Saimaa University of Applied Sciences
Technology, Lappeenranta
Degree Programme in Civil and Construction Engineering

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Constructive solutions of a sport complex roof structure

Bachelor’s Thesis 2017

ABSTRACT

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Degree Programme in Civil and Construction Engineering
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The main purpose of the thesis was to analyse and calculate two different types of steel arches for a roof structure according to Russian norms: tied arch with vertical suspensions and arch with suspended tie and V-shaped struts. In addition, it was required to choose a more optimal structural scheme for the roof structure with the least need for steel. The working process was organized with the help of Altcon company, specialized in metal constructions.

The first part is a review of general information about arch structures. For these goals, materials of the research papers, scientific works, textbooks and publications were analyzed.

The second part includes calculations of arches. To achieve this goal Lira-SAPR and Mathcad software programs have been used. During calculations formulas and values were taken in accordance with Russian norms.

As a result of this project, the comparison table with values of steel consumption for each arch was made. The step-by-step instruction of calculation was made as well. The results can be used by a student of technical specialities and construction designers as examples for understanding the primary structural behavior of arches.

Keywords: arch, tie, structure, stability

CONTENTS

1 INTRODUCTION

Nowadays combined arch-cable systems are the fastest growing progressive structural forms in Russia and abroad. These systems include structurally compound tension elements and also elements that work on the compression and bending (1). Application of these structures gives the possibility of creating long-span roof systems characterized by their lightness, high economy and architectural expressiveness (2). As a student, was always admired by long-span structures including arches and was interested in understanding their primary structural behavior.

The objects of the research are two structural schemes of the steel arch covering:

- Arch with tie and vertical suspensions
- Arch with suspended tie and V-shaped struts

The subject of the research is the stress-strain state of arch structures based on different design situations and models.

The aim of the research is to calculate and design an arched roof of long-span structures, and finally, to choose a more optimal structural scheme according to lower steel consumption. To achieve this goal the following tasks were determined:

- Creation for each arch the design model for calculating internal forces in program complex
- Checking and correction of the cross-section and stability
- Analysis and comparison of results between two types of arch
- Conclusions

Numerical studies were made with the help of the Lira-SAPR software program. The calculation algorithm for the determination of forces in the structure is based on finite element method (FEM) and methods of structural mechanics. To estimate the reliability of structures the method of limit states was used (3). This thesis provides the information needed for understanding the primary structural behavior of arches.

2 GENERAL INFORMATION

Basic data about arch structures including the main properties, application and classification are presented in this chapter.

2.1 Common definitions and properties

Arches have been used as a main structural component throughout the history of architecture from the prehistoric period to modern period. Originally, arch structures were built with masonry because of its durability. These structures were commonly used to span small distances ($\leq 5\mathrm{m}$). Modern arches are made of concrete or steel in which members are continuous and monolithic (4). It allows to span large area. Steel arches for bearing roof structures appeared in the 1940-1950 years after the invention of rolled and riveted joints (5).

The main idea of arch

Arch is a curved in shape structural element on two supports (Figure 1). Under influence of load at both supports appear not only vertical but also horizontal reactions. These horizontal reactions (H) are called thrust (5,6). This thrust creates in each section of the arch the moment which has the opposite sign relative to the moment from external loads, that allows to reduce them and in some cases, bring to zero (5).

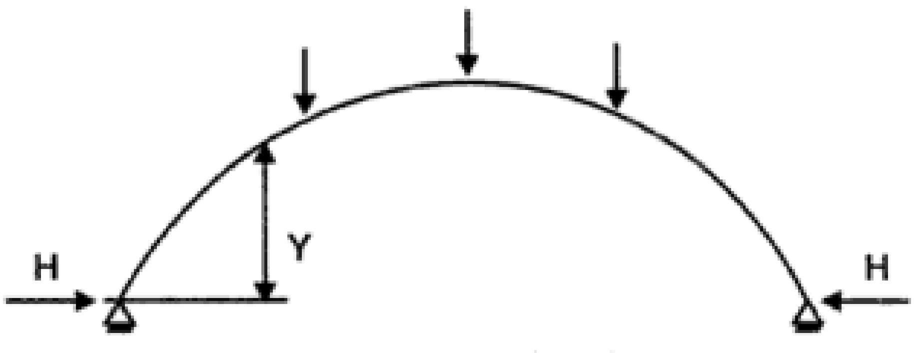


Figure 1. Design model of the arch (6)

The uniform distribution of bending moments is a distinctive feature of arch structures. It allows to create an economic structure according to material requirement that is essential for long-span systems.

2.2 Application of arch

Arches are used for sport complex roofing, vaults, arcades, shops, markets, exhibition halls, canopies for stadiums, stations, bridge constructions etc. They can span large distances from 15 m to 100 m and more. Below there are some examples of arches (Figures 2,3).

Figure 2. St. Pancras Station, London, England, U.K. (2010) (7)

Figure 3. Sydney Harbor Bridge, Sydney, New South Wales, Australia (2010) (7)

2.3 Classification of arches

In this part the general classification of arches will be considered.

Classification of arches by material:

- Stone arches
- Wood arches
- Concrete arches
- Metal arches

Classification of arches by shape (6):

- Polygonal
- Circular
- Parabolic
- Elliptical
- of any other curved shapes

Types of arches are presented chronologically, roughly in the order in which they were developed (Figure 4) (8):

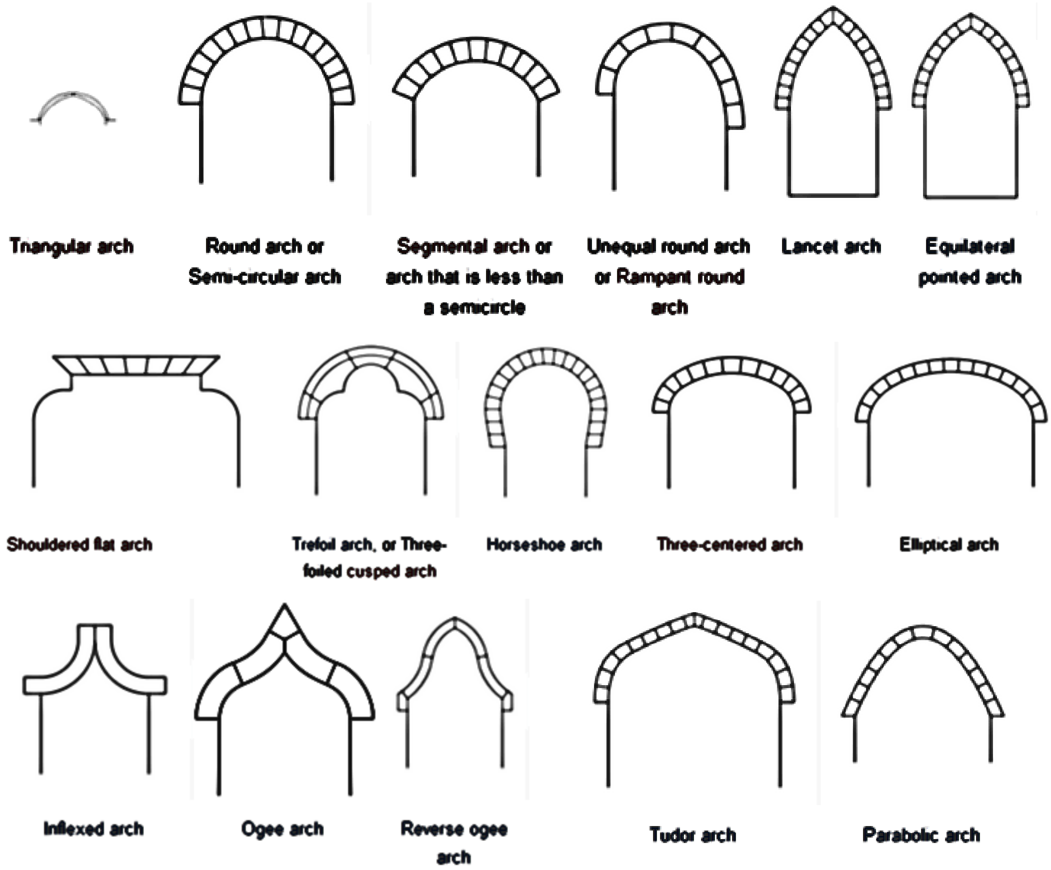


Figure 4. Types of arch (8)

Classification by structural behavior:

- Two-hinged arches
- Three-hinged arches
- Fixed arches

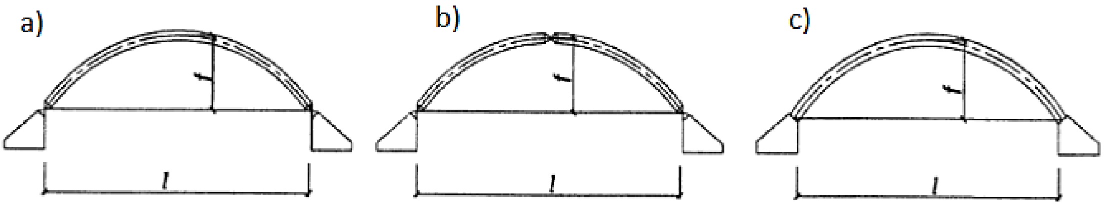


Figure 5. Systems of arches (5)

a- two-hinged; b- three-hinged; c- fixed

Three-hinged arch

The three-hinged arch is statically determinated (Figure 5, b). It is not hinged only at its base, but at the mid-span as well (5,9).

<u>Advantages:</u>

- Not sensitive to uneven settling of supports
- Not sensitive to fluctuations of temperature (10)

<u>Disadvantages:</u>

- The mid-span hinge makes the construction more difficult and expensive
- The value of bending moment at 1/4 of span is the largest compared to other arches (Figure 6). This makes the three-hinged arch the heaviest (5)

<u>Application:</u>

Three-hinged arches are not common nowadays. However, they are sometimes used for medium-span structures (11).

Two-hinged arch

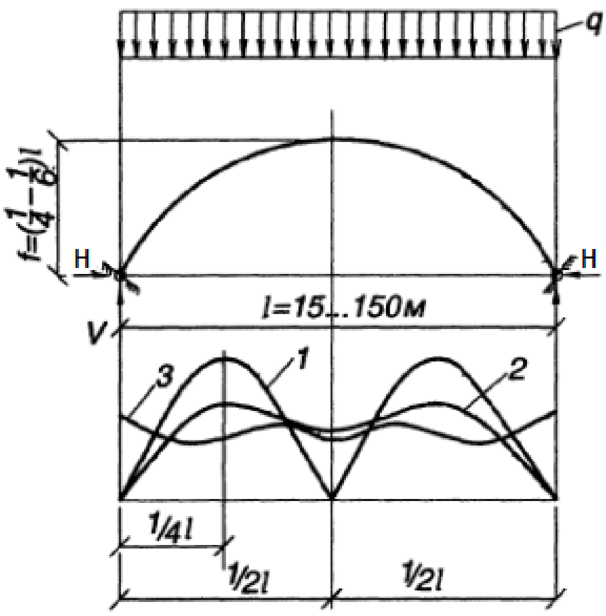
The two-hinged arch is once statically indeterminated (Figure 5, a).

<u>Advantages:</u>

- Due to its low stiffness per unit length the arch can easily be deformed without significant increasing of stresses that occur due to temperature fluctuations and settling of supports (5)
- More uniform distribution of bending moments in comparison with three-hinged arch (Figure 6) Therefore, two-hinged arches are quite economical in material consumption (10)
- Easy to manufacture and install

Application:

Two-hinged arches are often used in construction practice: bridge of long span, exhibition halls etc.



1-three-hinged; 2-two-hinged;
3-fixed

Figure 6. Distribution of bending moments (5)

Fixed arch

The fixed arch is three times statically indeterminated (Figure 5, c).

Advantages:

- The most uniform distribution of bending moments with local increase at supports (Figure 6). So, these arches are the most economical in material consumption

Disadvantages:

- In these arches, it is necessary to eliminate any possible settling of supports, which may require significant costs for foundations
- It is necessary to consider fluctuations of temperature in the calculation of arches (5)

Application:

The fixed arch is mostly used in reinforced concrete bridge and tunnel construction, where the spans are short (12).

Comparing the main advantages and disadvantages of different types of arches in most cases preference is given to the two-hinged arches.

2.4 General sizes of arch

General sizes of arch (Figure 5):

- Span – l
- Arch rise – f
- The height of the cross section of the arch – h

Span and arch rise are usually determined by technological and architectural requirements (10).

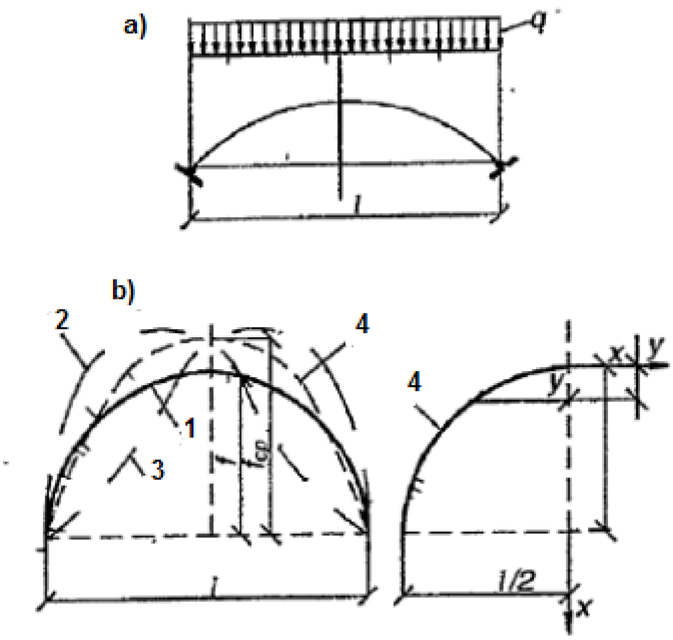
Depending on the ratio f/l, arches can be divided into *low angle* (f/l<1/4…1/10) and *raised* (f/l≥1/4…1) (5).

2.5 Configuration of arch

After determination of l and f it is necessary to choose the geometrical configuration of arch. A lot of curves can be made through three points (two supports and mid-span section). It is necessary to choose optimal curve. This optimal curve is called pressure curve, which depends on the type of loads (5).

For low angle arches (f/l<1/4…1/10):

Uniformly distributed dead load has the major influence on arch. The dead load is distributed along the span. Therefore, the most favorable geometrical configuration for this arch is quadratic parabola (Figure 7, a). But the curvature of the arch varies in length and it is difficult to manufacture. To make the manufacture of arch easier the parabola is replaced by a circular arc. The circular arc gives the close configuration to the parabola that has no effect on the structural behavior of arch (5,10).



a-low angle parabolic arch; b-raised arch;
1-original curve (catenary); 2,3- wind pressure
curves; 4-optimal curve

Figure 7. Geometrical configuration of arch (10)

<u>For raised arches (f/l≥1/4…1):</u>

Dead load is uniformly distributed along the arch axis. In this case, the most favorable configuration is catenary. However, wind load has the significant influence on the structural behavior of the arch. Wind load can act on arch in different directions, giving different pressure curves (2, 3 on Figure 7, b). Configuration of arch is taken by optimal average curve (4 on Figure 7, b) (10).

2.6 Thrust

Arches are thrust systems (Figure 1). The value of the thrust which depends on load, span and arch rise can vary considerably. To take the thrust special design solutions are required. In this thesis, the tie is used to take the trust. Vertical suspensions are used to avoid the sag of tie (Figure 8) (5).

In most cases, modern arches are designed as single-span structures with tie or without it. Some of them are presented on Figure 9 and will be considered as an example in this thesis.

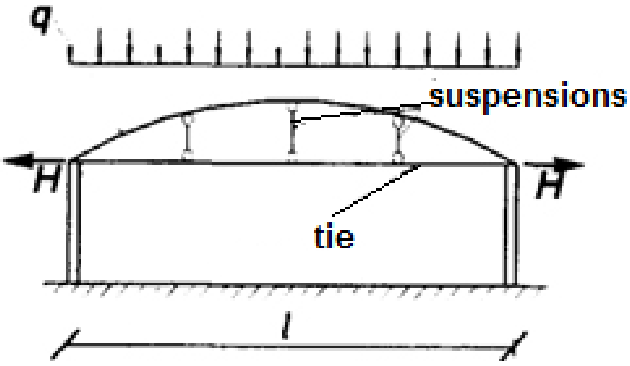


Figure 8. Arch with tie and vertical suspensions (5)

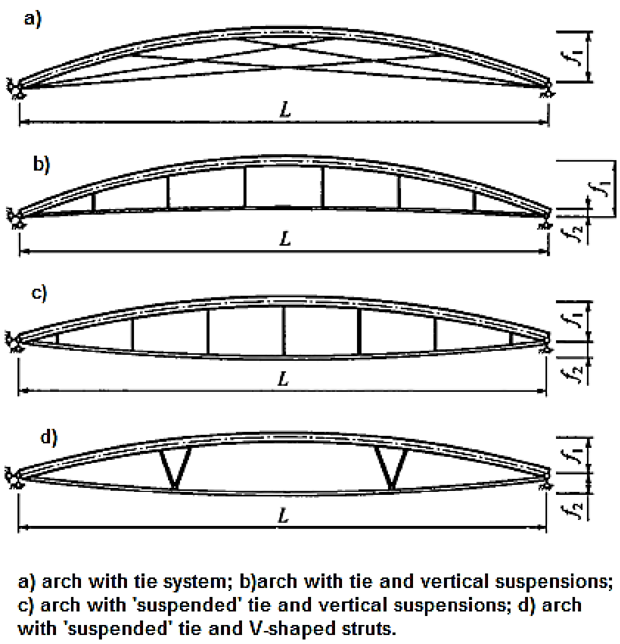


Figure 9. Variants of combined arch systems (1)

3 CALCULATION PART

The general principles of design and calculation of arches are presented in this chapter. Two types of arches were used as an example for understanding the primary structural behavior:

1. Arch with tie and vertical suspensions (Figure 9, b)
2. Arch with suspended tie and V-shaped struts (Figure 9, d)

All information complies with the theory of metal structures analysis according to Russian norms.

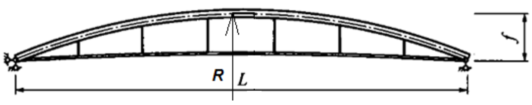
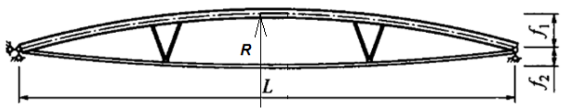
3.1 Initial data

Metal structures are designed in accordance with the following norms and regulations:

- SP 16.13330.2011 “Steel Structures”
- SP 20.13330.2011 “Loads and actions”
- STO 36554501-015-2008 “Loads and actions”

Recommendations from textbooks (1,5) were used for design and calculation as well. Initial data for designing are presented in Table 1.

Table 1. Initial data. Configuration of arches

1. Arch with tie and vertical suspensions	**2. Arch with suspended tie and V-shaped struts**
f/l=1/10=0.1	f/l=0.07
L=60m	
f=6m	f=4.35m; f1=2.25m; f2=2.1m;
R=78m	R=205.7m
Structural scheme 1	**Structural scheme 2**
	

Construction conditions: area of construction-Saint-Petersburg. According to SP 20.13330.2011 “Loads and actions”:

- Snow area – III – 180 kg/m^2 (design load)
- Wind area – II – 30 kg/m^2 (normal load)

3.2 Design solutions

Based on structural behavior arches are considered as two-hinged structures (left is immovable support and right is hinged movable support). Suspensions/V-shaped struts are connected with arch by hinge. Stability and spatial stiffness of the arch are provided by the system of bracings (on the top chord and on the bottom of the arch) and profiled deck. The step of arches is 6 m.

3.3 General principles

Calculation of arches was made with the help of the Lira-SAPR and Mathcad software programs.

LIRA (PK LIRA) - multifunctional software package for the calculation, research and design of structures of various purposes (13).

The theoretical basis of the LIRA (PK LIRA) is the finite element method (FEM). **The finite element method (FEM)** is a numerical technique for finding approximate solutions to boundary value problems for partial differential equations. FEM is based on division of a large problem into smaller parts. These smaller parts are called finite elements. The simple equations that model these finite elements are then assembled into a larger system of equations that models the entire problem (14).

Mathcad is a computer software primarily intended for the verification, validation, documentation and re-use of engineering calculations. This engineering math software allows to present calculations with plots, graphs, text, and images in a single document (15,16).

3.3.1 General principles of design

All loads and coefficients are defined in accordance with the requirements of SP 20.13330.2011 and STO 36554501-015-2008 “Loads and actions”. According to norms all loads are taken with an appropriate safety factor γ_f (17,18). Combination of loads is made with corresponding values of combination factor ψ .

Deflections are limited based on the aesthetic requirements in accordance with 15.1.5 in SP 20.13330.2011 “Loads and actions”.

3.3.2 General principles of calculation

In this part of the thesis the brief description of calculation is given:

1. Initially, it is necessary to make the compilation of loads

2. The calculation is made in two stages. The first stage is a calculation in the linear formulation for estimating the stability of arches. During the linear analysis, the general stability index is determined for the entire system. It should be ≥1.3 (1). Also during the linear calculation effective length factor (μ) for the top chord of the arch is determined (the top chord of the arch is under compression with bending). Then, the stability of the system is checked. If the stability of the system is OK, the system is continued to calculate in the geometrically nonlinear formulation (second stage of the calculation).
3. The calculation in geometrically nonlinear formulation (the constructive scheme including flexible elements and hinged movable support is sufficiently deformable) is done for estimating the strength of the system. Then the strength of elements is checked. If the strength of the system is OK, the calculation is finished.

To get a result of the calculation (with the required accuracy for engineering tasks), the following requirements should be made:

- Division the top chord of each arch into smaller finite elements: $\mathrm{d}s/S \leq 0.007$ ($\mathrm{d}s$ - the length of finite element, S - the total length of the arch axis) (see Figure 10)
- Numerical studies of structures, taking into account the geometrical nonlinear analysis, consider a calculation using the method of successive loading (19). The number of steps must be at least 40 (n≥40); also, should be satisfied the requirement: $\mathrm{n_i}/\mathrm{n} \geq 0.9$, ($\mathrm{n}$ - the total number of steps load, $\mathrm{n_i}$- the step of successive loading where the buckling of the system is fixed) (1)
- Difference between characteristics (maximum values of internal forces, horizontal and vertical deflections) from linear and non-linear analyses should be $\leq 10\%$.

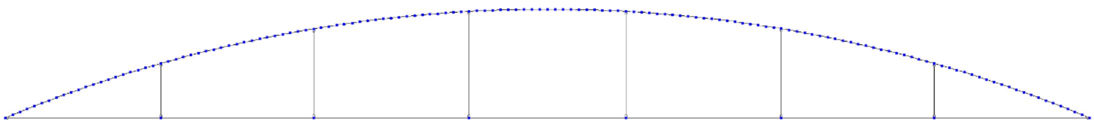


Figure 10. Division the structure into finite elements on example of the 1st type of arch

Lira-SAPR software program was used for the determination of internal forces in linear and nonlinear formulations. It was also used for the determination of general stability index for the entire system and effective length factors (μ). Mathcad was used for checking slenderness, stability and strength of the structure members according to the norms and regulations mentioned above. The calculation algorithm from Mathcad is given in Appendix 1.

3.4 Load compilation

Self-weight of structure

The self-weight of the arch is applied automatically by the Lira-SAPR with an appropriate safety factor γ_f and relates to permanent loads (see Figure 11).

$\gamma_f = 1.05$ for self-weight of metal structures (see Table 7.1 in (17))

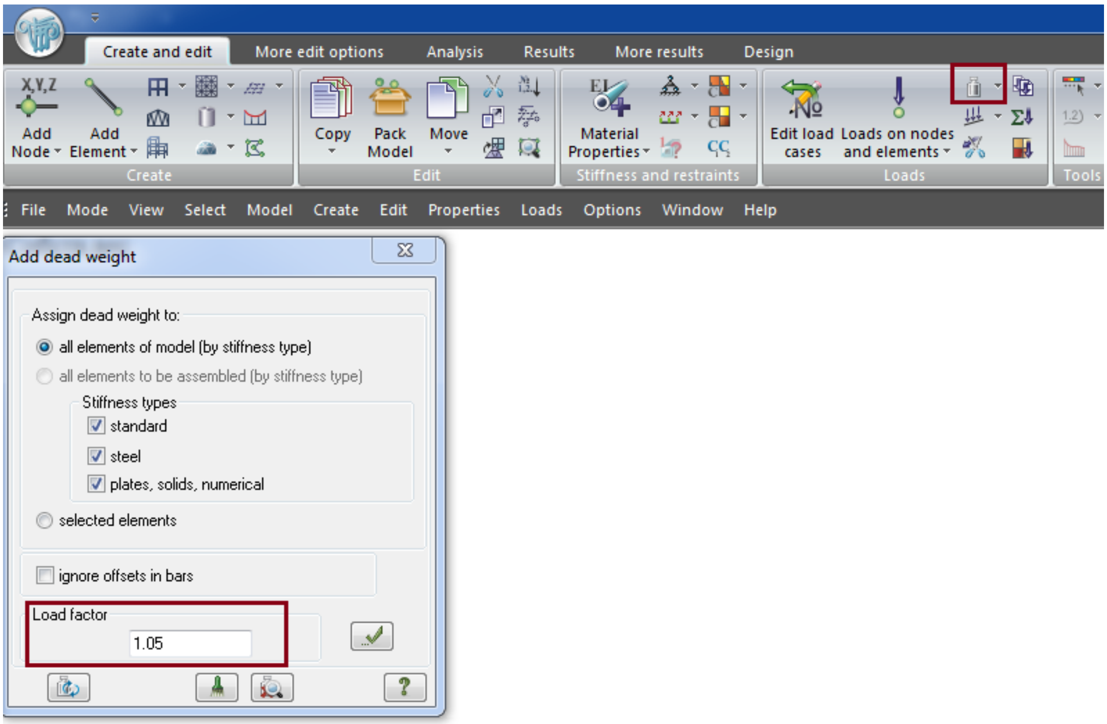


Figure 11. Applying of self-weight of the structure in Lira-SAPR

Snow load

The normal value of the snow load (snow weight) on the horizontal projection of the roof surface (see formula 10.1 in (17)):

$$S_0 = 0.7 \cdot c_e \cdot c_t \cdot \mu \cdot S_g$$

c_e – factor for demolition of the snow mass from roof surface by reason of wind load and other factors

c_t – thermal factor

μ – transfer factor that convents vale of the snow weight on the ground to weight on the horizontal projection of the roof surface

S_g – weight of the snow mass per 1m^2 on the horizontal ground surface

$S_0 = 0.7 \cdot 1 \cdot 1 \cdot 1 \cdot 1.8 = 1.26\ kN/m^2$

$c_e = 1.0$ (see requirements of 10.5-10.9 in (17))

$c_t = 1.0$ for roofs with thermal insulation (see requirements of 10.10 in (17))

$\mu = 1.0$ according to the Table G.1 of the Appendix G for roofs with slope angle $\leq 30°$ (17)

$S_g = 1.8\ kPa\ (1.8\ kN/m^2)$ for snow region III (see the Table 10.1 in (17))

$\gamma_f = 1.4$ (see 10.12 in (17))

According to SP 20.13330.2011 “Loads and actions” snow load on arches (f/l<1/8) is applied uniformly by the span of structure.

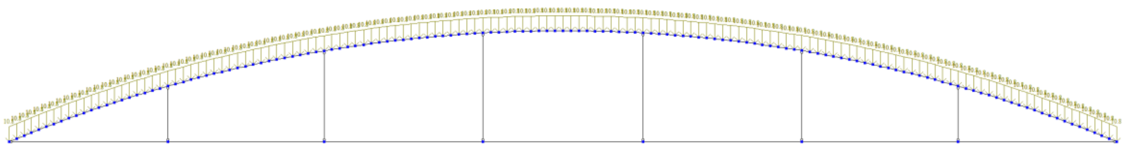


Figure 12. Distribution of snow load on example of the 1st type of arch

Wind load

Wind load is not taken into account in calculation according to STO 36554501-015-2008 and SP 20.13330 “Loads and Actions” because it creates negative pressure on the major part of the roof surface.

Load compilation is presented in Table 2. Note: value of a design load is obtained by multiplying normal load by an appropriate safety factor γ_f (17).

Table 2. Load compilation

Type of load	Normal load kN/m^2	safety factor γf	Design load kN/m2	Design load kN/m (step of arches =6m)
		Permanent load		
Profiled deck N75-750-0,9	0.125	1.05	0.131	**11.1**
Vapor barrier TechnoNIKOL(1,5x50m)	0.04	1	0.04	
Heat insulation 150 mm	1.3	1	1.3	
TechnoNIKOL PVC-membrane	0.04	1	0.04	
Technical load	0.2	1.05	0.21	
Total:	**1.505**		**1.721**	
Weight of bracings	**Chatacteristics load per 1metre (kN per 1m)**	**safety factor γf**	**Design load kN/m2**	
U-beam 14П (with parallel flanges) accroding to GOST 240-97	0.123	1.05	0.129	
Total:	**0.123**		**1.85**	
		Temporary load		
Snow load	1.26	1.4	1.8	**10.8**
Total:	**1.26**		**1.8**	

In Figure 12 is shown how to apply loads on structure in Lira-SAPR program.

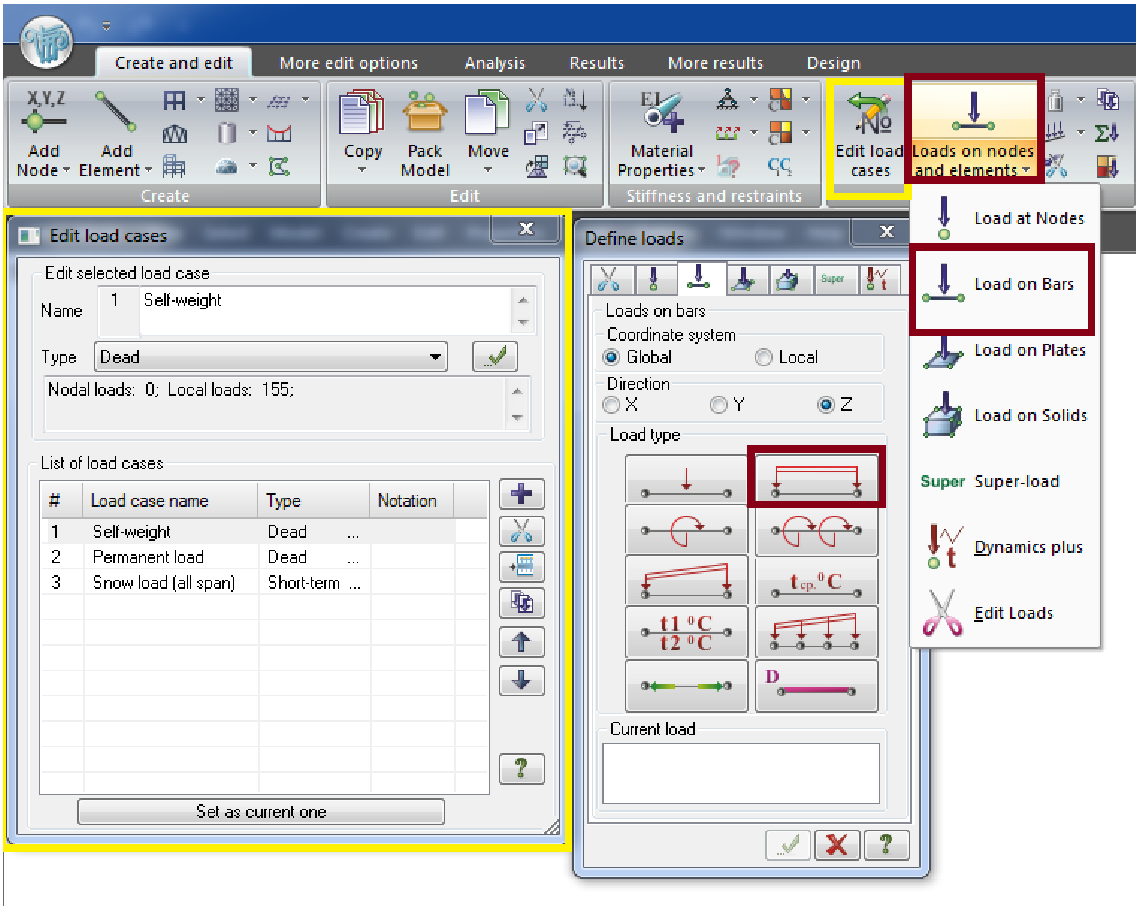


Figure 13. Applying of loads on the structure in Lira-SAPR

3.5 Load combinations

Structures are calculated according to the theory of ultimate limit state design. Thus, load combinations must be taken into account. These combinations are determined from the analysis of real variants of simultaneous action of different loads for the considered structure (17). The basic combination of loads consists of permanent and temporary loads (long-term and short-term).

Permanent load: self-weight of structure, soil pressure etc.
Temporary loads are divided into:

- Long-term loads – acting for a long period (people, equipment, storage load etc.).
- Short-term loads – acting for a short period (snow, wind, temperature load etc.)

Formula for basic combination of loads (see formula 6.1 in (17)):

$$C_m = P_d + \sum_i \psi_{li} P_{li} + \sum_i \psi_{ti} P_{ti}$$

P_d – design value of permanent load
P_l – design value of long-term temporary load
P_t – design value of short-term temporary load
ψ_{li} $(l = 1, 2, 3 \dots,)$ – combination factor for long-term temporary loads
ψ_{ti} $(l = 1, 2, 3 \dots,)$ – combination factor for short-term temporary loads

- Combination factors for long-term temporary loads (see 6.3 in (17)):

$$\psi_{l1} = 1.0;\ \psi_{l2} = \psi_{l3} = \ldots = 0.95$$

- Combination factors for short-term temporary loads (see 6.4 in (17)):

$$\psi_{t1} = 1.0;\ \psi_{t2} = 0.9, \psi_{t3} = \psi_{t4} = \ldots = 0.7$$

ψ_{li}, ψ_{ti} combination factors are determined in accordance with the degree of influence on structure.

Load combination from Lira-SAPR for linear calculation (1st stage) is presented below (Figure 14).

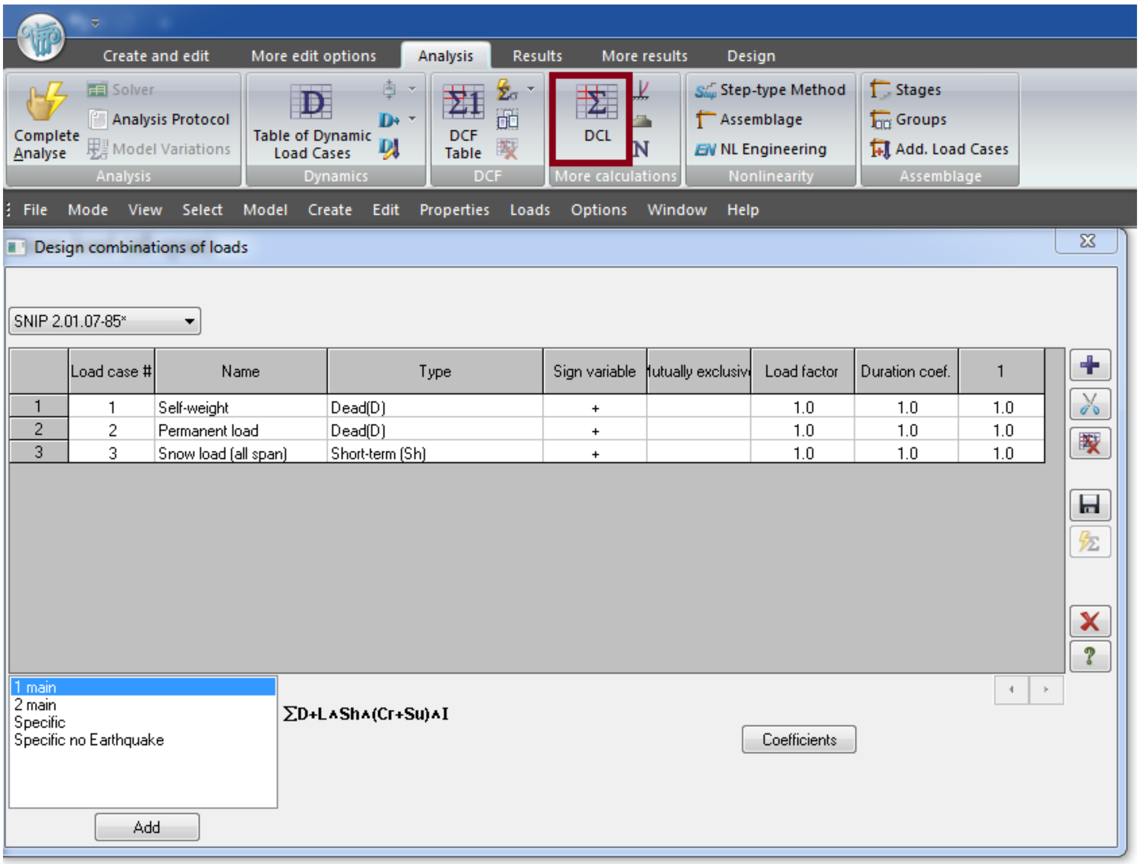


Figure 14. Combinations of loads in Lira-SAPR

Note: load factors on the Figure 14 $\gamma_f = 1.0$ because design loads were applied.

3.6 Calculation of the 1st type of arch

1st type of arch – arch with tie and vertical suspensions (see Figure 15).

Initially, the design model was done in Lira-SAPR program according to requirements that were given above (see paragraph 3.3.2). After that all loads were applied, combination of loads was done and the cross-section were selected, the calculation in linear formulation is started. Linear calculation – calculation of the non-deformed scheme.

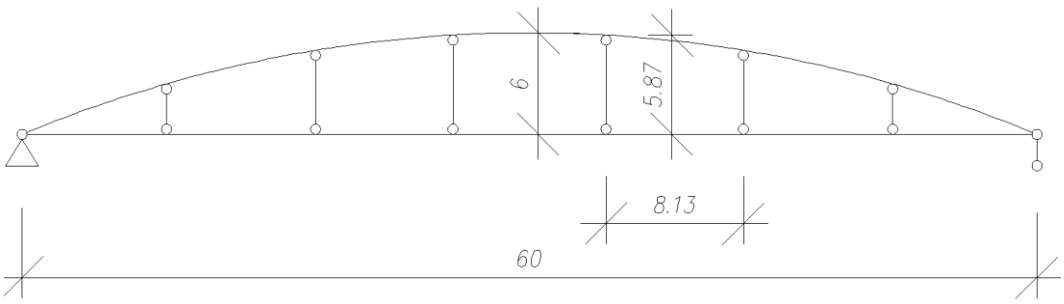


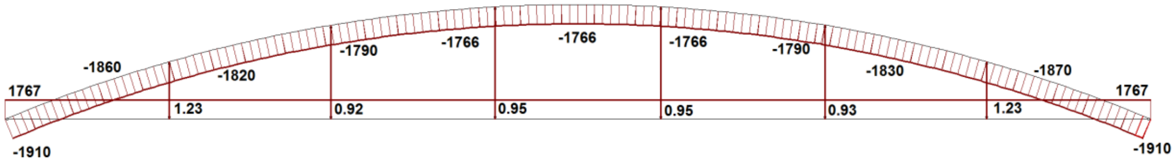
Figure 15. Scheme of the 1st type of arch

The step of bracings on the top chord of arch is 5 m, on the bottom is 8.13 m. The sections that were selected for calculation are the following:

- For the top chord of arch – 40K2 (Column I-section)
- For vertical suspensions – pipe section 89*4 mm
- For the tie – bar section 90*90 mm

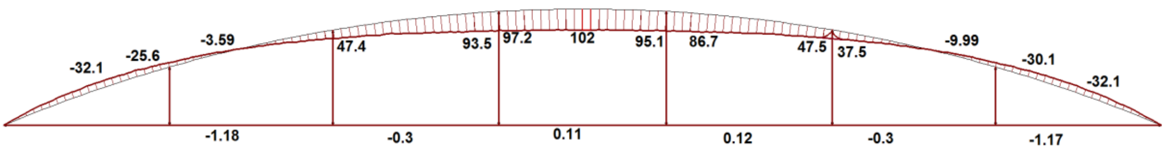
3.6.1 Determination of internal forces in Lira-SAPR (linear analysis)

At first internal forces in elements are determined using load combination (see Figure 14). The distribution of internal forces (M, N) in all elements of the scheme in linear formulation is given below (Figure 16,17).



max N=-1910kN

Figure 16. Distribution of axial force N, kN in the 1st type of arch



max M=102kN*m

Figure 17. Distribution of bending moment M, kN*m in the 1st type of arch

3.6.2 Stability calculation in Lira-SAPR

Then the stability calculation was made for the determination of the general index of stability (see Figure 18). The form of buckling for load combination is given below (Figure 19). The requirement that the general stability index for the entire system should be (stability safety factor) ≥1.3 is done. If the stability safety factor is≥1.3, it means that the system is stable. For this load combination it is 2.7.

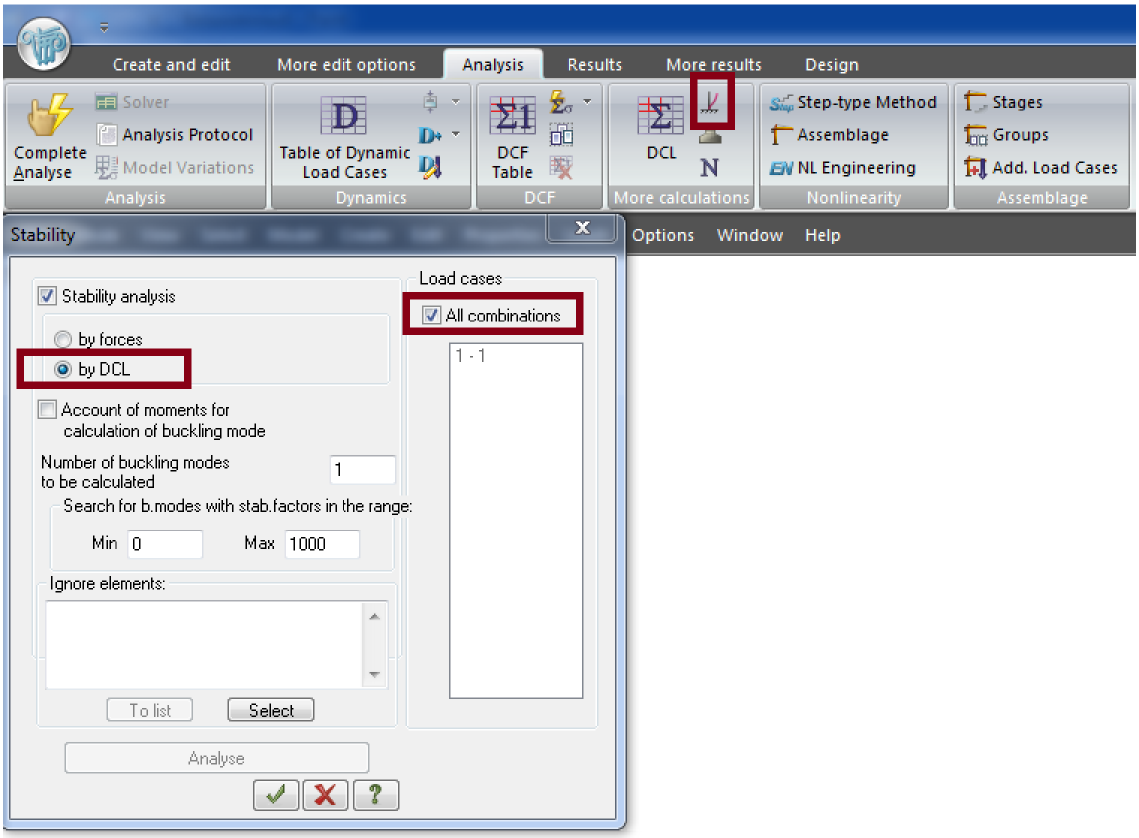


Figure 18. Stability calculation in Lira-SAPR

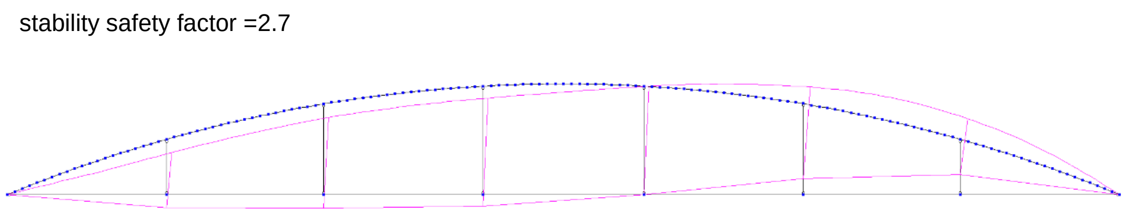


Figure 19. Form of buckling for load combination in Lira-SAPR

3.6.3 Determination of effective length factor

Then for this load combination the effective length factor of the top chord of the arch (μ) was determined. It is necessary to find the finite element that the first loses stability (with factor S_Q=1) (see Figure 20).

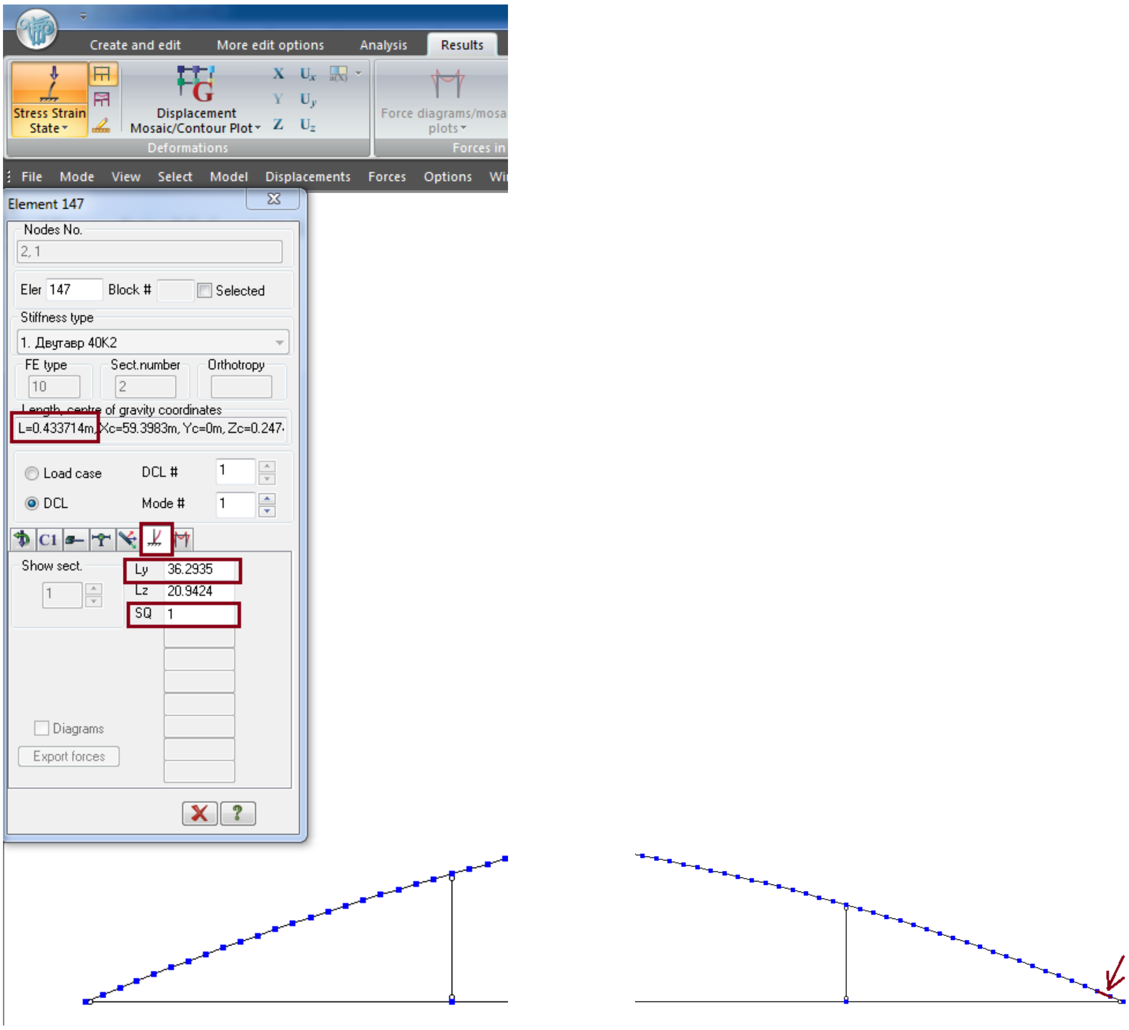


Figure 20. Description of the element that the first loses stability

Calculation of the effective length factor (μ) for the top chord of the arch (5,13):

$l_{ef} = L_y \cdot \sqrt{1.3} \cdot L_{fe}$– effective length

L_y – factor of effective length (from Lira-SAPR)

L_{fe} – length of the finite element

$l_{ef} = 36.2935 \cdot \sqrt{1.3} \cdot 0.433714 = 17.95\ m$

$\mu = \frac{l_{ef}}{L} = \frac{17.95}{60} = 0.299$ – the effective length factor for the top chord of the arch

After determination of the effective length factor and stability safety factor, the stability of the top chord of the arch and its selected cross-section were checked with the help of Mathcad (see Appendix 1). The unfavorable section for estimating stability and checking cross-section of the top chord of the arch was chosen: N=-1766kN, M=102kN*m (see Figure 16,17).

The slenderness of elements was also checked (see Appendix 1). If all the checks are correct, the calculation is continued in nonlinear formulation.

3.6.4 Calculation in geometrically nonlinear formulation

Calculation in geometrically nonlinear formulation is made for strength estimation of the selected cross-sections for the entire system. Geometrically non-linear analysis is a calculation that takes into account changes of the geometry when the system is deformed under the load. Nonlinear calculation is calculation of the deformed scheme.

Firstly, all linear finite elements are replaced by nonlinear. In this task the type of finite element №309 is used as non-liner element for the whole system (see Figure 21). This type of finite element is aimed at providing calculation for all kinds of rod systems taking into account the geometric nonlinearity. It is analogue of the universal linear finite element.

Then, it is necessary to make nonlinear loading using step-type method (13). This task considered the following nonlinear loading: self-weight + permanent load + snow load (see Figure 22).

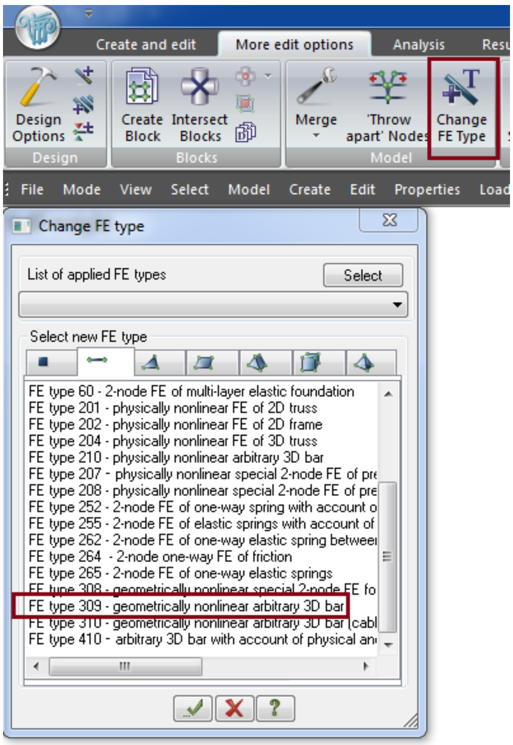


Figure 21. Types of finite elements

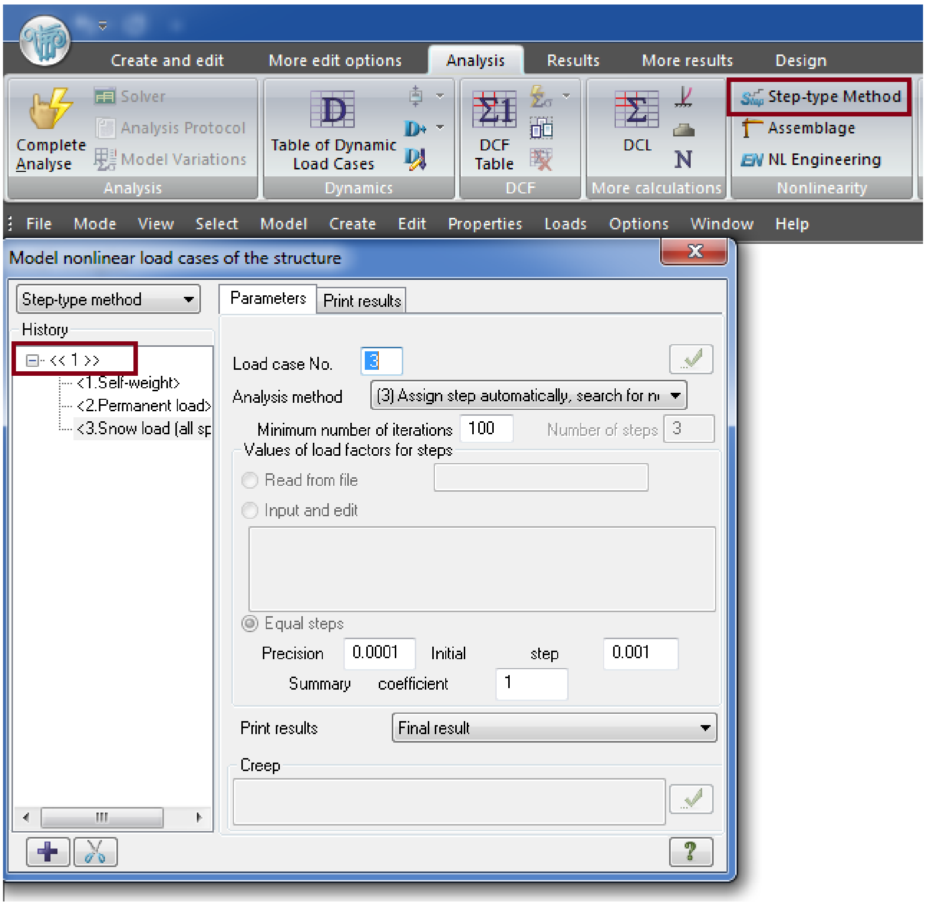
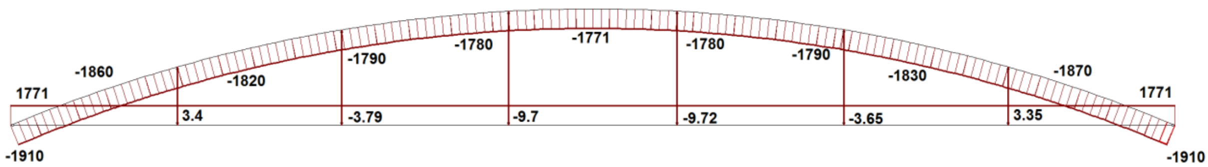


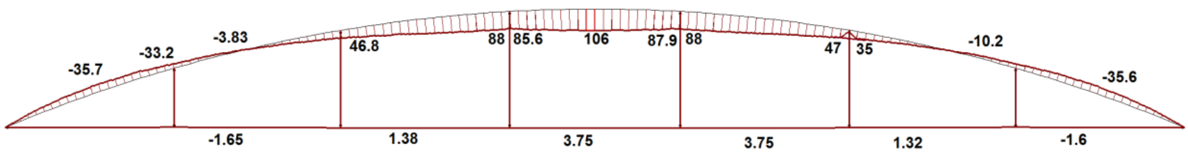
Figure 22. Non-linear loading

The distribution of internal forces (N, M) in geometrical nonlinear formulation is given below (see Figure 23,24).



max N=-1910kN

Figure 23. Distribution of axial force N, kN in the 1st type of arch, geometrically nonlinear analysis



max M=106kN*m

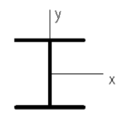
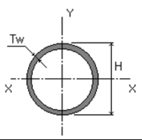
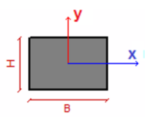
Figure 24. Distribution of bending moment M, kN*m, in the 1st type of arch, geometrically nonlinear analysis

After calculation in Lira-SAPR in geometrically nonlinear formulation, the strength of all elements was checked (see Appendix 1). If the check is correct, selected cross-sections are accepted.

3.6.5 Results

Cross-sections with corresponding characteristics of all elements that were accepted during calculation are shown in Table 3.

Table 3. Results of calculation for the 1st type of arch

Element	Internal forces kN, kN*m			Section		Steel	Effective length, cm		Slenderness		Limit		Strength	Stability calculation	
							lefx	lefy	λx	λy	λu		calculation	in plane	out of plane
Top chord	from linear analysis (stability)	N	-1766	A=218.67cm2	40K2 (Column I-section)	S-345	1795	500	102.865	49.407	136.44	150	-	0.726	0.428
		M	102	Wx=3331.2cm3											
	from nonlinear analysis (strength)	N	-1766	ix=17.45cm		Ry=320N/mm2	-	-	-	-	-	-	0.392	-	-
		M	106	iy=10.12cm											
Vertical suspensions	from nonlinear analysis (strength)	N	-9.72	A=9.331cm2	Pipe section 102*3mm	S-345	587	587	167.67	167.67	180	180	0.03	0.182	0.182
		M	-	ix=iy=3.501cm		Ry=320N/mm2									
Tie	from nonlinear analysis (strength)	N	1771	A=81cm2	Bar section 9*9cm	S-345	813	813	312.692	312.692	400	400	0.866	-	-
		M	3.75	Wx=121.5cm3		Ry=320N/mm2									
				ix=iy=2.6cm											

Comparison characteristics (maximum values of internal forces, horizontal and vertical deflection) between linear and non-linear analyses are presented in Table 4.

Table 4. Comparison characteristics from linear and nonlinear analyses

Characterisitc	Linear calculation	Nonlinear calculation	Difference, %
Max values of internal forces			
Max value of N, kN	-1910	-1910	0
Max value of M kN*m	102	106	3.8
Max horizontal deflection			
Displacement along x-axis, mm	63.7	62.4	2.0
Max vertical deflection			
Displacement along y-axis, mm	-195	-196	0.5

Difference between values of characteristics from linear and nonlinear calculations should is less than 10% (Table 4).

3.7 Calculation of the 2nd type of arch

2nd type of arch – arch with suspended tied and V-shaped struts (see Figure 25).

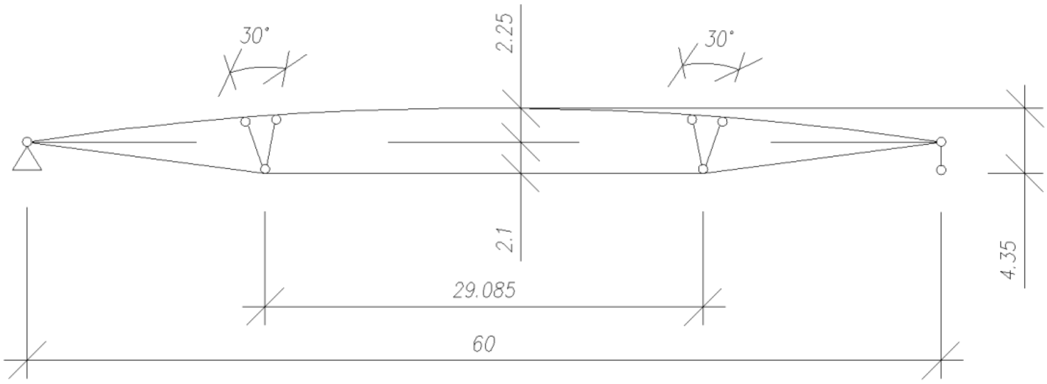


Figure 25. Scheme of 2nd type the arch

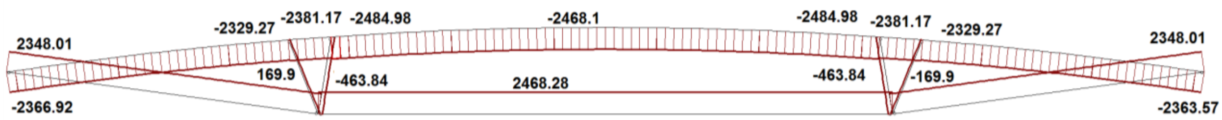
The step of bracings on the top chord of arch is 5 m, on the bottom is 8.13 m. The sections that were selected for calculation are following:

- For the top chord of arch – 40K4 (Column I-section)
- For vertical suspensions – pipe section 168*8 mm
- For the tie – bar section 54*300 mm

The algorithm of calculation is the same as for the 1st type of arch.

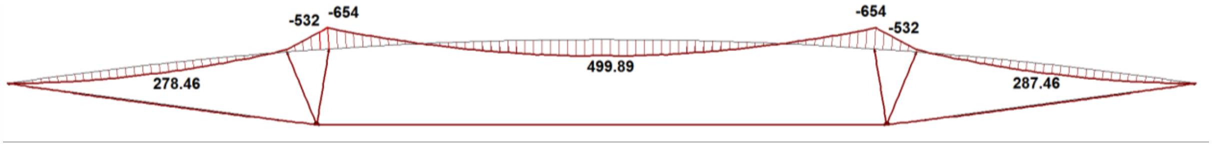
3.7.1 Determination of internal forces in Lira-SAPR (linear analysis)

The distribution of internal forces (N, M) from linear calculation is given below (see Figure 26,27).



max N=-2484.98kN

Figure 26. Distribution of axial force N, kN in the 2nd type of arch



Max M=-654kN*m

Figure 27. Distribution of bending moment M, kN*m in the 2nd type of arch

3.7.2 Stability calculation in Lira-SAPR

During calculation, the stability safety factor was determined. For this load combination (Figure 14) it is 3.0. The requirement that the general stability index for the entire system should be (stability safety factor) ≥1.3 is done. The form of buckling for load combination is given below (Figure 28).

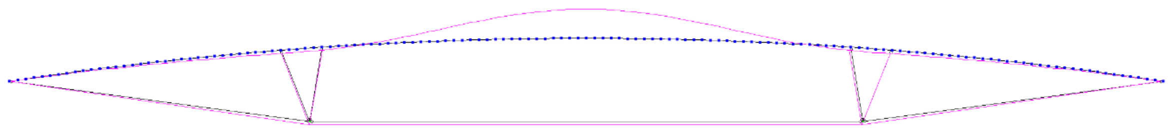


Figure 28. Form of buckling for load combination in Lira-SAPR

3.7.3 Determination of effective length factor

The finite element that the first loses stability (with factor S_Q=1) is shown on Figure 29 (in the middle of the top chord of the arch).

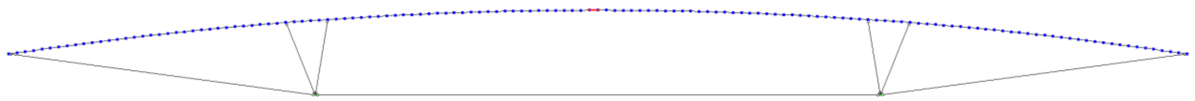


Figure 29. The finite element that the first loses stability

$l_{ef} = L_y \cdot \sqrt{1.3} \cdot L_{fe}$

For this finite element:

$L_y = 33.111$ – factor of effective length (from Lira-SAPR)

L_{fe} – length of the finite element

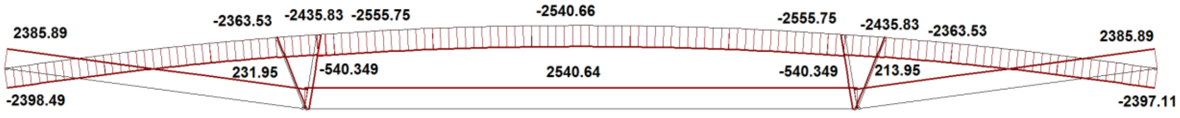
$l_{ef} = 33.111 \cdot \sqrt{1.3} \cdot 0.435 = 16.42\ m$

$\mu = \frac{l_{ef}}{L} = \frac{16.42}{60} = 0.274$ – the effective length factor for the top chord of the arch

After determination of the effective length factor and stability safety factor, the stability of the arch and its selected cross-sections were checked with the help of Mathcad (see Appendix 1). For this type of arch the unfavorable section for estimating stability and checking cross-section of the top chord of the arch was: N=-2468.1kN, M=499.89kN*m (see Figure 26,27). The slenderness of elements was checked as well (see Appendix 1). After all checks are correct, the calculation is continued in nonlinear formulation.

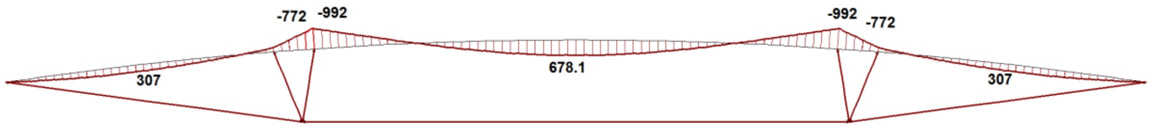
3.7.4 Calculation in geometrically nonlinear formulation

The distribution of internal forces (N, M) from geometrically nonlinear formulation is shown below (see Figure 30,31). Then strength estimation of the selected cross-sections for the entire system was made (see Appendix 1).



max N=-2555.75kN

Figure 30. Distribution of axial force N, kN in the 2nd type of arch, geometrically nonlinear analysis



max M=-992 kN*m

Figure 31. Distribution of bending moment M, kN*m in the 2nd type of arch, geometrically nonlinear analysis

3.7.5 Results

Cross-sections with corresponding characteristics of all elements that were accepted during calculation are shown in Table 5.

Comparison characteristics (maximum values of internal forces, horizontal and vertical deflection) between linear and non-linear analyses are presented in Table 6.

Table 5. Results of calculation for the 2nd type of arch

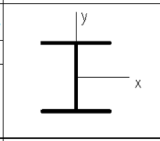
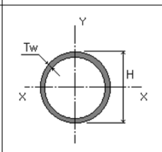
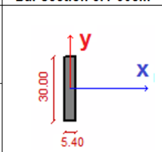
Element	Internal forces kN, kN*m			Section		Steel	Effective length, cm		Slenderness		Limit		Strength calculation	Stability calculation	
							lefx	lefy	λx	λy	λu			in plane	out of plane
Top chord	from linear analysis (stability)	N	-2468.1	A=295.59cm2	40K4 (Column I-section)	S-345	1642	500	92.66	48.78	119.82	139.62	-	1.003	0.673
		M	499.89	Wx=4481.4cm3		Ry=320N/mm2									
	from nonlinear analysis (strength)	N	-2540.7	ix=17.72cm			-	-	-	-	-	-	0.824	-	-
		M	678.1	iy=10.25cm											
Vertical suspensions	from linear analysis (stability)	N	-463.84	A=40.21cm2	Pipe section 168*8mm	S-345	395	395	69.79	69.79	177.90	177.90	0.467	0.535	0.535
	from linear analysis (stability)	N	-540.35	ix=iy=5.66cm		Ry=320N/mm2									
Tie	from nonlinear analysis (strength)	N	2540.66	A=100cm2	Bar section 5.4*30cm	S-345	2909	600	335.91	384.86	400.00	400.00	0.545	-	-
				ix=8.66cm		Ry=320N/mm2									
				iy=1.559cm											

Table 6. Comparison characteristics from linear and nonlinear analyses

Characterisitc	Linear calculation	Nonlinear calculation	Difference, %
Max values of internal forces			
Max value of N, kN	-2484.98	-2555.75	2.8
Max value of M kN*m	-654	-992	34.1
Max horizontal deflection			
Displacement along x-axis, mm	31	28	9.7
Max vertical deflection			
Displacement along y-axis, mm	-322	-289	10.2

Difference between values of characteristics from linear and nonlinear calculations almost $\leq 10\%$ (Table 6).

3.8 Deflections

Deformed shapes with values of deflections from nonlinear analysis for each type of arch are presented below. Deflections are limited based on the aesthetic requirements in accordance with 15.1.5 in SP 20.13330.2011 “Loads and actions”.

Deflections of the 1st type of arch

Deflections along the y-axis (vertical axis) are given below (see Figure 32).

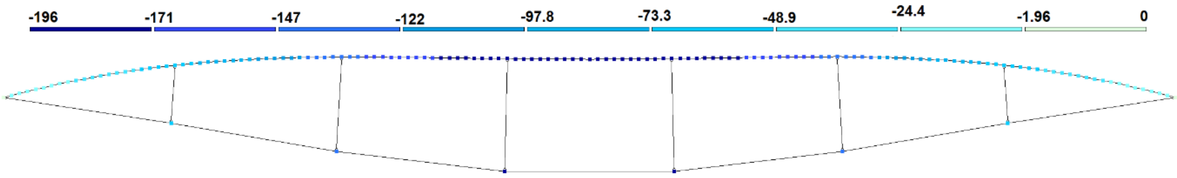


Figure 32. Deflections of the 1st type of arch along vertical axis from Lira-SAPR, mm

The maximum value of vertical deflection is in the middle of the top chord of arch. The deformations are uniformly and symmetrically from the middle to the edge of the top element. Deflections along x-axis (horizontal axis) are given below (see Figure 33).

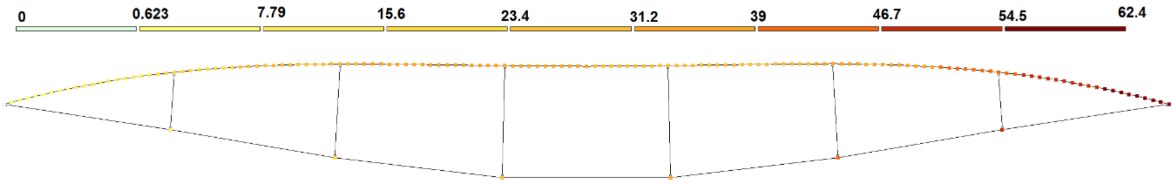


Figure 33. Deflections of the 1st type of arch along horizontal axis from Lira-SAPR, mm

The constructive scheme includes flexible elements and right hinged movable support is sufficiently deformable. Because of this, deflections uniformly distributed from left support to the right. The maximum horizontal deflection is on the right support.

<u>Deflections of the 2st type of arch</u>

Vertical and horizontal deflections are given below (see Figure 34,35)

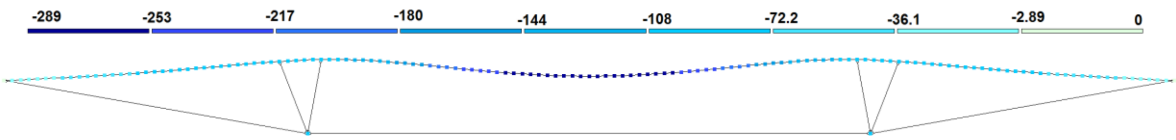


Figure 34. Deflections of the 2nd type of arch along vertical axis from Lira-SAPR, mm

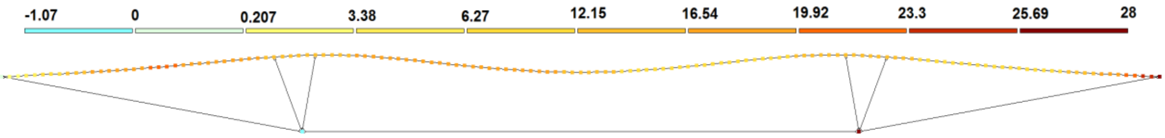


Figure 35. Deflections of the 2nd type of arch along horizontal axis from Lira-SAPR, mm

Distribution of deflections in the 2nd type of arch is the same as in the 1st.

Comparison of deflections between two types of arch

Vertical displacements in the 1st arch are less because the vertical elastic reactions of the suspensions, which are uniformly distributed along the entire length of the arch, prevent the movement of the top chord of the arch. In the 2nd arch, elastic suspensions are located only in two places closer to the supports. Because of this the arch has a large free span in the middle.

Horizontal displacements in the 1st arch are larger, because the horizontal movement of the arch is prevented only by the tightening force applied to the right support. And in the 2nd arch, the movement of the top chord is impeded by the tightening force applied to the right support and the tightening force applied to the bases of the V supports. Due to the rigidity of the formed triangle, the 2nd arch receives a flexible support along the horizontal axis.

3.9 Joints of arches

In this part of the thesis connections of vertical elements to the chord and to the tie for each type of arch are presented (see Figure 36,37).

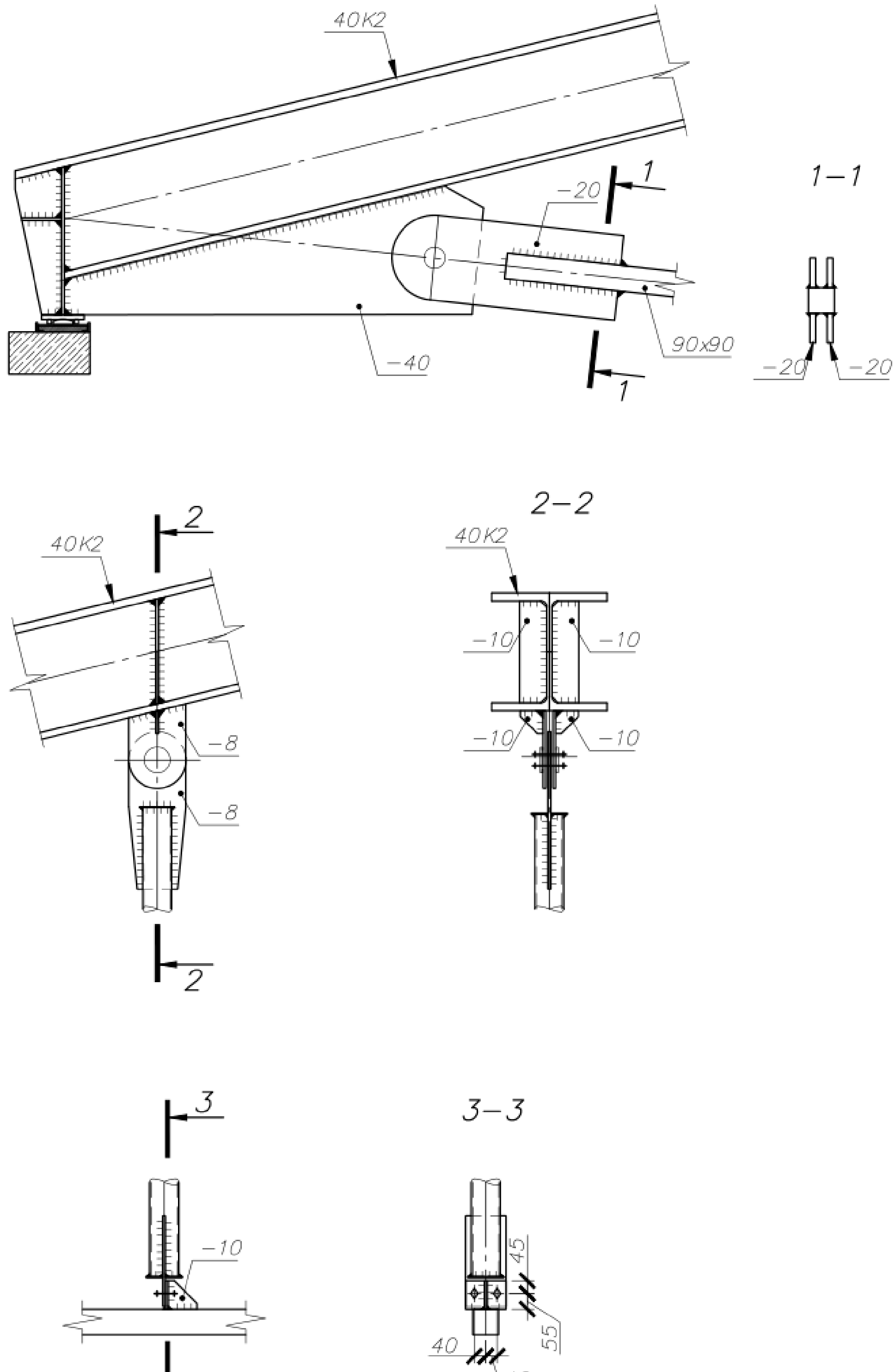


Figure 36. Connections for the 1st type of arch

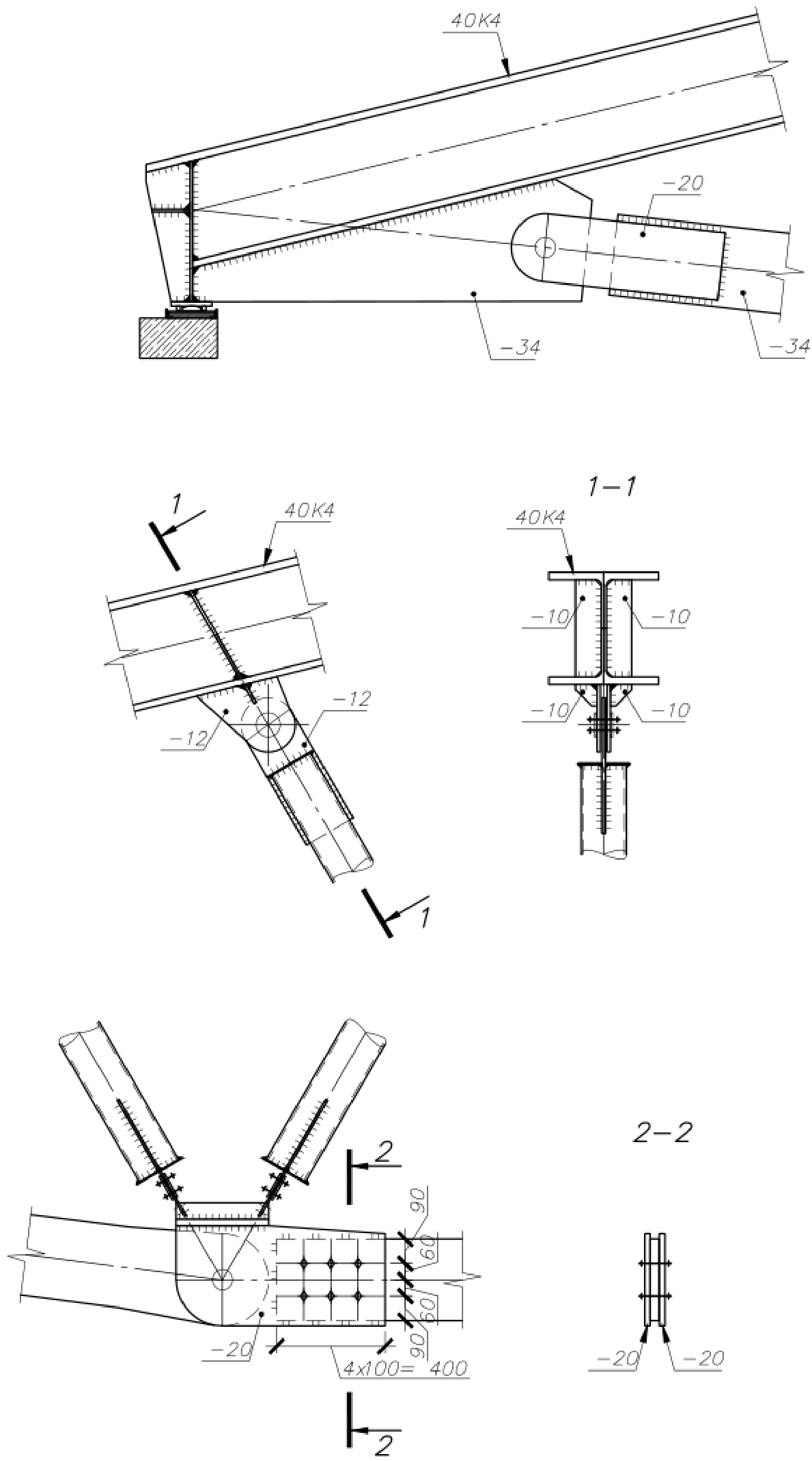


Figure 37. Connections for the 2nd type of arch

3.10 Comparison

To compare the constructive solutions the parameter of lower steel consumption was chosen. The comparison is given below in Table 7.

Table 7. Comparison of constructive solutions

Steel consumption, kg		
Element	1st type of arch: arch with tie and vertical suspensions	2nd type of arch: arch with tie and V-shaped struts
Top chord	10574.49	13985.66
Vertical suspensions/V-shaped struts	202.41	498.65
Tie	3790.80	7627.09
Joints/Connections	582.71	884.46
Total:	15150.40	22995.85
Difference between steel sconsumption between 1st arch and 2nd arch		34.1%

4 CONCLUSION

Nowadays, there is a need for large spans in construction. One way of covering long-span roof systems is to use modern combined arch-cable structures. Arches are used for different purposes from shops and markets to exhibition halls, stadiums and bridges. As arches are progressively developing structural forms, new methods of designing need to be explored.

In this study two constructive solutions for covering a roof system are considered:

1. Arch with tie and vertical suspensions
2. Arch with suspended tie and V-shaped struts

Calculation of such complex arched constructions was implemented by the method of finite element modeling using Lira-SAPR software program. Evaluation of arches strength and stability was made with the help of Mathcad.

The result of this research is a comparison of two structural schemes according to lower steel consumption. It shows that the steel consumption of the first arch is less than the second by 34 % with the same initial data. The margin of safety of the 1st type of arch is 20-30% more than the 2nd arch. Based on the calculation of the 2nd arch it is not allowed to exceed the design load.

This study provides the primary assessment of the structural behavior of arches.

During this research, the comparison algorithm of possible variants of vertical elements of arches was developed with the help of calculation. This method of calculating allows to consider different variants of lattice (vertical suspensions, struts etc.) to achieve the optimal constructive scheme.

FIGURES

TABLES

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Mathcad calculation of the 1st type of arch with tie and vertical suspensions

1. First part of the calculation was made in linear formulation for estimating stability.

CALCULATION OF STABILITY

Checking of the top chord of the arch . It is eccentrically compressed (compression with bending):

$$\frac{N}{\left(\varphi_e \cdot A \cdot Ry \cdot \gamma c\right)} \le 1$$ - formula for checking of the element's stability with acting of bending moment and compressive force **in the plane of action of the moment** (see formula 109 in (20))

$\gamma c := 0.9$ - operating conditions factor/partial safety factor (see table 1 (20))

Characteristics of the cross-section 40K2 (Column I-section) from STO 29-93:

$A := 218.67\,cm^2$ - cross-section area

$Af := 168\,cm^2$ - cross-section of flanges

$Aw := 50.67\,cm^2$ -cross-section of wedge

$Wx := 3331.2\,cm^3$ - section modulus of x-axis

$ix := 17.45\,cm$ - radius of gyration of x-axis

$iy := 10.12\,cm$ - radius of gyration of y-axis

$Ry := 320\,MPa$ - design value of steel's bending resistance by yeild strength (according to (20) Ry=320N/mm² or Ry=320MPa for steel S-345)

$E := 2 \cdot 10^4 \dfrac{kN}{cm^2}$ - elastic modulus

Forces from load combination (from calculation in Lira-SAPR software program):

$N := 1766\,kN$ - design max axial force in the section of the top chord of the arch

$M := 102\,kN \cdot m$ - design max bending moment in the section of the top chord of the arch (according to requirements 9.2.3 (20))

Calculation of φe.

φe - coefficient of stability in compression with bending

φe depends on values of non dimensionall slenderness $\lambda 1$ and modified relative eccentricity m_{ef} according to the table D3 (20) :

$m_{ef}=\eta m$ (see formula 110 (20));

m-relative eccentricity, η- factor of the cross-section shape influence

$$\lambda x1 = \lambda x \sqrt{\frac{Ry}{E}}$$

λx=lefx/ix-slenderness in x-axis plane; lefx-effective length (from LIra-SAPR), ix- radius of gyration of x-axis

$lefx := 1795cm$ - effective length (from Lira-SAPR)

$$\lambda x := \frac{lefx}{ix} = 102.865$$

$$\lambda x1 := \lambda x \cdot \sqrt{\frac{Ry}{E}} = 4.115$$

Calculation of relative eccentricity m:

m=eA/Wc

$Wc := Wx = 3.331 \times 10^{3} \cdot cm^{3}$ - (Wc-modulus section for the most compressed part of web. In this case Wc=Wx)

$e := \frac{M}{N} = 0.058\,m$ -eccentricity

$$m := \frac{e \cdot A}{Wc} = 0.379$$

Calculation of factor of the cross-section shape influence η (see table table 1 from Appendix 2).

η depends on non dimensional slenderness λ1 and relative eccentricity m

According to the table the type of cross-section is №5.

$$\frac{Af}{Aw} = 3.316$$

$$\lambda x1 = 4.115$$

Formula for calculation η-factor accroding to Table 1 (see Appendix 2):

$$\eta := (1.90 - 0.1 \cdot m) - 0.02 \cdot (6 - m) \cdot \lambda x1$$

$$\eta = 1.4$$

$$mef := m \cdot \eta = 0.531$$

$\varphi e := 0.386$ according to the table D2 (20)

$$\frac{N}{(\varphi e \cdot A \cdot Ry \cdot \gamma c)} = 0.726$$

CORRECT

$$\frac{N}{c1 \cdot \varphi y \cdot A \cdot Ry \cdot \gamma c} \le 1$$

- formula for checking of the element's stability with acting of bending moment and compressive force **<u>out of the plane of the moment</u>** (see formula 111 in (20))

Calculation of c1:

$$mx := \left(\frac{M}{N}\right) \cdot \left(\frac{A}{Wc}\right) = 0.379$$ -relative eccentricity

If mx<5 formula for calculating the factor c1:

c1=β/(1+αmx), it should be <1 (according to formula 112 (20))

α,β - coefficients that are determined according to the table 2 from Appendix 2

Type of section 2 according to the table 2 (Appendix 2)

$\alpha := 0.7$ because mx<1

β depends on non dimensional slenderness in plane of y-axis λy1

$lefy := 500cm$ - effective length in y-axis plane (500cm - distance between bracings)

$$\lambda y := \frac{lefy}{iy} = 49.407$$

$$\lambda y1 := \lambda y \cdot \sqrt{\frac{Ry}{E}} = 1.976$$ It is less than 3.14 (see the table 2 of Appendix 2)

$\beta := 1$

$$c1 := \frac{\beta}{(1 + \alpha \cdot mx)} = 0.79$$

Calculation of φy **.** It is determined as for central compressive elements according to 7.3.1 of (20))

From table 3 of Appendix 2:

$\alpha 1 := 0.04$

$\beta 1 := 0.09$

$\delta 1 := 9.87 \cdot (1 - \alpha 1 + \beta 1 \cdot \lambda y1) + \lambda y1^2$ (see formula 9 from (20))

$\delta 1 = 15.136$

$$\varphi y := 0.5 \cdot \frac{\left(\delta 1 - \sqrt{\delta 1^2 - 39.48 \cdot \lambda y1^2}\right)}{\lambda y1^2}$$ (see formula 8 from (20))

$\varphi y = 0.83$

$$\frac{N}{c1 \cdot \varphi y \cdot A \cdot Ry \cdot \gamma c} = 0.428$$ <u>CORRECT</u>

2.Second part of the calculation was made in nonlinear formulation for estimating strength.

CALCULATION OF STRENGTH

Characteristics of steel S-345:

$Ry := 320 \cdot MPa$

$E := 2 \cdot 10^4 \dfrac{kN}{cm^2}$

2.1 Stength calculation for the top chord of the arch. It is under compression with bending:

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} \leq 1$ - formula for checking the element's strength with acting of bending moment and compressive force (20)

$\gamma c := 0.9$

Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 1771kN$

$M := 106kN \cdot m$

Characteristics of the cross-section 40K2 (Colomn I-section) from STO 29-93:

$A := 218.67cm^2$

$W := 3331.2cm^3$

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} = 0.392$ CORRECT

2.2 Strength calculation for the tie of the arch. It is under tension with bending.

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} \leq 1$ $\gamma c := 0.9$

Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 1771kN$

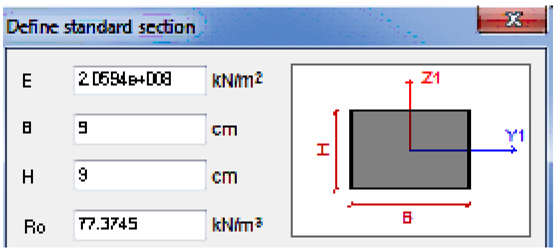
$M := 3.75kN \cdot m$

Characteristics of the cross-section 9*9cm (bar section):

$A := 81cm^2$

$W := 121.5cm^3$

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} = 0.866$ CORRECT



2.3 Strength calculation of vertical suspensions.
The calculation was made for compressive element (because its limit slenderness is less than for tension elements (20)) Formula for strength calculation for central compressive elements:

$$\frac{N}{A \cdot Ry \cdot \gamma c} \le 1 \qquad \gamma c := 0.9$$

Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 9.72 \cdot kN$

Characteristics of the cross-section 102*3mm (pipe section):

$A := 9.331 cm^2$

$ix := 3.501 cm$

$$\frac{N}{A \cdot Ry \cdot \gamma c} = 0.036$$

Vertical suspnesions should be also checked the stability:

$$\frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c} \le 1$$

- formula for checking of the element's stability with acting of central compressive force (see formula 7 in (20))

$$\lambda x1 = \lambda x \sqrt{\frac{Ry}{E}}$$

λx=lefx/ix-slenderness in x-axis plane; lefx-effective length, ix- radius of gyration of x-axis

$lefx := 587 cm$ - slenderness in plane of element

$$\lambda x := \frac{lefx}{ix} = 167.666$$

$$\lambda x1 := \lambda x \cdot \sqrt{\frac{Ry}{E}} = 6.707$$

$iy := ix = 3.501 \cdot cm$ - slenderness out of plane of element

$lefy := lefx = 587 \cdot cm$

$$\lambda y := \frac{lefy}{iy} = 167.666$$

$$\lambda y1 := \lambda y \cdot \sqrt{\frac{Ry}{E}} = 6.707$$

φ - **coefficient of stability**.It is determinate according to 7.3.1 of (20))

$\lambda 1$ is >0.4 the coefficient of stabilitiy φ is calculated according to formula 8 from (20):

From table 3 of Appendix 2:

$\alpha 1 := 0.03$

$\beta 1 := 0.06$

$\delta 1 := 9.87 \cdot (1 - \alpha 1 + \beta 1 \cdot \lambda x1) + \lambda x1^2$ (see formula 9 from (20))

$\delta 1 = 58.525$

$\varphi := 0.5 \cdot \frac{\left(\delta 1 - \sqrt{\delta 1^2 - 39.48 \cdot \lambda x1^2}\right)}{\lambda x1^2}$ (see formula 8 from (20))

$\varphi = 0.199$

$\frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c} = 0.182$ CORRECT (the element is stable in its plane and out of its plane)

CHECKING OF SLENDERNESS

Slenderness is limited by ultimate slenderness λu (according to the table 32 (20)):
For the top chord of the arch ultimate slenderness is :

$\lambda u=180-60\alpha$

For vertical suspensions ultimate slenderness is:

$\lambda u=210-60\alpha$

Where: $\alpha = \frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c}$ ($\varphi = \varphi e$ for the top chord of the arch)
According to the table 32 (20) α should be at least 0.5.

For the tie slenderness ultimate slenderness is:

$\lambda u=400$

Formula for calcultaion slenderness of elements:

λ=lef/i (according to requirement 10.4.1 (20))
lef - effective length
i-radius of gyration

Ultimate slenderness for the top chord of the arch

For the top chord of the arch (from linear calculation):

$\lambda x=102.865$

$\lambda y=49.407$

$\alpha y=0.428$

According to requirement of (20) α should be at least 0.5=> $\alpha y=0.5$

$\alpha x := 0.726$

$\alpha y := 0.5$

$\lambda ux := 180 - 60\cdot\alpha x = 136.44$

$\lambda uy := 180 - 60\alpha y = 150$

$\lambda x<\lambda ux=136.44$

$\lambda y<\lambda uy=150$ **CORRECT**

Ultimate slenderness for vertical suspensions

From nonlinear calculation:

$\lambda x=167.666$

$\lambda y=167.666$

$\alpha=0.182$

According to requirement of (20) α should be at least 0.5=> $\alpha=0.5$

$\lambda ux=\lambda uy=210-60*0.5=180$

$\lambda x<\lambda ux=180$

$\lambda y<\lambda uy=180$ **CORRECT**

Ultimate slenderness for the tie:

$ix := 2.6cm$

$iy := ix = 2.6\cdot cm$

$lefx := 813cm$ - distance between suspensions

$lefy := 813cm$ - distance between bracings

$$\lambda x := \frac{lefx}{ix} = 312.692$$

$$\lambda y := \frac{lefy}{iy} = 312.692$$

λx, $\lambda y<400$ **CORRECT**

Mathcad calculation of the 2nd type of arch with suspended tie and V-shaped struts

1. First part of the calculation was made in linear formulation for estimating <u>stability.</u>

CALCULATION OF STABILITY

1.1 Checking of the top chord of the arch . It is eccentrically compressed (compression with bending):

$$\frac{N}{\left(\varphi_e \cdot A \cdot Ry \cdot \gamma c\right)} \le 1$$

- formula for checking of the element's stability with acting of bending moment and compressive force **<u>in the plane of action of the moment</u>** (see formula 109 in (20))

$\gamma c := 0.9$

<u>Characteristics of the cross-section 40K4 (Column I-section) from STO 29-93:</u>

$A := 295.59 cm^2$

$Af := 226.8 cm^2$

$Aw := 68.79 cm^2$

$Wx := 4481.8 cm^3$

$ix := 17.72 cm$

$iy := 10.25 cm$

$Ry := 320 MPa$

$E := 2 \cdot 10^4 \frac{kN}{cm^2}$

<u>Forces from load combination (from calculation in Lira-SAPR software program):</u>

$N := 2468.1 kN$

$M := 499.89 kN \cdot m$

<u>Calculation of $\varphi e.$</u>

$lefx := 1642 cm$ - effective length (from Lira-SAPR)

$\lambda x := \frac{lefx}{ix} = 92.664$

$\lambda x1 := \lambda x \cdot \sqrt{\frac{Ry}{E}} = 3.707$

<u>Calculation of relative eccentricity m:</u>

m=eA/Wc

$$Wc := Wx = 4.482 \times 10^{3} \cdot cm^{3}$$

$$e := \frac{M}{N} = 0.203\,m$$

$$m := \frac{e \cdot A}{Wc} = 1.336$$

<u>Calculation of factor of the cross-section shape influence η (see table table 1 from Appendix 2).</u>

According to the table the type of cross-section is №5.

$$\frac{Af}{Aw} = 3.297$$

$$\lambda x1 = 3.707$$

Formula for calculation η-factor accroding to Table 1 (see Appendix 2):

$$\eta := (1.90 - 0.1 \cdot m) - 0.02 \cdot (6 - m) \cdot \lambda x1$$

$$\eta = 1.421$$

$$mef := m \cdot \eta = 1.898$$

$\varphi e := 0.289$ according to the table D2 (20)

$$\frac{N}{(\varphi e \cdot A \cdot Ry \cdot \gamma c)} = 1.003$$

<u>CORRECT</u>

$\frac{N}{c1 \cdot \varphi y \cdot A \cdot Ry \cdot \gamma c} \le 1$ - formula for checking of the element's stability with acting of bending moment and compressive force **<u>out of the plane of the moment</u>** (see formula 111 in (20))

Calculation of c1:

$mx := \left(\frac{M}{N}\right) \cdot \left(\frac{A}{Wc}\right) = 1.336$ -relative eccentricity

If mx<5 formula for calculating the factor c1:

c1=β/(1+αmx), it should be <1 (according to formula 112 (20))

α,β - coefficients that are determined according to the table 2 from Appendix 2

Type of section 2 according to the table 2 (Appendix 2)

$\alpha := 0.7$ because mx<1

β depends on non dimensional slenderness in plane of y-axis λy1

$lefy := 500cm$ - effective length in y-axis plane (500cm - distance between bracings)

$$\lambda y := \frac{lefy}{iy} = 48.78$$

$$\lambda y1 := \lambda y \cdot \sqrt{\frac{Ry}{E}} = 1.951$$ It is less than 3.14 (see the table 2 of Appendix 2)

$\beta := 1$

$$c1 := \frac{\beta}{(1 + \alpha \cdot mx)} = 0.517$$

Calculation of φy.

From table 3 of Appendix 2:

$\alpha 1 := 0.04$

$\beta 1 := 0.09$

$$\delta 1 := 9.87 \cdot (1 - \alpha 1 + \beta 1 \cdot \lambda y1) + \lambda y1^2$$ (see formula 9 from (20))

$\delta 1 = 15.016$

$$\varphi y := 0.5 \cdot \frac{\left(\delta 1 - \sqrt{\delta 1^2 - 39.48 \cdot \lambda y1^2}\right)}{\lambda y1^2}$$ (see formula 8 from (20))

$\varphi y = 0.833$

$$\frac{N}{c1 \cdot \varphi y \cdot A \cdot Ry \cdot \gamma c} = 0.673$$ CORRECT

1.2 Checking stability of vertical suspnesions:

$$\frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c} \le 1$$ - formula for checking of the element's stability with acting of central compressive force (see formula 7 in (20))

Characteristics of the pipe cross-section 168*8 :

$A := 40.21cm^2$

$ix := 5.66cm$

$iy := ix = 5.66 \cdot cm$

Forces from load combination (from calculation in Lira-SAPR software program):

$N := 463.84kN$

Effective length:

$lefx := 395cm$

$\lambda x := \frac{lefx}{ix} = 69.788$ - slenderness in its plane

$\lambda x1 := \lambda x \cdot \sqrt{\frac{Ry}{E}} = 2.792$

$lefy := lefx = 395 \cdot cm$

$\lambda y := \frac{lefy}{iy} = 69.788$ - slenderness out of plane

$\lambda y1 := \lambda y \cdot \sqrt{\frac{Ry}{E}} = 2.792$

φ - **coefficient of stability**.It is determinate according to 7.3.1 of (20))

$\lambda 1$ is >0.4 the coefficient of stabilitiy φ is calculated according to formula 8 from (20):

From table 3 of Appendix 2:

$\alpha 1 := 0.03$

$\beta 1 := 0.06$

$\delta 1 := 9.87 \cdot (1 - \alpha 1 + \beta 1 \cdot \lambda x1) + \lambda x1^2$ (see formula 9 from (20))

$\delta 1 = 19.02$

$\varphi := 0.5 \cdot \frac{\left(\delta 1 - \sqrt{\delta 1^2 - 39.48 \cdot \lambda x1^2}\right)}{\lambda x1^2}$ (see formula 8 from (20))

$\varphi = 0.748$

$\frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c} = 0.535$ CORRECT (the element is stable in its plane and out of its plane)

2.Second part of the calculation was made in nonlinear formulation for estimating strength.

CALCULATION OF STRENGTH

Characteristics of steel S-345:

$Ry := 320 \cdot MPa$

$E := 2 \cdot 10^4 \dfrac{kN}{cm^2}$

2.1 Stength calculation for the top chord of the arch. It is under compression with bending:

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} \leq 1$ - formula for checking the element's strength with acting of bending moment and compressive force (20)

$\gamma c := 0.9$

Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 2540.66kN$

$M := 678.1kN \cdot m$

Characteristics of the cross-section 40K4 (Colomn I-section) from STO 29-93:

$A := 295.39cm^2$

$W := 4481.8cm^3$

$\dfrac{N}{A \cdot Ry \cdot \gamma c} + \dfrac{M}{W \cdot Ry \cdot \gamma c} = 0.824$ CORRECT

2.2 Strength calculation for the tie of the arch. It is under tension.

$\dfrac{N}{A \cdot Ry \cdot \gamma c} \leq 1$ $\gamma c := 0.9$

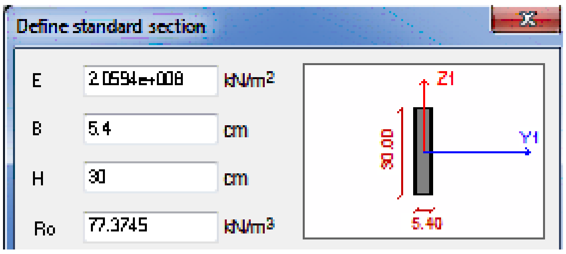
Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 2540.66kN$

Characteristics of the cross-section 5.4*30cm (bar section):

$A := 162cm^2$

$\dfrac{N}{A \cdot Ry \cdot \gamma c} = 0.545$ CORRECT



2.3 Strength calculation of vertical suspensions.
The calculation was made for compressive element (because its limit slenderness is less than for tension elements (20)) Formula for strength calculation for central compressive elements:

$$\frac{N}{A \cdot Ry \cdot \gamma c} \le 1 \qquad \gamma c := 0.9$$

Forces from nonlinear load combination (from calculation in Lira-SAPR software program):

$N := 540.35 \cdot kN$

Characteristics of the cross-section 168*8mm (pipe section):

$A := 40.21 cm^2$

$$\frac{N}{A \cdot Ry \cdot \gamma c} = 0.467$$

CHECKING OF SLENDERNESS

Slenderness is limited by ultimate slenderness λu (according to the table 32 (20)):
For the top chord of the arch ultimate slenderness is :

$\lambda u = 180 - 60\alpha$

For vertical suspensions ultimate slenderness is:

$\lambda u = 210 - 60\alpha$

Where: $\alpha = \frac{N}{\varphi \cdot A \cdot Ry \cdot \gamma c}$ ($\varphi = \varphi e$ for the top chord of the arch)
According to the table 32 (20) α should be at least 0.5.

For the tie slenderness ultimate slenderness is:

$\lambda u = 400$

Formula for calcultaion slenderness of elements:

λ=lef/i (according to requirement 10.4.1 (20))
lef - effective length
i-radius of gyration

Ultimate slenderness for the top chord of the arch
For the top chord of the arch (from linear calculation):
$\lambda x = 92.664$
$\lambda y = 48.78$

$\alpha x := 1.003$
$\alpha y := 0.673$

$\lambda ux := 180 - 60 \cdot \alpha x = 119.82$

$\lambda uy := 180 - 60 \alpha y = 139.62$

$\lambda x < \lambda ux = 119.82$
$\lambda y < \lambda uy = 139.62$ <u>CORRECT</u>

<u>Ultimate slenderness for vertical suspensions</u>
From nonlinear calculation (from linear calculation):
$\lambda x = 69.788$
$\lambda y = 69.788$
$\alpha = 0.535$

According to requirement of (20) α should be at least 0.5=> $\alpha = 0.5$

$\lambda ux = \lambda uy = 210 - 60 * 0.535 = 177.9$

$\lambda x < \lambda ux = 177.9$
$\lambda y < \lambda uy = 177.9$ <u>CORRECT</u>

<u>Ultimate skenderness for the tie:</u>

$ix := 8.66cm$
$iy := 1.559cm$

$lefx := 2909cm$ - distance between suspensions

$lefy := 600cm$ - distance between bracings

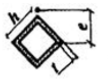
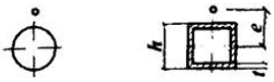
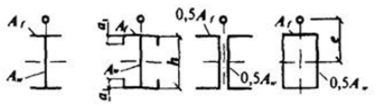
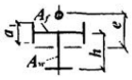
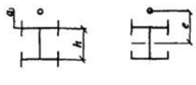
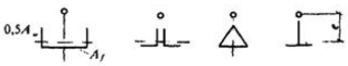
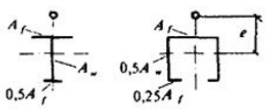
$$\lambda x := \frac{lefx}{ix} = 335.912$$

$$\lambda y := \frac{lefy}{iy} = 384.862$$

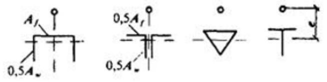
λx, $\lambda y < 400$ <u>CORRECT</u>

Appendix 2. Tables from SP 16.13330.2011 “Steel Structures”

Table 1. Determination value of factor of the cross-section shape influence (Appendix D.2 from (20))

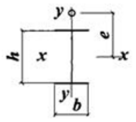
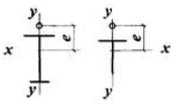
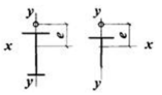
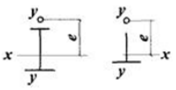
Type of section	Scheme	$\frac{A_f}{A_w}$	Value of η: $0 \le \bar{\lambda} \le 5$, $0,1 \le m \le 5$	$0 \le \bar{\lambda} \le 5$, $5 < m \le 20$	$\bar{\lambda} > 5$, $0,1 \le m \le 5$	$\bar{\lambda} > 5$, $5 < m \le 20$
1		-	1,0	1,0	1,0	
2	 $\frac{t}{h} = 0,25$	-	0,85	0,85	0,85	
3		-	$0,75 + 0,02\bar{\lambda}$	$0,75 + 0,02\bar{\lambda}$	0,85	
4	 $\frac{t}{h} = 0,25$	-	$(1,35 - 0,05m) - 0,01(5 - m)\bar{\lambda}$	1,1	1,1	
5	 $\frac{a_1}{h} \le 0,15$	0,25	$(1,45 - 0,05m) - 0,01(5 - m)\bar{\lambda}$	1,2	1,2	
		0,5	$(1,75 - 0,1m) - 0,02(5 - m)\bar{\lambda}$	1,25	1,25	
		≥ 1,0	$(1,90 - 0,1m) - 0,02(6 - m)\bar{\lambda}$	$1,4 - 0,02\bar{\lambda}$	1,3	
6	 $\frac{a_1}{h} \le 0,15$	-	$\eta_5\left[1 - 0,3(5 - m)\frac{a_1}{h}\right]$	η_5	η_5	
7	 $\frac{a_1}{h} \le 0,15$	-	$\eta_5\left(1 - 0,8\frac{a_1}{h}\right)$	$\eta_5\left(1 - 0,8\frac{a_1}{h}\right)$	$\eta_5\left(1 - 0,8\frac{a_1}{h}\right)$	
		0,25	$(0,75 + 0,05m) - 0,01(5 - m)\bar{\lambda}$	1,0	1,0	
		0,5	$(0,5 + 0,1m) + 0,02(5 - m)\bar{\lambda}$	1,0	1,0	
		≥ 1	$(0,25 + 0,15m) + 0,03(5 - m)\bar{\lambda}$	1,0	1,0	
9		0,5	$(1,25 - 0,05m) - 0,01(5 - m)\bar{\lambda}$	1,0	1,0	
		≥ 1	$(1,5 - 0,1m) - 0,02(5 - m)\bar{\lambda}$	1,0	1,0	
10		0,5	1,4	1,4	1,4	1,4
		1,0	$1,6 - 0,01(5 - m)\lambda$	1,6	$1,35 + 0,05m$	1,6
		2,0	$1,8 - 0,02(5 - m)\lambda$	1,8	$1,3 + 0,1m$	1,8

2 (2)

Type of section	Scheme	$\frac{A_f}{A_w}$	Value of η			
			$0 \le \bar{\lambda} \le 5$		$\bar{\lambda} > 5$	
			$0,1 \le m \le 5$	$5 < m \le 20$	$0,1 \le m \le 5$	$5 < m \le 20$
11		0,5	$1,45 + 0,04m$	1,65	$1,45 + 0,04m$	1,65
		1,0	$1,8 + 0,12m$	2,4	$1,8 + 0,12m$	2,4
		1,5	$2,0 + 0,25m + 0,1\lambda$	-	-	-
		2,0	$3,0 + 0,25m + 0,1\lambda$	-	-	-

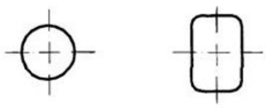
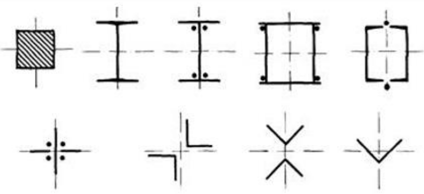
Note: In Mathcad calculation $\bar{\lambda} = \lambda 1$

Table 2. Determination of coefficients α, β (Table 21 from (20))

Type of section	Sheme	Values of coefficients			
		α when		β when	
		$m_x \le 1$	$1 < m_x \le 5$	$\bar{\lambda}_y \le 3{,}14$	$\bar{\lambda}_y > 3{,}14$
1		0,7	$0,65 + 0,05m_x$	1	$\sqrt{\varphi_c / \varphi_y}$
2					
3					
4		$1 - 0,3I_2/I_1$	$1 - (0,35 - 0,05m_x)I_2/I_1$	1	$1-\left(1-\sqrt{\varphi_c/\varphi_y}\right)\left(2I_2/I_1-1\right)$; β = 1 при $I_2/I_1 < 0,5$

Note: In Mathcad calculation $\overline{\lambda y} = \lambda y 1$

Table 3. Determination values of α, β (Table 7 from (20))

Type of section		Values of coefficients	
	Shape	α	β
a		0,03	0,06
b		0,04	0,09
c		0,04	0,14

Note: In Mathcad calculation $\alpha = \alpha 1$, $\beta = \beta 1$