

# Simulation-based analysis of performance of automated storage and retrieval system

Storage/retrieval (S/R) crane performance

Renata Kudryavtseva

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Jyväskylän ammattikorkeakoulu JAMK University of Applied Sciences

# jamk.fi

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Lähdevaara Hannu, Lehtola Pasi

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Abstract

Automated storage and retrieval system is one of the major material handling systems, which are extensively used in automated productions and distribution centers. AS/RS has been utilized as a part of advanced manufacturing and as an alternatives to traditional warehouses. AS/RS is greatly suitable for work-in-progress storage in factories. In addition, it can simplify inventory control and improve cost-effective utilization of time, as well as space and equipment.

Automated handling systems have been used in warehouses since 1950s, initially focusing on heavy pallet loads. However, with evolution of the AS/RS, types of automated storages have become smaller and are designed for any load sizes. The advantages of AS/RS are intended for moving loads with high volume and materials handling speed without interference of an operator. The main disadvantages are high investment costs, less flexibility, and higher investment costs in control system.

Statistical approaches are in trend nowadays. Statistics as science have developed in 17 century, but only now it has got tools to implement its principals. Due to development of ITtechnologies, statistical calculations have become easier with MS Excel, Simulation software and Mathematics software. For example, queueing theory's method had taken a lot of hours, if somebody would make this one on paper manually. However, now all calculations are done with computers and time required to develop simulation model have decreased significantly.

It's a good approach to apply statistical rules to analyze performance of AS/RS. The simulation approach will show better relationships between components of AS/RS and a dynamic system's parts.

Keywords/tags: Automated storage and retrieval system, warehouse design and control, simulationbased analysis, storage and retrieval machine

Miscellaneous (Confidential information)

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# 1 Introduction

Since information technologies have come in to the modern life, business environment was captured by massive data which requires a lot of technics for analyzing it. It is a difficult task for any successful manager who is able to work with this big quantity of data and make decisions by implementing different methods for information analyzing. *Nowadays, business tendency is being increased towards more product variety and a short life cycle of products*. Consequently, there are needs for automation of manufacturing, distribution centers and production. In addition, logistics operations and coordination between global supply chain parts is required.

One of the main a value-added figure of global supply chain is Warehousing. There are many types of warehouses which are used. Warehouse defines not only storage raw materials or finished products means, but also plays key role in relationships between other parts of global supply chain. In distribution center circumstances, for example, warehouse is a main figure in supply chain, but in manufacture it could be transitional point. There will not tell about warehousing principals or operations, but it will be discussed *how an automated storage and retrieval system is used instead of traditional racking*.

Automated storage and retrieval system is the major trend in material handling systems which have been utilized as alternative to traditional warehouse without interference of an operator. AS/RSs can play an essential role in modern factories for workin-process storage and offer the advantages of improved inventory control and costeffective utilization of time, space and equipment (Vasili & Tang, 2016, February). The AS/RS can lead to substantial savings for a company by reducing direct and indirect labor, energy, maintenance, and building costs (Sarker & Babu, 1995).

Evaluating **AS/RS** performance is challengeable due to the system operates in **high stochastic or dynamic environment**. According to Jean-Philippe Garliardi (Gagliardi, et al., 2012), there are various models for evaluating AS/RS performance, including static models and dynamic models. Static models are used to predict AS/RS steady-state performance using analytical expressions. On the other hand, simulation models reproduce the dynamics of AS/RS and performance may be stated by means of statistical tools.

*Statistical approaches are in trend nowadays*. Statistics as science have been developed in 17 century, but only now it has got tools to implement its principals. Due to development of IT-technologies, statistical calculations have become easier with MS Excel, Simulation software and Mathematics software. For example, queueing theory's method had taken a lot of hours, if somebody would make this one on paper manually. However, now all calculations are done with computers and time required to develop simulation model have decreased significantly.

It's a good approach to apply *statistical rules to analyze performance of AS/RS*. The simulation approach will show better relationships between components of AS/RS and a dynamic system's parts.

# 2 Research problem and methods

# 2.1 Research Problem

**Researched phenomenon** is an automated storage and retrieval system performance. To be more precisely a performance of storage and retrieval machines, because they evolve all AS/RS's operations.

Initially, *common warehouse principals* will be explained and in which part of traditional warehouse an AS/RS can be used. After that, methods which are applied for analysis of performance of warehouse will be revealed. Also, a structure and operation principals of an AS/RS will be shown.

The performance of AS/RS is the most important in design stage due to high interest of customers to achieve smallest investment costs possible. In design study, there is *no real existing system*, but parameters of AS/RS and customer's requirements of input/output production only. Without a real existing system, it is difficult to define size, speed, and quantity of aisles of AS/RS. For this reason and since an automated storage/retrieval system is a dynamic system, a good way to analyze performance is simulation model. The model will be made with *simulation software* "Enterprise dynam*ics, 10.1*" created by the company "Incontrol simulation solutions", the Netherlands. *Each simulation model should be validated*. The validation stage includes checking following parameters: a) is the travel time of cranes calculated right, b) is distribution laws of arriving products and server performance chosen correctly, and c) can we trust to simulation outputs, e.g. throughputs.

For the validation of the simulation model, analytical solution will be applied. *The analytical solution is "pencil and paper" method*, where close-form expressions, e.g. formulas will be used. Of course, in hi-tech century instead a pencil and a paper, there will be used *MS Excel and Mathcad*.

In case, if the *same results with errors of less than or equal to 10%* will be got from the simulation model and the analytical solution, it means that the simulation model is credible and can be applied in future.

In final stage it will identify advantages and drawbacks of the simulation model

# 2.2 Research questions

The research problem can be solved by asking the right research question (s). As the answers and solutions are based on the question (s), the form of the question is important. Typical forms are as follows: What? How? Why? How much? (Kananen, 2011, p. 19).

# 2.3 Research methods

The main types of research methods are *qualitative and quantitative approaches*. This division is based on types of data. Typically, data is called quantitative if it is in numerical form and qualitative if it is not. Notice that qualitative data could be much more than just words or text. Photographs, videos, sound recording, and so on, can be considered qualitative data. (Trochim & Donnelly, 2008, p. 11)

Therefore, *qualitative research* is a technique when it is needed to understand a phenomenon or an event. The qualitative approach leads to answer of question "What is the phenomena and what does it stand for". According to Kananen Jorma, (Kananen, 2011, p. 37) if the phenomena and its factors *("what")* and their interdependencies are unknown, the researcher must define the phenomena first. There may be no theories or models available on the phenomena, the aim is to acquire a deep understanding of the phenomena, and the aim is to create new theories and hypotheses.

*Quantitative research* is like a train proceeds stage by stage in accordance with statistical rules. Like in all other forms of research the process starts with establishing the research problem. There are clear phenomena and well understood, and the variables and the correlations between them are well-known. (Kananen, 2011, p. 72). Qualitative research approach responses answer *"So what?"* 

# 2.3.1 Quantitative research method

There is a quantitative research method used for solving problem, because all of *input and output data are numerical* and phenomena is known. To get problem solved *statistical tools* will be used. It will measure quantity of products through a system. The AS/RS system will be described in simulation model which always requires numbers.

The simulation specialist should use quantitative techniques whenever possible to test the validity of various components of the overall model. (Law & Kelton, 2000, p. 277)

# 2.3.2 Constructive research method

The aim of *constructive research* is *to solve practical problems* while producing an academically appreciated theoretical contribution. The solutions, that is, constructs, can be process, practices, tools or organization charts.

#### The research process involves the following:

- 1. Selecting a practically relevant problem.
- 2. Obtaining a comprehensive understanding of the study area.
- 3. Designing one or more applicable solutions to the problem.
- 4. Demonstrating the solution's feasibility.

5. Linking the results back to the theory and demonstrating their practical contribution.

6. Examining the general inability of the results. (Lehtiranta, et al., N.d.).

In research process theoretical principals of operation of automated storage and retrieval system will be studied and statistical tools will be applied to calculation of outputs of the system. Definitions of components of AS/RS will be reviewed. Furthermore, it will be explained how statistics tools to solve performance of AS/RS.

According to constructive research method principals, *case will illustrate performance of crane* of an automated storage and retrieval system in simulation software in *2D* as well as in *3D visions*. Applied solutions to the problem will be designed and throughput of AS/RS will be calculated and analyzed.

# 3 Warehousing fundamentals

# 3.1 Why have a Warehouse?

*Warehouses* play vital roles in the supply chain (see Figure 3-1).



Figure 3-1. The roles of a warehouse in logistics and supply chain management

*Row material and component warehouses* hold raw materials at or near the point of introduction into a manufacturing or assembly process.

*Work-in-progress warehouses* hold partially competed assemblies and products at various points along an assembly or production line.

*Finished goods warehouses* hold inventory used to balance and buffer the variation between production schedules and demand. For this purpose, the warehouse is usually located near the point of manufacture and is often characterized by the flow of full pallets in and full pallets out, assuming that product size and volume warrant pallet-sized loads.

**Distribution warehouses and distribution centers** accumulate and consolidate products from various points of manufacture within a single firm, or from several firms, for combined shipments to common customers.

*Fulfillment warehouses* and fulfillment centers receive, pick, and ship small orders for individual consumers.

**Local warehouses** are distributed in the field in order to shorten transportation distances to permit rapid response to customer demand. Frequently, single items are picked, and the same item may be shipped to the customer every day.

*Value-added service warehouses* serve as the facility where key product customization activities are executed, including packaging, labeling, marking, pricing, and returns processing. (Frazelle, 2002).

# 3.1.1 Common warehouse operations

Despite the name or role, warehouse operations have *fundamental activities* in common. According to Frazelle, (Frazelle, 2002, p. 8) the following list includes the activities found in most warehouses. These tasks, of functions, are also indicated on a flow line in Figure 3-2 to make it easier to visualize them in actual operation.

- 1. Receiving;
- 2. Prepackaging (optional);
- 3. Pick-up to storage;
- 4. Storage;
- 5. Order picking;

- 6. Packaging and/or pricing (optional);
- 7. Sortation and/or accumulation;
- 8. Unitizing and shipping.



Figure 3-2. Warehousing in logistics framework

#### 3.1.2 Logistics and Warehouse costs

Logistics costs account for a big part of supply chain costs. As a rule, logistics costs include transportation, warehousing, inventory carrying, customer service and order processing, administration, and other costs. Depending on activity of company, logistics costs vary 5-9%. In Table 3-1, example of logistics costs of The Establish, Inc. in 2010 (Logistics\_costs, 2010) are shown.



Table 3-1. Average percentage of logistics costs in % of sales

In any supply chain, Warehouse is a value-added segment, thus, percentage of value which will be added to common costs of supply chain depends directly on performance of warehouse. For reason of decreasing of global costs, a warehouse performance requires systematic analysis. The analysis allows to diagnose the problems in the current warehouse operations, to improve processes in warehouse and to decrease warehouse operation costs.

# 3.1.3 Warehouse performance measures

Warehouse measures are focused on the consumption of warehouse resources – people, space, and systems – to meet the mission of the warehouse: shipping perfect (right products, right quality, on time, damage-free, right paperwork) orders and storing product efficiently. The warehouse is accountable to the same competitive indicators the business is held to. Business competes on the basis of *financial, productivity, quality, and cycle time performance*. (Frazelle, 2002, p. 52) *Warehouse financial performance* establishes each warehouse as activity-based costing program. Each warehousing activity (receipt, putaway, store, pick, ship, and load) is established as costs for feasibility. An example appears in Table 3-2.

	Labor cost	Space cost	MHS cost	WMS cost	Total cost
Receving	1 963 055 €	238 125€	569 820€	218 333€	2 989 333 €
Putaway	1 090 534 €	0€	416 000€	240 333€	1 746 867€
Storage	999 640€	1 933 250 €	1 650 710€	123 833€	4 707 433 €
Picking	1 946 966 €	0€	1 830 782€	161 833€	3 939 581 €
Consolidation	287 188€	100 500 €	135 000 €	38 333€	561 021€
Delivery	68 225 €	50€	69 000 €	38 333€	175 608€
Marketing	3 534 218 €	105 000€	222 200€	113 833€	3 975 251 €
Returns	68 225 €	99 250 €	6 000 €	113 000€	286 475€
	9 958 051 €	2 476 175€	4 899 512€	1047831€	18 381 569€

Table 3-2. Warehouse financial performance

*Warehouse productivity performance* is the most popular and traditional measure. *Productivity* is the ratio of the output of a resource to the inputs required to achieve that output. Key assets in the warehouse productivity are labor, space, material handling systems, and warehouse management systems. In this case, labor productivity can be a very misleading indicator. Labor productivity is the ratio of units, orders, lines, or weight shipped out of the warehouse to the number of hours spent in operating, supervising, and managing the warehouse.

*Warehouse quality performance* according to Frazelle (Frazelle, 2002, pp. 54-55) has four key quality indicators – two for inbound handling and two for outbound handling:

*Pick-up accuracy* is the percent of items pick-up correctly.

*Inventory accuracy* is the percent of warehouse locations without inventory discrepancies.

Dispatching accuracy is the percent of order line picked without errors.

*Shipping accuracy* is the percent of order lines shipped without errors. (Frazelle, 2002, pp. 54-55).

Warehouse cycle time performance is measure in two key areas:

*Dock-to-Stock Time (DTS*) is the elapsed time from when a receipt arrives on the warehouse premises until it is ready for picking or shipping.

*Warehouse order cycle time (WOCT)* is the elapsed time from when an order is released to the warehouse floor unit it is picked, packed, and ready for shipping. (Frazelle, 2002, pp. 54-55)

Summary of indicators of warehouse performance have been presented in Table 3-3.

	Financial	Productivity	Utilization	Quality	Cycle Time
Receiving	Receiving cost per receiving line	Receipts per man-hour	% Dock door utilization	% Receipts processed accurately	Receipt processing time per receipt
Pick-up	Pick-up cost per pick-up line	Pick-up per man-hour	% Utilization of pick-up labor and equipment	% Perfect pick-up	Pick-up cycle time (per pick-up)
Storage	Storage space cost per item	Inventory per cost square foot	% Locations and cube occupied	% Locations without inventory discrepancies	Inventory days on hand
Order picking	Picking cost per order line	Order lines picked per man-hour	% Utilization of picking labor and equipment	% Perfect picking lines	Order picking cycle time (per order)
Shipping	Shipping cost per customer order	Orders prepeared for shipment per man-hour	% Utilization of shipping docks	% Perfect shipments	Warehouse order cycle time
Total	Total cost per order, line, and item	Total lines shipped per total man-hour	% Utilization of total throughput and storage capacity	% Perfect warehouse orders	Total warehouse cycle time= Dock-to-stock time+ Warehouse order cycle time

Table 3-3. Indicators summary of warehouse performance

#### 3.1.4 Warehouse pallet storage and retrieval systems

**Pallet storage and retrieval systems** selection decisions is an important stage in warehouse design. Independent solution can improve storage density, decrease inventory, save warehouse space and increase turn-over of warehouse.

Parameters which increase speed or cycle time of warehouse are different kind of transportations driving between storage racks. *The most popular retrieval systems* are: walkie stackers, counterbalance lift trucks, straddle trucks, straddle reach trucks, sideloader trucks, turret trucks, hybrid trucks, and automated storage and retrieval system (AS/RS). In table 3-4 retrieval system comparison. (Frazelle, 2002)

	Counter-balance	Straddle	Straddle Reach	Side-loader	Turret	Hybrid	AS/RS
Vehicle cost	\$30 000,00	\$35 000,00	\$40 000,00	\$75 000,00	\$95 000,00	\$125 000,00	\$200 000,00
Lift height capacity (m)	22	21	30	30	40	50	75
Aisle width (m)	10-12	7-9	6-8	5-7	5-7	5	4,5
Weight capacity (tonn)	2-10	2-6	2-5	2-10	3-4	2-4	2-5
Lift speed (km/h)	1,46	1,10	0,91	0,91	1,37	1,10	3,66
Travel speed (km/h)	10,06	8,60	8,96	8,05	8,96	8,96	12,80

Table 3-4. Pallet retrieval systems comparison

For research goals, not all warehouse operations will be chosen, only influence of an automated storage and retrieval system on warehouse performance. According to classification of warehouse operations, naturally, automated storage and retrieval system is not a warehouse in whole mean, but it is a part of warehousing. Actually, an AS/RS contains three warehousing operations: putaway or pick up to storage, storage and order picking or dispatching from warehouse.

# 3.2 Automated storage and retrieval system

Automated storage and retrieval systems (AS/RSs) can be defined as a combination of equipment and controls which automatically handle, store and retrieve materials with great speed and accuracy, without direct handling by a human worker. (Vasili & Tang, 2016, February). In automated storage the control of system occurs under computer and a human participation is only needed to input data to the computer. Automated storage systems divide in two general types: fixed-aisle automated storage/retrieval system (AS/RS) and carousel storage system (Groover, 2016, p. 333). In a thesis will be a talk about only automated storage and retrieval system (see Figure 3-3) known also like highly automated storage and for the thesis aims it will be used the AS/RS name.



Figure 3-3. Automated storage and retrieval system structure

# 3.2.1 The AS/RS definitions and components

At the beginning, it must be said that automated storage systems have an own terminology which will used further. AS/RS is storage system consisting of one or more aisles of storage racks attended by *storage/retrieval machines (S/R machines)*, usually one S/R machine per aisle. The S/R machines (sometimes referred to as cranes) are used to deliver materials to the storage racks and to retrieve units from the racks. Each AS/RS aisle has one or more input/output stations where units a delivered into the storage system and withdrawn from it. The input/output stations are called *pickupand-deposit stations (P/D stations)* (Groover, 2016, p. 337).

An automated storage system consists of following components shown in Figure 3-4 such as *storage space or structure, S/R machines, storage racks and P/D stations*.



Figure 3-4. Automated storage and retrieval system components

Made of steel *the storage structure* supports the units contained in AS/RS, it may also be used to support the roof of building of location an automated storage system. Another function of the storage structure is support *the aisle hardware* required to align the S/R machines. The hardware includes guide rails at the top and bottom of the structure as well as end stops, and other features required for safe operation (Groover, 2016, p. 338).

*S/R machine, also known as crane or stacker crane, and shuttle system* (see Figure 3-5), is used to perform storage operations, transferring loads among the storage from the input station to the storage and from storage to the output station. To perform tasks of AS/RS, the S/R machine must perform of horizontal and vertical travels, so for these aims S/R machine has of a rigid mast on which includes an elevator system for vertical motion and wheels to permit horizontal travel along a rail system. According to Mikell P. Groover (Groover, 2016, p. 339) modern S/R machines are available with horizontal speeds up to 3, 33 m/s and vertical or lift speeds up to around 0.83 m/s. These speeds determine the time required for the S/R machine to travel from P/D station to a particular location in the storage aisle. The S/R machine is accomplished by any of several mechanisms, including forks for pallet loads and friction devices for flatbottom tote pans.



Figure 3-5. Storage and retrieval machine structure

*The pickup-and-deposit station, also known as input/output (I/O) point,* is some point where the shuttle system takes in to move to storage or bring loads from storage. The P/D station generally is located in ends of aisles and to handle loads at the P/D station can used manual load/unload, forklift truck, conveyor, and automated guided vehicle system (AGVS).

From mechanics point of view, the structure of an automated storage and retrieval system does not seem to be complicated, but it is a very challengeable mechanism for operations due to difficult computer (information technology) control. The system includes a huge number of sensors which send signals to logic controllers, design must be responded to aims, also automated storage usually connected with a Warehouse management system (WMS) to receive (process) tasks to perform the basic functions.

Load identification is the primary role of automatic identification of AS/RS. The scanners are located at the induction or transfer location, to scan a product identification code. The data are sent to AS/RS computer, which upon receipt of load identification, assigns and directs the load of the storage location (Vasili & Tang, 2016, February).

Designers must first configure the storage rack in terms of their numbers, length, height and depth; these racks define the system's storage capacity and required floor space. In addition, the number of cranes and their degree of freedom must be set. Another crucially important decision that designers have to make concerns the number and location of P/D stations, where the cranes can interface with upstream and downstream nodes of the material-handling system. All these design decisions greatly

influence the required initial investments and, later, determine the maximum throughput that the system can achieve. (Gagliardi, et al., 2012).

# 3.2.2 AS/RS performance

Since the automated storage and retrieval system requires a significant investment and high operating costs, it must be used as efficient as possible. Design of warehouse has to be designed prior to its implementation and subsequent use to have efficient utilization of storage areas. In the design stage it should be decided that it can handle current and future demand requirements, avoiding overcapacity and bottlenecks. As a matter of fact, for making such decision it must be determined the AS/RS performance.

As it was being said in subchapter 3.5, an AS/RS perform a part of all warehouse operations. For analyzing, three warehouse operations are needed: pick-up to storage, storage and dispatching from storage. As results, it will be calculated *dock-to-stock time*, *e.g. single- and dual-command cycle time*, *warehouse order cycle time*, *e.g. average products storage in warehouse*, *products quantity or throughputs over time*.

#### 3.2.3 Travel time model of AS/RS crane

One an important measure in AS/RS performance is *crane's* **travel time** which is the service time for a transaction including both stacker crane travel time and pickup/deposit time. The pickup/deposit time is generally independent of the rack shape and travel velocity of the S/R machine. Therefore, the travel time for an AS/RS is the time used by S/R machine to move from its P/D station to the location of requested item and lastly return to its I/O station position. (Vasili & Tang, 2016, February)

The performance of AS/RS depends mainly on the operating policy of the S/R machine's *cycle*, which can be *a single-command and a dual-command cycle*. The two types of cycle used in AS/RS design are shown in Figure 3-6. According by Sarker (Sarker & Babu, 1995) the two connecting arms between I/O and storage or retrieval point are called *the legs*, and the arm(s) between the storage (retrieval) and retrieval (storage) points are called *time-between (TB)*. For a single command (SC) cycle there is no time-between but two legs. 1. *Single command cycle (SC)* is a single storage or retrieval is performed and the storage/retrieval cycle time is equal to the sum of times for picking up a load at the I/O station (at the end of the aisle), traveling to the storage location, depositing the load in the rack, and retuning to the I/O station.



Figure 3-6. Single-command and Dual-command cycles

2. *Dual command cycle (DC)* is a storage and a retrieval are performed in a cycle, and the cycle time is the sum of pick-up time, travel time to storage locations, time to unload, travel time to retrieval point, time to load and the time to return to I/O station and the unloading time.

# 4 Simulation

*Simulation* is one of the most widely used operations-research and management-science techniques, if not the most widely used. One indication of this is the Winter Simulation Conference, which attracts 600 to 700 people every year. In addition, there are several simulation vendor users' conferences with more than 100 participants per year. (Law & Kelton, 2000)

*Simulation modeling* is an important tool in research and development for predicting and characterizing, analyzing and understanding, and solving real-world problems. All of modeling theories use statistical or other mathematical methods which describe an

optimal decision making. Modeling, particularly simulation, is a logical process concentrated (focused) on outcomes and used to derive the decision based on mathematics patterns. (Sokolowski & Banks, 2012)

According to Law A. M. (Law & Kelton, 2000, p. 2) application areas for simulation are numerous and diverse. Below is a list of some *particular kinds of problems* which simulation has been found to be a useful and powerful tool:

- Designing and analyzing manufacturing systems;
- Evaluating military weapons systems or their logistics requirements;
- Determining hard ware requirements or protocols for communications networks;
- Determining hardware and software requirements for a computer system;
- Designing and operating transportation systems such as airports, freeways, ports, and subways;
- Evaluating designs for service organizations such as call centers, fast-food restaurants, hospitals, and post offices;
- Reengineering of business processes;
- Determining ordering policies for an inventory system;
- Analyzing financial and economic systems.

# 4.1 Discrete-event simulation

**Discrete-event simulation** by Law A.M. (Law & Kelton, 2000) is the modeling of a system as it evolves over time by a representation in which the state variables change instantaneously at separate point in time. In more mathematical terms, the system can change at only a countable number of points in time. These points in time are the ones at which an *event* occurs, where an event is defined as an instantaneous occurrence that may change the state of the system.

Discrete – event simulation is always based **on the queueing theory principals**. Simple explanation is presented below (see Figure 4-1).



#### Figure 4-1. Queueing model example

The queueing system analysis is usually based on three types of estimates: the expected average delay in queue of the n customers completing their delay during the simulation, quantity of customers in system over given time and server idle/busy status. Time of arriving customers and server(s) are independent variables in statistical terms. Customers delay and number of customers in system are dependent variables, respectively. Independent variables follow some distribution which is defined by statistical laws.

# 4.2 Validation of simulation model

Example. An organization paid a consulting company € 500 000 go perform "simulation study". After the study was supposedly competed, a person from the client organization called and asked, "*Can you tell me in five minutes on the phone how to validate our model?*" (Law & Kelton, 2000, p. 265)

One of the most *difficult problems* by Averill M. Law (Law & Kelton, 2000, p. 264) facing a simulation analyst is that on trying to determine whether a simulation is an accurate representation of the actual system being studied, i.e. whether the model is valid. A simulation model and its results have *credibility* if the manager and other key project personnel accept them as "correct".

Model assumptions fall into two general classes: structural assumptions and data assumptions. Structural assumptions involve questions of how the system operates and usually involve simplifications and abstractions of reality. For example, consider the customer queueing and service facility in a bank. Customers can from one line, or there can be an individual line for each teller. If there are many lines, customers could be served strictly on a first-come-first-served basis, or some customers could change lines if one line is moving faster. The number of tellers could be fixed and variable. *Structural assumptions* should be verified by actual observation during appropriate time periods and by discussions with managers and tellers regarding bank policies and actual implementation of these policies. (Banks, et al., 2010, p. 415)

**Data assumptions** should be based on the collection of reliable data and correct statistical analysis of the data. The reliability of the data was verified by consultation with bank manager, who identified typical rush hours and typical slack times. When combining two or more data sets collected at different time, data reliability can be further enhanced by objective statistical tests for homogeneity of data. (Banks, et al., 2010, p. 415).

For the reason, that performance of AS/RS is computed on design study and there is **no a real existing system**, **the validation** will be done with a use of **analytical solution** (see explanation in subchapter 4.2.1.). The analytical solution includes the Travel time model and throughput based on travel time. The Travel time model will be used for validation of calculations of single- and dual-command cycle time of the crane.

# 4.2.1 Travel time model by mean time

By using *the Travel time model method*, single- and dual-command cycles of an AS/RS can be computed. This approach contains *closed-form expressions* (AS/RS\_throughput\_analysis, 2003). There are following assumptions used:

*L* = length of storage (m), *H* = high of storage (m), *Sh* = horizontal velocity (m/s), *Sv* = vertical velocity (m/s), *Th* = time in horizontal direction, *Tv* = time in vertical direction, *T* = max (*Sh*,*Sv*), *Q* =  $\frac{min(Sh,Sv)}{T}$ , *T<sub>pd</sub>*=pick-up/deposit time.

#### Single-command cycle's expected time with random storage

$$E(SC) = T \cdot \left(1 + \frac{Q^2}{3}\right) + 2 \cdot Tpd$$

Dual-command cycle's expected time with random storage

$$E(DC) = \frac{T}{30} \cdot \left(40 + 15 \cdot Q^2 - Q^3\right) + 4 \cdot Tpd$$

The idea of the validation is that if the simulation model's and the analytical solution's outputs will be the same (with errors of less than or equal to 10%), consequently, the simulation model is a credible and it is possible to trust the results.

# 5 Selecting probability

# 5.1 Review of basic probability and statistics

An experiment is a process whose outcome is not known with certainty. The set of all possible outcomes of an experiment is called the *sample space* and is denoted by *S*. The outcomes themselves are called the *sample points* in the sample space.

A random variable is a function (or rule) that assigns a real number (any number greater than  $-\infty$  and less than  $\infty$ ) to each point in the sample space S. In general, random variables are denoted by capital letters such as X, Y, Z and the values that random variables take on by lowercase letters such as x, y, z.

A random variables X is said to be *discrete* if it can take on at most a countable number of values. Countable means that the set of possible values can be put in a one-to-one correspondence with the set of positive integers. *Continuous variables* are uncountable and for themselves the probability that X lies in the interval [a, b] is given by:

$$P(a \le X \le b) = \int_{a}^{b} f(x) \, \mathrm{d}x$$

The probability density function (PDF) means, that all statements about X can be computed from f(x) for all continuous random variables X equal x.

*The distribution function* (sometimes **cumulative distribution function (CDF)**) F (*x*) of the random variable X is defined for each real number x as follows:

$$F(x) = P(X \le x)$$
 for  $-\infty < x < \infty$ 

where P ( $X \le x$ ) means the probability associated with the event [ $X \le x$ ]. Thus, F (x) is the probability that, when the experiment is done, the random variables X will have taken on a value no larger than the number x. (Law & Kelton, 2000, p. 237) In this paper, discrete random variables are not considered, thus, there are listed popular distributions of continuous variables.

The aim of the next subchapter is to discuss distributions that have been found to be useful in simulation modeling and to provide properties of these distributions. All distributions definitions have been given in Averill M. Law' book, chapter 6 (Law & Kelton, 2000, pp. 298-329)

# 5.2 Continuous probability distributions

# 5.2.1 Uniform distribution

**Uniform distribution** is one where a random variable X is uniformly distributed on the interval [a, b] if its pdf and cdf (Figure 5-1) are given by:

$$PDF = f(x) = \frac{1}{b-a}$$

$$CDF = F(x) = \frac{x-a}{b-a}$$

The uniform distribution is used as a "first" model for a quantity that is felt to be randomly varying between a and b, but about which little else in known.



Figure 5-1. Uniform pdf and cdf

5.2.2 Exponential distribution

**Exponential distribution** has a random variable X is said to be exponentially distributed with parameter  $\lambda > 0$  if its pdf and cdf (Figure 5-2) are given by:

$$PDF = f(x) = \lambda \cdot e^{\lambda x}$$

$$CDF = F(x) = 1 - e^{-\lambda x}$$

Possible applications is interarrival times of "customers" to a system that occur at constant rate, or time to failure of a piece of equipment.





Figure 5-2. Exponential pdf and cdf

# 5.2.3 Normal distribution

**Normal distribution** (Figure 5-3) is continuous for all variables of X between  $-\infty < x < \infty$ . It has characteristic symmetrical shape, which means that the mean, the median and the mode have the same numerical value. (Kumar, et al., 2010).

$$PDF = f(x) = \frac{1}{\sqrt{2 \pi \sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$CDF = F(x) = \frac{1}{2} \left[ 1 + \frac{erf(x-\mu)}{\sigma \sqrt{2}} \right]$$



Figure 5-3. Normal pdf and cdf

# 5.2.4 Lognormal distribution

Lognormal distribution LN ( $\mu$ ,  $\delta^2$ ) is possible to define time to perform some task or quantities that are the product of a large number of other quantities.

$$egin{aligned} &f(x) \coloneqq rac{1}{\left(x \cdot \sigma \cdot \sqrt{2 \cdot \pi}
ight)} \cdot e^{rac{-\left(\ln\left(x
ight) - \mu
ight)^2}{2 \cdot \sigma^2}} \ &F(x) \coloneqq rac{1}{2} + rac{1}{2} \, rac{\left(\ln\left(x
ight) - \mu
ight)}{\sqrt{2 \cdot \sigma}} \end{aligned}$$

Plots of Lognormal distribution pdf and cdf are presented in Figure 5-4.



Figure 5-4. Lognormal pdf and cdf

#### 5.2.5 Weibull distribution

*Weibull distribution's* application is time to compete some task, time to failure of a piece equipment. Plots of Weibull distribution pdf and cdf are presented in Figure 5-5.

$$PDF = f(x) = \frac{\kappa}{\lambda} \left(\frac{x}{\lambda}\right)^{\kappa - 1} e^{-\left(\frac{x}{\lambda}\right)^{\kappa}}$$

$$CDF = F(x) = 1 - e^{-\left(\frac{x}{\lambda}\right)^{\kappa}}$$



Figure 5-5. Weibull pdf and cdf

#### 5.2.6 Gamma distribution

*Gamma distribution* can be used for time to compete some task, e.g., customer service or machine repair. Plots of Gamma distribution pdf and cdf are presented in Figure 5-6.

$$PDF = f(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha - 1} e^{-\beta x}$$

$$CDF = F(x) = \frac{1}{\Gamma(\alpha)} \gamma(\alpha, \beta x)$$



Figure 5-6. Gamma pdf and cdf

# 5.3 Hypothesizing a distributional form

A major task in simulation is the collection and analysis of input data. One of the first steps in this task is hypothesizing distributional form. There are steps in the development of a useful model of input data:

1. *Collect data from the real system of interest*. This often requires a substantial time and resource commitment. Unfortunately, in some situations it is not possible to collect data (for example, when time is extremely limited, when the input process does not yet exist, or when laws or rules prohibit the collection of data). When data are not available, expert opinion and knowledge of the process must be used to make educated guesses.

2. *Identify a probability distribution* to represent the input process. When data are available, this step typically begins with the development of a histogram data. Give the histogram and a structural knowledge of the process, a family of distributions is cho-

sen. *Histograms or frequency distribution* is useful in identifying the shape of a distributions. Examples of the exponential and normal distribution histograms are shown in Figures 5-7 and 5-8.



Figure 5-7. Exponential distribution histogram example



Figure 5-8. Normal distribution histogram example

3. Evaluate the chosen distribution for goodness of fit. *Goodness of fit* may be evaluated informally, via graphical methods, or formally, via statistical tests. The *chi-square and the Kolmogorov-Smirnov tests* are standard goodness-of-fit tests. (Banks, et al., 2010, pp. 353-363)

# 6 Simulation model

# 6.1 Entities parameters

For analysis goal it will be used following entities parameters:

- A single-aisle storage where S/R machine performs on one side of storage rack.
- I/O point is located at the lower left-hand corner

- The S/R machine operates dual-command bases, i.e., multiple stops in the aisle are no allowed.
- The single-aisle storage's dimensions are length L = 20 m and height H = 10 m.
- The S/R machine speed is in horizontal  $S_h = 1.8$  m/s and in vertical  $S_{v=} 0.5$  m/s. In calculating the travel time, velocities are constant.
- The S/R machine travels simultaneously in the horizontal and vertical directions.
- Randomized storage is used.
- Pick-up and deposit time associated with load handling is  $T_{pd} = 20$  sec.

# 6.2 Simulation model

On this stage it doesn't need to define arrival products distribution, because travel time depends only on stacker crane's parameters. The model has following atoms:

Source atom "Product" produces arrival products for system,

Multitransform atom which divide incoming products into groups,

# Input accumulating conveyor,

# <u>Warehouse atom,</u>

<u>Portal crane</u> which takes out products from Input queue atom and pick-up them to Warehouse atom, and then it receives product from Warehouse to WMS Queue,

# Output accumulating conveyor,

Server atom which gets Orders from Portal crane,

Atom Sink "Client" shows how many orders walked through the system.

<u>Travel-time atom</u> will record how many time portal crane needs to perform dual-command cycle (3.3.1).

<u>History viewer</u> follows what kind of operations the Portal Crane will do and how many time, it will need for each operation in percentage.

<u>Summary report</u> records Inputs / Outputs through all atoms, counts average stay of products and average content in each atom.

Below whole the simulation model is represented in 2D also in 3D vision (Figures 6-1 and 6-2). Connecting lines reflects sequence of a product travels through chain of atoms in the model.



Figure 6-1. 2D Simulation model



Figure 6-2. 3D Simulation model



Video 6-1. Randomized storage (by double click it is possible to watch the model's work)

# 6.2.1 Disciplines for atoms

# 6.2.2 Arrival products atom

*Arrival products atom* has inter-arrival time (s): Lognormal (30, 10), time till first product =0 s and Trigger on exit (Figure 6-3) is set so that the software understands that the Warehouse atom has row and columns and product must be put to any column or row at random pattern.

Source	e - Product		
General	Visualization		
Atom	name:	Product	
Setting	Is		
Inter-a	arrival time [s]:	4DS LogNormal(80, 10)	~
Time t	ill first product [s]:	4DS 0	~
Numb	er of products:	1. Unlimited	~
Send	to:	1	~
Trigger	5		
Trigge	r on creation:	10. Do Nothing	~
Trigge	r on exit:	4DS do(SetLabel([row], dUniform(1,10	$\sim$

#### Figure 6-3. Atom Product discipline

Trigger on exit:

do( Label([column],i):= dUniform(1,20), Label([row],i):= dUniform(1,10) )

6.2.3 Multitransform atom

<u>Multitransform atom</u> divides all products into 3 groups. First group which is 60% has color Red, second is 25 % and color Blue, and third is 15 % from all products and color Green. Discipline of Multitransfrom atom is shown in Figure 6-4.

K Multi	Transform - Mul	ltiTransform	×
General	Labels		
Atom	name:	MultiTransform	
Selecti	on rules		
Atom	selection rule:	2. transform all atoms	$\sim$
Row s	election rule:	6. empirical distribution> the row nu	~
Miscell	leanous		
Edit t	ransformation s	ettings	

Figure 6-4. Multitransform atom discipline

🗶 Table	of MultiTrans	form			
File Edit	View				
Dimensior	ns				
<u>R</u> ows:	3	<u>C</u> olum	ns: 7		<u>S</u> et
Row	Selector	new name	new icon	new color	new X s
1		Red		colorred	
2		Blue		colorblue	
3		Green		colorgreen	

Transformation settings are presented in Figure 6-5.

# Figure 6-5. Transformation settings

Row selection rule (see Figure 6-6): Serieral Labels MultiTransform Atom name: Selection rules 2. transform all atoms Atom selection rule: Row selection rule: Close the row number in the ^ ~ transformation table is defined by the empirical distribution Inputstrategy Miscelleanous (values are rounded). Edit transformation settings

Figure 6-6. Row selection rule

34

Empirical distribution must be created in a different atom. Below in Figure 6-7 is shown

how it is possible to do own distribution in .

Compirical Distribution	ution - Inputs	strategy	
General			
Atom name:	Inputs	trategy	
Settings			
Distribution name	e: Inputs	trategy	
Number of classe	s: 3		
Return a string:			
Miscellaneous			
Edit classes	Table o	of Inputstrategy	
Eure classes	File Edit	View	
	Dimension	s	
Help	<u>R</u> ows:	3	<u>C</u> olumns:
_		Cumulative%	Value
	1	60	1
	2	85	2
	3	100	3

Figure 6-7. Empirical distribution of Input strategy

6.2.4 Input accumulating conveyor

<u>Input accumulating conveyor</u> contains only trigger on exit (Figure 6-8), which is considered as <u>point of start for travel from Input conveyor to Warehouse.</u>

Seneral	Specific	Visualization	
Atom	name:	Input accumulating conveyor	
Setting	gs		
Capad	city:	10000	
Send	to:	1	~
Speed	i [m/s]:	4DS 1	~
Trigger	rs		
Trigge	er on entry:	4DS 0	~
Trigge	r on exit:	4DS Label([inputstart], i) := Time	~

Figure 6-8. Input accumulating conveyor atom discipline

Trigger on exit: Label([inputstart], i) := Time

# 6.2.5 Warehouse atom

<u>Warehouse atom</u> has Trigger on entry that warehouse changes product's label from name "Product" to name "Order". It is required for the Portal crane to identify what kind of product should be retrieved from storage, also Trigger on entry is considered as <u>point of stop for travel from Input conveyor to Warehouse</u>. Label "Input" helps to atom "Travel time" to count Travel time from Input conveyor to Warehouse.

Also, the Warehouse atom has <u>Trigger on exit which is the start point for travel from</u> <u>Warehouse to Output Conveyor</u>.

#### Trigger on entry:

do(

Label([inputstop],i) := Time, Label([Input],i):= Label([inputstop],i) - Label([inputstart],i), Set(name(i),[Order]))

Trigger on exit:

do( SetLabel([row],2,i), SetLabel([column],0,i), Label([outputstart]):= Time)

Warehouse miscellaneous (see Figure 6-9) helps to the Portal crane to identify columns

and rows.

< Warel	Warehouse - Warehouse			
General	Miscellaneous	Visualization		
Setting	gs			
Put in	row:	4DS label([row],i)	~	
Put in	column:	4DS label([column],i)	~	
If occu	upied search in:	Rows ascending	~	
Z size	[m]:	10		
Numb	er of rows:	10		
Numb	er of columns:	20		
Viev	v table			

Figure 6-9. Warehouse miscellaneous

# 6.2.6 Portal crane

Portal crane has a lot of settings, because it excuses a main function in model.

In General settings Input strategy is set as:

"Largest queue: Accept product from the atom with the largest queue (if all queues are empty then open all channels and wait)".

#### Output strategy is:

**Portal Crane** × General Movement Specific Transportation Visualization General Atom Name: Portal Crane Dimensions 20 X size [m]: 4 Y size [m]: 10 Z size [m]: X direction:  $\sim$ Settings 2. Largest queue: Accept product f Input strategy: Close 6. By atom name: if the  $\sim$ Send to: atom name matches Order then send to channel 2 else 1 Triggers 4DS 0  $\sim$ Trigger on entry: 4DS 0  $\sim$ Trigger on exit:

"Send to: queue WMS if atom name is order". (See Figure 6-10)

# Figure 6-10. Portal crane atom discipline

In specific settings it is ordered that Time of pick-up/deposit is 20 s (see Figure 6-11).

ieneral	Movement	Specific	Transportation	Visualization
Specifi	c			
Load	quantity:	4	IDS 1	
Load	time (s):	4	DS 20	
Unioa	d time [s]:	4	IDS 20	
Go to	parking loca	tion:	]	
Detail	ed destinatio	ns: 🗹	]	

Figure 6-11. Portal crane specific settings

In transportation settings (Figure 6-12) horizontal and vertical speed are set, as well as length and high of the Portal crane. The simulation software calculates itself timeframe within which the Portal crane performs operations. Acceleration and deceleration are ignored.

Portal Cra	ine				>
General	Movement	Specific	Transportation	Visualization	
Parame	eters				
X Spe	ed [m/s]:	[	4DS 1.8		
Y Spee	ed [m/s]:	[	4DS 0.5		
Z Spe	ed [m/s]:	[	4DS 0.5		
X Acce	eleration [m/s <sup>2</sup>	)፡ [	4DS 0		
Y Acce	Y Acceleration [m/s <sup>2</sup> ]:		4DS 0		
Z Acce	eleration [m/s <sup>2</sup>	ን፡ [	4DS 0		
X Dec	eleration [m/s	²]: [	4DS 0		
YDece	eleration [m/s	ין: [	4DS 0		
Z Deceleration [m/s <sup>2</sup> ]:		²]: [	4DS 0		
Transp	oort height [m	]: [	4DS 10		
Fixed	X Value [m]:	[	4DS 20		
Fixed	Y Value [m]:	[	4DS0		

Figure 6-12. Transport settings

# 6.2.7 Output accumulating conveyor

<u>Output accumulating conveyor</u> contains only trigger on entry, which is considered as point of stop for travel from Input conveyor to Warehouse.

Trigger on entry:

do(

Label([outputstop],i):= Time,

Label([Output],i):= Label([outputstop],i) - Label([outputstart],i))

6.2.8 Server WMS

**Server WMS** atom requires nothing else than cycle time's distribution (Figure 6-13). In particular case, distribution reflects how often orders are forwarded to customers. Definition distribution laws will be described in further chapters.

Settings		
Setup time [s]:	0	~
Cycletime [s]:	4DS negexp(9)	~
Send to:	1	~
Input strategy:	Any inputchannel	~

# Figure 6-13. Server WMS atom discipline

# 6.2.9 Traveltime atom

*To calculate travel time,* it was necessary to break down dual-command cycle into two parts:

1. from input accumulating conveyor to warehouse

2. from warehouse to output accumulating conveyor.

The reason to proceed this way is that the <u>software is meant to calculate product life</u> in the system, but it is not capable to calculate cycle time of machineries. If it would be ordered to calculate whole cycle from input conveyor to output conveyor, then the results would be: product travel time to Warehouse + product travel time from Warehouse + product waiting time in Warehouse. However, this task doesn't meet the requirements of research. Still, product waiting time is an excessive link in calculation of cycle time of the crane and for this reason calculation of cycle time was broken down into two parts.

To define travel time Conveyor-Warehouse, Trigger on entry in Warehouse settings is:

```
Label([Input],i):= Label([inputstop],i) - Label([inputstart],i).
```

Time Warehouse-Conveyour is in settings of Output Conveyor Trigger on entry:

Label([Output],i):= Label([Outputstop],i) - Label([Outputstart],i).

Thus, atom Traveltime computes time of Label Input and Label Output.

From setting "Variables" "Value or string of Input and Output" should be chosen (Figure 6-14 and 6-15).

🗧 Data	Recorder	- Traveltin	ne	>
General	Excel	Variables		
Settin	as			
1	Input		{.Label([~1],ı) 1. Value or strii	
< V	/ariable Da	ata Record	er	
Se	ttings			
N	ame:		mou	
Ð	pression:		1. Value or string of Input	

Figure 6-14. Data recorder variable Input

🗧 Data	Recorder	- Traveltim	e		×
General	Excel	Variables			
Settin	qs				
					[
1	Innut		(Labal)(, 11-i)(1. Value er striv		
2	Output		{.Label([~1],i)[1. Value of strif		
长 Va	ariable Da	ta Recorder	r		
Set	tings				
	-				
Na	me:		Output		
Exp	pression:		1. Value or string of Output		~
-					
				_	
			OK		Cance

Figure 6-15. Data recorder variable Output

6.2.10 Travel time calculations

In software the result of input/output travel time looks like in Figure 6-16, but for convenience, all calculations will be done in MS Excel.

Table of Traveltime								
File Edit	View							
Dimensions	Dimensions							
<u>R</u> ows:	30	<u>C</u> olumns:	2					
	Input	Output						
1	32	30						
2	32.7777777	30.1666666						
3	38	36						
4	30	27.6666666						
5	31.6666666	29.1666666						
6	36	34						
7	36	34						
8	26.6666666	26.0000000						
9	24	26						
10	32.7777777	30.1666666						
11	32	30						
12	34	32						
13	29.4444444	27.1666666						
14	27.7777777	26						
15	36	34						
16	26.6666666	26						
17	36	34						
18	28	26						

Figure 6-16. Input and Output travel times

Still, load time from the input conveyor to the Portal crane and, consequently, from the Warehouse to the Portal crane was excluded from calculations of dual-command cycle time. It's two times excluded due to that the software counts travel time from the point when product already left an atom, but no time of loading to the Portal crane. But in our calculations their loading times are a part of dual-command cycle. For this reason, loading times (40s) should be added manually.

In addition, the time in warehouse "Travel Empty" is excluded from calculations. For this reason, tool "Charts" should be used from where it is possible to see percentage of operations of the Portal crane. Toolbar --> Results --> Graphs (Figure 6-17).





summar	y rep	ort			
name	conte current	nt average	throu input	output	staytime average
Inputstrategy	0	0.000	0	0	0.000
Portal Crane	1	1.000	0	0	0.000
Traveltime	0	0.000	30	30	0.000
Product	1	0.900	34	33	97.819
Warehouse	1	1.207	31	30	141.413
WMS	0	0.063	30	30	7.525
Client	0	0.000	30	0	0.000
Product12	0	0.000	0	0	0.000
Input accumulat	1	1.000	33	32	111.780
Output Accumul	a 0	0.042	30	30	5.000
History of Port	0	0.000	0	0	0.000
System report	0	0.000	0	0	0.000
MultiTransform	0	0.000	33	33	0.000
Model start time lauantai, toukokuu 05 2018 12.24.42 Model end time lauantai, toukokuu 05 2018 13.24.42 Bunlenoth (seconds) 3600 00					

Figure 6-18. Summary report of the system working over 1 hour

Simulation run length was 3600 s (1h) and from summary report (Figure 6-18) quantity of products which are gone through the full cycle are 30 items.

Now it is possible to calculate a dual-command cycle of the Portal crane. There are time: Input Conveyor-Warehouse, Warehouse – Output Conveyor, "Travel empty" time between locations of Warehouse and should be added 40 s which the system doesn't consider.

Table 6-1. Dual-command cycle calculation

Simulation time	<u>3 600</u>	
Travel full	18,70 %	673,20
Travel empty	7,44 %	267,84
Idle	1,27 %	45,72
Load time	38,39 %	1 382,04
Unload time	36,11 %	1 299,96
Quantity	32	
Dual-command cy	cle time (s)	
Travel empty	8,37	
Average sum I/O	60,70	
Load/Unload	40	
	<u>109,07</u>	

The table of results (Table 6-1) helps to calculate mean of Travel-time. After calculations are done, the Portal crane travel time is estimated with mean figure of **109,07 s**. Full output numbers are contained in Appendix 1. From summary report (see Figure 6-18) the crane performed **30 full cycles**, and **picked-up** and **retrieved 30 pallets**.

# 6.3 Validation by Travel-time model with random storage

According to entities parameters in chapter 3.2. It is possible to calculate travel-time for given S/R machine.

$$\begin{array}{l} \hline Travel-time \ calculation \\ L \coloneqq 20 \ m \quad H \coloneqq 10 \ m \quad Sh \coloneqq 1.8 \ \frac{m}{s} \quad Sv \coloneqq 0.5 \ \frac{m}{s} \quad Tpd \coloneqq 20 \ s \\ \hline Th \coloneqq \frac{L}{Sh} = 11.11 \ s \quad Tv \coloneqq \frac{H}{Sv} = 20 \ s \\ T \coloneqq \max(Th, Tv) = 20 \ s \\ Q \coloneqq \frac{\min(Th, Tv)}{T} = 0.56 \\ \hline E(SC) \coloneqq T \cdot \left(1 + \frac{Q^2}{3}\right) + 2 \cdot Tpd = 62.06 \ s \\ E(DC) \coloneqq \frac{T}{30} \cdot (40 + 15 \cdot Q^2 - Q^3) + 4 \cdot Tpd = 109.64 \ s \end{array}$$

Figure 6-19. Analytical solution

In Figure 6-19 example single-command cycle takes 62,06s and dual-command cycle is 109,64s. Furthermore, for comparison with other approaches and for throughputs' calculation dual-command cycle's time will be used.

*The total average time for full cycle* (3 operations=travel to storage, travel to retrieval load, travel to I/O point) is *109,64s*,

# The crane performs 32 cycles for 1 hour, storages and retrievals 32 pallets.

# 6.4 Selecting Input distribution

As a matter of fact, in the following thesis work input distribution doesn't influence Travel time of the Crane, which is one of the basics in creating warehouse design model.

However, to successfully implement simulation model in real cases, it's necessary to make adjustments to the model applying input distribution. This step would allow a manager to get access to variety of information. Among this information following aspects could be found: how fast incoming flow can fill warehouse, how many products are left in the queue, how many aisles are needed for precise distribution in order to avoid queues on conveyors.

Now, it's going to be demonstrated how distribution is defined with a help of MS Excel and simulation software. Furthermore, it is possible to play with changing of disciplines of model atoms.

For definition of distribution which follows input products, it is good way to use histograms. Approximately, it could be defined what kind of shape does histogram have and, then after it could be checked with Chi test or Kolmogorov-Smirnov test.

Assume that arrival list has been got from a client. Actually, create arrival list from the head. Example of arrival list in Figure 6-20. Full text of arrival list is Appendix 2.

	Α	В	С	D	E
1					
2					
3			Arrival time	Difference in time	Real numbers
4			8:00		
5		1	8:07	0:07	7
6		2	8:09	0:02	2
7		3	8:13	0:04	4
8		4	8:24	0:11	11
9		5	8:30	0:06	6
10		6	8:32	0:02	2
11		7	8:35	0:03	3
12		8	8:39	0:04	4
13		9	8:45	0:06	6
14		10	8:49	0:04	4
15		11	8:53	0:04	4
16		12	8:57	0:04	4
17		13	9:01	0:04	4
18		14	9:09	0:08	8
19		15	9:17	0:08	8
20		16	9:25	0:08	8
21		17	9:33	0:08	8

Figure 6-20. Arrival list example

# 6.4.1 Selecting of distribution with MS Excel

E	F	G	н
Arrivals			
difference		min	2
7		max	13
2			
4		Bins	
11		1	
6		2	
2		3	
3		4	
4		5	
6		6	
4		7	
4		8	
4		9	
4		10	
8		11	
8		12	
8		13	
8			

Step	1.	Create	bins	of	<sup>:</sup> histoaram	as	in	Figure	6-21.
		0.00.00		_					• ==-

Figure 6-21. Classes/bins for Histogram

# Step 2. Create histogram and shape plot

Use MS Excel tool "Histogram" (data analysis --> Histogram). How columns in Histogram tool should be fill is presented in Figure 6-22.

* Show Detail	📄 Data Analysis	
Histogram		? ×
Input Input Range: Bin Range:	SES5:SES54	OK Cancel <u>H</u> elp
Output options	SL\$20	

Figure 6-22. MS Excel histogram tool

As result MS excel observes frequency of bins and creates histogram. If curve is added to histogram chart, it is possible to recognize shape of distribution as in Figure 6-23.



Figure 6-23. Histogram of input probability

From practice and experience it is possible to say that this distribution follows *lognormal distribution* rules.

# Step 3. Kolmogorov-Smirnov Goodness-of-Fit Test for Lognormal distribution

However, it is possible to validate suggestion that is the Lognormal distribution. As Goodness-of-Fit will be applied Kolmogorov-Smirnov test (K-S test). If *test value* will *smaller* than critical value *null hypothesis will not rejected*.

# Hypothesis:

# H<sub>0</sub>: the data follows Lognormal distribution

# H1: the data doesn't follow Lognormal distribution

1. Sort real numbers from smallest to largest and found their cumulative probability *F(x)*.

2. Theoretical/empirical probability of "*n*" of *N* observations. Also *N* is degree of freedom (*d.f.*) in K-S test.

3. Compute

$$Dmax = max \left(\frac{1}{N} - F(x)\right)$$

$$Dmin = max\left(F(x) - \frac{(i-1)}{N}\right)$$

4. Compute

D = max(Dmax, Dmin)

5. *Critical value* from Table (Appendix 3) for significance level 0.05 and degree of freedom 50 *is 0.18845*.

In MS Excel it is necessary to create a table as in Figure 6-24. Full calculations are in Appendix 4).

	А	В	С	D	E	F	G	н	1	J	
1	Real	<u>F(xi)</u>	<u>i</u>	<u>i/N</u>	<u>i/N-F(xi)</u>	<u>(i-1)/N</u>	<u>F(xi)-(i-1)/N</u>		1		
2	numbers	<u>CDF</u>		Empirical CDF	<u>Dmax</u>		<u>Dmin</u>		   		
3	2	0,062	1	0,02	-0,04	0	0,062		<u>N= d.f.</u>	50	
4	2	0,062	2	0,04	-0,02	0,02	0,042		<u>mean</u>	5,94	
5	3	0,125	3	0,06	-0,07	0,04	0,085		<u>st. dev</u>	2,56	
6	3	0,125	4	0,08	-0,05	0,06	0,065		1		
7	4	0,224	5	0,1	-0,12	0,08	0,144		1		
8	4	0,224	6	0,12	-0,10	0,1	0,124		1		
9	4	0,224	7	0,14	-0,08	0,12	0,104		Dmax	0,1833	
10	4	0,224	8	0,16	-0,06	0,14	0,084		Dmin	0,144173	
11	4	0,224	9	0,18	-0,04	0,16	0,064		1		
12	4	0,224	10	0,2	-0,02	0,18	0,044				

Figure 6-24. Kolmogorov-Smirnov test example

From test **D** max = **0.1833**, the value doesn't exceed the table value (0.18845) for 50 degree of freedom and level of significance = 0.05, therefore the null hypothesis is not rejected and arrival time is Lognormal distributed.

# 6.4.2 Selecting of distribution with Autofit tool

Enterprise Dynamics 10.1 allows to define given distribution with "Autofit" tool. This function requires only to connect MS Excel file with data to the software. The Autofit does all tests for given input data and offers options distributions itself.

In both methods with MS Excel and Autofit (Figure 6-25), the results were the same. Arrival list of products follows to Lognormal distribution with **mean = 5, 94** and standard deviation **=2,56**. It means that product should arrive to a system every 5.94 min  $\pm$  2.56 min.



Figure 6-25. Autofit histogram. Simulation software's tool

# 7.1 Class-based storage

If in practice *class-based storage* is needed, it possible to change arrival products discipline.

In the new model products arrive according ABC analysis principals. *Red* products have 80%, *Blue* products have 15% and *Green* products have 5% of all products. Red products are pick-up in *columns 1-14*, blues are *15-18* and greens are *19-20*.

Delete atom multi transform and add to Trigger on exit of Product atom follow command:

> do( SetLabel([row],dUniform(1,10),i), var([Product], vbValue), Product := Uniform(0,1), Case( WhichIsTrue( Product <= 0.8, Product <= 0.95, Product <=1), do ( Color(i) := ColorRed, Icon(i) := IconByName([circlered]), Label([Product],i) :=1, SetLabel([column],dUniform(1,14),i)), do( color(i) := ColorBlue, icon (i) := IconByName([circleblue]), Label([Product],i) :=2, SetLabel([column],dUniform(15,18),i)), do( color(i) :=ColorGreen,

```
Icon(i) := IconByName([circlegreen]),
Label ([Product], i) := 3,
SetLabel([column],dUniform(19,20),i)
)))
```



Video 7-1. Class-based storage (by double click it is possible to watch the model's work)

As results, Travel time of the Crane is 99s (Table 7-1). Therefore, it can be obtained, that Class-based storage allows to the Crane performs faster and throughputs are 37 incoming and out coming pallets.

Table 7-1. Class-based storage travel time

Simulation time	<u>3 600</u>	
Travel full	16,19 %	582,84
Travel empty	8,39 %	302,04
Idle	2,08 %	74,88
Load time	36,67 %	1 320,12
Unload time	36,67 %	1 320,12
Quantity	37	
Dual-command cycl	e time (s)	
Travel empty	1,61	
Average sum I/O	57,25	
Load/Unload	40	
	98,86	

# 7.2 Depended on Warehouse content the crane cycle

If someone want to order to the crane to make output from warehouse only if warehouse is 50% full, it is possible to use Bernoulli trials.

In the crane's setting should be changed input strategy:

do( var([atmQueue], vbAtom, in(1, c)), var([atmWarehouse], vbAtom, in(2, c)), var([valThresholdUnloadWarehouse], vbValue, 0.5), lf(

Content(atmWarehouse) / (Att([Columns], atmWarehouse) \* Att([Rows], atmWarehouse)) > valThresholdUnloadWarehouse,

OpenIc(Bernoulli(50, 1, 2), c),

OpenIc(1, c) ))



only fill storage.mp<sup>2</sup>

Video 7-2. Bernoulli trials for fill a storage (by double click it is possible to watch the model's work)

# 8 Conclusion

This paper has presented a based- simulation performance analysis of an End-of-Aisle automated storage and retrieval system, as well as a method of validation of the simulation model. A reader may familiarize themselves with primary stages in simulation modeling and working of simulation software.

8.1 Shortly "Why simulation model is based on travel time measures"

In simulation model the travel time of the Portal crane is calculated as:

Travel time from Input conveyor to Warehouse + Travel time from Warehouse to Output Conveyor + "Empty travel" (time between warehouse locations) + Loading time from the Input conveyor to the Portal crane (20s) + Loading time from the Warehouse to the Portal crane (20s). Two loading times are added manually (Explanation in chapter 6.2.10)

It might seem that calculations of travel time by simulation looks complexly, but travel time is good measure for validation of the simulation model. As it was said, simulation model should be validated, without validation the manager cannot prove that a model gives right results. In this case, the simplest validation way is a travel time method which is based on close-form expressions.

Altogether, travel time was calculated by simulation results and by analytical solution. In both cases results were quite alike to each other. As conclusion, the simulation model is credible and can be used in more advanced, complex systems.

# 8.2 Summary about Simulation

It's quite easy to calculate capacity and cycle time with close-form expressions, but simulation model provides more opportunities in AS/RS's performance researching.

Firstly, this simulation model is ready to be used further as a part of more complex system. For instance, creating a model of production facility.

Secondly, model has a lot of different settings which could be adjusted very fast. For example, to calculate cycle time it was ordered for the crane to pick up products from the largest queue. From the largest queue means that the Portal crane picks-up two products in columns in warehouse and after that alternately retrieves and pick-up to warehouse. But we can change commands for the Crane input strategy. After changes (see chapter 7.2) during experiment the Crane will operates that storage is to be filled according to Bernoulli law, meaning that products will be retrieved from warehouse once it's 50% filled.

In practice that is important, if in a real existing system product came from production for the first two hours of plant working. In this case the crane is set to fill only. For example, after of beginning of working of plant that it could operate a combination of 2 actions.

Thirdly, simulation model unlike traditional methods shows how successfully the crane copes with an input flow and how many products are in queue on average. Based on received results, there is a possibility to change speed of the crane or adjust time of loading/unloading. Simulation model allows to make those changes quite easy.

# 8.3 Warehouse cost with AS/RS

What is the difference between traditional racks and AS/RS in term of warehouse costs. In table 3-2 (Chapter 3.1) warehouse costs are given. AS/RS decreases pick-up, storage and retrieval operations' costs by 8%. Consider in table below (Table 8-1), how many times warehouse costs are decreased with integrated AS/RS.

Traditional racks					
	Labor cost	Space cost	MHS cost	WMS cost	Total
Receving	1 963 055 €	238 125€	569 820€	218 333€	
Putaway	1 090 534 €	0€	416 000 €	240 333€	
Storage	999 640€	1 933 250€	1 650 710€	123 833€	
Picking	1 946 966 €	0€	1 830 782 €	161 833€	
Consolidation	287 188€	100 500 €	135 000 €	38 333€	
Delivery	68 225 €	50€	69 000 €	38 333 €	
Marketing	3 534 218 €	105 000 €	222 200€	113 833€	
Returns	68 225 €	99 250 €	6 000 €	113 000 €	
Total	9 958 051 €	2 476 175€	4 899 512€	1 047 831€	<u>18 381 569 €</u>

Table 8-1. Warehouse costs with AS/RS

AS/RS					
	Labor cost	Space cost	MHS cost	WMS cost	Total
Receving	1 963 055 €	238 125€	569 820€	218 333€	
Putaway	1 003 291 €	0€	382 720€	240 333€	
Storage	919 669€	1 778 590€	1 518 653€	123 833€	
Picking	1 791 209 €	0€	1 684 319€	161 833€	
Consolidation	287 188€	100 500 €	135 000 €	38 333 €	
Delivery	68 225 €	50€	69 000 €	38 333€	
Marketing	3 534 218€	105 000 €	222 200€	113 833€	
Returns	68 225 €	99 250 €	6 000 €	113 000 €	
Total	9 635 080 €	2 321 515€	4 587 713€	1 047 831€	<u>17 592 138 €</u>

After calculations, it is clear that an AS/RS decreases three warehouse operations by 8 % decreases all warehouse costs by 4% (1- total of cost with traditional racks/total of costs with AS/RS).

An AS/RS can decrease warehouse costs significantly, but it's necessary to consider, that AS/RS is required large investment costs and maintenance costs.

# 8.4 Is there any reason to have AS/RS?

As recommendation, in first step of decision making about AS/RS, it would be reasonable to calculate input probability with method which have been presented in this thesis. For example, input probability from chapter 6.4.1 was not so intensive (products arrival each 5 min) for AS/RS performance. Forklifts would be enough to cope with pace. Workforce expenses are not high enough in this case too.

Once input probability and results were calculated, it was defined that from production arrival to storage more than 50 items of products arrive and their' arrival distribution equals to 5s per product. Thus, AS/RS is a great option to consider.

The main advantage property of AS/RS is space savings. If warehouse has a huge volume of products and for that you need a lot of racks, bravely begin to make decision about AS/RS.

# 9 Discussion

Major difficulty in thesis project development was this software which was used to create the simulation model. From my point of view, to utilize such software it is required to have strong IT-programming skills and knowledge, or field of logistics should pay more attention mathematics approaches in solving problems.

Creators of software have developed extremely complicated programming language with a help of which commands are generated. Most time of thesis writing was consumed to construct the model especially due to a complexity of programming code.

Advantages of the software are that clients can get a clear picture of operations which are going to be established in warehouse, once it is ready to be used. Software allows to simulate in 3D (three dimensions). 3D is much easier to understand than naked statistical numbers and technical drawings for unskilled or person who doesn't have deep knowledge in mathematics simulation. With presentation of 3D model takes less time to convince a client. For this reasons companies have to employ IT-specialists for creating models. In most cases IT-specialists don't have expertise in logistics, it is inevitable to have both employers: logistics and IT. A great way to solve this issue is to introduce more quantitative approaches for logistics study.

# References

AS/RS\_throughput\_analysis, 2003. *AS/RS\_throughput\_analysis*. Available at: <u>https://www2.isye.gatech.edu/~shackman/isye6202/ASRS\_ThroughputAnalysis.pdf</u>

Banks, J., Carson, J., Nelson, B. & Nicol, D., 2010. *Discrete-Event System Simulation*. 5th ed. Upper Saddle River: Pearson education.

Frazelle, E., 2002. *World-class Warehousing and Material Handling*. USA: R.R. Donelley and Sons Company.

Gagliardi, J.-P., Renaurd, J. & Ruiz, A., 2012. Models for automated storage and retrieval system: a literature review. *International journal of Production Research,* Issue 24, pp. 7110-7125.

Groover, M., 2016. *Automation, Production Systems, and Computer-integrated manufacturing.* Fourth edition ed. Essex: Person education limited.

Kananen, J., 2011. *Rafting through the thesis process: step by step guide to thesis research.* Jyvaskyla: Jyvaskylan ammattikorkeakoulu.

Kumar, U., Crocker, J., Chitra, T. & Saranga, H., 2010. *Reliability and Six sigma*. New York: Springer Science + Business Media Inc. .

Law, A. & Kelton, W., 2000. *Simulation modeling and analysis.* 3rd ed. Singapore: McGraw-Hill Book Co.

Lehtiranta, L., Junnonen, J., Kärnä, S. & Pekuri, L., N.d.. *The constructive research approach: Problem solving for complex projects.* Available at: <a href="http://www.gpmfirst.com/books/designs-methods-and-practices-research-project-management/constructive-research-approach">http://www.gpmfirst.com/books/designs-methods-and-practices-research-project-management/constructive-research-approach</a>

Logistics\_costs, 2010. Logistics Cost and Service. Available at: <a href="http://logisticsportal.iadb.org/sites/default/files/2010\_logistics\_cost\_and\_service\_pr">http://logisticsportal.iadb.org/sites/default/files/2010\_logistics\_cost\_and\_service\_pr</a> esentation.pdf

Sarker, B. R. & Babu, P., 1995. Travel time models in automated storage/retrieval systems: a critical review. *International Journal of production economics,* Issue 40, pp. 173-184.

Sokolowski, J. A. & Banks, C. M., 2012. *Handbook of real-world applications in modeling and simulation*. Hoboken, New Jersey: John Wiley & Sons, Inc..

Trochim, W. M. & Donnelly, J. P., 2008. *Research methods knowledge base.* 3rd ed. Mason, Ohio: Atomic Dog/Cengage Learning.

Vasili, M. & Tang, S., 2016, February. Travel Time Analysis of an Open-Rack Miniload AS/RS under Class-Based Storage Assignments. *IACSIT International Journal of Engineering and Technology*, Issue 1, pp. 70-75.

# Appendices

# Appendix 1.

Calculation of dual-command cycle time (s)

	<u>Travel time</u>		
	<u>Input</u>	<u>Output</u>	Sum of I/O
1	36,00	34,00	70,00
2	30,00	30,00	60,00
3	32,78	30,17	62,94
4	30,00	28,00	58,00
5	28,00	26,00	54,00
6	28,00	26,00	54,00
7	34,00	32,00	66,00
8	33,33	30,67	64,00
9	32,00	30,00	62,00
10	28,89	26,67	55,56
11	23,89	26,00	49,89
12	36,00	34,00	70,00
13	29,44	27,17	56,61
14	31,67	29,17	60,83
15	28,00	26,00	54,00
16	32,78	30,17	62,94
17	31,11	28,67	59,78
18	28,89	26,67	55,56
19	38,00	36,00	74,00
20	23,89	26,00	49,89
21	31,11	28,67	59,78
22	34,00	32,00	66,00
23	31,11	28,67	59,78
24	32,78	30,17	62,94
25	34,00	34,00	68,00
26	32,00	32,00	64,00
27	31,67	29,17	60,83

28	36,00	34,00	70,00
29	26,67	26,00	52,67
30	34,00	32,00	66,00
31	32,78	30,17	62,94
32	23,33	26,00	49,33

Appendix 2.

	<u>Arrival time</u>	Difference in	<u>Real numbers</u>
		<u>time</u>	
	8:00		
1	8:07	0:07	7
2	8:09	0:02	2
3	8:13	0:04	4
4	8:24	0:11	11
5	8:30	0:06	6
6	8:32	0:02	2
7	8:35	0:03	3
8	8:39	0:04	4
9	8:45	0:06	6
10	8:49	0:04	4
11	8:53	0:04	4
12	8:57	0:04	4
13	9:01	0:04	4
14	9:09	0:08	8
15	9:17	0:08	8
16	9:25	0:08	8
17	9:33	0:08	8
18	9:37	0:04	4
19	9:41	0:04	4
20	9:53	0:12	12
21	10:03	0:10	10
22	10:11	0:08	8
23	10:16	0:05	5
24	10:27	0:11	11
25	10:31	0:04	4
26	10:37	0:06	6

27	10:41	0:04	4
28	10:46	0:05	5
29	10:51	0:04	4
30	11:01	0:10	10
31	11:08	0:07	7
32	11:13	0:05	5
33	11:19	0:06	6
34	11:23	0:04	4
35	11:28	0:05	5
36	11:34	0:06	6
37	11:38	0:04	4
38	11:43	0:05	5
39	11:52	0:09	9
40	11:59	0:07	7
41	12:05	0:06	6
42	12:10	0:05	5
43	12:13	0:03	3
44	12:21	0:08	8
45	12:34	0:13	13
46	12:39	0:05	5
47	12:44	0:05	5
48	12:49	0:05	5
49	12:54	0:05	5
50	12:59	0:05	5

Appendix 3.

Kolmogorov-Smirnov test. Table of critical values

n∖ <sup>α</sup>	0.001	0.01	0.02	0.05	0.1	0.15	0.2
1		0.99500	0.99000	0.97500	0.95000	0.92500	0.90000
2	0.97764	0.92930	0.90000	0.84189	0.77639	0.72614	0.68377
3	0.92063	0.82900	0.78456	0.70760	0.63604	0.59582	0.56481
4	0.85046	0.73421	0.68887	0.62394	0.56522	0.52476	0.49265
5	0.78137	0.66855	0.62718	0.56327	0.50945	0.47439	0.44697
6	0.72479	0.61660	0.57741	0.51926	0.46799	0.43526	0.41035
7	0.67930	0.57580	0.53844	0.48343	0.43607	0.40497	0.38145
8	0.64098	0.54180	0.50654	0.45427	0.40962	0.38062	0.35828
9	0.60846	0.51330	0.47960	0.43001	0.38746	0.36006	0.33907
10	0.58042	0.48895	0.45662	0.40925	0.36866	0.34250	0.32257
11	0.55588	0.46770	0.43670	0.39122	0.35242	0.32734	0.30826
12	0.53422	0.44905	0.41918	0.37543	0.33815	0.31408	0.29573
13	0.51490	0.43246	0.40362	0.36143	0.32548	0.30233	0.28466
14	0.49753	0.41760	0.38970	0.34890	0.31417	0.29181	0.27477
15	0.48182	0.40420	0.37713	0.33760	0.30397	0.28233	0.26585
16	0.46750	0.39200	0.36571	0.32733	0.29471	0.27372	0.25774
17	0.45440	0.38085	0.35528	0.31796	0.28627	0.26587	0.25035
18	0.44234	0.37063	0.34569	0.30936	0.27851	0.25867	0.24356
19	0.43119	0.36116	0.33685	0.30142	0.27135	0.25202	0.23731
20	0.42085	0.35240	0.32866	0.29407	0.26473	0.24587	0.23152
25	0.37843	0.31656	0.30349	0.26404	0.23767	0.22074	0.20786
30	0.34672	0.28988	0.27704	0.24170	0.21756	0.20207	0.19029
35	0.32187	0.26898	0.25649	0.22424	0.20184	0.18748	0.17655
40	0.30169	0.25188	0.23993	0.21017	0.18939	0.17610	0.16601
45	0.28482	0.23780	0.22621	0.19842	0.17881	0.16626	0.15673
50	0.27051	0.22585	0.21460	0.18845	0.16982	0.15790	0.14886
	1.94947	1.62762	1.51743	1.35810	1.22385	1.13795	1.07275
OVER 50	√n						

Appendix 4.

Kolmogorov-Smirnov test's full calculation

<u>Arri-</u>	<u>F(xi)</u>	<u>i</u>	<u>i/N</u>	<u>i/N-F(xi)</u>	<u>(i-1)/N</u>	<u>F(xi)-(i-1)/N</u>
<u>vals</u>	<u>CDF</u>	-	<u>Empirical</u>	<u>Dmax</u>	-	<u>Dmin</u>
<u>differ-</u>			<u>CDF</u>			
<u>ence</u>						
2	0,062	1	0,02	-0,04	0	0,062
2	0,062	2	0,04	-0,02	0,02	0,042
3	0,125	3	0,06	-0,07	0,04	0,085
3	0,125	4	0,08	-0,05	0,06	0,065
4	0,224	5	0,1	-0,12	0,08	0,144
4	0,224	6	0,12	-0,10	0,1	0,124
4	0,224	7	0,14	-0,08	0,12	0,104
4	0,224	8	0,16	-0,06	0,14	0,084
4	0,224	9	0,18	-0,04	0,16	0,064
4	0,224	10	0,2	-0,02	0,18	0,044
4	0,224	11	0,22	0,00	0,2	0,024
4	0,224	12	0,24	0,02	0,22	0,004
4	0,224	13	0,26	0,04	0,24	-0,016
4	0,224	14	0,28	0,06	0,26	-0,036
4	0,224	15	0,3	0,08	0,28	-0,056
4	0,224	16	0,32	0,10	0,3	-0,076
4	0,224	17	0,34	0,12	0,32	-0,096
4	0,224	18	0,36	0,14	0,34	-0,116
4	0,224	19	0,38	0,16	0,36	-0,136
5	0,357	20	0,4	0,04	0,38	-0,023
5	0,357	21	0,42	0,06	0,4	-0,043
5	0,357	22	0,44	0,08	0,42	-0,063
5	0,357	23	0,46	0,10	0,44	-0,083
5	0,357	24	0,48	0,12	0,46	-0,103
5	0,357	25	0,5	0,14	0,48	-0,123
5	0,357	26	0,52	0,16	0,5	-0,143
5	0,357	27	0,54	0,18	0,52	-0,163

6	0,509	28	0,56	0,05	0,54	-0,031
6	0,509	29	0,58	0,07	0,56	-0,051
6	0,509	30	0,6	0,09	0,58	-0,071
6	0,509	31	0,62	0,11	0,6	-0,091
6	0,509	32	0,64	0,13	0,62	-0,111
6	0,509	33	0,66	0,15	0,64	-0,131
6	0,509	34	0,68	0,17	0,66	-0,151
7	0,661	35	0,7	0,04	0,68	-0,019
7	0,661	36	0,72	0,06	0,7	-0,039
7	0,661	37	0,74	0,08	0,72	-0,059
8	0,790	38	0,76	-0,03	0,74	0,050
8	0,790	39	0,78	-0,01	0,76	0,030
8	0,790	40	0,8	0,01	0,78	0,010
8	0,790	41	0,82	0,03	0,8	-0,010
8	0,790	42	0,84	0,05	0,82	-0,030
8	0,790	43	0,86	0,07	0,84	-0,050
9	0,884	44	0,88	0,00	0,86	0,024
10	0,944	45	0,9	-0,04	0,88	0,064
10	0,944	46	0,92	-0,02	0,9	0,044
11	0,976	47	0,94	-0,04	0,92	0,056
11	0,976	48	0,96	-0,02	0,94	0,036
12	0,991	49	0,98	-0,01	0,96	0,031
13	0,997	50	1	0,00	0,98	0,017