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ADVANCING TECHNOLOGY OF AUTOMATED REFRACTION – DEVELOPING AN ALGORITHM FOR AUTOMATED SUBJECTIVE REFRACTOMETER

An Innovation Project

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Master Thesis
Spring 2022
Master of Healthcare, Clinical Optometry
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ABSTRACT

Oulu University of Applied Sciences
Master of Healthcare Clinical Optometry

Title of thesis: Advancing Technology of Automated Refraction – Developing an Automated Subjective Refractometer
Supervisor: Arto Hartikainen
Term and year: Spring 2022
Number of pages: 63

Purpose
The purpose of this master thesis was to develop an algorithm for an automatic subjective refractometer with which the refractive status of an eye could be measured. It was an innovation project to Topcon Healthcare Solution Emea Oy, later called a “Company”. This master thesis was focused only on the distance refraction. It did not include binocular vision problems and near vision examination.

Methods
Distance refraction was explained by searching information from highly respected optometry text books and evidence-based studies. After that, there was an analysis of what must be considered when different parts of the examination are programmed to an algorithm. The greatest concern was about the Jackson Crossed-Cylinder examination. That examination was opened by mathematical calculations. Especially, the axis examination was carefully explained with the calculations.

Results
As results, distance refraction was found to include seven different parts: A starting point, the first spherical adjustment, an examination of astigmatism, the last adjustment of the spherical power, a binocular balancing, a binocular control of accommodation, and finally, the distance refraction of both eyes. Two new proposals of axis examination were made from mathematical calculations about cylinder axis examination. A couple of algorithms about different parts of the distance refraction were made together with the Company’s optometrist.

Conclusions
This thesis was to help the Company in planning the automatic refractometer. In addition to that, this thesis was a kind of reminder to every optometrist how the subjective distance refraction should be performed. This thesis gave new information about the cross-cylinder examination, especially the axis examination. All the necessary information to understand obliquely crossed cylinders effects was explained. A couple of tables from calculations explained various features of cylinder axis errors.

Keywords: subjective distance refraction, automatic refractometer, cross cylinder, astigmatism, obliquely crossed cylinders, gold standard of refraction.

CONTENTS

1 Background

Uncorrected refractive error is the leading cause of moderate and severe visual impairment (MSVI meaning visual acuity between 0.32 and 0.05) globally. The total amount is 86,1 million people having MSVI because of uncorrected refractive error. Among the blindness group (visual acuity worse than 0.05), uncorrected refractive error is the third cause, globally 2,2 million people, after cataract (15,2 million) and glaucoma (3,6 million). (Steinmetz, J et al. 2020).

Uncorrected refractive error is an available cause of vision impairment. Therefore, there is a considerable need for more eye examinations and spectacle corrections. There have been automated objective refractometers for decades. The newest step in eye examination is an automated subjective refractometer, where the examination is done by a specific algorithm instead of an eye care practitioner. (Otero, C. 2019). This master thesis is an innovation project about developing an algorithm for subjective refraction by the needs of Topcon Healthcare Solutions Emea Oy, later called a "Company".

Subjective refraction is a series of questions asked from the patient concerning various test charts viewed to the patient. This thesis explains different parts needed to perform subjective distance refraction. Only the distance refraction is included in this thesis. Near vision and examination of binocular vision problems are ruled out. Different parts of the distance refraction examination are analyzed, taking into account that an automated algorithm is doing the examination. A particular interest will be about the Jackson Crossed-Cylinder (JCC) examination. That part of the eye examination is probably the most challenging to the patient.

2 Theoretical Background

The human eye forms the image from the outside world to the back of the eye (retina). This formation of the image into the retina is the first step of the whole process that results in vision in the visual cortex. (Atchison, DA & Smith, G. 2000, pp. 3-7). Subjective refraction refers to examining the eye where the examiner uses different techniques to find out if there is a need to correct the refraction status of the eye. (Millodot, M. 2000 p. 260).

2.1 A Human Eye as an Optical Instrument

The purpose of the human eye as an optical instrument is to form a sharp image from the point of interest into the bottom (i.e., retina) of the eye. In creating a sharp image, the eye acts just like any other optical system. The eye can deviate rays of light. In an optimal situation, the eye deviates parallel light from infinity to the central part of the retina, called foveola. That kind of eye is called an emmetropic eye. (Atchison, DA and Smith, G. 2000, p. 57).

Several eye models explain the optical structure of an emmetropic eye. One of the best-known eye models is Gullstrand number 1 (exact) eye (Atchison, DA and Smith, G. 2000, p. 44). In that eye model, there are six refracting surfaces and six cardinal points for the calculations. Many times it is not necessary to use complicated exact eye models. Therefore, there are also simplified eye models, such as Emsley's reduced schematic eye (ERE). In that model, there is just one refracting surface (cornea) with a refracting power of 60 DPT, a total length of the eye 22,2 millimeters, and distance from the nodal point (N´) into the foveola (F´) 16,67 millimeters (Atchison, DA and Smith, G. 2000, p. 45) (*Figure 1*). In this thesis, the optical properties of the patient's eye are represented as the eye had only one refracting surface, just as is in ERE. The only addition to this eye model used in this thesis, is the capability of the eye to accommodate. That is, the eye has an ability to increase it's refracting power in order to focus objects that are nearer from the eye than being in infinity. (Atchison, DA and Smith, G. 2000, p. 45).

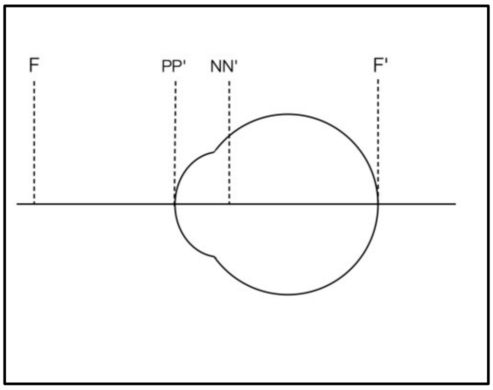


Figure 1: Emsley's reduced schematic eye. Picture: Arto Hartikainen.

When a human eye refracts the parallel light from infinity like Emsley's reduced eye without accommodating power, the eye is said to be an emmetrope. A point object in the infinity will be focused as a point in the retina in that situation (Rosenfield, M. 2006, p 27.).

If the human eye in a relaxed accommodative state doesn't refract parallel light rays from infinity into the retina, the eye is said to have a refractive error. In those situations, the eye is not emmetropic; instead, it is an ametropic eye. The three refractive errors are myopia, hyperopia, and astigmatism. (Rosenfield, M. 2006, pp. 27-31).

2.1.1 Myopia

Myopia means a refractive error, where the eye has too much refracting power with respect to the length of the eye. In myopia, parallel light from infinity will be focused on the vitreous instead of being focused on the retina. Parallel light rays must diverge before being deviated within the eye from being focused in the retina. Myopia may be caused either because the eye is too long compared to the ERE or because the eye has too much refracting power compared to ERE. These two possible conditions are called axial myopia or refractive myopia, respectively. (Rosenfield, M. 2006, p. 27).

The far point is a distance from the eye, from where the rays of light will conjugate with the retina during no accommodation. In emmetropia, the far point is in infinity. In myopia, the far point is nearer than in infinity. Light rays from the far point will diverge when they come inside the refracting surface of the eye and then will be focused in the retina. The power needed to correct the amount of myopia is the inverse value of the distance in meters from the correcting lens of the myopia into the eye’s far point (*Figure 2*). In these calculations, distances in front of the correcting lens are negative. (Sebag, M. 2020).

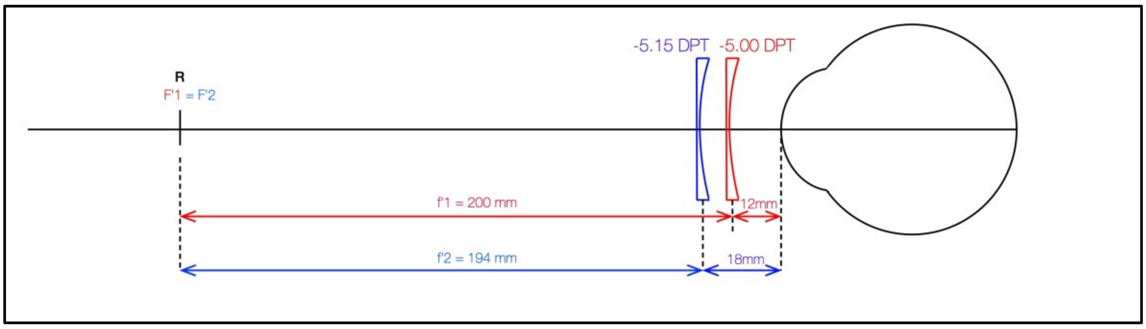


Figure 2: Far point (R) of a myopic eye and the negative power needed to correct the refracting error. Picture: Arto Hartikainen

The power needed to correct myopia depends on the distance of the correcting lens from the eye, as shown in *Figure 2*. When the correcting lens is at the corneal plane, refracting error is named as an ocular refraction. When performing subjective refraction, refraction is corrected with trial lenses. Lenses are placed before the refracted eye to some distance from the eye. Lens distance from the eye (vertex distance) may usually differ from 5 to 20 millimeters (Distometer and Conversion Disc Instructions). In myopia, the negative power increases when the vertex distance increases (*Picture 2*). The higher the myopia is, the more critical it is to measure vertex distance. (Sebag, M. 2020). In this thesis, a term called “an eye refraction” is used. It means the exact spherocylinderical correction that the eye needs in the trial lens distance from the cornea to correct the refraction error.

2.1.2 Hyperopia

Hyperopia means a refractive error where the eye has too little refracting power with respect to the length of the eye. In hyperopia parallel rays of light from infinity will be focused behind the retina, that is, into the choroid. Light rays must diverge from being focused in the retina. Hyperopia may be caused either because the eye is too short compared to the ERE or because the eye has too little refracting power compared to ERE. These two possible conditions are called axial hyperopia or refractive hyperopia, respectively. (Rosenfield, M. 2006, p 37).

In hyperopia, there isn't any real distance from where the rays of light would conjugate in the retina. For that reason, the far point is said to be behind the eye. Light rays must converge towards the far point when they come inside the refracting surface of the eye and then will be focused in the retina. The power needed to correct hyperopia is the inverse value of the distance in meters from the correcting lens of the eye into the eye's far point *(Figure 3*). In these calculations, distances behind the cornea are positive. (Sebag, M. 2020).

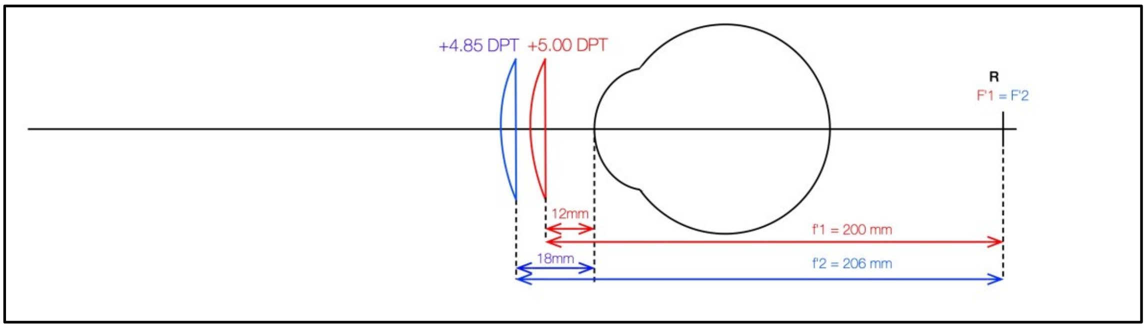


Figure 3: Far point (R) of a hyperopic eye and the positive power needed to correct the refracting error. Picture: Arto Hartikainen

Just like in myopia, the power needed to correct hyperopia depends on the distance of the correcting lens from the eye, as shown in *Picture 3*. In hyperopia, the positive power decreases when the vertex distance increases. In high hyperopic ametropias, it is critical to measure vertex distance (Sebag, M. 2020).

2.1.3 Astigmatism

The word astigmatism comes from two words; "a," meaning "lacking," and "stigma", meaning "a point." These two words explain well what astigmatism is. Astigmatism means a refractive error where the eye doesn't form a point picture from a point object. Instead of one point picture, an astigmatic eye creates two lines from a single-point object. The eye has two different refracting powers that make those two focal lines previously mentioned. In regular astigmatism those refracting powers are perpendicular to each other. If those two refracting powers are not perpendicular to each other, astigmatism is said to be irregular. (Rosenfield, M. 2006, pp. 39-41).

An example of uncorrected astigmatism is given in *Figure 4*. In that figure, both lines of foci are in front of the retina. In this case, astigmatism is said to be compound myopic astigmatism. If only the first line of foci is myopic and the second foci lies in the retina, astigmatism is said to be simple myopic astigmatism. Suppose both lines of foci are behind the retina. In that case, astigmatism is said to be compound hyperopic astigmatism. If only the second foci is behind the retina and the first lies in the retina, astigmatism is said to be simple hyperopic astigmatism. When the first line of foci is in front of the retina and the second line of foci is behind the retina, astigmatism is fixed. (Atchison, DA & Smith, G. 2000, p. 60).

Astigmatism is also classified based on the direction of its axis. When an astigmatic eye has the greatest refracting power in the vertical direction or within ±30º from the vertical meridian, it is said to be with the rule astigmatism. When the greatest refracting power of the eye is within ±30º from the horizontal direction, it is said to be against the rule astigmatism. In *Figure 4*, there is against the rule astigmatism. The horizontal meridian creates a vertical focal line with the shortest focal length. The least refracting meridian is the vertical meridian. That meridian creates the horizontal focal line. (Atchison, DA and Smith, G. 2000, p. 60).

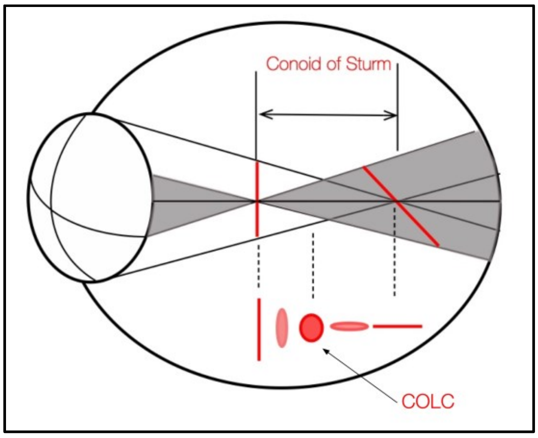


Figure 4: An astigmatic eye error (compound myopic astigmatism, against the rule astigmatism). Conoid of Sturm and a circle of the least confusion (COLC) are shown. Picture: Arto Hartikainen

Of great importance is the distance between the two lines of foci in astigmatic eye error. That distance is called the conoid of Sturm (Donders, FC. 1864, p. 485). In the dioptric midpoint of the two lines of the foci is a special place from the point of subjective refraction, "a circle of least confusion" (COLC) (*Figure 4)*. If there is uncorrected astigmatism in the eye examined during the subjective refraction, the best visual acuity is achieved when the COLC is in the retina. In that situation the cross-cylinder technique will work in a best way (Hartikainen, A. and Seppänen, M. 2022).

2.2 Subjective Distance Refraction of the Eye

An eye examination, where the examiner finds out if the patient has a refractive error, is called refraction. Refraction can be made both objectively and subjectively. During objective refraction, the patient doesn't give any judgments about his or her vision. The subjective refraction is based on the patient's choice when comparing one lens against another to get maximal visual acuity with a suitable lens combination. (Millodot, M. 2000, p. 260).

The eye's refractive status is defined as how the light rays from infinity refract when passing onto the eye. For that reason, test marks showed to the patient must be far away. In theory, the test marks should be in infinity, but in an actual situation, this is not possible. Optical infinity is the term used in clinical optometry. That is a distance (6 meters) from where the light rays come practically parallelly onto the eye (Millodot, M. 2000, p. 140). Distance refraction should be measured in that distance. On the other hand, standard distance for visual acuity measurements is 4 meters (Enoch, JM et al. 1984).

Commonly, subjective refraction is performed at shorter distance than recommended optical infinity, 6 meters. Many times the refraction examination distance is five or four meters. When the refraction is measured in distancies like 5 or 4 meters, it is essential to understand and remember the dioptric difference between refraction distance and actual infinity. For example, the dioptric difference between four meters and infinity is 0,25 DPT. When prescribing eyeglasses, that difference must be taken into account in those situations when it is imperative to get the absolute best visual acuity in infinity. If the refractive error measured to four meters is sf -2,50, the refractive error to infinity is sf -2,75. In the same way, if the refraction error measured to 4 meters is sf +2,50, the refraction error to infinity is +sf 2,25. The dioptric difference between 5 meters and optical infinity is 0,20 DPT. Even from optical infinity (6 meters), the difference to actual infinity is 0,167 DPT. (Enoch, JM et al. 1984).

Subjective refraction is usually said to be the gold standard of measuring a refraction error of the eye. All other refraction techniques (autorefraction, retinoscopy) regarding their validity are compared to subjective refraction. Surprisingly, the procedure of gold standard of the refraction is poorly explained, if not explained at all. In clinical studies where the term gold standard of refraction is used, subjective refraction may have been performed in many different ways. There is a need to explain the requirements of various tests that have to be done when the subjective refraction is done like the gold standard of refraction (Elliott, D. B. 2017).

2.2.1 Visual Acuity and Subjective Distance Refraction

The subjective distance refraction aims to find a suitable lens combination from both spherical and cylindrical lenses so that the parallel light rays from infinity should conjugate in the retina. The procedure is controlled by showing smaller and smaller optotypes to the patient (alphabets, numbers, or picture symbols) and finding out how good visual acuity the patient will get. The best possible visual acuity is achieved when astigmatism is corrected fully and parallel light rays conjugate in the retina, and to be exact, in the retina's outer limiting membrane (OLM) (Chen, B et al. 1993). One must remember that the retina's thickness is in the center of the fovea about 245 micrometers. At the same time, one dioptre difference in the eye's refracting power corresponds to a 300 micrometer difference in the distance of the eye's focal point. So, there is about 0.75 dioptrical difference from the first retinal layer, the inner limiting membrane, to the last layer, the retinal pigment epithelial layer. For that reason, it can be said precisely to what layer of the retina must parallel light conjugate (Benjamin, WJ. 2006, p 1034). In *Figure 5*, there is an OCT image of the retina with different layers. OLM is shown with the red arrow. A dioptrical scale shows where the light will conjugate if there is a minimal refractive error.

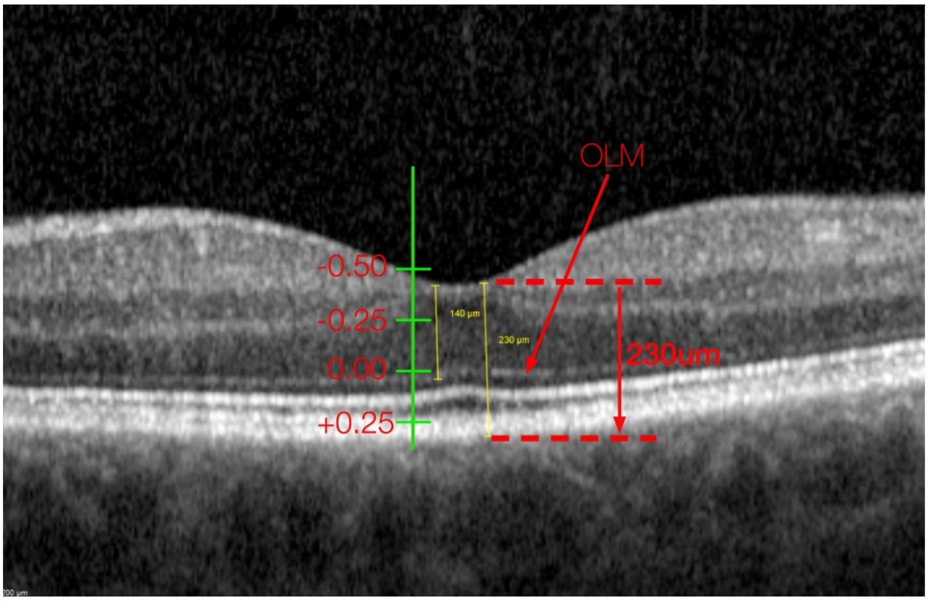


Figure 5: An OCT picture of the retina. OLM represents the layer, where the light must conjugate. Green scale shows where the light will conjugate in slight refractive errors. Thickness of the foveola is 230 micrometer in this picture. Figure: Arto Hartikainen

So-called normal visual acuity is the ability to see two points separately when they are visible in one minute of arc in an object plane. This acuity is referred to as visual acuity 1.0 (or 6/6 or 20/20) (Visual Standards, Aspects and Ranges of Vision Loss. 2002). When parallel light rays are not conjugated in the outer limiting membrane of the retina but are defocused, visual acuity will be lower than maximal possible. When the uncorrected refraction error is 1 DPT, visual acuity is lowered from 1.0 to about the 0.6 level, and in the case of 2 DPT of uncorrected refraction error, visual acuity is about 0.2. (Westheimer, G. 2010). This lowering of visual acuity can be used to estimate the refractive error during subjective refraction.

2.2.2 Starting Point of the Subjective Distance Refraction

Subjective distance refraction is a procedure where the examiner finds a suitable spherocylinderical lens combination that gives the patient maximal visual acuity in a minimal accommodation state. That method is called maximum plus or minimal ms power for maximum visual acuity (Benjamin, WJ. 2006, p. 1263). During the examination, the examiner must control how the visual acuity changes while changing different lenses (spherical or cylindrical lenses) to the patient. At first, the examiner must decide the starting lens combination that the patient uses at the beginning of subjective refraction. There are a couple of lens possibilities that can be used. Those possibilities are the result of retinoscopy, the result of an automated objective refractometer (autorefractometer), previous distance correction powers, or with no lenses at all (Franklin, A. 2007).

Retinoscopy and autorefractometry are both objective measurements of current refractive error. Using retinoscopy needs quite a lot of experience from the examiner. Autorefractometers are easier to use compared to retinoscopy. Both methods are susceptible to accommodation, which is good to keep in mind when looking at the results. Despite that, if a result of either one is possible to get, it is a good starting lens combination in subjective refraction (Mukash, SN et al. 2021). Previous eyeglass powers for distance vision can also be starting power of subjective refraction. That is an option if there is not for some reason an objective measurement of refraction available (Wilkinson, M. 2016). Sometimes it is not possible to get objective refraction, or there is no result of previous glasses, or the patient has no eyeglasses earlier. In those situations, subjective refraction must be started without any lens powers (Benjamin, WJ. 2006, p. 1261).

2.2.3 The First Adjustment of the Spherical Power

During monocular subjective refraction, the spherical power must be adjusted two times to give the maximum visual acuity to the patient. The first spherical adjustment is done at the beginning of the refraction, just before examining astigmatism with the cross-cylinder. The second time to adjust the spherical power is after the cross-cylinder examination. In a concise way of explaining subjective refraction, we can say that refraction is first adjusting sphere, then cross-cylinder examination, and finally adjusting sphere again. (Hartikainen, A. and Seppänen, M. (2022).

The first spherical power adjustment is started by looking at visual acuity with the starting point powers. After getting the starting point visual acuity, sphere power is changed by either adding +0,25 DPT or -0,25 DPT and finding whether either lens change would increase the visual acuity compared to the starting point acuity. (Hartikainen, A. and Seppänen, M. (2022). The examiner has to keep in mind that accommodation can affect the results in a way that there is too high minus or too low plus spherical power in the result. For that reason, it is recommended to use a fogging technique in the first adjustment of the spherical power. Fogging means that the examiner adds spherical plus power to the starting power (or decreases spherical minus power). The visual acuity decreases at least two to three lines compared to the starting point visual acuity. In that situation, light rays from a distance are focused in front of the retina, into the vitreous. In other words, the eye has too much refracting power. If the eye accommodates in that situation, it increases the refracting power of the eye more, leading to even poorer visual acuity. Accommodation does not help in that situation; it worsens the situation. That is why the eye is in a relaxed accommodative position when being fogged (Wilkinson, M. 2016).

When the visual acuity has been lowered, the extra plus power (or too lower minus) is taken away 0,25 DPT by 0,25 DPT, and visual acuity is checked in every step of changing power. Power is changed so long that it improves visual acuity. When visual acuity doesn't improve by changing the power towards minus, the COLC is in the OLM of the retina. That is the endpoint of the first spherical power adjustment. (Borish, IM. and Benjamin, WJ. 2006, p. 1263).

The other usual way of examining the first spherical endpoint is to exploit the natural longitudinal (or axial) chromatic aberration of the eye. The shorter the wavelength of the visible light is, the more will that wavelength deviate within the eye. The mean difference in dioptres between the wavelengths 480 – 700 nm is about 1.50 DPT (Vinas, M et al., 2015). In subjective refraction examination, this phenomenon is used in the Duochrome (or Bichrome) test. In Duochrome test, there is a red area and a green area, side by side. Inside the color area, there are some black symbols (numbers, letters etc.) on each side. A dioptical difference within the eye between green light (535nm) and red light (620nm) is about 0.50 DPT and in the same time dioptrically at the same distance (0.25 DPT) from the yellow light (570nm). (Gantz, L. 2015). A green light will deviate more inside the eye than a red light. The spherical endpoint will be found by comparing the symbols' sharpness in the green and red areas. If the spherical lens is too weak minus (or too strong plus), the red area will be focused nearer the OLM than the green area, and thus the symbols within the red area appear sharper than symbols in the green area. If the spherical lens is too strong minus (or too weak plus), opposite happens. The symbols will appear sharper in the green area than the symbols in the red area. When the spherical lens is adjusted to the right power, symbols will be equal in sharpness within both colors, *Figure 8.* In some cases, it may happen that there is not any spherical lens that will make both red and green sides to be equal in clarity. In those cases, one lens change will make green sharper after the previous lens where the red was better. If that happens, the spherical endpoint is 0.25 DPT lower minus or 0.25 DPT stronger plus power than the spherical lens in the situation of the first time green side better ("first green"). (Borish IM. and Benjamin, WJ. 2006, 1239).

Figure 7: Duochrome test in a situation of optimal spherical correction. Both green and red sides are at equal distance from OLM and thus appear equally sharp. Figure: Arto Hartikainen

Duochrome test has been found to be a reliable test in finding the first spherical end point lens. It works confidentially also in situations where there may be uncorrected astigmatism during the first spherical end point lens examination. (Gantz, L. 2015).

2.2.4 Examination of Astigmatism

There may be a cylindrical lens in the starting point of refraction or not. During the first spherical adjustment, the possible cylindrical power is kept in the trial lens frame as it was measured during objective refraction or measured from the previous glasses. Only the spherical power is changed. After the first spherical adjustment, the possible astigmatism correction must be checked that it is fully corrected to its amount and direction. If there is no cylindrical lens in the starting point lens combination, possible astigmatism must be checked after the first spherical adjustment. (Hartikainen, A. and Seppänen, M. 2022).

Astigmatism can be examined in many different ways, but probably the most common technique is Jackson Crossed-Cylinder (JCC) method. In this thesis, only JCC-technique is explained in the correction of astigmatism. JCC means a unique spherocylinderical lens with which both the amount of astigmatism and the axis of astigmatism can be examined accurately. The most common JCC lens is $\pm$ 0,25 lens, which means sf +0,25 cyl -0,50 lens combination. There are also JCC-lenses $\pm$ 0,50 and $\pm$ 1,0. Those lenses may be used when the visual acuity is worse than 0,50 in the situation of astigmatism examination (Harris, WF. 2007).

When JCC is used, the patient is recommended to look at something round-shaped picture (Borish IM. and Benjamin, WJ. 2006, p 1224). There is usually a unique astigmatism test mark for this purpose included in projectors (*Figure 7*). The JCC will be rotated in front of the patient´s eye so that the axis of the JCC will rotate 90 degrees. This rotation gives two positions to the JCC, and, at the same time, two alternatives to the patient to compare, usually named “number one” and “number two”. These two positions of JCC shift the interval of Sturm in the retina without changing the place of the COLC in the retina. That is the fundamental feature of JCC. Astigmatism varies by flipping the lens, but the circle of least confusion does not change its place. (Puntenney, I. 1954).

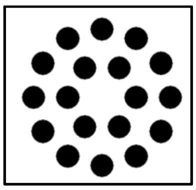


Figure 6: Cross cylinder test chart. Figure: Arto Hartikainen

Both the amount and the axis of astigmatism can be determined by using JCC. The procedure is the same in both examinations (examination of the amount and the axis of astigmatism). The only difference in measuring the power and axis of the astigmatic correction is the orientation of the JCC during measurement. The JCC cylinder axis must be kept in the same direction as the correction cylinder axis in power examination. In astigmatism axis measurement, the cylinder axis of the JCC is kept 45 degrees to the axis of the correcting cylinder. (Borish IM. and Benjamin, WJ. 2006, pp. 1213-1215).

The examiner shows two alternative views by rotating the JCC so that the axis of the JCC will change 90 degrees between numbers one and two. The patient must tell which alternative is clearer or are those alternatives equal in clarity. If either of the alternatives is clearer, the examiner changes the amount of the cylinder or the cylinder's axis depending on whether the examiner is determining the power or the direction of the cylinder, respectively. Testing is continued until both alternatives are equal in sharpness. (Borish IM. and Benjamin, WJ. 2006, pp. 1213-1215).

When there is a cylinder power at the beginning of the JCC examination, the exact axis of the correcting cylinder is adjusted first, and the fine adjustment of the cylinder power after axis adjustment (Hartikainen, A. and Seppänen, M. (2022). In the axis adjustment, the axis of the JCC is kept 45 degrees from the correcting cylinder axis. With the JCC, two alternatives are given by rotating 90 degrees the JCC. The patient answers if there is a difference in clarity between the alternatives. If either alternative is sharper, the correction cylinder must be rotated to the direction of the preference JCC axis. For example: the correcting power is Sph +1.00 Cyl -0.50 ax 0. The patient prefers a clearer alternative when the JCC minus cylinder axis is in 45 degrees. The correcting cylinder must be rotated from 0 degrees towards an angle of 45 degrees. (Borish IM. and Benjamin, WJ. 2006, pp. 1213-1215). Questioning about possible clarity differences is continued in the same way until both alternatives look the same. The axis's rotation depends on the correcting cylinder's power. The more powerful the correcting cylinder is, the less is the amount of rotation of the cylinder. One recommendation has been given for the first rotation of the cylinder axis based on the power of the correcting cylinder. (Del Priore LV, Guyton DL. 1986.). (*Table 1*).

Table 1: The first rotation of the correcting cylinder in JCC axis alignment. Table adapted from: Del Priore LV, Guyton DL. 1986. The Jackson cross cylinder, a reappraisal. Ophthalmology 93:1461–146

Power of the Correcting Cylinder	The First Rotation Required
≤ 0.25	30º
0.50	15º
0.75	10º
1.00 - 1.75	5º
2.00 – 2.75	3º
3.00 – 4.75	2º
≥ 5.00	1º

After adjusting the cylinder axis, the cylinder power is adjusted to full correction. If the astigmatism is corrected using the minus cylinder, the reference cylinder axis in the JCC is also minus. Now, JCC is placed so that the JCC axis and correction cylinder axis coincides. Two alternatives to compare are given to the patient by rotating the JCC 90 degrees. If the patient prefers a clearer alternative where the correcting cylinder axis and the JCC cylinder axis are in the same direction, the correction cylinder must be added. Addition of cylinder is 0,25 DPT, when 0,25 DPT JCC is used. After adding the more powerful cylinder, the examiner shows again two alternatives, and the patient answers which one is better. If the patient prefers once again the same alternative as earlier, correcting cylinder is again added by 0.25 DPT. Once the power of the correcting cylinder has been added 0.50 DPT, the correcting spherical power must be changed to the opposite direction compared to the power of the correcting cylinder by 0,25 DPT. In that way, the COLC will stay in the retina. For example: at the beginning of the cylinder power examination, the correction power is sf +1.0. cyl -0.50 ax 0º. The patient prefers clearer that alternative where the JCC minus axis is also in 0º. The correcting power of the cylinder must be added to cyl -0.75. Now the correcting power is sf +1.00 cyl -0.75 ax 0º. After that, the patient prefers once again a clearer alternative where the JCC minus axis is in 0º. Now the cylinder power must be added to cyl -1.00, but simultaneously, the spherical power of the correcting lens must be changed from +1.00 to +1.25. In general, when the minus cylinder is changed to "more minus"-direction by 0.50 DPT, spherical power must be changed to "more plus"-direction 0.25 DPT. The correction power would be sf +1.25 cyl -1.00 ax 0º. (Borish, IM. and Benjamin, WJ. 2006, p. 1216).

Suppose the correcting cylinder is too strong at the beginning of the JCC examination of the power. In that case, the patient prefers a clearer alternative where the JCC axis of reference is 90º degrees away from the correction cylinder. In that situation, the cylinder power of the correction cylinder is decreased 0.25 DPT at a time (when using 0.25 DPT JCC) until both alternatives given in JCC are equal. The spherical component of the correcting lens must be changed 0.25 DPT in every 0.50 DPT cylinder reduction in the same principle that it is when adding the correcting cylinder. That is when the minus cylinder is changed to "less minus"-direction by 0.50 DPT, spherical power must be changed 0.25 DPT to "more minus"-direction. (Hartikainen, A. and Seppänen, M. 2022).

Sometimes, there is no cylinder correction in the starting point of the subjective refraction. Objective refraction has shown only spherical refraction error, or the starting point is without any spectacle correction. It is still possible that the patient also has an astigmatic refraction error. If there is only a spherical lens correction after the first adjustment of the spherical power, a JCC examination is still performed on the patient. First, the JCC is placed with its cylinder axes at 0º and 90º degrees. If there are some differences in clarity, it means that the patient has astigmatism in either direction, and examination goes just in the way it goes in typical power determination. In this case, the axis examination is after the power examination. If both 0º and 90º-degree orientations are equal, there can still be astigmatism in an oblique direction. For that reason, the JCC cylinder axel is turned so that two alternatives are shown, now in 45º and 135º degrees. If there is some difference, the amount of astigmatism is examined first, and after power examination the fine adjustment of the axis is made. If all directions 0º and 90º and 45º and 135º are equal in clarity without any cylinder correction, there is no astigmatism in that eye examined. (Hartikainen, A. and Seppänen, M. 2022).

2.2.5 Final Adjustment of the Spherical Power

After cylinder examination with JCC, a final adjustment of the spherical power has to be done. The eye has only one focal point when astigmatism is fully corrected with its power and axial alignment. The purpose of the final spherical adjustment is to find a spherical lens that will set the eyes focal point to the OLM (Borish, IM. and Benjamin, WJ. 2006, p. 1234).

One usual way to find the final spherical power is fogging technique. Fogging is done just in the same way as it was done in the first spherical adjustment. The amount of extra plus power is typically between 1 to 2 DPT. Then the extra plus power is removed by 0.25 DPT at a time. Plus power is removed as long as the visual acuity gets better. The first plus power reduction, which does not improve visual acuity from the previous one, is the fogging method's stopping point. The previous spherical power is the power that will locate the image into the OLM. (Benjamin, WJ. 2006, p. 1236). For example, lens power after JCC examination is sf -2.75 cyl -1.00 ax 20. The spherical endpoint will be examined using fogging lens sf +1.50. Fogging is started by changing the spherical -2.75 lens to the lens of sf -1.25, and after that the visual acuity will be checked. Then the spherical lens will be changed step by step to stronger minus power, -0.25 DPT at a time. After every lens change, visual acuity will be examined. Let us assume that with the lens sf -2.50 cyl -1.00 ax 20, the patient has visual acuity of 1.25. Then the spherical lens will be changed from -2.50 to -2.75. With the lens -2.75, the patient can't see any letters from the visual acuity row of 1.60. The last lens change did not improve visual acuity. The spherical endpoint is, in this case, sf -2.50, and the whole refraction in this example would be sf -2.50 cyl -1.00 ax 20

The other usual way of examining the spherical endpoint is to use again the Duochrome test. Is is used at the same way as it was used during the first adjustment of the spherical power. The purpose od the Duochrome test is to find a lens, where from both the green and red side the optotypes can be seen equally sharp. If that situation doesn't exist, then the endpoint is 0.25 DPT lower minus or stronger plus that the "first green" lens power is. (Borish IM. and Benjamin, WJ. 2006, 1239).

2.2.6 Binocular Balancing

After monocular refraction of the right and left eye, the eyes have to be balanced binocularly. It is possible that during monocular refraction, accommodation occurs to some extent in one of the eyes or both eyes. Binocular balancing aims to achieve the same accommodative state in both eyes. Balancing of the right and the left eye is in practice finding out, what will be the spherical power difference between right and left eye. Once the dioptrical difference has been found, it will be kept the same till the end of the subjective refraction. In binocular balancing, cylinder powers are not changed at all. Only spherical lens powers will be fixed. As the name "binocular balancing" states, both eyes are being tested simultaneously; that is, neither eye is occluded. Usually, the test chart shown to the patient is dissociated totally by vertical prisms so that the right eye and the left eye can be tested monocularly under binocular conditions. (Benjamin, WJ. 2006, pp. 1250-1251).

Dissociation can also be made by polarizing lenses and using polarizing charts. In polarizing binocular charts, there is often something that can be seen with both eyes. That can be, for example, one row of numbers or a circle that can be seen with both eyes. Those objects, seen by both eyes, are so-called fusion locks that help maintain binocular fusion and prevent phorias or tropias from manifesting (Hartikainen, A. and Seppänen, M. 2022.).

One of the most frequently used binocular balancing tests is the Bichrome-Balance test. That is the same test that is used in monocular refraction, named Duochrome, but now used in binocular viewing conditions. When testing binocularly, same amount of vertical prism (usually 3 or 4 prism dioptres) is placed before each eye. One prism is placed with its base down (usually before the right eye) and another prism is placed base up (usually before the left eye). The prisms have to be of same power before both eyes so that the optical distortion caused by prisms are the same to both eyes. There may also be a specific chart in the projector where there is a red and green test separated to both eyes by a polarization filter. (Borish, IM. and Benjamin, WJ. 2006, p. 1253).

Binocular balancing starts with the patient using that spherocylinderical power obtained in monocular refraction. Bichrome-Balance test is performed so that the examiner finds out first that spherical power when optotypes in the red side are seen last time sharper than in the green side ("last red"). Then after that lens power, with the addition of 0.25 DPT more minus (or 0.25 DPT less plus), comes a situation where optotypes in both red and green parts are equal in clarity ("equality"). After the "equality" lens, by adding the next -0.25 DPT lens power, comes a situation where the optotypes in the green side are the first time sharper than in the red side ("first green"). It is possible that after the "last red" lens comes the "first green" without the "equality" lens. When the spherical power "first green" in the right eye is found, the power is changed to 0.25 DPT less minus (or 0.25 DPT more plus) to the right eye. The same is done to the left eye: after finding the "first green" lens, spherical power is changed to 0.25 DPT less minus. Now there is in both eyes that spherical lens that the next stronger minus lens (or next lower plus lens) will change the green side the first time better than the left side. This is the situation of the same binocular accommodative state of the right and left eyes and same time an end point of binocular balancing. At this point, the spherical power difference between the right and left lens can be "locked". It will be kept the same the rest of the examination. The best feature about the Bichrome-Balance test is that it can be used in situations of similar visual acuities in both eyes and the situation of different maximal visual acuities in the right and the left eye. (Borish, IM. and Benjamin, WJ. 2006, p. 1253).

One other generally used binocular balancing test is to compare visual acuities of both eyes in a binocular condition. When using this system, both eyes should have the same visual acuity. Otherwise, the eye with the better visual acuity would be easily fogged wrongly to the level with worse eye. Once again, a vertical prism of 3 or 4 prism dioptres is used to dissociate the right and the left eye. One prism base down is placed before the right eye, and another prism is placed base up before the left eye. When the prisms are placed, the patient sees two equal visual acuity lines. It is recommended that a certain amount of fogging plus lens is placed before the monocular spherocylinderical powers when starting the test. That extra fogging power will place the eye's focal point in front of the retina. That situation prevents accommodation. The row of letters shown to the patient must be adjusted to the visual acuity so that the patient can see the optotypes, although the optotypes may be unclear. Now, under the fog, the patient compares the relative clarity of the acuity row seen by the right eye (upper row, if the prism in that eye is base down) to the row seen by the left eye (lower row, if the prism in that eye is base up). If the optotypes are clearer in one of the eyes, +0.25 DPT lens is added to the eye. Then, a comparison of the clarity between the upper and lower row is repeated. The accommodative balance between the eyes is achieved when the optotypes are as equally clear (or unclear) as possible. Then the prisms are taken away, and fogging plus powers are taken away by 0.25 DPT at a time as long as the binocular visual acuity improves. When there is no more prolonged improvement in visual acuity, the final binocularly balanced spherocylinderical power has been found (Borish, IM. and Benjamin, WJ. 2006, p. 1250).

2.2.7 Control of Accommodation and Final Distance Refraction of the Eye

The last part of distance refraction of the eyes is to control once more the accommodation and find the maximal plus power or minimum minus spherical power with which the patient can achieve maximal visual acuity. The eyes have been examined first monocularly, and astigmatism is fully corrected in both eyes; both eyes have been balanced binocularly so that the accommodative effort is equal. In this last part of the refraction, spherical extra plus power is added to the binocularly balanced spherocylinderical powers, usually about 1.50 DPT to both eyes. The cylinder powers are kept the same as they were found after the JCC examination. The fogging is removed step by step, and visual acuity is checked in the same way it is done monocularly, explained earlier. In this final part, both eyes are used, so the examination is done in as natural seeing conditions as possible. Fogging lenses are removed as long as the visual acuity improves. When visual acuity doesn’t improve anymore, final spherical powers are found. The final spherical power is the strongest plus power or the lowest minus power with which the maximal visual acuity is achieved. In that situation, the whole lens combination with the spherical and cylindrical power is the distance refraction of the eye (Borish, IM. and Benjamin, WJ. 2006, p. 1260).

2.3 Automated Refraction

Refraction error of the eye can also be detected automatically. There have been objective autorefractometers available for decades. Autorefractometers usually give a good starting point for subjective examination. Control of accommodation has been one of the main problems with these types of equipment. (Rodriguez-Lopez, V & Dorronsoro, C. 2022). The last development in autorefractors is an automated subjective refractometer. The development started in 2004 when Topcon BV-1000 was launched. It was the first refractometer that included an automated subjective examination. (Trusit D, Yasufumi F. 2004). In recent years, a couple of companies have launched several refractometers that include automated subjective refraction. All of those refractors include an objective refractometer. The result from the objective refractometer is in a key role in a subjective part of that refractometer. Accuracy of the results by automated subjective refractometers is usually fairly good compared to subjective refraction, the gold standard of refraction. The main advantage of these automated subjective refractometers is saving examination time. Almost all automated subjective refractometers are faster to get the result compared to the examination time that an optometrist needs to get the result. The other benefit of an automated subjective refractometer is that it can be used in areas with no eye care practitioners. (Rodriguez-Lopez V. and Dorronsoro C. 2022)

3 The Purpose, Objectives, and Tasks of the Research Development Work and of the Different Stages

3.1 Purpose of the Study Statement

The purpose of this master thesis was to develop a distance refraction algorithm for an automatic subjective refractometer measuring refractive status.

3.2 Statement of the Research Question

The research question of this thesis was to find out what is the right way of performing an automated subjective distance refraction to be most reliable and comparable to subjective refraction done by an eye care practitioner?

3.3 Summary Description of the Experimental Design

This thesis was written between February 2021 and March 2022. At first, theoretical information of subjective distance refraction was collected from PubMed searches (keywords: subjective refraction, automatic subjective refraction, examination of astigmatism, Jackson crossed cylinder) and from some optometry textbooks, such as "Borish's Clinical Optometry". Mathematical calculations were made to explain, what happens in cylinder axis examination. Evidence-based information of subjective distance refraction was converted to the algorithm. This project did not include validity and reliability testing of the created clinical algorithm. Therefore, statistical hypothesis testing was not performed. This research project did not require IRB review.

3.4 Specific Aims

Most parts of the subjective distance refraction include spherical lens changing from one lens to another lens. Only one part of the examination includes cylinder lens changes. For that reason, there were two specific aims in this thesis. The first was to find out how the spherical lens chance will be controlled. The second aim was a control of cylinder lens change during the JCC examination. Both aims are lens change controls, but examining spherical and cylinder lenses differ a lot, so a controlling process is different in each part

3.5 Methodology

3.5.1 Project

The Company had already started to develop an algorithm of subjective refraction when this innovation project started early in 2021. A couple of tests were asked to develop further and other tests to check that they performed as they should.

Methodology in this thesis was a qualitative research method, meaning here, extrapolating the theoretical knowledge of subjective refraction to specific commands for the software controlling the program. Evidence-based information of subjective refraction was converted to the algorithm. An existing data of subjective refraction was used to make recommendations for the Company. In addition to that, new knowledge was obtained from mathematical calculations concerning axis examination.

When the recommendations were ready, these were introduced to the Topcon Healthcare optometrist. Together with the Company's optometrist, recommendations were changed to programmable algorithms that could be used in the subjective refractometer. The Company decides afterward will those algorithms be used with the refractometer or not.

3.5.2 Outcomes

All data about different parts in subjective refraction was obtained from optometry textbooks and studies. Examination methods described in this thesis are generally accepted in optometry. Examination of cylinder axis was needed to be explained in a vectorial and trigonometrical way. Based on mathematical calculations, the writer of this thesis made two new proposals for axis examinations (Proposals 1 and 2, see later).

Developing an algorithm for subjective refraction was in practice, at first, thinking the right questions in each part of the examination. Then, all the possible answers had to be thought. In a subjective refraction, there are usually only three possible answers from the patient. One is better, two is better, or, they are the same. In Duochrome test, it is the same. Red is better, green is better, or, they look the same. When all possible answers are known, each of them will produce their own path. A single path will consider all the questions and possible answers that will be in that path. Lastly, a stopping point of each path had to be defined.

An automatic refractometer can be developed in many ways, depending on the needs of the product. For instance, the target may be to be as fast refractor as possible. On the other hand, it can be developed to examine as detailed as possible. Those expectations would lead to different algorithms used in the program. This thesis can be used to those needs that the Company will have. There is information about all the parts of the subjective distance refraction.

3.5.3 Co-Operation

Co-operation with the Company's optometrist was very good. The optometrist explained the needs of the Company. Those needs were combined with the evidence-based information of the subjective refraction. Co-operation was done by video meetings and emails. A couple of exact algorithms about some parts of the examination were done with the Company’s optometrist.

3.5.4 Framework

This thesis was a part of commercial product development. For that reason, there were trade secrets including this thesis. There is no permission to tell if some parts of this thesis results will be used in the end products. This thesis gave the Company general information and recommendations about performing different parts of the subjective distance refraction.

4 Implementation of the Research Development Work

4.1 Methods

In the theoretical background, all the parts that have to be done during subjective distance refraction were explained. If we summarize all the parts, where the examiner asks questions from the patient and gets feedback from changing lenses before eyes, those steps are:

- The first spherical adjustment.
- Examination of astigmatism.
- The second spherical adjustment.
- Binocular adjustment of right and left lens powers.
- Binocular fogging and reducing fog to achieve the distance refraction result eventually.

There are four parts where only spherical lenses are used. Cylinder lenses are used only in one part of the examination. Each part of the spherical lens and cylinder lens examination will be explained by the facts what will happen during the subjective refraction in the retina level. Explanations make possible to convert that information to an algorithm to be used in an automatic subjective refractometer.

4.2 Results

4.2.1 Control of the Spherical Lens Examination

Changing a spherical lens from one power to another will change the place of the focal point, or the conoid of Sturm, within the eye, depending on whether the astigmatism is fully corrected or not, respectively. Adding minus power in front of the eye causes the focal point to move posteriorly inside the eye. For instance, if the focal point of the eye lies in the vitreous, adding minus power moves the focal point from the vitreous towards the retina and further to the choroid. Adding plus power causes an opposite phenomenon. When the focal point or the COLC of the eye comes closer to the OLM of the retina, either from the vitreous or from the choroid, the patient can see smaller and smaller optotypes, thus improving the visual acuity (Borish, IM. and Benjamin, WJ. 2006, p. 1187).

The first spherical lens adjustment, the second spherical lens adjustment and also the final binocular fogging lens removal are exactly the same procedures. That is, the purpose is to get as good visual acuity as possible. The control of that kind of examination is done by showing smaller and smaller optotypes.

In a binocular balancing part, if a comparison method is used, control is done by the patients answers when there is an equality in sharpness (or equality in blurriness) of each eye. Controlling should be easy because the are just three simple alternatives that the patient can answer: right eye is clearer, left eye is clearer or both are equal clear.

Spherical lenses are also used in the Duochrome or Bichrome Balance examination. This examination aims to find a spherical lens with equally sharp both red and green sides. The other possibility is to find the spherical lens that is 0.25 DPT to the plus direction from the first green lens. Both the question to the patient and a next step from the answer are easily programmed into an algorithm (boorish, IM. and Benjamin, WJ. 2006, p. 1238). When using red and green charts, the only thing that can be a problem is accommodation. The red side of the Bichrome Balance will deviate behind the OLM and causes accommodation. It is recommended to tell the patient that they have to look mainly at the green side and take a quick view to the red side. Looking that way minimizes accommodation to happen.

4.2.2 Control of the Jackson Crossed-Cylinder Power Examination

When the JCC examination of astigmatism begins, the previous, first adjustment of the spherical power, must be done precisely. The first spherical adjustment aims to get the COLC to the OLM. In that situation, both focal lines of the uncorrected (or uncorrected or overcorrected) astigmatism are at equal distances from the OLM. Suppose we have an eye with the distance refraction of sf -2.50 cyl -1.00 ax 0º. If we take the cylinder power away, there is uncorrected astigmatism, and therefore the eye creates a conoid of Sturm and two focal lines instead of one focal point. When a spherical power sf -3.00 is before the eye, COLC will lie in the OLM. The least refracting meridian (the horizontal meridian) creates a vertical focal line. That line is 0.50 DPT behind the OLM. The strongest refracting meridian, the vertical meridian, will create a horizontal focal line, 0.50 DPT in front of the retina (*Figure 9*).

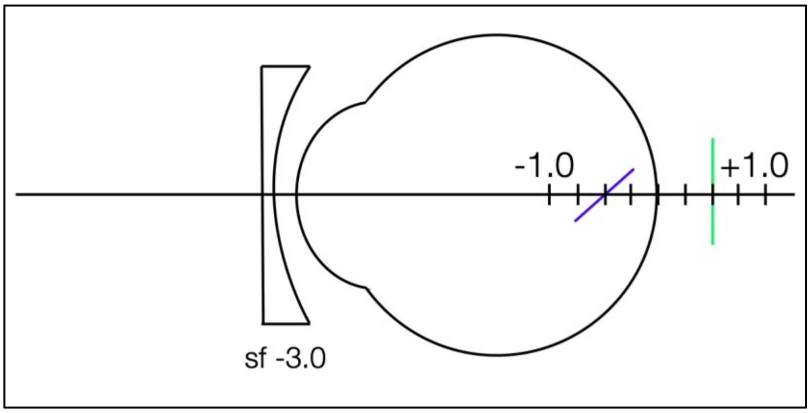


Figure 6: A spherical lens -3.0 in front of an eye with the distance refraction sf -2.50 cyl -1.0 ax 0. The vertical focal line is 0.50 DPT behind the OLM (green line), and the horizontal focal line is 0.50 DPT in front of the OLM (blue line). Dioptrical distances are exaggerated in the picture. Figure: Arto Hartikainen

In this situation, the JCC will function in its best. The COLC doesn't move anywhere from the OLM when alternatives one and two are given with the cross-cylinder. Only the distance of the focal lines from the OLM will vary. That means from the clinical point of view, the sharpness of the astigmatism chart will vary when the distances of the focal lines from the OLM vary. The closer the focal lines are, the smaller the circle of the least confusion is in the OLM and, therefore, the sharper the view of the astigmatic chart is. And also opposite happens, that is; the further the focal lines are from each other, the bigger is the circle of the least confusion in the OLM. Therefore the picture of the astigmatic chart is more unclear (Hartikainen, A. and Seppänen, M. 2022.).

Cylinder power and also axis examination are done by comparing two views shown with the JCC. A question to the patient is easy: "Compare two alternatives, one and two. Tell me which one is sharper or are those equal". Alternatives to answers are simple, too: "Number one," "number two," or "equal." The patient has to look at some test chart during the examination. There are many possibilities to look at; numbers, alphabets, or unique astigmatic charts designed to look. In comparing different charts, one must understand what happens during the examination. There will be a different amount of residual astigmatism in different axis orientations based on the direction of eye astigmatism and correction cylinder axis. If alphabets or numbers are used, it may cause difficulties to the patient to compare different residual astigmatisms. For instance, in some situations, residual astigmatism can be oriented to "with the rule" astigmatism. In that situation, vertical lines of the chart alphabets or numbers are focused behind the retina. Those lines can be accommodated to the retina. Suppose the patient sees alphabets that include vertical lines. In that case, the patient may feel that the alternative, which makes the alphabet's vertical lines look sharp, is better than the other, where those vertical lines are less sharp. The choice is based on the vertical lines, not the overall sharpness, what it should be (Gantz, L. 2015). When round shapes are used, there will never be any situation where some residual astigmatism would cause that kind of over representative of some direction. For that reason, a round shape is the most recommended chart when using the JCC (see *Figure 6*).

Whenever the cross-cylinder is placed in front of the eye so that the axis of the eye astigmatism and cross-cylinder coincides, cross-cylinder will move the focal lines of the eye in the following way: the minus axis of the cross cylinder will move the focal line of the eye parallel to the minus axis of the JCC "to the minus direction", that is, towards the posterior part of the eye. Likewise, the plus cylinder of the cross-cylinder will move the focal line of the eye parallel to the plus axis of the JCC "towards the plus direction", that is, towards the anterior part of the eye (Benjamin, WJ. 2006, p 1197). Let us go back to the previous example sf -2.50 cyl -1.0 ax 0 and the situation when there is just a spherical lens -3.00 in front of the eye. Two alternatives are given with the JCC. Number one, when the minus axis of the JCC is in direction 0º degrees, and number two, when JCC minus axis is in direction 90º degrees. In the number one situation, the horizontal focal line in front of the retina moves 0.25 DPT towards the retina (to the minus direction). Simultaneously, the vertical focal line behind the retina moves also towards the retina (to the plus direction) by 0.25 DPT. Focal lines are now at 0.50 DPT distance from each other, and the circle of the least confusion lies in the OLM (*Figure 10*).

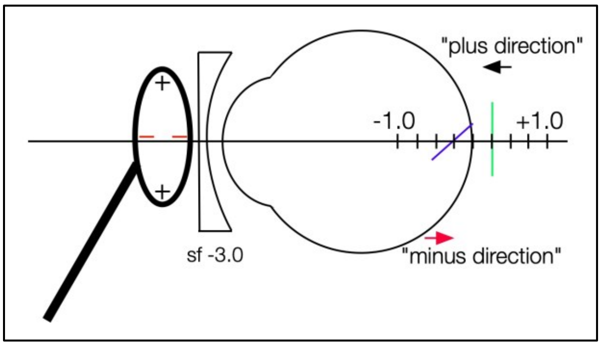


Figure 7: A Jackson Crossed-Cylinder ± 0.25 is placed with its minus axis at 0 degrees. The horizontal focal line (blue) will move to "minus direction" and the vertical focal line to "plus direction". Uncorrected astigmatism is 0.50 DPT. Figure: Arto Hartikainen

In number two, minus axis of the JCC moves the vertical focal line behind the retina further behind the retina. Same time, plus axis of the JCC moves the horizontal focal line further behind the OLM. In this situation, focal lines are at 1.50 DPT distance from each other (*Figure 11*).

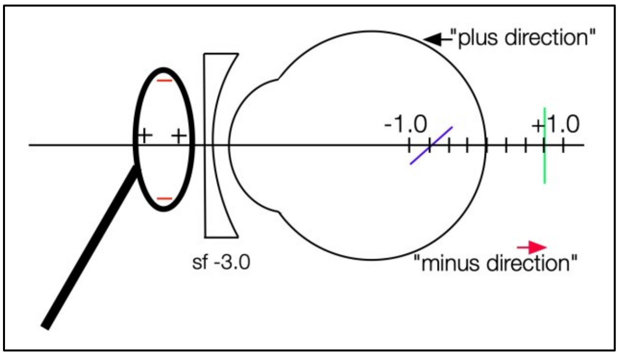


Figure 8: A Jackson Crossed-Cylinder ± 0.25 is placed with its minus axis at 90 degrees. The vertical focal line (green) will move to "minus direction" and the horizontal focal line (blue) to "plus direction". Uncorrected astigmatism is 1.50 DPT. Figure: Arto Hartikainen

The patient compares the sharpness of the image between these two alternatives. It should be accessible to the patient to say that number one is clearer than number two. The dioptrical cylindrical difference between numbers 1 and 2 is so big. That is, 0.50 DPT of uncorrected cylinder compared to 1.50 DPT of the uncorrected cylinder.

When astigmatism is fully corrected, there are no more two focal lines. Instead, one focal point in the eye is formed by the spherocylinderical correction lens. In that situation, both alternatives of the JCC produce 0.50 DPT of uncorrected astigmatism, and therefore both alternatives are equal in sharpness. To be exact, both alternatives are equally blurred rather than equally sharp because there is 0.50 DPT uncorrected cylinder in both alternatives. That is the end point of the cylinder power examination.

There is one particular point that is important to know, generally concerning the cylinder power examination. That is when there is either an undercorrection or overcorrection of the cylinder by 0.50 DPT. An example of the undercorrection case is given next. The spherocylinderical correcting lens is sf -2.75 cyl -0.50 ax 0º, and there is under correction of half dioptre. In number one (JCC minus cyl ax 0º), JCC will correct all the cylinder needed, so both focal lines move to the OLM and creates one focal point. The eye is fully corrected in that situation, and the focal point lies in the OLM. In number two (JCC minus ax 90º), uncorrected astigmatism is 1.0 DPT. The patient will probably say that number one is very sharp, the sharpest of all, during the JCC examination. After that answer, the examiner adds cylinder to the 0.75 DPT and shows again two alternatives with the JCC. Again, number one is sharper than number two, but now the patient may say that previously the picture was clearer; these are not so good anymore. The patient can be confused because the examination continues from a sharp view towards less sharp views. The same phenomenon happens if there was at the beginning of the JCC power examination, 0.50 DPT overcorrection of astigmatism. In that situation, one alternative would give a full cylinder correction compared to the other, forming 1.0 DPT overcorrection. The examiner must know this phenomenon happens sometimes. It is important to explain to the patient that they must compare only one lens against the other, not to think if the view is sharp or not or was it sharper in the previous comparisons.

Another thing is also important to know if an examiner uses plus cylinders during refraction. When there is a need to increase the plus cylinder during the JCC examination by 0.25 DPT, it moves the focal line behind the retina toward the OLM. Simultaneously, the COLC moves 0.12 DPT in front of the OLM, towards the vitreous. So, the COLC is slightly on a myopic side from the OLM. In that situation, the sharper alternative given with the JCC is not as sharp as if the COLC would be in the OLM. If the first spherical adjustment had also resulted in too low minus power, the COLC is even more further away from OLM, toward the vitreous. That situation can make it difficult for the patient to decide which alternative is clearer. One alternative that should be the better has not much astigmatic blur but has spherical blur because the COLC is in front of the OLM. The other alternative has more astigmatic blur. It may lead, in some cases, to under-correction of astigmatism. It is essential to avoid undercorrection of spherical myopia if plus cylinders are used during the JCC examination to avoid the previous situation. It is also essential to change the spherical lens to a "more minus" direction by 0.25 DPT simultaneously when the plus cylinder has been changed 0.50 DPT to a "more plus" correction. Otherwise, the COLC will move to the myopic position. When minus cylinders are used in the JCC examination, it is not so harmful to forget to change the spherical lens compared to the situation where plus cylinders are used. If the examiner does not change the spherical lens to a plus direction by 0.25 DPT after adding minus cylinder by 0.50 DPT, the COLC will move 0.25 DPT behind the OLM. Accommodation moves the COLC easily back into the OLM. But when plus cylinders are used, accommodation can't move COLC from the vitreous side into the OLM.

Overall, in cylinder power examination, the patient compares two blurred images caused by different amounts of uncorrected astigmatism. The comparison should be easy because the differences are significant and easily distinguishable. The patient must know this examination only to be a comparison of two blurred images. The endpoint is equal blurriness, not a sharp image as it is with the spherical lens examination.

4.2.3 Control of the JCC Axis Examination

In the cylinder axis determination, the JCC is placed so that its axis of astigmatism is 45º from the correcting cylinder axis. In addition to this, the correcting cylinder in the trial lens may not be coefficient to eye astigmatism. Because of that, there are at least two or even three different cylinder axes that are obliquely crossed to each other.

When the algorithm is developed for cylinder axis examination, one problem is with the amount of rotation of the correcting cylinder axis. Another problem is when to stop the examination. A mathematical analysis of obliquely crossed cylinders was used in this thesis to solve these problems.

Mathematics about obliquely crossed cylinders is more complicated than combining cylinders parallel or perpendicular to each other. Two cylinders can be combined as they were mathematical vectors, when both cylinders are plus cylinders or minus cylinders and when all the axis orientations are drawn as doubled (Gartner, W F. 1965). Instead of drawing vectors to the paper and measuring the results from the drawings, vectorial combinations can be changed to trigonometric calculations. Those calculations include both right angle and oblique triangle calculations. Trigonometric calculations are more precise than measurements from drawings of combined vectors. For that reason, all calculations in this thesis are made trigonometrically.

4.2.4 From Vectors to Trigonometry

In explaining cylinder axis examination, the first thing is to realize that there are three different cylinders and axis angles. The first is the eye astigmatism, the second is the correction cylinder and the third is cylinder from the JCC. We can split the combination of these three different cylinders into two separate parts. First, we will combine the correcting cylinder and the JCC cylinder. After that, we combine the result from the first combination to the eye astigmatism power and axis direction. We don't have to calculate any spherical powers included in correcting power of the JCC or the correcting lens or the eye refraction, because the COLC is in the OLM in the starting point of the JCC examination, and JCC doesn't move that place. The only thing that has to be calculated is the amount of the cylinder of these three component. In *Figure 13,* there is an example of a vectorial combining of two cylinders. Note, that all agles of the cylinders are doubled in vectorial combinations. For that reason, for instance an angle of 45 degrees is pointing up in the picture. In *Figure 13*, the correcting cylinder is cyl -0.50, and axis is in 0º position (green arrow in the picture). JCC of ± 0.25 is placed first so that its minus cylinder is in 45º (JCC "number1", a blue arrow pointing up). The amount of cylinder in the ± 0.25 JCC is 0.50 DPT. JCC is then rotated so that the minus cylinder is in 135º (JCC "number2", blue arrow pointing down). The vectorial combination of these JCC alternatives are red arrows (comb1 and comb2). The length of the red arrow is the amount of the combination cylinder.

From the picture, we can measure the length of the comb1 and comb2 vectors, but it is appropriate to change this vectorial drawing to trigonometry.

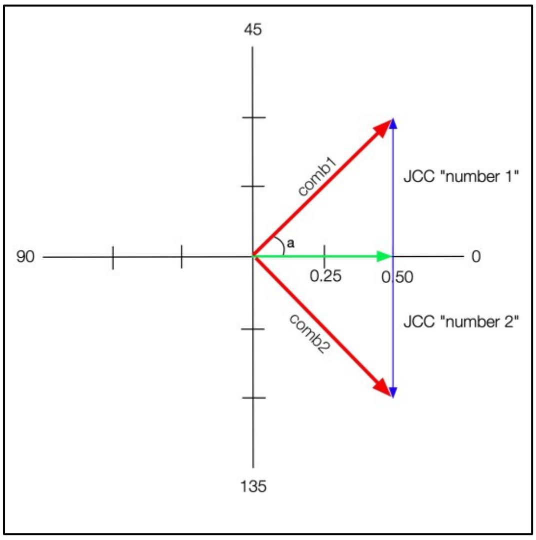


Figure 9: Combination of correcting cylinder -0.50 (green arrow) and Jackson Crossed-Cylinder positioned first at 45º (JCC "number 1", blue arrow upwards) and 135º (JCC "number 2" blue arrow downwards) and combination of these two cylinders, red arrows comb1 and comb2. Figure: Arto Hartikainen

Trigonometrically, this is a case of a right-angled triangle. In a right-angled triangle, we have three components; so-called "adjacent" (side A), "opposite" (side B), and "hypotenuse" (side C). A green arrow is, in this case, is a side A, a blue arrow is a side B, and a red arrow, the combination cylinder power, is the hypotenuse, a side C. The hypotenuse C is calculated from the A and B as follow:

$$C = \sqrt{A^2 + B^2}$$

An angle “*a*” can be calculated from the formula:

$$a = \tan^{-1}\left(\frac{B}{A}\right)$$

In the example above in *Figure 13,* the combination cylinder power is

$C = \sqrt{0.5^2 + 0.5^2}$ = 0.71 DPT

An angle “a” in the example above:

$a = \tan^{-1}\left(\frac{0.5}{0.5}\right) = 45°.$

In the example above, an alternative, number 1, given with the JCC with the minus cylinder in 45º, results in comb1 cyl -0.71. An angle “a” is 45º upward from the horizontal x-axis. The second alternative, number 2, also results in comb2 cyl -0.71, but now the angle is 45º downwards from the horizontal x-axis.

Next, the comb1 and comb2 cylinders can be combined with eye astigmatism and angle. That combination will be explained with an example, where the eye astigmatism is 0.50 DPT, and the axis of the eye astigmatism would be 10º.

When the correction of astigmatism is explained theoretically, eye astigmatism can be thought to be a plus cylinder that will be neutralized with correcting minus cylinder. In this example, the eye astigmatism can be said to be +0.50 DPT, axis 10º. That should be corrected with cyl -0.50 ax 10º. When combining oblique cylinders by vectors, all cylinders must be positive or negative. For this reason, the eye cylinder +0.50 ax 10º will be changed to minus cylinder format. Then the eye cylinder will be cyl -0.50 ax 100º. In this format, the eye cylinder can be combined as a vector to the combination of the correcting cylinder and JCC cylinder (comb1 and comb2).

In the *Figure 14*, there is a vectorial picture of an eye astigmatism of cyl -0.50 ax 100º, meaning that cyl correction would be needed to angle of 10º, shown as a green arrow. An angle “b” is in the picture 20º, because of need for doubling all the angles. Comb1 cylinder from *Figure 13*, a red line in the picture, is combined as a vector to the eye astigmatism. An angle "c" can be calculated to be as follow:

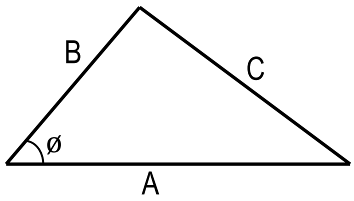
$c = a - b,$

where "a" is an angle of comb1 cylinder and "b" is an angle of eye cylinder.

In this situation, there is a triangle with no right angle. Instead, there is an oblique triangle with two known sides of the triangle and one known angle between the known sides. In this situation, the law of cosines can solve the third side of the triangle.

The law of cosines:

$C^2 = A^2 + B^2 - 2ABcosø$



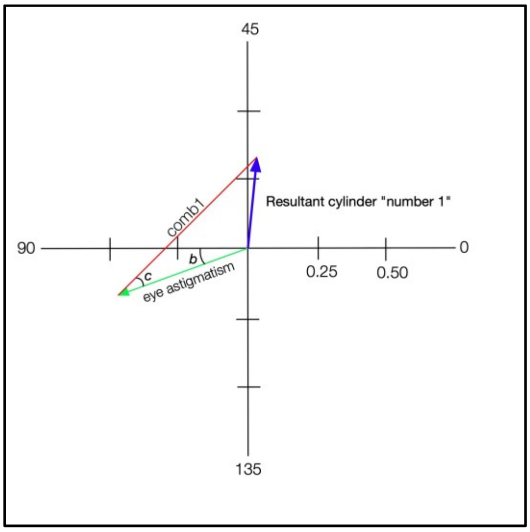


Figure 10: Vectorial explanation of the situation, where eye astigmatism -0.50 DPT is in an angle of 10º, but the correcting cylinder is in 0º and JCC is in 45º. Green arrow = eye astigmatism. Red line = comb1 cylinder. Blue arrow = the amount of astigmatism with which the patient sees the "number 1" alternative. Figure: Arto Hartikainen

In this example (*Figure 14*) the eye astigmatism (0.50 DPT) is one known side of the triangle, comb1 cylinder is the other known side (0.71 DPT) and an angle between the two known sides, "c", is 45º - 20º = 25º.

The resultant cylinder "number1" = $\sqrt{0.5^2 + 0.71^2 - 2 \times 0.5 \times 0.71 \times \cos 25}$ = 0.33 DPT

In *Figure 15*, the situation is drawn when the other alternative (number 2) is given with the JCC in the previous example. Again, eye astigmatism (0.50) and comb2 astigmatism (0.71) are known sides in the triangle. An angle enclosing those sides (d) is in this case

$d = a + b$, that is; $d = 45 + 20 = 65$.

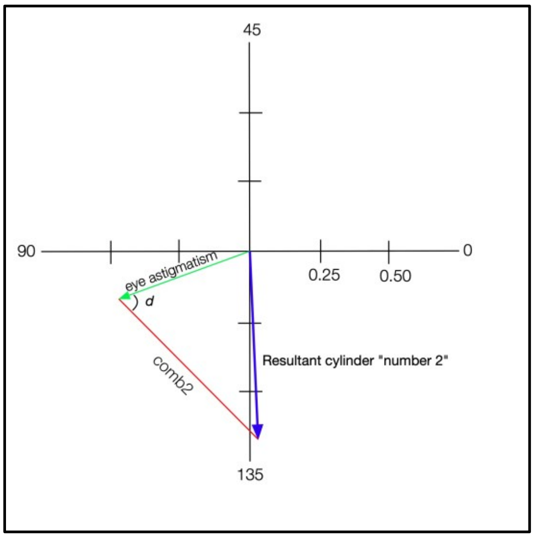


Figure 11: Vectorial explanation of JCC alternative "number 2". Explanation of various sides of the triangle is given in picture 11. Figure: Arto Hartikainen

The resultant cylinder "number 2":

= $\sqrt{0.5^2 + 0.71^2 - 2 \times 0.5 \times 0.71 \times \cos 65}$ = 0.67 DPT.

In this example, the patient has a cylinder correction lens cyl -0.50 ax 0º. Two alternatives are given with the JCC: in number 1, the JCC minus cylinder is in 45º, and in number 2, JCC minus cylinder is in 135º. The comparison of the sharpness of the astigmatic chart (*Figure 7*) is, in reality, comparing astigmatic blurriness caused by 0.33 DPT against astigmatic blurriness of 0.67 DPT. There is a dioptrical difference of 0.34 DPT (0.67 – 0.33) between blurriness. The difference in the blurriness is the main thing during the JCC axis examination. The examiner asks from the patient, showing both alternatives with the JCC, which one is the clearer (or sharper) or are the alternatives the same. When the axis of eye astigmatism and correction cylinder axis is not in the same direction, there is some difference in astigmatic blurriness. When the eye astigmatism axis and correction cylinder axis are parallel, both alternatives create 0.50 DPT astigmatic blurriness and thus are equal in sharpness. To be more realistic, they both are equally a little bit blurred, not sharp.

4.2.5 Dioptrical Explanation of the Alternatives During JCC Axis Examination

Trigonometric calculations can be made to any cylinder powers and axis misalignments. These calculations help to understand what is happening during axis examination with the JCC. It also helps to estimate and define how precisely it is possible to examine the exact cylinder axis. In *Table 2,* there are calculations of different cylinders and different misalignments of correcting cylinder axis. The results in the *Table2* are dioptrical cylinder power differences from JCC alternatives number one and two in given cylinders axis misalignments. All calculations are made assuming that the correcting cylinder power is the same as eye astigmatism.

Table 2: Cylinder power differences with different amounts of astigmatism (grey row) and axis misalignments (green column). Calculations: Arto Hartikainen.

Axis misalignment	Astigmatism						
	0.25	**0.50**	**0.75**	**1.0**	**1.50**	**2.00**	**3.00**
1°	0.02	0.04	0.05	0.06	0.10	0.14	0.20
2°	0.04	0.06	0.10	0.14	0.20	0.28	0.42
5°	0.08	0.18	0.26	0.34	0.52	0.69	0.97
7°	0.12	0.24	0.36	0.48	0.72	0.93	0.99
10°	0.18	0.34	0.51	0.67	0.93	0.97	0.98
20°	0.32	0.62	0.82	0.90	0.93	0.94	0.94

In a closer analysis, one can notice that when the axis misalignment begins to grow from 1º, the difference between alternatives also grows. But, it extends only to a certain amount of axis misalignment, and after that, the difference is about the same. When using ± 0.25 JCC the maximal difference is slightly under one dioptre. The bigger the astigmatism is, the less is the amount of misalignment needed to achieve this maximal difference.

When the axis misalignment is small, the difference between alternatives is also small. For instance, in 0.25 DPT astigmatism and misalignment of 1º, one alternative produces total astigmatism of 0.49 DPT, and the other alternative creates astigmatism of 0.51 DPT. The difference is minimal, only 0.02 dioptres of an astigmatic blur. An interesting question is, what is the minimal noticeable astigmatic difference that the patient with normal visual acuity can have. There are several studies concerning the minimal astigmatic error that can have an influence to vision. But these studies do not measure excactly that phenomenon that is happening during JCC axis examination. In JCC axis examination the patient compares two views that both have an astigmatic error, but there is sometimes very little difference between those views; see above example of cyl 0.25 and 1-degree axis misalignment. For that reason we will use in this thesis the fact that astigmatic error decreases visual acuity by about half compared to the same amount of spherical error (Benjamin, WJ. 2006, p. 1193). In a numeric form, for instance, 2 DPT of an astigmatic blur will cause about the same effect on vision as 1 DPT of a spherical blur. Using this fact is reasonable to think that the difference (in the case of 0.25 DPT astigmatism and 1º axis misalignment) of 0.02 DPT in astigmatic defocus would be the same as that of 0.01 DPT in spherical defocus. One can easily deduce that so little difference is most likely impossible to notice.

4.2.6 Accuracy of Cylinder Axis Examination

The cylinder axis examination aims to find the exact direction of astigmatism. The example above shows that it is nearly impossible to examine low cylinders with one degree of accuracy. It is necessary to create a suitable recommendation of the accuracy to different cylinder powers. That recommendation must be based on the eye's limit to recognize slight astigmatic differences. There is no sense to continue the examination of the axis if the alternatives are so close to each other that a comparison would be impossible to make.

The mathematical understanding of combining oblique cylinders makes it possible to create a proposal to the accuracy of axis examination. At first, it must be estimated what the eye's ability to detect small cylindrical differences is. In the first proposal, that ability is estimated to be 0.12 DPT. That detection ability is about the same as 0.06 DPT spherical difference detection ability. The reason for the amount of 0.12 DPT is nothing else than it is a rounded half of the clinically used minimal power difference in lens powers. The writer of this thesis estimates that power represents “a small” difference.

In *table 3*, there are calculations about the minimum axis misalignment required that the astigmatic difference between JCC alternatives one and two is 0.12 DPT.

Table 3: Minimum axis misalignment that causes 0.12 DPT difference in cylinder powers when JCC alternatives 1 and 2 are given in axis examination. Calculations: Arto Hartikainen.

	0.25	**0.50**	**0.75**	**1.0**	**1.25**	**1.50**	**1.75**
Min.	7º	3,5º	2,3º	1,7º	1,4º	1,1º	1º

An example will explain what this table means in an actual situation. Let us assume that patient has an eye astigmatism of 0.25 DPT, and the correcting cylinder cyl -0.25 is in 0º. In this situation, two alternatives are given with the JCC; the eye astigmatism has to be at least 7º away from the 0º direction so that the patient could notice any difference between given alternatives. If the eye astigmatism is somewhere between the angle of 174º - 6º, the patient would not notice any difference in sharpness because the difference between the alternatives is under the limit of detection (in this proposal, cyl difference 0.12DPT).

The purpose of the proposal is to give a guideline on how to continue the axis examination. In this example of 0.25 DPT of astigmatism, the recommended amount of rotating the axis is at first 20º. That amount of rotation is continued until both alternatives are the same or when the patient prefers the alternative that tells to rotate back the axis. In turning the axis back, the amount of rotation is half of the original amount of rotation. In the case of cyl 0.25, it is 10º. After the back rotation of 10º, the examination is recommended to be stopped. Shortly; in a case of cyl 0.25 DPT, if needed to rotate, rotate 20º until both alternatives are the same, or it must be rotated back. If it is necessary to rotate back, rotate once 10º, and the examination is finished.

An explanation for this recommendation is as follows. In the starting point, the correction cyl 0.25 is in 0º. When the patient can detect a difference, the eye astigmatism must be at least 7º or more from 0º. Then the axis will be rotated to 20º and again, two alternatives are given with the JCC. Now let us assume that the patient prefers better the direction of rotating back. Again, the eye astigmatism must be at least 7º from the axis 20º. That is, 13º, 12º, 11º, 10º, 9º 8º or 7º. Then, the axis is rotated to 10º. If the ocular astigmatism is not in 10º, the remaining possible axis positions could be 7º, 8º, 9º, 11º, 12º, or 13º. These axis orientations are under the detection limit, so the patient can't n notice any difference if alternatives were shown. That is why it is useless to show any alternatives to the patient. The examination of the axis can be stopped in this situation. The accuracy of the axis examination will be 3º in this case. And, if the patient hadn't noticed any difference in the starting point of the examination, the maximal error could be, in that case, 6º. Calculation about these axis errors will be given in chapter 4.2.9. in this thesis.

When the cylinder power increases, it is possible to examine the cylinder axis more precisely. Following the same principle of 0.12 DPT minimum detectable possibility with every cylinder power, Proposal number 1 of cylinder axis rotation examination is given in *Table 4*

Table 4: Proposal number 1of the of cylinder axis examination in different cylinder powers, based on the cyl 0.12 DPT of detection ability. Proposal: Arto Hartikainen.

	The first rotation until same or need to rotate back	**Rotating back**		
0.25	20º	back 10º	Finished	
0.50	20º	back 10º	back or forth 5 º	Finished
0.75 and 1.00	10º	back 5º	back or forth 3º	Finished
1.25 and more	5 º	back or forth 3 º	1º until the same	Finished

Let us take another example of combining information from both tables 3 and 4. In this example, there is eye astigmatism of 1.00 DPT, and in the starting point, the correction cylinder is in 0º. If the patient notices any difference, the axis must be at least 2º away from position 0º, the starting point. Then axis would be rotated to 10º. Next, the patient prefers rotating back. Axis is rotated to 5º (half of the original rotation). Then the patient still notices a difference, and the axis is rotated 3º back. Now, the axis is in 2º. In this situation, the amount of axis error can be only 1º. Only the axis orientations of 1ºand 3º are directions that the eye can't detect any differences when the correction cylinder is in 2º. Accuracy is excellent, 1º, although the examination will be stopped at this point.

Proposal number 1 is based on the idea that the minimal detectable dioptric difference in astigmatic blurriness is 0.12 DPT. Because the detection ability is only an assumption, another assumption is also taken into account. That is 0.06 DPT detection ability in astigmatic differences. In that case, the minimal axis misalignment to be detected would be half of what can be seen in *Table 3*. Then it would mean that the cylinder axis could be examined twice as accurately as it is recommended in *Table 4*. This 0.06 DPT detection ability will make a difference to the proposal about cylinders of 0.25, 0.50, 0.75, and 1.00. Cylinders that are greater in power than 1.00 would be examined in any case in the accuracy of 1º. In *Table 5*, Proposal number 2 is given of axis rotation during axis examination. In that proposal, it is assumed that the eye has an outstanding capability of detecting slight astigmatic differences, that is, 0.06 DPT differences.

Table 5: Proposal number 2 of the amount of cylinder axis rotation during the examination in different cylinder powers, based on the cyl 0.06 DPT od detection ability. Proposal: Arto Hartikainen.

Cylinder	**The first rotation until same or rotating back**	**Rotating back**		
0.25	20º	back 10º	back or forth 5º	Finished
0.50	10º	back 5º	back or forth 3 º	Finished
0.75 and more	5º	back 3º	back or forth 1º until the same	Finished

The Proposals number 1 and 2 will shorten the examination time and in the same time, makes the examination easier for the patient. But, these proposals are made on an assumption than the patient answers correctly to the questions. If that happens, then the examination can be stopped before the patient tells that there is no differencies between the alternatives. But, if we take account the possibility that the patient gives an uncorrect answer by accident, then the examination must be continued until the patient reports equality in alternatives. This kind of examination can be started just like it is started in the Proposals 1 or 2. Always, when the direction of rotation must be changed, the next rotation is half of the previous rotation. The examination is continued as long as the patient can detect any differencies or when the amount of rotation would be under 1º. The only exceptions to this would be cylinders 0.25 and 0.50. In cyl 0.25, rotation should always be at least 5º and in cyl 0.50 2-3º, respectively. An explanation to these exceptions will be given in the chapter 3.2.9.

4.2.7 Blurriness Induced in the Cylinder Axis Examination

In cylinder axis examination, the two alternatives give different amounts of astigmatic blurriness based on the amount of the misalignment. The difference between the alternatives grows to a certain maximal difference as the misalignment grows. In addition to that, another phenomenon happens also. At first, from the misalignment of 1º to more misalignment, the better alternative of the two possible produces less and less astigmatic view to the patient. That is, when the misalignment grows, the better alternative is more spherical and thus sharper. Then comes a certain amount of misalignment where the better alternative is the least astigmatic, so the most spherical. We may call this amount of misalignment “the sharpest point”. After the sharpest point, the better alternative begins to increase in astigmatic blurriness. At the same time, the worse alternative given with the JCC grows all the time in blurriness from the slightest misalignment. An example: the amount of astigmatism is 1.00 DPT. When the axis of the correcting cylinder is 2º off-axis, the better alternative will produce 0.43 DPT of astigmatic blurriness and the worse alternative 0.57 DPT, respectively. The difference is just noticeable (0.57 – 0.43) 0.14 DPT. When the off-axis is 10º, the better alternative produces 0.17 DPT and the worse 0.84 DPT, and the difference grows to 0.67 DPT. If the off-axis is 13º, the better alternative creates only 0.12 DPT blurriness and the worse alternative 0.94 DPT. This is “the sharpest point” of cylinder 1.00. When misalignment of correcting cylinder grows from that sharpest point, also the better alternative produces bigger and bigger astigmatic blurriness. In 15º of axis misalignment, better 0,13 DPT compared to 1.01 DPT, produced by the worse alternative. In 20º off-axis, better 0.27 DPT, and worse 1.17 DPT.

It is essential to know this phenomenon of “the sharpest point” to happen. In that point, the patient will probably inform that “now the view is very sharp”. When the axis is rotated toward the correct position, neither alternative given then are not anymore so sharp as it was in “the sharpest point”. The patient may be confused because the examination continues toward an unsharp view from the previous, sharp view. It is essential to tell the patient that this examination is going toward an equally blurred view, not toward the sharp view. In *Table 6*, there are some cylinder powers and “the sharpest point” for each cylinder. From that table, it can be seen that “the sharpest point” varies depending on the amount of cylinder. Another thing to notice is that there are more than one axis direction in low cylinder powers where this “sharpest point” exists. Also, the amount of astigmatic blurriness is different in different cylinder powers. The greater astigmatism will produce a sharper view in “the sharpest point”.

Table 6: The amount of cylinder axis misalignment in the "the sharpest point". "The better" shows the total astigmatic blurriness amount so it describes how sharp the view is in "the sharpest point". Calculations: Arto Hartikainen.

Cylinder	**0.25**	**0.50**	**0.75**	**1.00**	**1.50**	**2.00**	**2.50**	**3.00**
The sharpest point	28-36º	20-25º	16-18º	13-14º	9º	7º	6º	5º
The better: DPT	0.31	0.21	0.15	0.12	0.08	0.07	0.06	0.05

4.2.8 Using ± 0.50 JCC Instead of ± 0.25 in Axis Examination

An important thing to know, what is the difference if JCC ± 0.50 is used instead of ± 0.25 during the axis examination. When the two alternatives are shown with the ± 0.50 JCC, the absolute cylindrical powers will be stronger than those shown with the ± 0.25 JCC. For example, in a situation of 0.25 astigmatism and 1º of cylinder misalignment, the ± 0.50 JCC gives 0.99 DPT and 1.01 DPT of astigmatism, when the better alternative and the worse alternative are shown, respectively. The difference between the alternatives is 0.02 DPT. Another example is shown in a *Figure 16*. There is a situation of 1.00 DPT astigmatism and 5º of cylinder misalignment. The ± 0.50 JCC gives 0.83 DPT and 1.17 DPT of astigmatism, when the better alternative and the worse alternative are shown, respectively. The difference between the alternatives is 0.34 DPT. We can see from the *Table 2* that the difference is the same in both previous examples if the ± 0.25 JCC is used. In generally, in most cases the difference remains the same regardless of using ± 0.25 or ± 0.50 JCC. Only in the situation of big cylinders (> 2.00) and significant misalignments, the dioptrical difference is bigger with ± 0.50 JCC compared to ± 0.25 JCC. But in these situations, ± 0.25 JCC also gives such significant differences that are very easily compared. It can be said that there is no benefit of using bigger JCC powers than ± 0.25 in terms of axis accuracy.

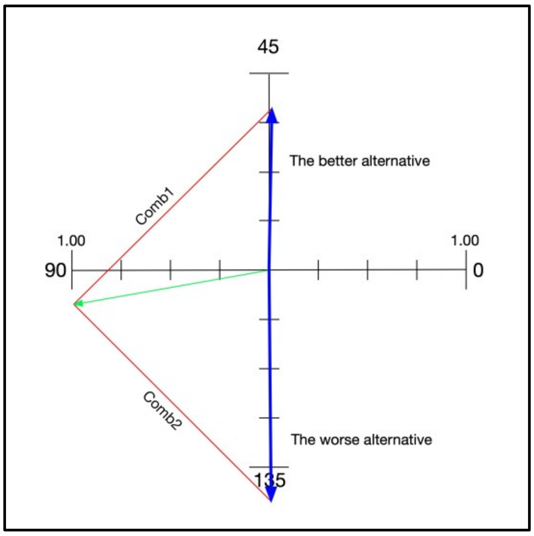


Figure 12: An astigmatism of 1.00 DPT and axis error of 5º between eye astigmatism (green arrow) and correction cylinder. JCC 0.50 is used. The better alternative gives 0.83 DPT astigmatism, and the worse alternative 1.17 DPT. Figure: Arto Hartikainen

In developing an algorithm of cylinder axis examination, the amount of rotation must be fitted to the amount of correction cylinder. The smaller the cylinder is, the bigger should be the axis rotation. Also, the examination hasn't necessarily to be continued to the situation where the patient can't see any difference. It can be stopped when the minimal possible difference detection has been achieved. The mathematical calculations enable us to find these critical points of the examination.

4.2.9 Cylinder Axis Misalignment

If the correcting cylinder axis is not at the same direction compared to the eye astigmatism, those two cylinders will form resultant cylinder. The power of the resultant cylinder can be calculated from the vectorial combination and trigonometrical calculations, as mentioned earlier. *Figure 17* gives a mathematical explanation of this principle. In that picture, eye astigmatism and the correcting cylinder are the same, 1.00 DPT. There is a 10º misalignment of correcting cylinder. The green arrow represents the eye cylinder and axis, here 10º. In the picture, it must be drawn as being 20º. The blue arrow is the correcting cylinder, in 0º position. A letter “a” is the angle between those two cylinders, thus 20º. The resultant cylinder (R) caused by the 10º misalignment of the correcting cylinder is the third arrow, colored as red. Mathematically this can be calculated from the law of cosines as follow:

$R = \sqrt{1.00^2 + 1.00^2 - 2 \times 1.00 \times 1.00 \times \cos 20}$ = 0.35 DPT

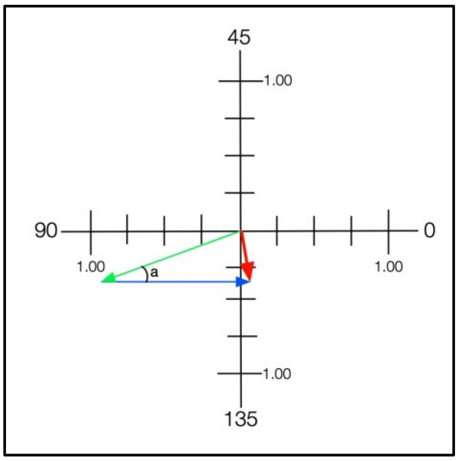


Figure 13: The resultant cylinder, 0.34 DPT (red arrow) from 10º misalignment of the cyl 1.00 astigmatic correction. Green arrow = 1.00 DPT of eye astigmatism in 10º. Blue arrow = 1.00 DPT of cylinder correction in 0º. The letter "a" = angle 20º in the vectorial combination drawing. Picture: Arto Hartikainen

As the misalignment grows, so grows also the resultant cylinder. The maximal resultant cylinder is in the situation where the misalignment is 90º. Then the resultant cylinder is double what the ocular astigmatism is.

There are calculations in *table 7* of some residual cylinder powers from different misalignment of correcting cylinders. All the calculations assume that both correcting cylinder and ocular astigmatism are the same.

Table 7: Residual cylinder caused by different amount of misalignment of different cylinder powers. Calculations: Arto Hartikainen.

		CYL 0.25	**0.50**	**0.75**	**1.00**	**1.50**	**2.00**	**2.50**	**3.00**
Misalignment	**1º**	0.01	0.02	0.03	0.03	0.05	0.07	0.09	0.10
	2º	0.02	0.04	0.05	0.07	0.10	0.14	0.17	0.21
	5º	0.04	0.09	0.13	0.17	0.26	0.35	0.44	0.52
	10º	0.08	0.17	0.26	0.35	0.52	0.69	0.86	1.04
	15º	0.13	0.26	0.39	0.52	0.78	1.03	1.29	1.55

Table 7 helps to understand how accurate the axis must be examined in different powers. A good example is the lowest cylinder, cyl 0.25 DPT. From *Table 7* can be seen that there is no need to try to find 0.25 DPT cyl axis in 1-5º of accuracy because the residual astigmatism is practically zero

in axis misalignments under 5º. At the same time, it is most likely impossible to recognize any differences given with the JCC. If we accept the 0.12 DPT as the minimum difference to be recognized, the axis has to be at least 7º off-axis. This situation can, at most, cause 6º of axis error. That error would cause 0.05 DPT of residual, uncorrected astigmatism. On the other hand, if the eye can recognize a JCC difference of 0.06 DPT, the maximal axis error could be 3º, and residual astigmatism would be 0.026 DPT. Even though it is unknown if the minimal detection ability is 0.12 DPT or 0.06 DPT, one can use either proposal given *in Tables 4 and 5*. Residual cylinders from a possible axis errors are insignificantly low in both cases. One of the most important things to understand from these calculations is that with the low cylinders, it is both unnecessary and most likely impossible to try to examine cylinder axis 1º by 1º.

There are a couple of exciting misalignments that are essential to know. The first one is the situation when the misalignment is 30º. *Figure 18* shows this situation as an example cylinder of 1.00 DPT.

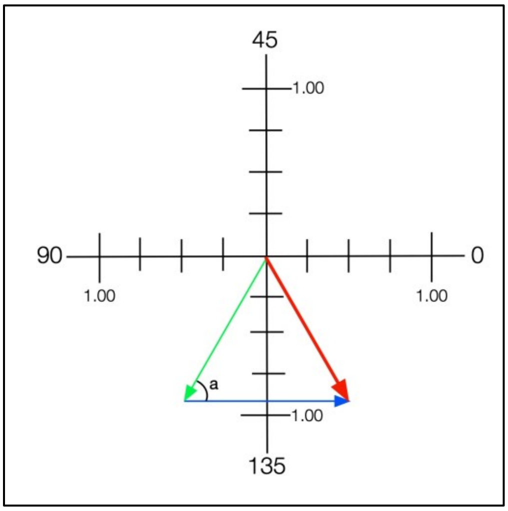


Figure 14: Resultant cylinder R (red arrow) in 30º misalignment of cylinder 1.00. Figure: Arto Hartikainen

$R = \sqrt{1.00^2 + 1.00^2 - 2 \times 1.00 \times 1.00 \times \cos 60}$ = *1.00 DPT*

With all cylinder powers, assuming that both eye astigmatism and correction cylinders are the same power, when the misalignment is 30º, the resultant cylinder will be the same as the correcting cylinder. In practice, that means there is no real correcting effect from the correcting cylinder. Although the cylinder is in front of the astigmatic eye, the axis error makes the residual cylinder as big as the amount of uncorrected astigmatism. The other important misalignments of the correcting cylinder are misalignment of 15º and 10º. In 15º misalignment, the residual cylinder is about half of the correction (and eye astigmatism) cylinder. In 10º misalignment, the residual cylinder is about 1/3 of the correcting cylinder.

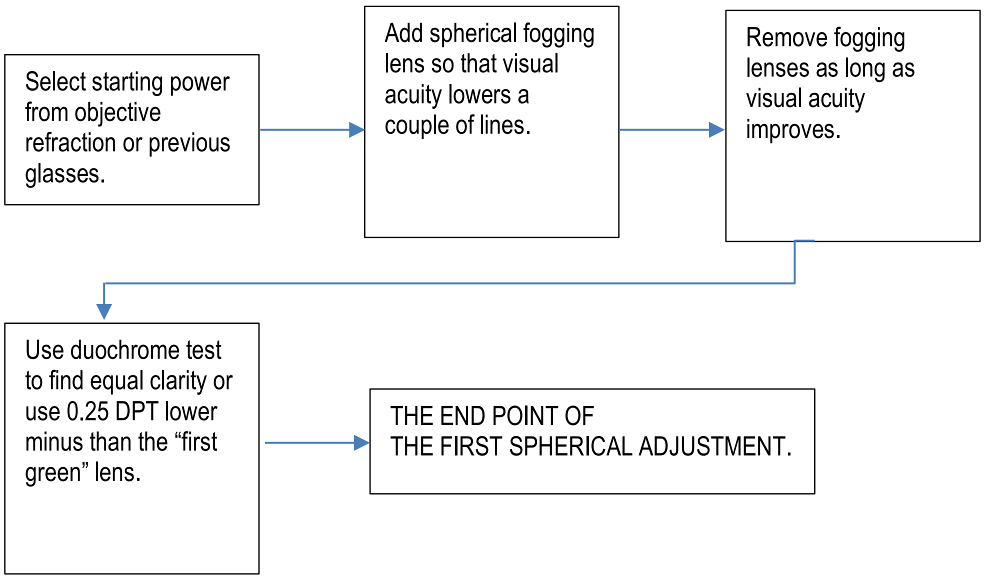
4.2.10 The Main Difference Between Spherical and Cylinder Power Examinations

The principle of the spherical lens and cylinder lens controls has been told in previous chapters. The critical thing is to realize the main difference in these examinations. In spherical lens examination, the examination goes from an unclear view to a more sharp view. The endpoint is the sharpest vision. In the cylinder examination, the patient is seeing more or less unclear views all the time. Both the power and the axis examination end to the situation of equally blurred view in both alternatives. That is how the cylinder examination differs from any spherical lens change part of examination. There may be once or twice a situation during cylinder examination when the view is sharp, for instance, the “sharpest point” situation during axis examination. But, soon after that, view will be unclear again. This kind of examination can confuse the patient and lead even to some mistakes concerning the answers to the comparisons. The second spherical lens adjustment is also an essential part because, in that part, the examiner can check that the cylinder examination had gone as it should go. Checking is done by comparing visual acuities obtained before and after the JCC examination. If there had been done any changes to the cylinder, both to the power or/and the axis, visual acuity after the JCC examination must be at least as good or most likely better compared to the acuity before the JCC exam. If it is worse than before the the JCC examination, then JCC examination has gone wrong and must be examined again. It is also essential to tell the patient that during the JCC exam, the view is intentionally blurred all the time.

4.3 Results as Flowcharts

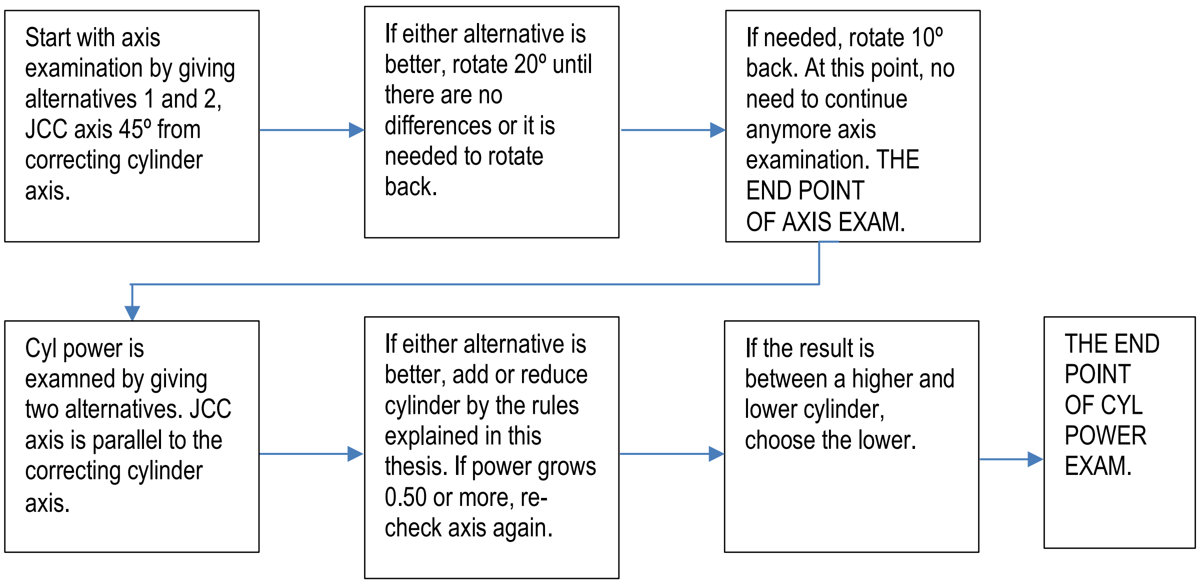
All recommendations of different parts of the subjective distance refraction will be shown here as flowcharts. The results are as follows:

The first spherical adjustment, monocular examination:



Examination of astigmatism; based on the <u>Proposal 1</u>

Cyl -0.25 dpt in the starting power



Cyl -0.50 dpt in the starting power

Start with axis examination by giving alternatives 1 and 2, JCC axis 45º from correcting cylinder axis.

If either alternative is better, rotate 20º until there are no differencies or it is needed to rotate back.

Rotate 10º back. Give alternatives again. If rotation is still needed, rotate 5º. At this point, no need to continue anymore. THE END OF AXIS EXAM.

Cyl power exam is done in the same way as in earlier (cyl 0.25) case.

Cyl -0.75 or -1.00 dpt in the starting power

Start with axis examination by giving alternatives 1 and 2, JCC axis 45º from correcting cylinder axis.

If either alternative is better, rotate 10º until ther are no differencies or it is needed to rotate back.

If needed, rotate 5º back. Give alternatives again. If rotation is still needed, rotate 3º. At this point, no need to continue anymore. THE END OF AXIS EXAM.

Cyl power is examined in the same way as in earlier (cyl 0.25) case.

Cyl more than 1.00 dpt in the starting power

Start with axis examination by giving alternatives 1 and 2, JCC axis 45º from correcting cylinder axis.

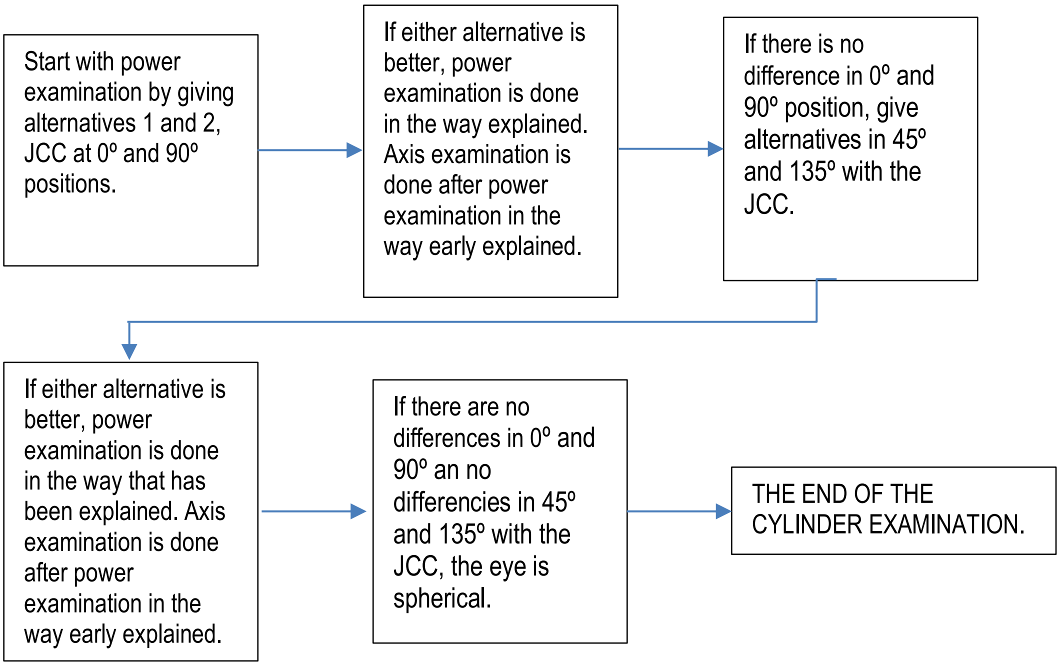
If either alternative is better, rotate 5º until there are no differencies or it is needed to rotate back.

If needed, rotate 3º back. Give alternatives again. If rotation is still needed, rotate 1 º until there are no differencies. THE END OF AXIS EXAM.

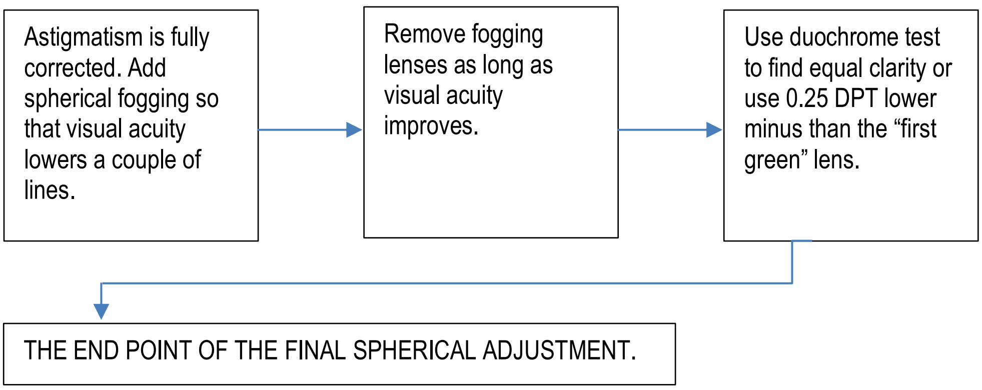
Cyl power is examined in the same way as in earlier (cyl 0.25) case.

If a Proposal 2 is used, then the examination of astigmatism is done in the way that can be found from *Table 5*, page 41.

Examination of astigmatism, no cylinder lens in the starting point of the subjective refraction.



The final spherical adjustment, monocular examination:



- Note! This examination is just the same as is the first spherical adjustment, but now astigmatism is fully corrected.

Binocular balancing, Bichrome Balance test

The test is done in a binocular viewing condition. Polarizing filters or vertical prisms are used.

First the right eye: add -0.25 dpt spherical power until the "first green" is found. Then remove the last -0.25 lens.

After right eye, same procedure is done to the left eye. Find the "first green" and then remove the last sph -0.25.

The difference of the spherical powers between the right and left eye is found. At the same time, accommodative balance is found.

THE END POINT OF THE BINOCULAR BALANCING.

- Visual acuites doesn't have to be the same using this method.

Binocular balancing, visual acuity comparing test

The test is done in a binocular viewing condition. Polarizing filters or vertical prisms are used.

Add fogging lens, about 0.50-0.75, so that the visual acuity decreases a couple of lines.

Compare the clarity between the right and left eye. If either is clearer, add spherical +0.25 lens to that eye. Compare again.

Comparison is continued as long as there is an equality in clarity in both eyes. The difference between the right and left spherical powers have been found.

Remove binocularly fogging lenses until there is no improvement in visual acuity. THE END POINT OF THE BINOCULAR BALANCING.

- In this balancing method, both eyes should have the same maximal visual acuity.

Final control of accommodation and the result of the distance refraction

ADD binocularly 1.00 – 1.50 DPT spherical fogging power. Neither eye is occluded.

Remove fogging lenses and control visual acuity improvement. Continue as long as visual acuity improves.

When the fogging lens removal doesn't improve visual acuity, the pervious spherical lens is THE END POINT OF THE DISTANCE REFRACTION.

DISTANCE REFRACTION = THE STRONGEST PLUS / THE WEAKEST MINUS SPHEROCYLINDE-RICAL POWER THAT GIVES THE MAXIMAL VISUAL ACUITY.

- Note! The distance refraction is measured to the distance of 4-6 meters. There is a little dioptical difference (0.25 – 0.16 DPT) from the measuring distance to the real infinity.

5 Discussion

Subjective refraction is generally said to be the “gold standard” of refraction. Studies about automatic refractometers usually compare the results of the autorefractometer to the subjective refraction result. Surprisingly, the content of the subjective refraction that has been done in each study can vary considerably from each other. The term “gold standard” in the context of subjective refraction has not been defined, what the examination should include. The writer of this thesis proposes that the term “gold standard” of subjective refraction means that all seven parts explained in this master thesis must be included in that examination. The reason is apparent. The first step of the seven steps is a starting point. It is a natural point. The next point, the first spherical adjustment, is crucial because it makes the following part, JCC examination, function properly. If the first spherical lens is wrong, the JCC examination will not work as it should be. The third part, the JCC examination, is essential to make the eye free of astigmatism. Then the eye has only one focal point, and it can achieve the best possible visual acuity. After JCC examination, the visual acuity must be checked once more to make sure, that the JCC examination has gone as it should go. The next part is to balance the accommodative status between both eyes. After that, the eyes must be relaxed by fogging. Finally, an extra plus power is removed, and the eyes can achieve the best possible visual acuity with the minimal possible accommodative state. When we are talking about a “gold standard” of some procedure, it is the best possible way of performing it. The examination of the eye's refractive status must include all those seven parts to be called a gold standard of subjective refraction.

Subjective refraction is a procedure of precise order. Seven different parts must be done in the order earlier mentioned. All questions to the patient are short and straightforward. “Can you see better with the lens or without?” The answer can be controlled with optotypes. Usually, there are three different answer possibilities. Every possible answer will lead to one and only one procedure to do next. The whole examination is a series of questions, known in advance. The procedure to every answer is also known in advance. It is reasonable that this kind of mechanical series of questions and answers can be programmed into an algorithm. If the algorithm is carefully planned, it does just the same steps that an optometrist does in an eye examination.

Optometrists very commonly use Jackson Crossed-Cylinder examination, but it still may be poorly understood. We use it as we are taught to be used, mechanically following red or white dot at the edge of the JCC. In this thesis, the JCC examination was explained by mathematical calculations. Calculations were needed to make any recommendations on developing an algorithm for that part of the examination. These calculations also help all optometrists understand what to expect from the patient during the JCC exam. Two different recommendations (called as Proposal 1 and 2) of the axis examination were given in this thesis because there is no exact knowledge of how small differences the eye can recognize. The writer of this thesis has never found this kind of recommendation before based on the capability of recognizing slight astigmatic differences.

One of the most noticeable information from these calculations is that with the lowest cylinders, 0.25 and 0.50 DPT, there is no need to examine the axis in 1º of accuracy. At the same time, it is also impossible to examine so precisely. If the axis of the correction cylinder must be rotated during axis examination, the rotation must be big enough. Some patients try to give answers to every possible question even though the question would be impossible to answer. The patient wants to help the examiner and may imagine to see some difference during every question. The examiner must know, when to stop the examination. With low cylinders, rotation is bigger, and rotation can be smaller with higher cylinders. Cylinders stronger than 1.00 DPT can be examined usually in 1º of accuracy.

An optometrist wants to prescribe the best possible powers to the patient. To be an expert in prescribing optimal eyeglasses also means that the examiner knows the theory of each part of the examination. Mathematical explanations of the obliquely crossed cylinders help to realize when it is important to keep examining and when the examination can be stopped. There are several tables in this thesis that may help understand the different phenomenons happening during the JCC examination.

It seems that automatic subjective refractometers will develop better and better soon. One part of an eye exam that would benefit from this development is an examination of binocular vision problems. Especially if the manufacturers begin to use some kind of eye-tracking in the refractometers, that tracking system could be used in examining phorias or tropias. The prism cover test is one of the most crucial examinations in phoria and tropia cases. In that test, the examiner watches how the patient's eye is moving while covering and uncovering one eye. Sometimes it isn't easy to see the exact movement during this exam. It is reasonable to think that an eye-tracking system can be much better in recognizing this kind of eye movement compared to the optometrist. If it were possible to record a video from those eye movements simultaneously, that kind of automatic examiner would be beneficial to the optometrist. This kind of development in automatic eye examination is welcome.

Subjective refraction is a series of known questions and known responses. That is the reason why an automated machine can do this procedure. Everything will probably go well as long as the patient has normal visual acuity, a normal level of communicating abilities and has not any eye diseases that would affect to optics of the eye. The problems will begin with the automated examination if the patient can't easily answer the questions or can’t understand the question well enough. The controller of the examination must be aware of that kind of situation and stop the examination when necessary. It will be seen how much artificial intelligence will be in the automatic refraction programs to solve these problematic situations. When an optometrist examines the patient, these kinds of problems don't exist. At first, an optometrist does the mechanical series of questions and responses to the answers. But when problems about answers exist, the examiner can discuss with the patient and continue the examination in another way. Prescribing optimal spherocylinderical powers to the patient is combining the result of an eye examination and the needs and expectations of the patient. It is also explaining to the patient, what is possible to achieve with the prescribed powers, and what is not. That information can be given only after a comprehensive eye examination, that is, a questionnaire to the patient, subjective refraction, and a slit-lamp examination. An automated subjective refractometer can't do all these parts at the moment. In optimal circumstances, it can most likely perform subjective refraction that gives a very similar result compared to the examination done by an eye care professional.

6 Conclusion

An uncorrected refraction error is the biggest reason for impaired vision globally. Eye care professionals can't reach every patient who need an eye exam. Suppose a technical device can perform a reliable refractive status examination of the eye. That could help achieve more patients who need a spectacle correction compared to a situation where an eye care professional is required. A subjective automatic refractometer can also be helpful to the optometrist. It will shorten the examination time, and thus more time is left for other examinations, such as eye health exams.

This master thesis was an innovation project to Topcon Healthcare Solutions Emea Oy, concerning about an algorithm for automatic subjective refraction. When an algorithm of subjective refraction is developed, the first thing was to analyze all the parts of the refraction. In this thesis, all parts were explained carefully based on evidence-based knowledge and generally accepted rules. Subjective distance refraction was found to include seven different parts: A starting point, the first spherical adjustment, an examination of astigmatism, the second adjustment of the spherical power, a binocular balancing, a binocular control of accommodation, and finally, the distance refraction of both eyes. Most parts of the subjective refraction include adjusting spherical lenses. Changing spherical lenses moves the focal point or the Sturm of conoid inside the eye, closer or further from the OLM of the retina, the place where the eye's focal point should settle. The patient responds to the spherical lens changes are detected by checking visual acuities. The longitudinal chromatic aberration of the eye is utilized for Duochrome test. Control of the lens changes is done by knowing whether the red or the green side is sharper. Both visual acuity and color-based responses from the patient are quite easily programmable into the algorithm of the subjective refractometer. The visual acuity-based protocol must be done with the principle of the mildest minus power or the strongest plus power that gives the best visual acuity. The Duochrome test is used done to find "equal clarity in red and green" or 0.25 DPT less minus than the "first green" lens.

Probably the most challenging part of the subjective refraction is the examination of astigmatism. The Jackson Crossed-Cylinder method is mainly used by many practitioners . At the same time, it can be a confusing test to the customer and sometimes also to the examiner. A mathematical explanation was used both for power examination and especially the axis examination. Combining oblique cylinders by vectors was modified to trigonometric triangle calculations. Calculations explained that the patient compares two, more or less, blurred views and tries to notice if there is a difference in blurriness. The examination can be stopped to the point where the difference in blurriness is too little to be recognized. An amount of residual cylinder caused by an axis error was also calculated, helping to understand the need for axis accuracy of different cylinder powers. Based on the mathematical calculations, two proposals of cylinder axis examination were created. Those proposals are shown in *Table 4 and 5.* Following either of the tables, it is possible to shorten the examination time and make the examination easy for the patient. When either of these recommendations is used, axis examination can usually be done by asking alternatives only from two to four times, regardless of the amount of astigmatism. Following this protocol from *Table 4 or 5*, a significant benefit will be with examining the lowest cylinders, 0.25 and 0.50. Stopping the examination after just a few questions can be done without waiting for “both alternatives are the same”. It is enough to find an axis that is reasonably close to the axis of eye astigmatism. Mathematical calculations show that errors of a couple of degrees in cylinder axis 0.25 and 0.50 are insignificant.

Binocular balancing can be performed either by binocular Duochrome test, called Bichrome-Balance, or by comparing visual acuities of the right and left eye under a suitable fogging condition. Both eyes should have an equal maximal visual acuity in the comparison method. The Bichrome-Balance test was found to be more versatile than the comparison method because the Bichrome-Balance test can also be used in situations where maximal visual acuities are different in right and left eyes.

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