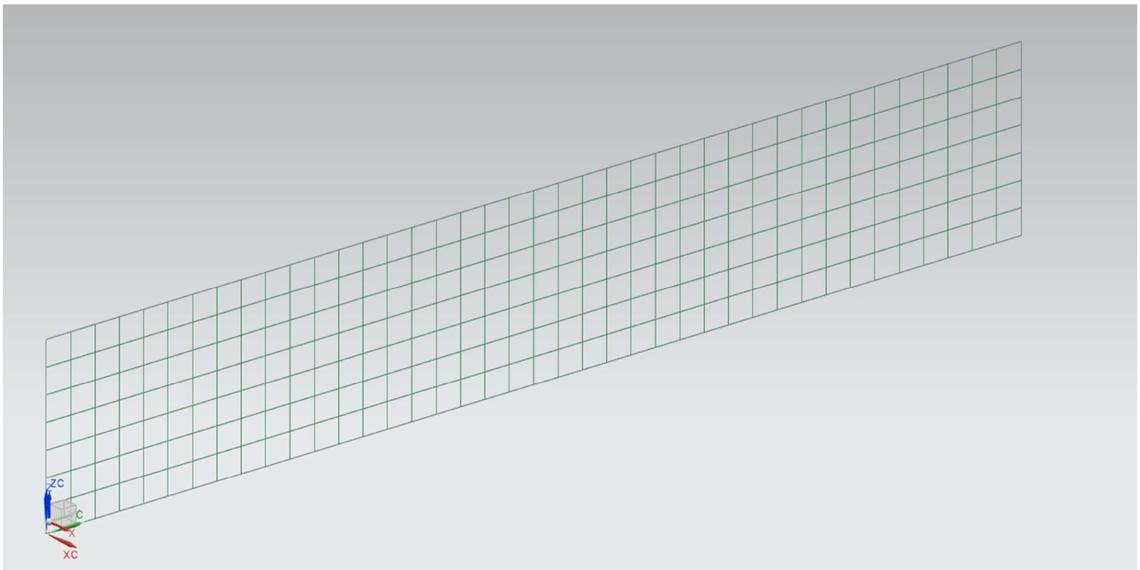

Figure 8: CQUAD4 mesh

For constraints, the structure is fixed in all degrees of freedom at horizontal plane (Y-coordinate) with a 1N force applied in the - Z direction as shown in figure 8

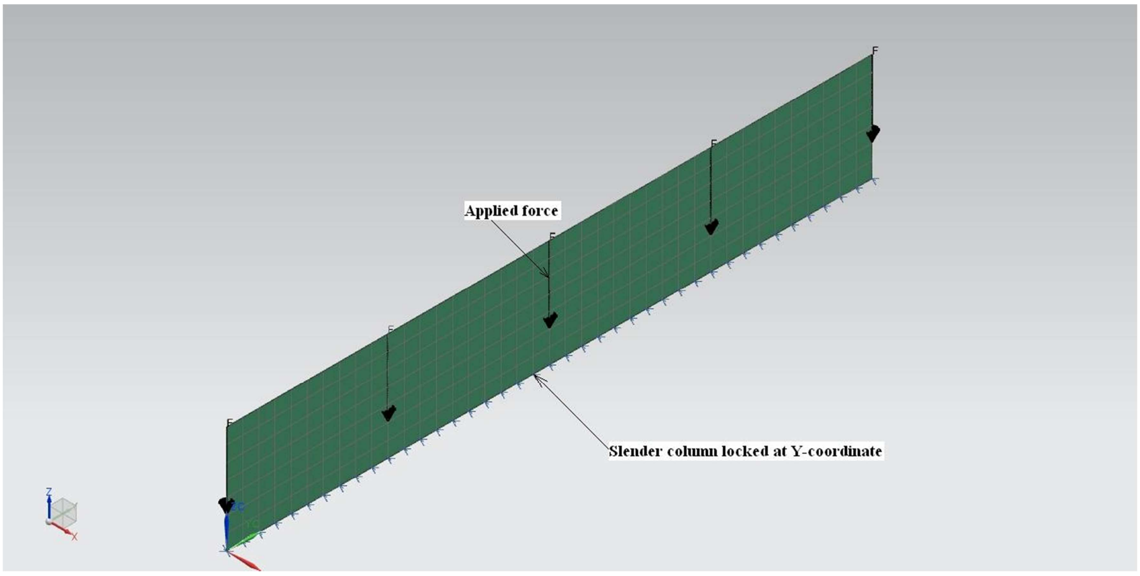

Figure 9: Slender column subjected to boundary conditions (locked at Y-coordinate)

A similar analysis is done but this time for constraints, the structure is fixed in all degrees of freedom at the horizontal plane (Y-coordinate) and the two vertical planes (Z -coordinate) with a 1N force applied in the - Z direction as shown in figure 9.

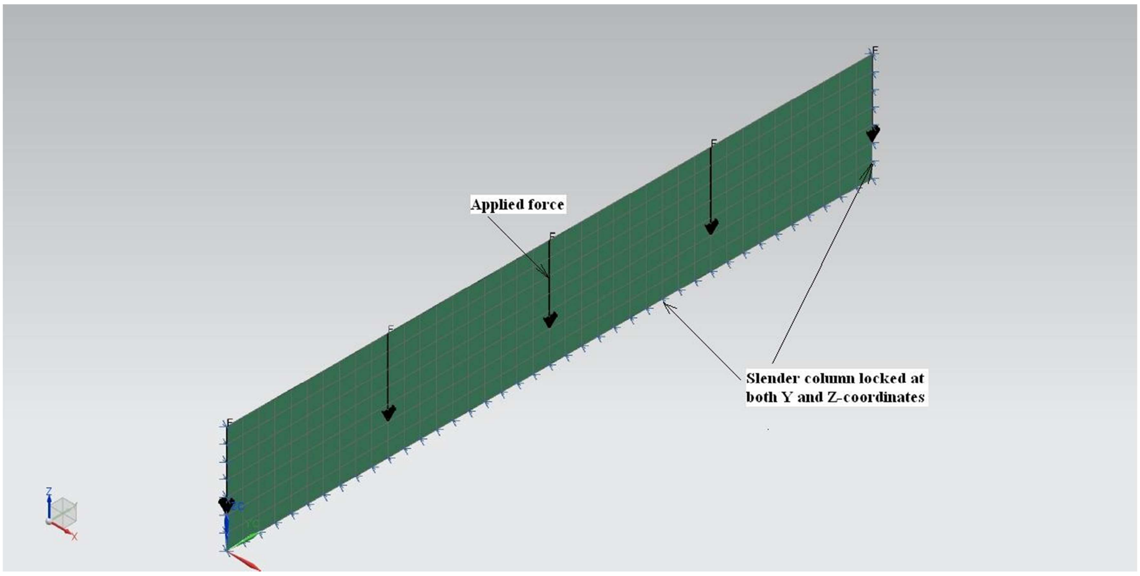

Figure 10: Slender column subjected to boundary conditions (locked at both Y and Z-coordinates)

3.1.2 Simulation Results

As compressive loads applied on the beam keeps increasing, a load is reached at which the slender column becomes unstable and suddenly bows out sideways. These sideways deformations are normally too large to be acceptable; hence, the slender column is considered to have failed. As shown in figure 10.

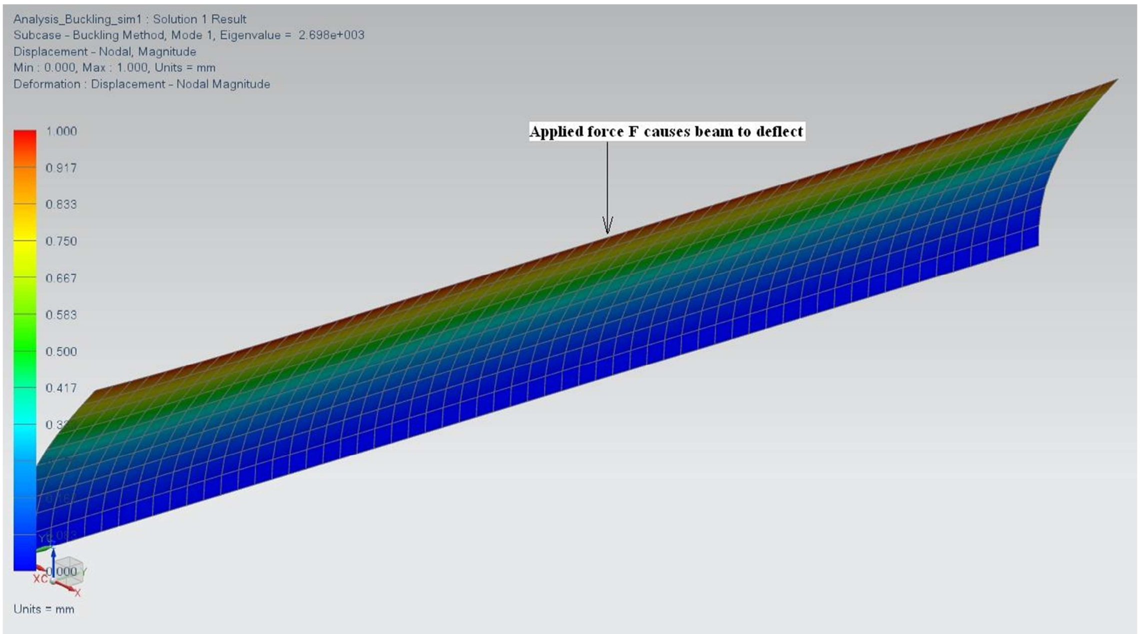

Figure 11: Buckling of the stiffening beam

3.1.3 Investigation Process

The principle is that the stiffener is analysed in FEM with only the bottom surface locked, as shown in figure 12

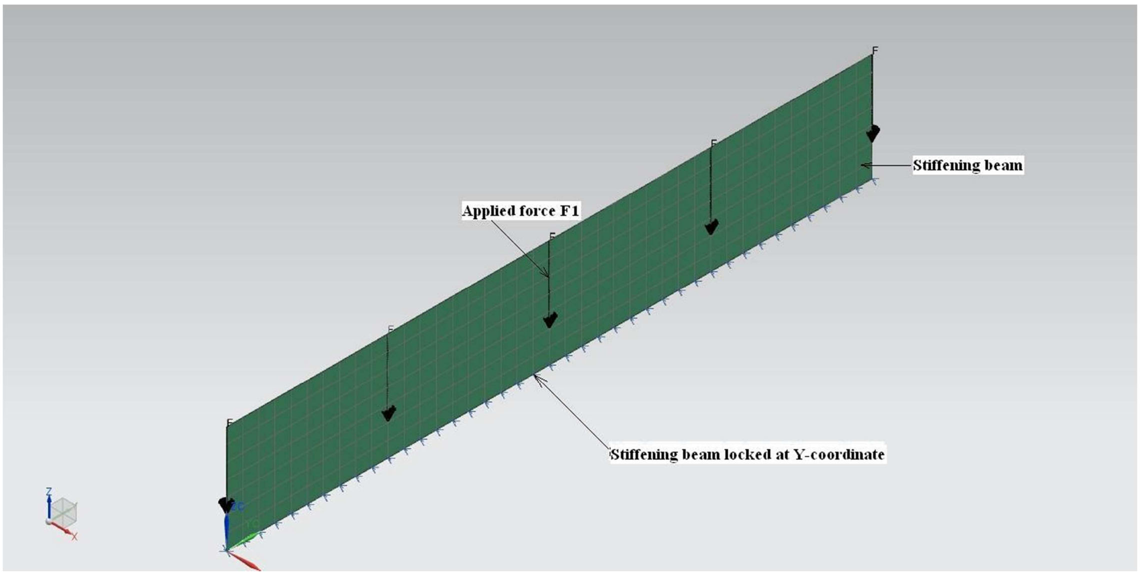

Figure 12: Stiffening beam locked at Y coordinate

After thoroughly going through all the stimulation process the buckling force F1 is then determined. This is done by multiplying the first eigen value for this particular slenderness stimulation by the applied force on the stiffener.

The procedure is redone but this time with two added buckling preventers that would lock also the sides of the stiffening beam. As shown in figure 13.

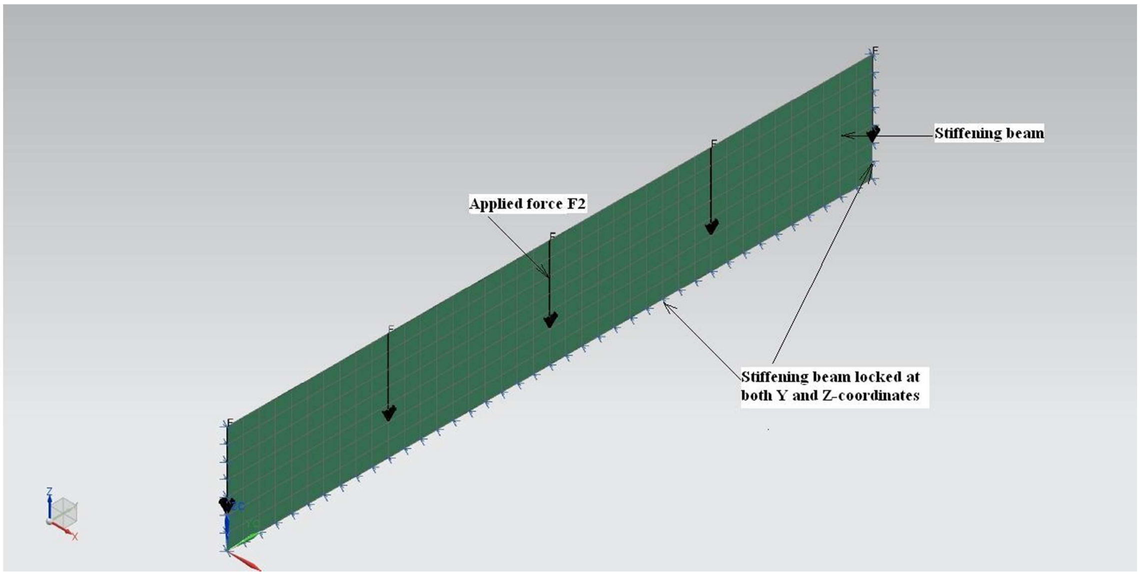

Figure 13: Stiffening beam locked at both Y and Z coordinates

Once again after thoroughly going through all the stimulation process the new buckling force F2 is determined.

When the beam becomes longer and longer the ratio of these two forces F2/F1= parameter will approach 1.

At this point a buckling preventer will no more prevent the buckling of the beam, therefore it will be absurd to place a buckling preventer at this position.This means that the critical length (*Lc*) at which a buckling preventer becomes useless for a given slenderness ratio of the stiffening beam has been detremined.

The reader should note that this study will not provide information as to how densely the buckling preventer has to be placed but instead at which critical length it is no more preventing buckling.This implies that in other to prevent buckling in the stiffening beam the buckling preventer needs to be placed at a length much smaller than this critical length. Figure 14, below demonstrates the critical length (*Lc*) at which two nose point buckling preventers are placed.

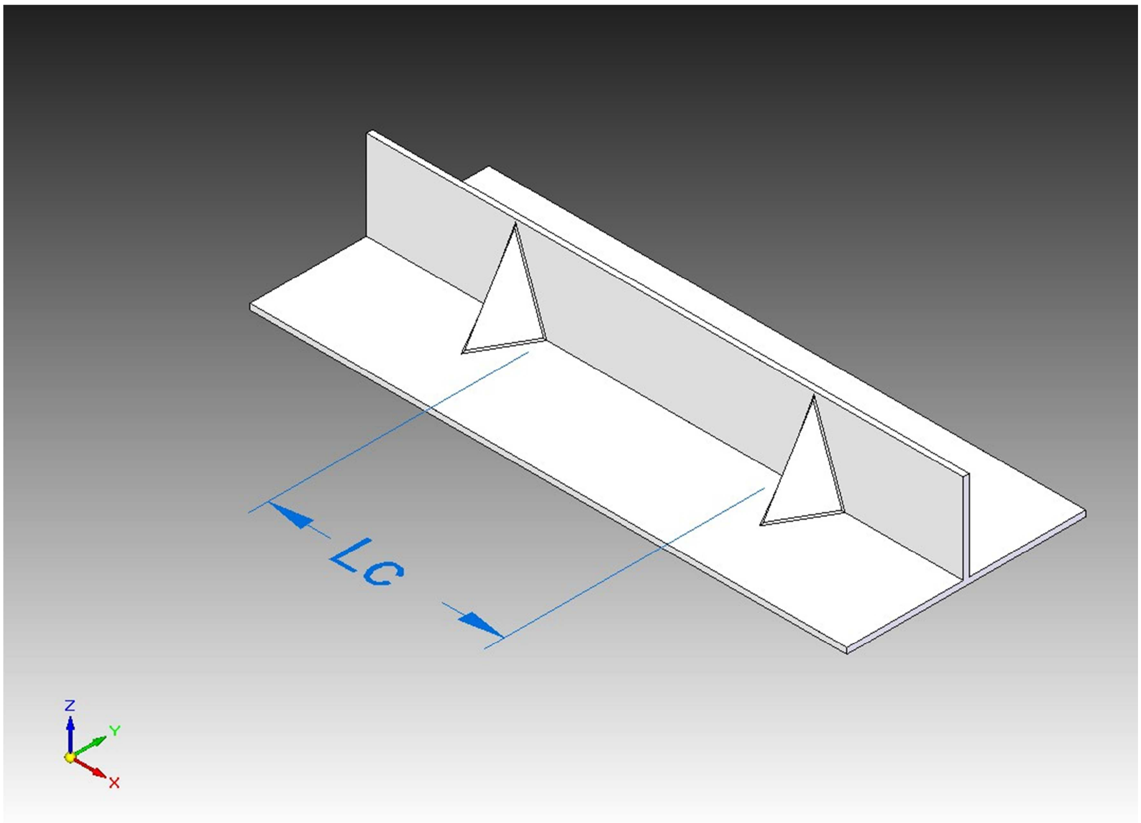

Figure 14: Thin walled stiffened panel with buckling preventers spaced at a critical length (Lc)

For practical cases a study must be performed on what kind of buckling preventer is suitable for the given position, for the purpose of this thesis a nose shaped buckling preventer was used. The figures below (Figures 15 and 16) show views of the nose shape buckling preventer.

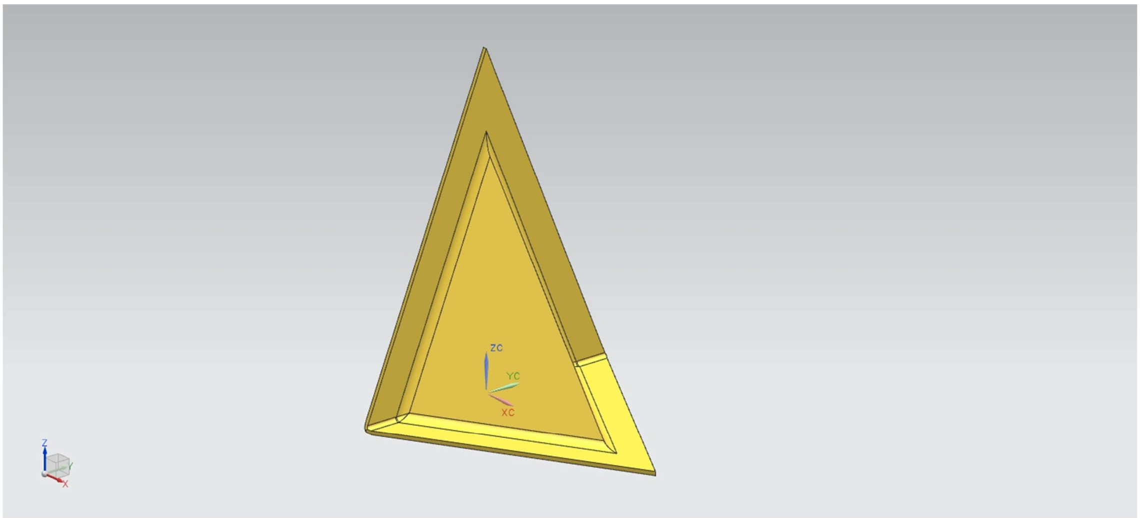

Figure 15: Isometric view of nose shape buckling preventer

The nose shape design was chosen because it’s unique and acts at right angles to the stiffening beam providing maximum bending resistance at both the stiffener and panel interface, as well as helping transmit induced compressive forces.

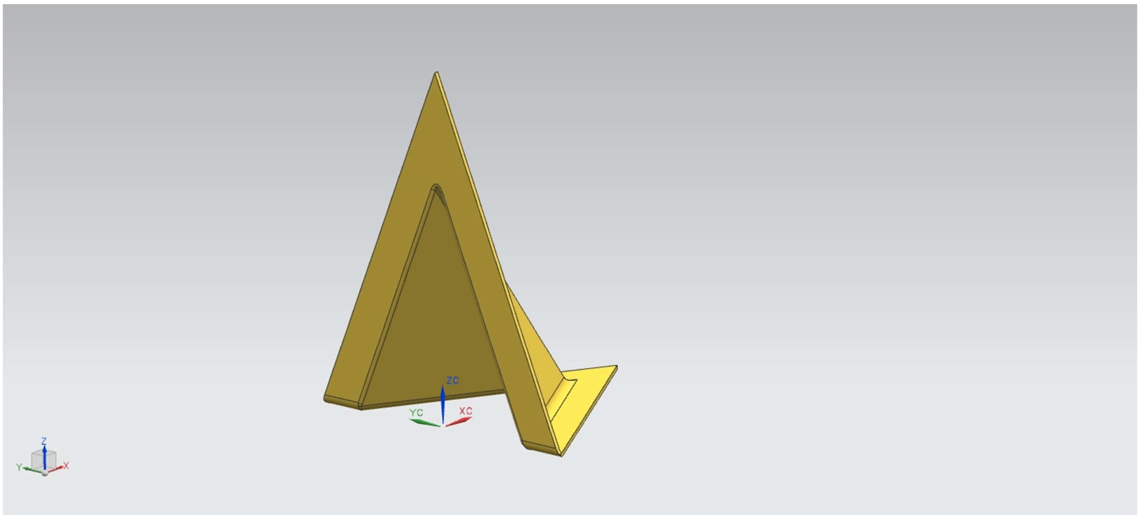

Figure 16: Showing the hollow section of the nose shape buckling preventer

4 RESULTS

The first analysis was done by varying the dimensions of the slender column (stiffening beam) but maintaining the same slenderness ratio of 200, after successfully completing the FEM analysis the following results were obtained.

Table 3: Results from FEM analysis on the slender column with a slenderness ratio of 200.

L_p(mm)	F1(N)	F2(N)	F2/F1	S_r	h(mm)	t(mm)
200	1,036	17100	16505,79151	200	2000	10
250	1,297	13090	10092,5212	200	2000	10
400	2,087	7953	3810,73311	200	2000	10
500	2,619	6354	2426,116838	200	2000	10
700	3,693	4548	1231,51909	200	2000	10
1000	5,326	3194	599,6995869	200	2000	10
2500	136,8	1316	9,619883041	200	2000	10
3000	164,9	1113	6,749545179	200	2000	10
4000	221,4	847,6	3,82836495	200	2000	10
6000	334,3	655,4	1,960514508	200	2000	10
8000	447,3	651,8	1,45718757	200	2000	10
10000	506,3	708,8	1,399960498	200	2000	10
12000	673,3	789,8	1,173028368	200	2000	10
25000	1413	1463	1,035385704	200	2000	10
40000	2264	2295	1,01369258	200	2000	10
42000	2378	2408	1,012615643	200	2000	10
45000	2548	2576	1,010989011	200	2000	10

Where

L_p (mm) = the distance between buckling preventers

F1 (N) = the applied force when the stiffening beam is locked at the bottom surface

F2 (N) = the applied force when the stiffening beam is locked at both bottom and side surfaces

S_r = slenderness ratio of the beam with regards to its height and thickness

h = height of slender column = (effective height ℓ)

t = thickness of slender column

By plotting F2/F1 against length L_p for slenderness ratio of 200 generates a single cruciform curve.

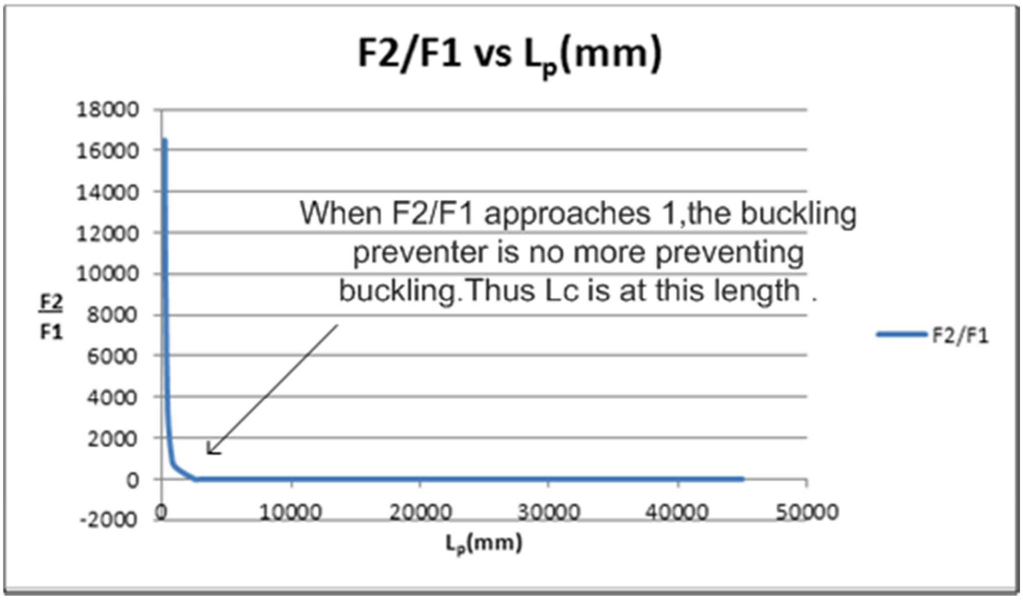

Figure 17: F2/F1-L_p (mm) graph

Combining the results for all five slenderness ratios and plotting F2/F1 against length L_p (mm) generates multiple cruciform curves with each slenderness ratio defined by a specific curve in descending order.

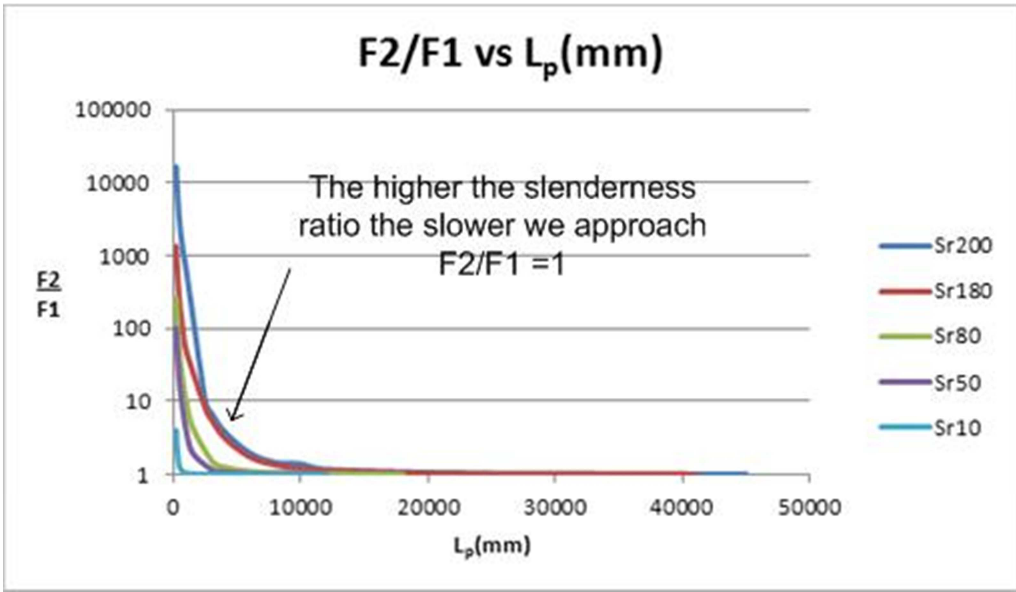

Figure 18: F2/F1 vs L_p (mm) graph

4.1 Critical Length (*Lc*)

It can be seen from Table: 3 how the ratio of F2/F1 approaches 1 for a stiffening beam with slenderness ratio of 200. As discussed earlier, this is where the buckling preventer becomes irrelevant. Therefore the critical length $Lc = L_p$ at the row for which F2/F1=1. Meaning at this point it will be absurd to place a buckling preventer at this position.
This implies that with regards to Table: 3 the critical length (*Lc*) at which the buckling preventer becomes useless is *Lc*=45000 (mm) ± 1% because it gives an F2/F1 ratio which is more closer to 1 compared to other lengths from the analysis.

The above stated procedure is then repeated for determing the critical lengths for the remaing four slenderness ratios that is for 180, 80, 50 and 10. The results obtained is presented in Table 4 below.

Table 4: critical length results for all five slenderness ratios investigated for this thesis

S_r	Lcmin(mm)	Lcmax(mm)	Lc(mm)
10	2482,5	2517,5	2500
50	11880	12120	12000
80	19816	20184	20000
180	40557,2	41442,8	41000
200	44509,5	45490,5	45000

To illustrate the position at which buckling can no more be prevented with regards to a given slenderness ratio S_r a graph of *Lc* (mm) versus S_r is plotted as shown in Figure 19.

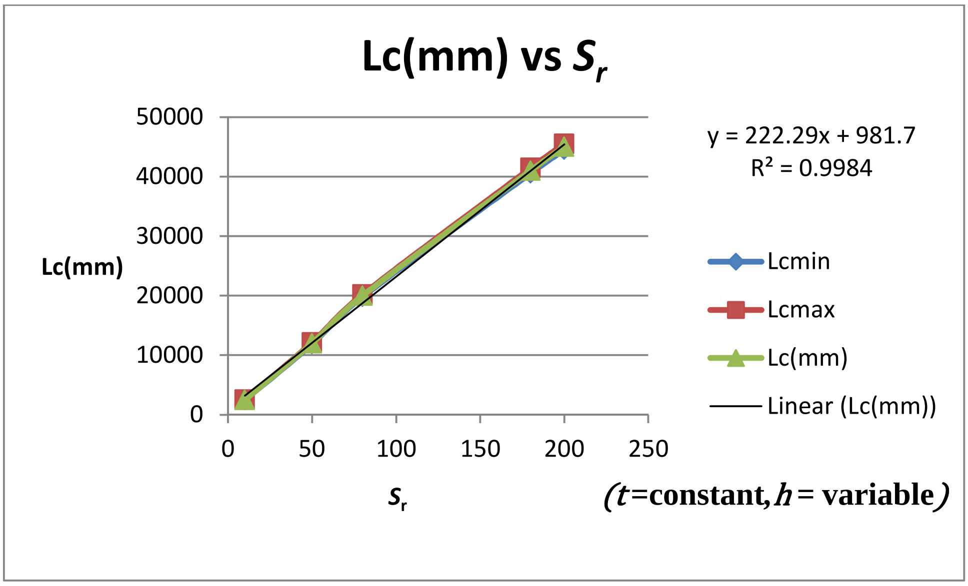

Figure 20: Correlations between critical length and slenderness ratio

Please refer to Appendix A for results for the four remaining slenderness ratios used in the analysis.

The critical length *Lc* is shown in the graph above and illustrate the position at which buckling can no more be prevented in relation to the stated slenderness ratio parameters for the investigated slender beam.

A column is described as *long* or *short* depending on the value of its slenderness ratio S_r, not its absolute length. As discussed earlier the greater the slenderness ratio, the greater the tendency for the column to fail by buckling, and thus the lower the loadbearing capacity of the column.

Therefore reading from the graph (in figure 20) as S_r is increasing the *Lc* is almost increasing linearly. This is because the loadbearing capacity of a slender column with a slenderness ratio of say 10 will be much higher as compared to that with a slenderness ratio of say 180.

To prove this concept, assume the slender column in figure 1 is hinged at both ends and centrally loaded.

Where

F = applied load

l = length of column

t = thickness

h = height

$c = \frac{h}{2}$

Then using the classic formula for determining the bending stress in a beam under simple bending [52]

$$\sigma_{max} = \frac{Mc}{I} = \frac{M}{Z} \qquad \text{Eq.4.0}$$

Where

σ_{max} = the maximum normal stress in the column

M = the resultant internal moment

c = the perpendicular distance from the neutral axis to a point farthest away from the neutral axis.

I = the moment of inertia about the centroidal axis

Z = Young's modulus

$$I = \frac{th^3}{12} \quad \text{Eq.5.0}$$

$$Z = \frac{I}{C} = \frac{th^2}{6} \quad \text{Eq.6.0}$$

Therefore putting Eq.6.0 into Eq.4.0

$$\sigma_{max} = \frac{M6}{th^2} \quad \text{Eq.7.0}$$

But since $M = Fl$

Then Eq.7.0 becomes

$$\sigma_{max} = \frac{6F\,max\ l}{th^2}$$ Hence solving for $F\ max$

$$F_{max} = \frac{th^2 \sigma max}{6l} \quad \text{Eq.8.0}$$

Substituting $S_r = \frac{h}{t}$ for h in Eq.8.0

$$F_{max} = \frac{{S_r}^2\ t^3 \sigma\ max}{6l} \quad \text{Eq.9.0}$$

Therefore from Eq.9.0 it can be deduced that as the slenderness ratio increases, so the loadbearing capacity of the beam increases.

This implies that the distance at which a buckling preventer needs to be placed to prevent a stiffening beam with a slenderness ratio of 10 from buckling is evidently going to be much smaller than that with a S_r of 180.

In spite of that, the energy criteria analysis of beam deflection is however beyond the scope of this thesis.

5 DISCUSSION

The model described in this thesis has been performed in NASTRAN NX 8.0 and analysed with a linear buckling analysis module.

Stiffness as a material property is very important in structural engineering since it gives the extent to which materials resist deformation in response to applied load. Hence the modulus of elasticity E and Poisson ratio was assumed to be 1000 (MPa) and 0.3 respectively.

From the graph (in figure 20) as S_r is increasing the Lc is almost increasing linearly. This resulted in a straight line graph with the equation (Eq.10) stated below:

$$y = 222{,}3x + 981{,}7 \quad \text{[mm]} \qquad \text{Eq.10}$$

R^2 = 0, 9984

This can be re-written as:

$$Lc = 222{,}3S_r + 981{,}7 \text{ [mm]} \qquad \text{Eq.11}$$

R^2 = 0, 9984 $10 < S_r < 200$; t =10, h = variable

Where R^2 (coefficient of determination) gives how close the values estimated from the prediction function are to the actual data values.

Assuming a prismatic beam, with a cross-sectional area A and length l is subjected to an axial load F (which passes through the centroid of the cross-section) as illustrated in figure 21. Then the stress (σ) in the column is given as: applied force per cross-sectional area (A) perpendicular to the force as shown in the equation below

$$\text{Stress} \quad \sigma = \frac{F}{A} \qquad \text{Eq.12}$$

The well-known equations of stress and strain energy is derived and reported elsewhere [53]. Thus the total strain energy in the beam is

$$U = \frac{F^2 l}{2AE} \qquad \text{Eq.13}$$

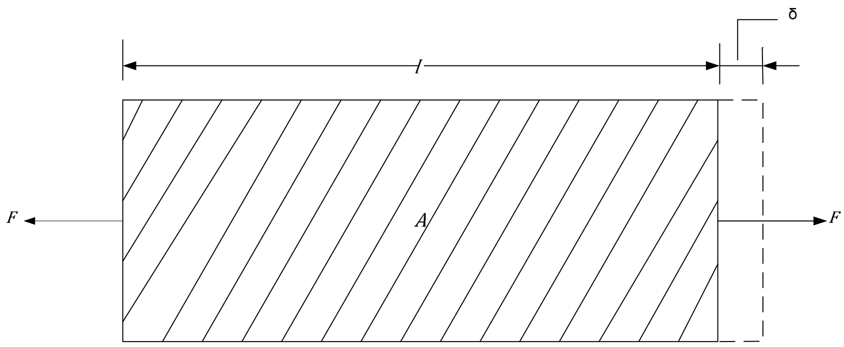

Figure 21: Prismatic beam subjected to an axial load (James M. Gere, Barry J. Goodno, 2009:140)

Where U = Strain energy

E = Young's modulus

A = Cross-sectional area

From the above equation (Eq.13), increase in the length of the beam (which is this case will be *Lc* the distance between each buckling preventer) increases the strain energy capacity even though the load is unchanged (because more material is being strained by the load).

On the other hand, increasing either the Young's modulus or the cross-sectional area decreases the strain energy because the strains in the beam are reduced.

Given that the area of a rectangle $= l * h$ Eq.14

Where l = length and h = height

Then increase in the cross-sectional area *A* (which therefore means height *h* will be increasing) will decrease the strain energy.

And since height h is increased then S_r will inevitably increase. Therefore increase in S_r will decrease strain energy capacity of the beam.

As stated earlier, when a slender column is subjected to small compressive loads, the column axially shortens according to

$$\delta = \frac{Fl}{AE} \qquad \text{Eq.15}$$

Where δ is the axial shortening, F is the applied load, A is the cross-sectional area, E is the Young's modulus of the material and l is the column length. [54] If continually larger loads are applied, a load is reached at which the column suddenly bows out sideways. This load is referred to as the critical or buckling load of the column. This sideways deformation occurs because the slender column would have reached its energy absorption capacity and therefore fails catastrophically by buckling.

Therefore we conclude that as the buckling occurs, energy absorption decreases significantly. Hence attaching buckling preventers on the sides of the stiffening beam enhances its ability to absorb energy.

6 CONCLUSION

The objective of this thesis is to find the critical length *Lc* at which buckling preventers becomes useless and no more prevents buckling when attached to the sides of stiffeners. The Finite Element Analysis carried out on the stiffening beam has given general results from which it is concluded that:

- Plotting the values of *Lc* (mm) versus S_r gives a straight line graph with the imperial finding of the critical length *Lc* as

  $Lc = 222{,}3S_r + 981{,}7$ [mm]
  R^2 = 0, 9984 10 <$S_r < 200$; $t = 10$, $h =$ variable
  Where R^2 is the coefficient of determination

- As buckling occurs the stiffening beam loses its strain energy capacity, therefore attaching buckling preventers on its sides will enhance its ability to absorb energy.
- Increase in *Lc* increases the strain energy capacity of the beam.
- Increase in *E* decreases the strain energy capacity of the beam.
- Increases in S_r due to the fact that the height h of the beam increases will decrease the strain energy capacity of the beam.

6.1 Recommendations

This thesis concentrated only on the finding of the critical length at which it becomes absurd to place buckling preventers since it will no longer be preventing buckling.

Although the finite element model used for this thesis had been validated analytically a full-scale test specimen should be validated experimentally.

Lastly the proposed design for the buckling preventer should be tested experimentally.

REFERENCES

1. Leissa, A.W. and Ayoub, E.F, 1989, Tension buckling of rectangular sheets due to concentrated forces. ASCE journal of Engineering Mechanics, 115, 2749-62.
2. Hibbeler .R.C, 2011, Mechanics of Materials, 8th ed. Pearson Prentice Hall South Asia Pte Ltd. Singapore.
3. Bryan, G.H. 1891. The stability of a plane plate under thrust in its own plane, with applications to the buckling, Proc. London Math. Soc., vol.22, pp.54–67.
4. Timoshenko S.P. 1936. The stability of the stiffened plates, Der Eisenbau (in German), vol.12, and pp.147–163.
5. Timoshenko, S.P.; Goodier, J.N.1951. Theory of Elasticity, 2nd ed., McGraw-Hill Book Company, New York.
6. Seide, P. 1953. The effect of longitudinal stiffeners located on one side of a plate on the compressive buckling stress of the plate stiffener combination, Report No.NASA TN 2873.
7. Troitsky, M.S. 1976. Stiffened Plates, Buckling, Stability and Vibration. Elsevier, Amsterdam.
8. Kapur, K.; Hartz, B.1966. Stability of plates using the finite element method. Eng. Mech. Div, ASCE, vol.92, pp.177–195.
9. Dawe, D.J.1969. Application of the discrete element method to the buckling analysis of rectangular plates under arbitrary membrane loadings, Aeronaut. Quart. vol.20, pp.114–128.
10. Shastry, B.P.; Venkateswara Rao, G.; Reddy, M.N. 1976. Stability of stiffened plates using high precision finite elements, Nuclear Engineering and Design, vol.36, pp.91–95.
11. Shen Peng-Cheng; Huang Dade; Wang Zongmu 1987. Static, vibration and stability analysis of stiffened plates using B spline functions, Computers and Structures, vol.27, pp.73–78.
12. Madhujit Mukhopadhyay; Abhijit Mukherjee 1990. Finite element buckling analysis of stiffened plates, Computers and Structures, vol. 34, pp.795–803.
13. Meiwen Guo; Issam E. Harik .1992. Stability of Eccentrically Stiffened Plates, Thin-Walled Structures, vol.14, pp.1–20.

14. Sabir, A.B.; Djoudi, M.S.1995. Elastic buckling of stiffened plates by the finite element method, Energy Sources Tech. Conf. Exh. PD, 70, ASME, pp.191–198.
15. Grondin, G.Y.; Elwi, A.E.; Cheng, J.J.R.1999. Buckling of stiffened steel plates – a parametric study, Journal of Constructional Steel Research, vol.50, pp.151–175.
16. Sheikh, I.A.; Elwi, A.E.; Grondin, G.Y. 2003. Stiffened steel plates under combined compression and bending, Journal of Constructional Steel Research, vol.59, pp.911–930.
17. Vörös, G.M. 2007. An improved formulation of space stiffeners, Computers and Structures, vol.85, pp.350–359.
18. Vörös, G.M. 2007. Buckling and vibration of stiffened plates, International Review of Mechanical Engineering, I.RE.M.E. vol.1. n.1., pp.49–60.
19. Sapountzakis E.J.; Mokos V.G. 2009 A Displacement Solution to Transverse Shear Loading of Composite Beams by BEM, CMC: Computers, Materials & Continua, vol.10 (1), pp.1-39.
20. Tan, C. L.; Shiah, Y.C.; Lin, C.W. 2009. Stress Analysis of 3D Generally Anisotropic Elastic Solids Using the Boundary Element Method, CMES: Computer Modeling in Engineering & Sciences, vol. 41, No. 3, pp. 195-214.
21. Liu, C.S. 2007. Elastic Torsion Bar with Arbitrary Cross-Section Using the Fred-holm Integral Equations, CMC: Computers, Materials & Continua, vol.5 (1), pp.31-42.
22. Sapountzakis, E.J.; Tsiatas, G.C. 2007. Flexural-Torsional Buckling and Vibration Analysis of Composite Beams, CMC: Computers, Materials & Continua, vol.6 (2), pp. 103-116.
23. Dziatkiewicz, G.; Fedelinski, P. 2007. Dynamic Analysis of Piezoelectric Structures by the Dual Reciprocity Boundary Element Method, CMES: Computer Modeling in Engineering & Sciences, vol. 17, No. 1, pp. 35-46.
24. Wang, P. B.; Yao, Z. H.; Lei, T. 2006. Analysis of Solids with Numerous Micro cracks Using the Fast Multi pole DBEM, CMC: Computers, Materials & Continua, vol.3(2), pp. 65-76.
25. Sanz, J.A., Solis, M.; Dominguez, J. 2006. Hyper singular BEM for Piezoelectric Solids: Formulation and Applications for Fracture Mechanics, CMES: Computer Modeling in Engineering & Sciences, vol. 17, No. 3, pp. 215-230.

26. Zhou, J. X.; Koziara, T.; Davies, T. G. 2006. A Fast Space-Time BEM Method for 3D Elastodynamics, CMES: Computer Modeling in Engineering & Sciences, vol. 16, No. 2, pp. 131-140.
27. Fernandes, G.R.; Venturini, W.S. 2005. Building Floor Analysis by the Boundary Element Method, Computational Mechanics, vol. 35, pp. 277-291.
28. Botta, A. S.; Venturini, W. S. 2005. Reinforced 2d Domain Analysis Using BEM and Regularized BEM/FEM Combination, CMES: Computer Modeling in Engineering & Sciences, vol. 8, No. 1, pp. 15-28.
29. Divo, E; Kassab, A.J. 2005. Transient Non-linear Heat Conduction Solution by a Dual Reciprocity Boundary Element Method with an Effective Posteriori Error Estimator, CMC: Computers, Materials & Continua, vol.2 (4), pp. 277-288.
30. Miers, L. S.; Telles, J. C. F. 2004. A General Tangent Operator Procedure for Implicit Elastoplastic BEM Analysis, CMES: Computer Modeling in Engineering& Sciences, vol. 6, No. 5, pp. 431-440.
31. Rashed, Y.F. 2004. Green's First Identity Method for Boundary-Only Solution of Self-Weight in BEM Formulation for Thick Slabs, CMC: Computers, Materials& Continua, vol.1(4), pp. 319-326.
32. Zhang, Ch.; Savaidis, A. 2003. 3-D Transient Dynamic Crack Analysis by a Novel Time-Domain BEM, CMES: Computer Modeling in Engineering & Sciences, vol. 4, No. 5, pp. 603.
33. Hatzigeorgiou, G.D.; Beskos, D.E. 2002. Dynamic Response of 3-D Damaged Solids and Structures by BEM, CMES: Computer Modeling in Engineering & Sciences, vol. 3, No. 6, pp. 791-802.
34. Ochiai, Y. 2001: Steady Heat Conduction Analysis in Orthotropic Bodies by Triple-reciprocity BEM, CMES: Computer Modeling in Engineering & Sciences, vol. 2, No. 4, pp. 435-446.
35. Providakis, C.P. 2000. BEM / FEM Comparison Studies for the Inelastic Dynamic Analysis of Thick Plates on Elastic Foundation, CMES: Computer Modeling in Engineering & Sciences, vol. 1, No. 3, pp. 123-130.
36. Shiah, Y.C.; Tan, C.L. 2000. Fracture Mechanics Analysis in 2-D Anisotropic Thermo elasticity Using BEM, CMES: Computer Modeling in Engineering & Sciences, vol.1, No. 3, pp. 91-99.

37. De Paiva, J.B. 1996. Boundary Element Formulation of Building Slabs, Engineering Analysis with Boundary Elements, vol. 17, pp. 105-110.
38. Katsikadelis J.T.; Sapountzakis E.J. 1991. A BEM Solution to Dynamic Analysis of Plates with Variable Thickness, Computational Mechanics, vol.7, pp. 369-379.
39. Katsikadelis J.T.; Sapountzakis E.J. 1985. Torsion of Composite Bars by the Boundary Element Method, Journal of Engineering Mechanics, ASCE, vol.51, pp.1197-1210.
40. Boobnov. I.G .1902. The Stresses in a Ship's Bottom Plating Due to Water Pressure, Transactions of the institute of Naval Architects,London,Vol.44 and Theory of Structures of Ships,Vol.1 and 2. St. Petersburg, 1912-14.
41. Timoshenko. S.P, 1953. History of Strength of Materials, McGraw-Hill Book Companying. N.Y.pp.439
42. Wikipedia.2004.Buckling. [online] Available at: http://en.wikipedia.org/wiki/Buckling[Accessed 24 February 12].
43. Frederick A. Leckie, Dominic J. Dal Bello, 2009, Strength and Stiffness of Engineering Systems, Springer Science + Business Media, LLC 233 Spring Street, New York 10013 US.
44. Nautiyal. B.D, 2001. Introduction to Structural Analysis. New Age International Ltd. Publishers. 4835/24, Ansari Road Daryaganj New Delhi-110002.
45. Stanley W. Crawley; Robert Morton, 1993. Steel buildings: analysis and design. 4th ed. John Wiley & Sons, Inc. Toronto, Canada.
46. Gambhir Murari Lal, 2004. Stability analysis and design of structures, Springer-Verlag Berlin Heidelberg. Germany.
47. Bleich Friedrich, 1952, buckling strength of metal structures, McGraw-Hill Book Company, Inc. New York.
48. Jeom Kee Paik, Anil Kumar Thayamballi, 2003. Ultimate limit state design of steel plated structures, John Wiley & Son, Ltd. West Sussex. England. pp. 226-228.
49. Wikipedia.2001.Finite element method. [online] Available at: http://en.wikipedia.org/wiki/Finite_element_method. [Accessed 24 February 12].

50. Robert M. Jones, 2006. Buckling of Bars, Plates, and Shells, Bull Ridge Publishing, Blacksburg, Virginia.pp3-4.
51. Wikipedia.2004.Proof by contradiction.[online] Available at: http://en.wikipedia.org/wiki/Proof_by_contradiction[Accessed 24 February 12]
52. Hibbeler. R.C, 2011, Mechanics of Materials, 8th ed. Pearson Prentice Hall South Asia Pte Ltd. Singapore.pp287-289.
53. Lawrence N. Virgin, 2007, Vibration of Axially Loaded Structures, Cambridge University Press.32 Avenue of the Americas. New York .NY 10013-2473 USA. pp 20-21.
54. James M. Gere, Barry J. Goodno, 2009, Mechanics of Materials, 7th ed. Cengage Learning, 1120 Birchmount Road Toronto, Ontario M1K 5G4 Canada. pp 140.
55. Byron D. Tapley, Thurman R. Poston,1990, Eshbach's handbook of engineering fundamentals,4th ed. John Wiley & Sons, N.Y

APPENDICES

APPENDIX A

RESULTS FROM THE ANALYSIS OF THE FOUR REMAINING SLENDERNESS RATIOS

Below is the table and graphs showing the behaviour of the stiffening beam at four different slenderness ratios.

The result for analysis with slenderness ratio of 180 is given below:

Table 5: Results from FEM analysis on the slender column with a slenderness ratio of 180

L_p(mm)	F1(N)	F2(N)	F2/F1	S_r	*h*(mm)	*t*(mm)
200	1,28	1710	1335,9375	180	1800	10
250	1,603	1309	816,5938865	180	1800	10
400	2,582	795,3	308,0170411	180	1800	10
500	3,24	635,4	196,1111111	180	1800	10
700	4,572	454,8	99,47506562	180	1800	10
1000	6,596	319,4	48,42328684	180	1800	10
2500	16,94	132,7	7,833530106	180	1800	10
3000	20,42	111,8	5,475024486	180	1800	10
4000	27,38	86,05	3,142804967	180	1800	10
6000	41,33	71,43	1,728284539	180	1800	10
8000	55,28	74,75	1,352206946	180	1800	10
10000	69,23	83,54	1,206702297	180	1800	10
12000	83,18	94,51	1,136210628	180	1800	10
20000	1390	1453	1,045323741	180	1800	10
26000	1803	1856	1,029395452	180	1800	10
32000	2226	2266	1,017969452	180	1800	10
40000	2784	2832	1,017241379	180	1800	10
41000	2870	2901	1,010801394	180	1800	10

By plotting F2/F1 against length L_p (mm) for slenderness ratio of 180 generates a single cruciform curve.

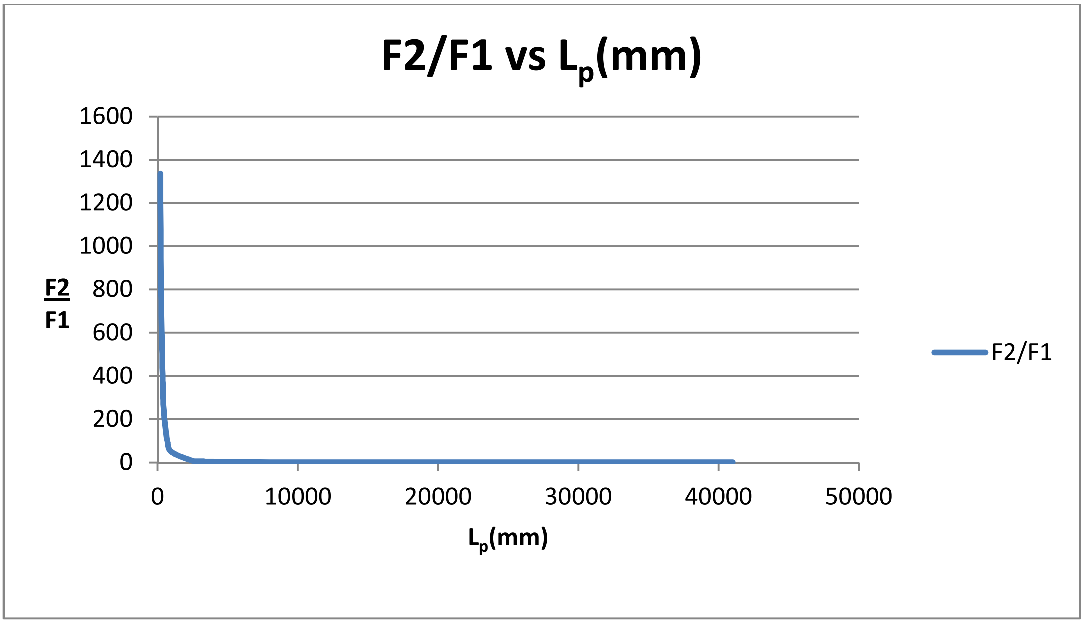

Figure 22: F2/F1-L_p (mm) graph for a slenderness ratio of 180

The result for analysis with slenderness ratio of 80 is given below:

Table 6: Results from FEM analysis on the slender column with a slenderness ratio of 80.

L_p(mm)	F1(N)	F2(N)	F2/F1	S_r	*h*(mm)	*t*(mm)
200	65,51	17100	261,0288506	80	800	10
250	82,26	13090	159,1295891	80	800	10
400	133,2	7953	59,70720721	80	800	10
500	167,6	6355	37,9176611	80	800	10
700	237	4560	19,24050633	80	800	10
1000	342,1	3273	9,56737796	80	800	10
1500	518,4	2243	4,326774691	80	800	10
3000	1048	1609	1,535305344	80	800	10
4000	1402	1771	1,263195435	80	800	10
6000	2108	2328	1,104364326	80	800	10
8000	2815	2973	1,056127886	80	800	10

10000	3521	3646	1,035501278	80	800	10
12000	4227	4331	1,024603738	80	800	10
14000	4934	5023	1,018038103	80	800	10
16000	5640	5720	1,014184397	80	800	10
18000	6490	6557	1,010323575	80	800	10
20000	7053	7118	1,009215936	80	800	10

By plotting F2/F1 against length L_p (mm) for slenderness ratio of 80 generates a single cruciform curve.

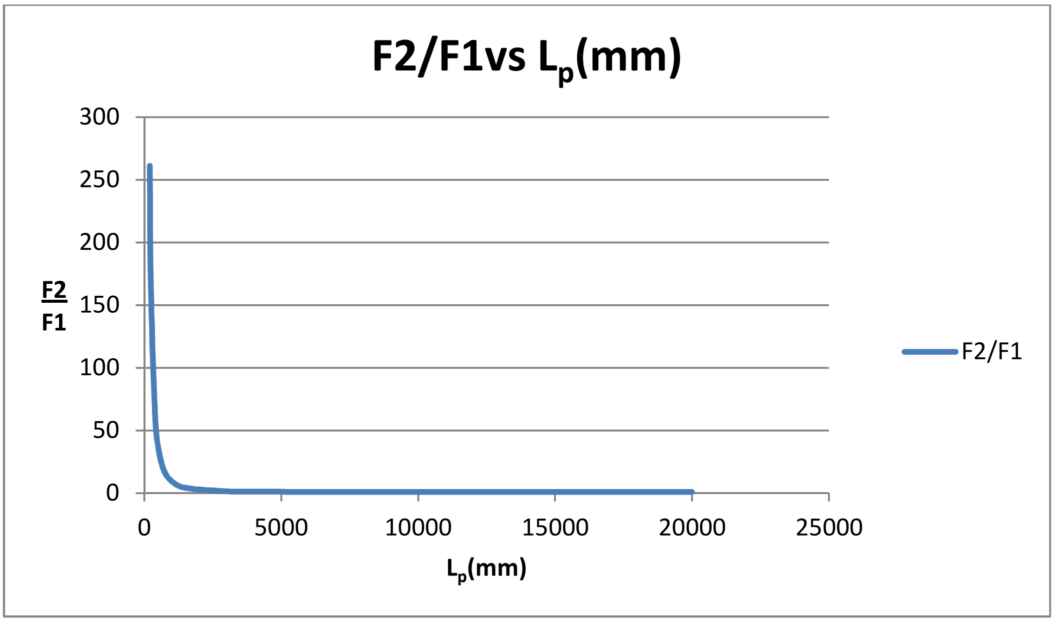

Figure 23: F2/F1-L_p (mm) graph for a slenderness ratio of 80

The result for analysis with slenderness ratio of 50 is given below:

Table 7: Results from FEM analysis on the slender column with a slenderness ratio of 50.

L_p(mm)	F1(N)	F2(N)	F2/F1	S_r	h(mm)	t(mm)
200	169,7	17100	100,7660577	50	500	10
250	213,4	13040	61,1059044	50	500	10
400	346,2	7960	22,99248989	50	500	10
500	436,6	6400	14,65872652	50	500	10
700	615,6	4710	7,651072125	50	500	10
1000	886,8	3372	3,802435724	50	500	10
1500	1339	2611	1,949962659	50	500	10
3000	2698	3158	1,170496664	50	500	10
4000	3603	3926	1,089647516	50	500	10
6000	5413	5620	1,038241271	50	500	10
8000	7224	7379	1,021456257	50	500	10
10000	9034	9481	1,049479743	50	500	10
12000	10840	10950	1,010147601	50	500	10

By plotting F2/F1 against length L_p (mm) for slenderness ratio of 50 generates a single cruciform curve.

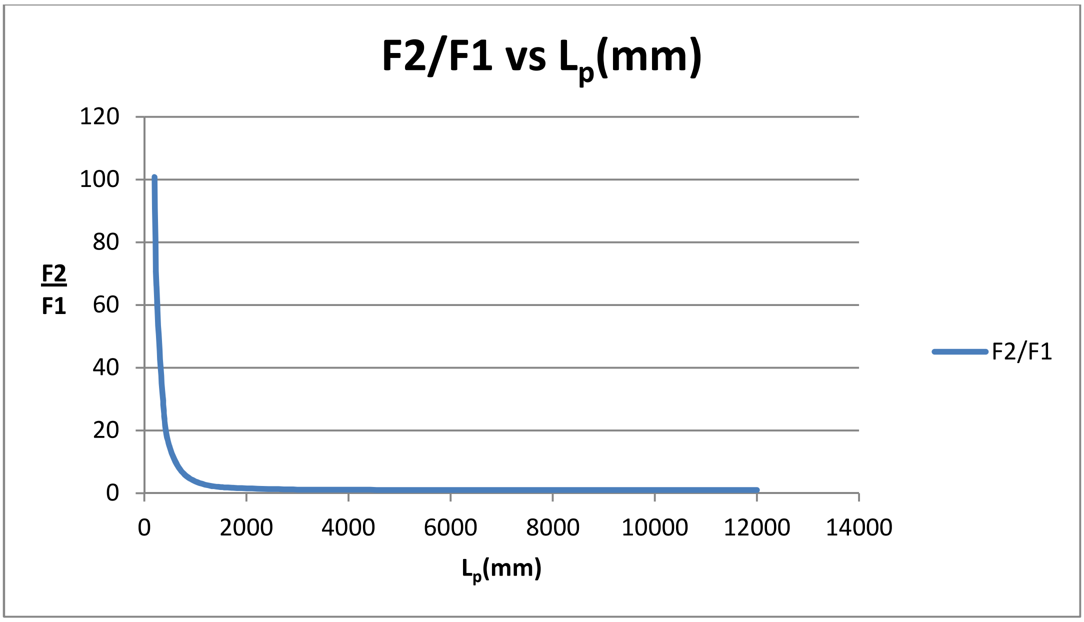

Figure 24: F2/F1-L_p (mm) graph for a slenderness ratio of 50

The result for analysis with slenderness ratio of 10 is given below:

Table 8: Results from FEM analysis on the slender column with a slenderness ratio of 10.

L_p(mm)	F1(N)	F2(N)	F2/F1	S_r	*h*(mm)	*t*(mm)
200	4634	18420	3,974967631	10	100	10
250	5814	14570	2,506019952	10	100	10
400	9356	13050	1,394826849	10	100	10
500	11720	14320	1,221843003	10	100	10
700	16440	18080	1,099756691	10	100	10
1000	23520	24590	1,045493197	10	100	10
2500	58910	59360	1,007638771	10	100	10
3000	70710	71100	1,005515486	10	100	10
4000	94310	94630	1,003393065	10	100	10
6000	141500	141800	1,002120141	10	100	10

8000	188700	188900	1,001059883	10	100	10
10000	235800	236100	1,001272265	10	100	10
12000	283000	283300	1,001060071	10	100	10

By plotting F2/F1 against length L_p (mm) for slenderness ratio of 10 generates a single cruciform curve.

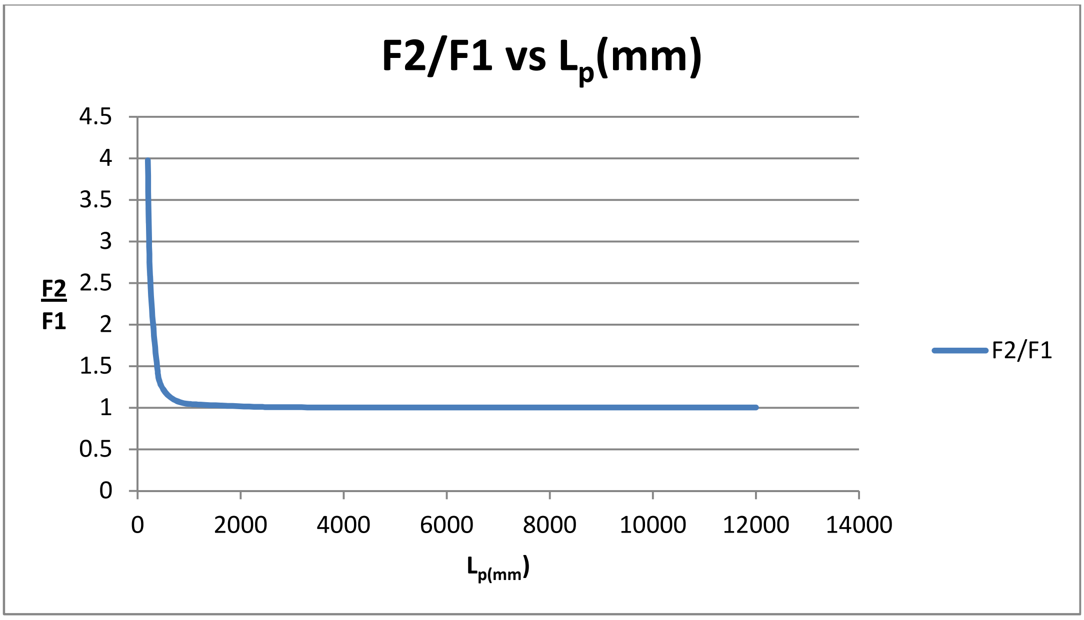

Figure 25: F2/F1-L_p (mm) graph for a slenderness ratio of 10