Tribology and machine elements

Bachelor's thesis

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ABSTRACT

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ABSTRACT

The general purpose of this thesis was to explain the fundamental theory of tribology and to study the behavior of machine elements behavior with tribology. This thesis was commissioned by HAMK University of Applied Sciences with the aim to provide additional studying material for this science area. The initial background was based on machine elements theory and the thesis results are essential knowledge for machinery related design.

This work aims to define theory and results behind friction phenomena. Also, to determine different wear types and reasons for wear failure. This thesis also focuses on different aspects of lubrication and the influence of viscosity and lubrication regimes on machine elements performance. An additional target of this thesis work was to present tribological behavior of gears and bearings elements.

The starting theory used in this work was based on machine elements and strength of materials studies but it required a further knowledge of other fields such fluid and solids mechanics, thermodynamics, physics and material science. The thesis focuses only on tribological functions of gears and bearings and requires previous knowledge of these elements.

The studies presented in this work allowed analysis of different phenomena causing friction and the definition of different formulas to evaluate friction. The work also provided methods of calculations to predict wear losses under non-lubricated and lubricated conditions. Furthermore lubrication regimes for operating machine elements and viscosity response for different working conditions were presented. Finally, equations quantifying bearing and gears tribological behavior were determined.

Keywords Contact, friction, wear, lubrication, regimes.

Pages 58.

Contents

1 INTRODUCTION

The term tribology emerged in the 1960s and is defined as “the science of rubbing” from Greek translation. Tribology is the engineering science of moving interacting surfaces. The science covers friction, wear and lubrication for interacting machine elements. It has been related to human invention since ancient history from the creation of wheels to inclusion of liquids while building pyramids to avoid friction. Throughout time, beside Da Vinci who established the first model of friction, many scientists contributed in the establishment this science area. In modern industry, Tribology has significant importance since it is responsible for reliability, performance and durability of moving machine components in all machinery domains. Actually, adequate application in this domain can procure enormous savings estimated as 50 times the research cost.

Friction is the phenomena of motion opposition. In reality, it is considered that any moving elements endure friction response. Also friction causes undesirable effects and enormous losses in industry. Thus friction is fundamental to comprehend tribology mechanisms, causes and results. Wear is the degradation of material surfaces and it is common damage mode for machine elements. It is a defective system response caused by different factors. Meanwhile, lubrication is the most common techniques applied to avoid friction and wear effects and it is applied in all forms of modern design. However, the understanding of tribology behavior of interacting surfaces requires knowledge of several sciences including chemistry, fluid and solid mechanics and applied mathematics.

The study of tribology is crucial for numerous components in modern design especially with bearings and gears applications. A bearing is a machine element that constrains relative motion and reduces friction between moving parts to only the desired motion. Bearings enhance the functionality of machinery and help to save energy and they are essential for the stable operation of machinery and for ensuring its top performance. Because of their wide use and their relation to shafts, bearings are standardized parts. While, gears are one of the most important machine elements and are widely used especially in power transmission machinery and applications. These toothed wheels are meant to keep a specified ratio of angular velocities and shaft relations while transmitting powers between these components. Due to the high efficiency the implication of gears has emerged in various domains that involve high speed engines and tool boxes. Basically a gear is a toothed wheel having a specified space between teeth enabling it to form a mesh with different gears. The engagement between the gears enables power transmission between shafts based on different relations. The evaluation of tribology for these two components, involves prior knowledge of their different aspects.

The aim of this thesis is to define the fundamentals of tribology design. In fact, the idea is to analyze origins and different process of friction and also by quantifying friction and friction losses in machine elements. The aim is also to define different wear mechanisms, to understand the theory behind it and predict wear damage estimations. Similarly, the goal is to present theory for lubrication applications and different regimes. Moreover it will cover tribological analysis for bearings and gears.

2 SURFACE CHARACTEISTCS

2.1 2.1-Solid surfaces

The properties of surfaces are important to estimate the performance of any material for thermal or electrical applications. The understanding of the interaction between machine elements is based on the knowledge of different properties of the solid surfaces present in these elements. Hence, the characteristics of solid surfaces are critical for the determination of the tribological functions of machine elements since they affect the area of contact, wear, friction and lubrication features. Different factors contribute to the diversity of surface properties as the material of the solid, the interaction with the outer environment and preparation mode applied to the material’s surface.

A common aspect about different materials surfaces is that they always contain impurities and irregularities despite the formation mode. In fact, even the most precise preparation leaves casual deviations on the surface. In addition to that, the forming process can influence the properties of the zones that constitute the solid surface, and the mechanical behavior of the material is based on the amount of deformation of the layers in these zones. The bulk material of each metal is responsible for the reaction of these layers to deformation and hence altering their mechanical properties and their tribological functions. For example the forming of deformed layers from grinding, machining, polishing or lapping affects the metallurgical features of the surface such as strain or residual stresses, also the reaction of the solid surfaces to the environment may produce chemically reacted layers or physisorbed layers resulting in some natural oily or greasy films.

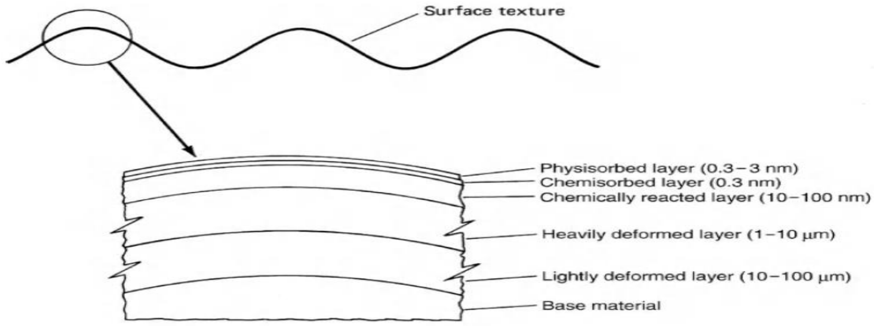

Figure 2 Example of a solid surfaces composition (Bhushan, 2013)

The interaction of solid surfaces with the environment may lead to the formation of new layers on the surfaces since most of metals are chemically reactive, for instance, the presence of oxygen may cause the formation of an oxide layer, also vapor, hydrocarbons and other environmental elements may induce the formation of natural films on the solid surfaces. Actually, these films affect the friction and wear of the solid surface and therefore the understanding of different layers is crucial for tribology study (Nosonovsky and Bhushan 2008, 13-17).

2.2 Surface roughness:

Surface roughness is a very important property affecting the behavior of mating surfaces when they come in contact, it results from the irregularities left by the cutting or shaping tools used to form the machine element; thus the definition of roughness as the deviation of the surface from ideal form, or the variation in surface height measured from a reference plane. It is considered as a texture of a surface including roughness height, waviness, lay and flaws. These features are formed according to different factors such as process, tools and the materials. Surface roughness can be evaluated in 2-D or 3-D models on a profile or an area, and ISO 25178 is the surface roughness measurement standard.

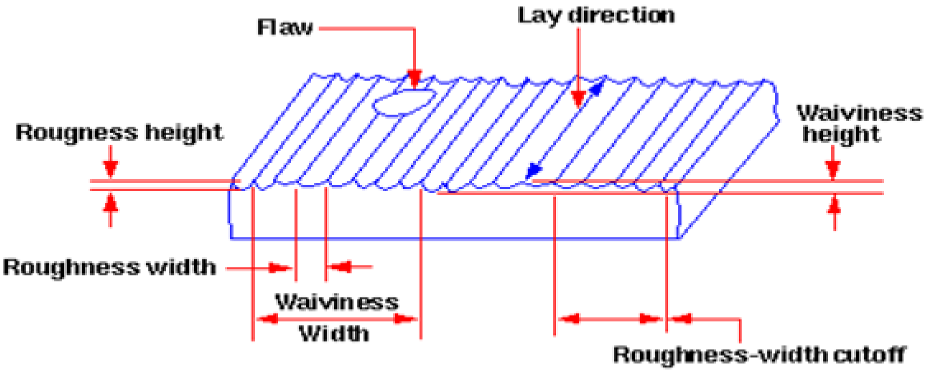

Figure 3 surface topography (ANSI/ASME 46.1:2009, 2)

Surface roughness is characterized by several parameters like R_z (average peak height), and R_p (maximum peak height) that are used in specific industries in certain countries, but the most used parameter are R_a which is the arithmetic average of absolute values(center line average) and R_q the root mean square(standard deviation of asperity heights). These two parameters are defined as follows

$$R_a = \frac{1}{n}\sum_{i=1}^{n}|y_i| \tag{1}$$

$$R_q = \sqrt{\frac{1}{n}\sum_{i=1}^{n} y_i^2} \tag{2}$$

Where n is the number of points equally spaced along the trace and y_i is the vertical distance from point I to the mean line. The mean line is a virtual line dividing the height of the profile.

Generally R_a and R_q values are measured in micro-meters and R_a is used to give roughness grades where these grades increase as the R_a value increase. All of the roughness parameters are evaluated in an arithmetical and statistical way, yet, spatial features are required for isotropic surfaces and Gaussian distribution is used to anticipate the behavior of these parameters. Also, for machines the roughness of surfaces can be predicted using machining factors like feed rate, working angle and tool edge roundness.

Low roughness values increases the manufacturing costs, but it is fundamental for predicting the material performance and the contact between elements since it has a primary role in tribology. Actually, rough surfaces tend to wear rapidly and have higher coefficients of friction compared to smooth ones. Besides, diverse measurement techniques are used to estimate surface roughness like optical, specular or scanning techniques (Bhushan, 2013).

2.3 Surface contact

The interaction between two engineering surfaces and the surface roughness initiates contact spots that form the area of contact between these two surfaces, this area is basically composed of asperities having different heights. In fact, this contact causes deformation that can be elastic or plastic depending on the load applied and the metals hardness. Contacting surfaces can be either conformal or non-conformal, for conformal contact the surfaces fit into each other so the load is carried by a large area, while for non-conformal the load is situated at high concentrated spots such as point or line contacts. In order to understand the type of contact between 2 bodies, the resultant geometry should be analyzed. The contact geometry results in a composite modulus and a composite curvature of interacting solids. The following figure shows the curvature calculation mode (Schmid, Hamrock and Jacobson 2014, 198-200).

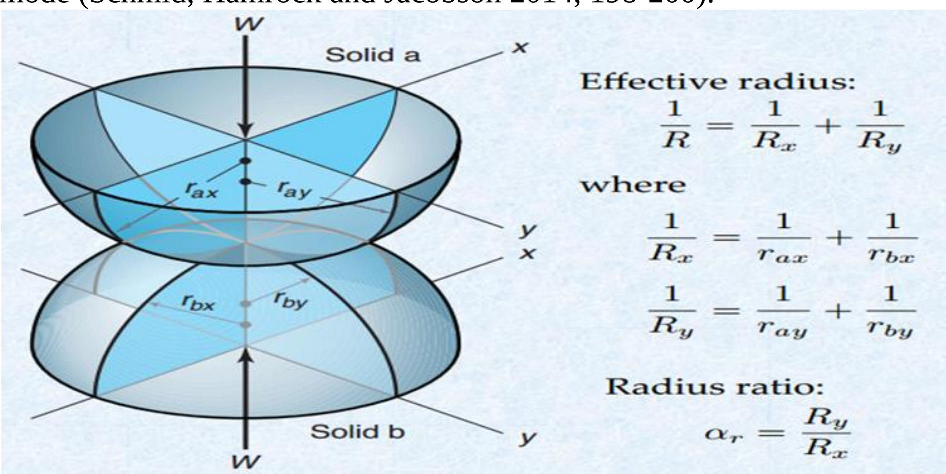

Figure 4 Geometry of contacting elastic solids (Schmid, Hamrock and Jacobson 2014, 199)

However, the analysis of the stresses and deformation in the area of contact depends mainly on the asperities geometry and requires deep mathematical analysis, actually there are different theories established to simplify this subject. The most efficient analyses was based on the Hertzian contact theory for smooth non-conformal surfaces, the following table summarize different formulas used for surface contact calculations for elastic solids where p_0 is maximum contact pressure, p_m mean contact pressure, Y is yield stress and K is Tresca stress criterion.

Parameter	Circular contact (Diameter = $2a$, Load = W)	Line contact (Width = $2a$, Load = W'/unit length along y axis)
Semi-contact radius or width	$a = \left(\frac{3WR}{4E^*}\right)^{1/3}$	$a = 2\left(\frac{W'R}{\pi E^*}\right)^{1/2}$
Normal approach	$\delta = \frac{a^2}{R} = \left(\frac{9W^2}{16RE^{*2}}\right)^{1/3}$	$\delta = \frac{2W'}{\pi} \left\{\frac{1-\nu_1^2}{E_1}\left(\ln\left(\frac{4R_1}{a}\right) - \frac{1}{2}\right) + \frac{1-\nu_2^2}{E_2}\left(\ln\left(\frac{4R_2}{a}\right) - \frac{1}{2}\right)\right\}$
Contact Pressure	$p = p_0\left\{1-\left(\frac{r}{a}\right)^2\right\}^{1/2}$ $p_0 = \frac{3}{2}p_m = \frac{3W}{2\pi a^2} = \left(\frac{6WE^{*2}}{\pi^3 R^2}\right)^{1/3}$	$p = p_0\left\{1-\left(\frac{x}{a}\right)^2\right\}^{1/2}$ $p_0 = \frac{4}{\pi}p_m = \frac{4W'}{\pi a} = \left(\frac{W'E^*}{\pi R}\right)^{1/2}$
Maximum tensile stress	$(1-2\nu)\,p_0/3$ at $r = a$ (on the contact plane, $z = 0$)	Zero
Maximum shear stress	$0.31\,p_0$ at $r = 0$ and $z = 0.48a$ for $\nu = 0.3$	$0.30\,p_0$ at $x = 0$ and $z = 0.78a$ for all ν
Limit of elastic deformation	$(p_0)_y = 1.60Y = 3.2k$, Tresca criterion $= 1.60Y = 2.8k$, von Mises criterion	$(p_0)_y = 1.67Y = 3.3k$ Tresca criterion $= 1.79Y = 3.1k$, von Mises criterion ($\nu = 0.3$) (von Mises depends on ν)

Composite curvature, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ where R_1 and R_2 are the principal radii of curvature of the two bodies (convex positive).

Composite modulus $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$ where E and ν are Young's modulus and Poisson's ratio, respectively.

Figure 5 deformation formulas for elastic solids (Bhushan 2013, 99)

When the asperity tip is assumed to be spherical the deformation problem is simplified and the load to initiate yield is calculated using the following formula

$$Wy = 21{,}17R^2Y\left(\frac{Y}{E'}\right)^2 \quad (3)$$

Where R is composite curvature, Y is yielding stress of softer material and E' is composite modulus. (Bhushan 2013, 98-101)

Conventionally, yielding occurs in the solid with lower yield stress or hardness. The deformation establishes stresses opposing the applied load and if this load is big enough the material existing in the contact area undergoes a full plastic deformation. Generally, the deformation mode of two surfaces in contact can be predicted using the plasticity index

$$\psi = \frac{E'}{H}\left(\frac{\sigma}{\beta}\right)^{\frac{1}{2}} \tag{4}$$

Where E' is composite modulus, H hardness of softer material, β is tip radius of asperity and σ is standard deviation of asperity heights. If ψ is smaller than 0,6 then the deformation is classified as elastic while if ψ is greater than 1 then the deformation mode is set to be plastic, if it is between these two values then it is called an intermediary area. The plasticity index simplifies the matter since it is calculation is only based on parameters than can be easily measured from the surfaces of the materials. The real area of contact for elastic deformation can be evaluated using the following formula

$$Ae = C\left(\frac{W}{E'}\right)^{\frac{1}{2}} \tag{5}$$

Where C is proportionality constant, w is load and E' is composite modulus.(Rahnejat 2010,49-50)
For real area of plastic contact the following equation is used

$$Ap = \frac{W}{H} \tag{6}$$

3 FRICTION

3.1 Definition

Friction is a component of tribology science, defined as the resistance force opposing relative motion between solid surfaces sliding or rolling against each other. It is the origin of energy loss and wear of moving contact surfaces of machine elements. There are two principal types of friction: dry friction which exists between two dry surfaces in contact with each other, and fluid friction where fluid is present between two surfaces in relative motion. Friction is a response and it is always opposing the motion direction, hence the force required to keep the relative motion of a body is kinetic force while static friction is the force required to initiate motion.

3.2 Sliding friction

When two machine elements are sliding against each other, sliding obeys two rules, the first states that the friction force F is directly proportional to the normal load W as follows

$$F=\mu W \tag{7}$$

Depending on motion presence, μ can be expressed as a coefficient of kinetic friction μ_k or a coefficient of static friction μ_s. While the second rule states the friction is independent of the contact area. Generally the coefficient of kinetic friction is independent of sliding velocity with some exceptions (SAHOO 2005, 49).

3.3 Adhesion

One on the most common phenomena responsible for friction is adhesion. Adhesive bonds are generated either by chemical or physical interaction of the bordering asperities, hence an additional force is needed to shear the contact generated. In fact, adhesion increases the real area of contact between surfaces, and the real area of contact can be evaluated as follows

$$Ar = Ap[1+\alpha(\frac{F}{W})^2]^{1/2} \tag{8}$$

Where Ap is the area for plastic contact, w is normal load, f is tangential load and α is factor (9 for metals).

Therefore friction force depends on bulk shear strength and the following equations are valid:

For dry contact

$$F_a = A_r \tau_a \tag{9}$$

$$\mu_a = \frac{\tau_a}{p_r} \tag{10}$$

For contact with partial liquid film

$$F_a = A_r[\alpha\tau_a + (1-\alpha)\tau_l] \tag{11}$$

For plastic contact

$$\mu_a = \frac{\tau_a}{H} \tag{12}$$

Where F_a is adhesion friction force, τ_a is average shear strength of dry surface, τ_l is shear strength of lubricant film,α is fraction of unlubricated area, p_r is real mean pressure and H is hardness of the weakest material. Recently, some progression in fracture investigation allowed a new formula to estimate the adhesion friction factor

$$\mu_a = C\frac{\sigma_{12}\sigma_c}{n^2(WH)^{1/2}} \tag{13}$$

Where σ_{12} is interfacial tensile strength, σ_c is critical crack opening, n is hardening factor and H is hardness and C is proportionality constant (Bhushan 2013, 208-214; Stolarski 2000, 15).

3.4 Ploughing and deformation

Ploughing is a process of grooves generation resulting from penetration of asperities on the higher hardness surfaces into a less hard surface in the other body, it is initiated either by asperities or hard wear particles in the contact area. The friction coefficient then is

$$\mu_p = C\frac{K}{E(WH)^{1/2}} \tag{14}$$

Where K is fracture toughness, however it was found that in this case the friction is related to the radius of curvature r and penetration depth b as follows

$$\mu_p = \frac{2}{\pi}[(\frac{2r}{b})^2 \sin^{-1}(\frac{b}{2r}) - ((\frac{2r}{b})^2 - 1)^{0.5}] \qquad (15)$$

Also when deformation occurs mechanical energy is dissipated and the work done by friction force can be considered equal to the work done by plastic deformation hence the coefficient of friction resulting from deformation is

$$\mu_d = \frac{Ar}{W}\tau_{\max}[1 - 2\frac{\ln(1+\frac{\tau_s}{\tau_{\max}}) - (\frac{\tau_s}{\tau_{\max}})}{\ln(1-(\frac{\tau_s}{\tau_{\max}})^2)}] \qquad (16)$$

Where τ_{max} is the ultimate shear strength and τ_s is the average interfacial shear strength (Bhushan 2013, 215-220; Stolarski 2000, 16-17).

3.5 Rolling friction

Rolling friction happens when a body of a near perfect shape is rolled over the surface of another body; it usually requires surfaces with low roughness. Rolling friction is usually small compared to sliding friction. Rolling friction is considered as a combination of slip, rolling and spinning where slip is the major factor responsible of friction generation. However, adhesion in rolling is characterized by limited micro-contacts in a micro-slip area, therefore adhesive bonds are not likely to form and if they exist the rupture of the contact would be in tension and not in shear. So the main factor responsible for friction is deformation with the exception of some cases.

Rolling can be classified as free or tractive. Rolling is considered free when no tangential or sliding can occur, the coefficient of free rolling friction can be obtained depending on the geometry of rolling body as example

For a cylinder

$$\mu_r = \frac{2\alpha a}{3\pi R} \qquad (17)$$

For a sphere

$$\mu_r = \frac{3\alpha a}{16R} \qquad (18)$$

Where R is radius of cylinder or sphere, a is half-width of contact and α is hysteresis loss factor.

Tractive rolling is rolling in presence of slip and friction force. Thus the coefficient of friction resulting from slip phenomena can be calculated as

$$\mu_r = \frac{V_s}{V_r}\mu_k \qquad (19)$$

Where V_s is slip velocity V_r is rolling velocity and μ_k is coefficient of sliding kinetic friction.
Also for some machine elements such as wheels or variable speed gears it is practical to use the following equation to estimate rolling friction coefficient

$$\mu_r = \frac{P'}{VW} \tag{20}$$

Where W is the normal load, P' is rate of energy dissipation and V is distance travelled (Stolarski 2000, 234-236).

3.6 Liquid contact friction

The presence of any kind of liquid film between contacting machine elements alters the nature of the friction force, actually the total friction force becomes the sum of intrinsic friction and the stiction force rising from meniscus and viscous influence, thus the total friction force F can be estimated as

$$F = \mu_r(W + F_m) + F_v \tag{21}$$

Where μ_r is coefficient of friction in meniscus absence, F_m is meniscus force in normal direction, F_v viscous force in sliding direction. And the friction coefficient including meniscus viscosity is F/w (Cai and Bhushan 2008, 100). Meniscus and viscous forces can be calculated using the following figure

Force		Flat-on-flat	Sphere-on-flat
Static meniscus force		$F_m = \frac{\pi x_n^2 \gamma(\cos\theta_1 + \cos\theta_2)}{h} + 2\pi\gamma x_n \sin\theta_{1,2}$ (1 and 2 for lower and upper surface, respectively)	$F_m = 2\pi R\gamma(\cos\theta_1 + \cos\theta_2) + 2\pi R\gamma \sin\varphi \sin(\varphi + \theta_2)$ (sphere in contact with flat) $\sim 2\pi R\gamma(\cos\theta_1 + \cos\theta_2)$ (for small ϕ) $\sim \frac{2\pi R\gamma(\cos\theta_1 + \cos\theta_2)}{1 + D/(s - D)}$ (sphere close to flat and for small ϕ) $\sim 2\pi R\gamma(1 + \cos\theta)$ (sphere close to a flat with a continuous liquid film and for small ϕ)
Viscous force	Normal separation	$F_{V\perp} \sim -\frac{3\pi\eta x_{n0}^4}{4t_s h_0^2}$ (for $h_s \sim \infty$)	$F_{V\perp} \sim -\frac{6\pi\eta R^2}{t_s} \ln\left[\frac{(4RD_0 + x_{n0}^2)^2}{8RD_0(x_{n0}^2 + 2RD_0)}\right]$ (for $D_s \sim \infty$)
	Tangential separation	$F_{v\parallel} = \frac{8\eta x_n'^3}{3t_s h_0}$	$F_{v\parallel} = \frac{8\eta[2R(s - D_0)]^{3/2}}{3t_s s}$

Figure 6 meniscus and viscosity calculations (Cai and Bhushan 2008, 99)

3.7 Friction and materials

Friction depends on the nature of the materials in contact and also on their mode of preparation and forming as well as factors such as operating environment ; in addition, the presence of natural films or surface preparations have a direct effect on the friction coefficient values. The table below gives friction coefficient values of different engineering materials.

Coefficient of friction for a range of material combinations				
combination	Static		Dynamic	
	dry	lubricated	dry	lubricated
steel-steel	0.5...0.6	0.15	0.4...0.6	0.15
copper-steel	-	-	0.5...0.8	0.15
steel-cast iron	0.2	0.1	0.2	0.05
cast iron - cast iron	0.25	0.15	0.2	0.15
friction material - steel	-	-	0.5-0.6	-
steel-ice	0.03	-	0.015	-
steel-wood	0.5-0.6	0.1	0.2-0.5	0.05
wood-wood	0.4-0.6	0.15...0.2	0.2...0.4	0.15
leather-metal	0.6	0.2	0.2...0.25	0.12
rubber-metal	1	-	0.5	
plastic-metal	0.25...0.4	-	0.1...0.3	0.04...0.1
plastic-plastic	0.3-0.4	-	0.2...0.4	0.04...0.1

Figure 7 materials friction coefficients (tribology-abc,2010)

4 WEAR

4.1 4.1-Definition

Wear is another component of tribology science. It is the action of material loss from contacting surfaces in relative motion. When contact happens during sliding, rolling or impact motion, the interaction between asperities of the contacting surfaces causes wear based on the mechanical behavior and the chemical nature of the these surfaces, in fact, wear starts by altering the properties of surfaces during contact. Next, wear particles start to form until actual material loss occurs. Wear is an independent response to friction and in some cases these two tribology elements have inverse behavior such as the case of ceramics having low wear and high friction. The wear process is used in industry in many applications such as shaving, grinding, machining and polishing, but it is an undesirable for machine elements such as bearings and gears. There are different types of wear based on the working conditions and the materials properties (Collins, Busby and Staab 2010, 59-60).

4.2 Adhesive wear

Adhesive wear occurs when adhesive junctions created in the contact area are sheared by sliding. Depending on the physical and the chemical properties of the material, asperities undergo several steps in order that final wear happens, these steps are initiated by deformations of contact asperities of the material and as sliding contact remain the surface films are eliminated. Then under load, instantaneous adhesive bonds are created but after further motion sliding shears this contact junctions and material is transferred. These fragments are modified and transferred until final creation of wear particles. Adhesive wear is one of the most common types of

wear and it is ruled by the material properties such as grain boundaries, in fact, materials containing high grain boundaries generate elevated rates of wear compared to materials with low grain boundaries.
Usually in order to estimate the amount of the adhesive material wear, the Archard model is used.

$$V_{ad} = k\frac{W}{H}L \tag{22}$$

Where V_{ad} is the volume removed, k is wear coefficient, W is normal load, L is sliding distance and H is the hardness of the softer material in contact. However recently Bhushan has showed that this formula is valid for plastic contacts while for elastic ones the following can be also used

$$V_{ad} = k\frac{W}{E'(\frac{\sigma'}{\beta'})}L \tag{23}$$

Where W, k and L are the same as previously, E is composite modulus, α' is composite standard deviation of heights and β′ is correlation height (Bhushan 2001, 279).

4.3 Abrasive wear

Abrasive wear occurs when two materials with significant hardness difference are in contact; it results from plastic deformation after penetration of the asperities of the hard material into the surface of the weaker one. Abrasive wear depends on different factors and it is a process ruled by several mechanisms such as plowing when the material is moved to the side resulting in the development of adjacent ridges and grooves, and cutting when the material is cut from the surface as microchips and debris without contacting to side of grooves, also fragmentation when the material cut from the surface and indentation start local fracture and hence the propagation of the crack results in significant loss of material. Abrasive wear is frequent and it is a grave type of wear compared to the other types because of the amount of material loss that can occur.
In order to estimate the volume of abrasive wear the same Archard approach can be used

$$V_{ab} = k_{ab}\frac{W}{H}L \tag{24}$$

Where W,H,L are the same as previously and k_{ab} is wear coefficient taking into account the geometry of asperity and typically ranges between 10^{-6} and 10^{-1}.
Because of the relation between fracture and abrasive wear the following equation has been set

$$V_{ab} = n^2\frac{P_y E W^{3/2}}{k_c{}^2 H^{3/2}}L \tag{25}$$

Where k_c is fracture toughness, P_y is yield strength, H is hardness of the softer material, n is the work hardening factor, W normal load and E is modulus of elasticity (Bhushan 2001, 282).

4.4 Surface fatigue wear

Contact surfaces are acting under different loads in most machine elements such as gears, bearings and friction drives. These elements are subject to repeated cycles of loading and unloading that lead to the initiation of cracks in their contact area and failure. Hence the classification as fatigue failure since it is related to stress contact. Wear fatigue is generated through transfer of stress to the contact spots and development of deformation in cycles, then after crack nucleation at subsurfaces, cracks are generated and propagated until the creation of wear particles. However, it is irrelevant to quantify fatigue wear as mass or volume, instead life cycles are used. For the analytical approach of fatigue wear several traditional theories are still applied and they are adapted to describe precisely the fatigue wear life of each machine element that undergoes sliding and rolling contacts. For sliding contacts, volume of material removed due to fatigue is calculated as

$$V_f = C\frac{\eta\gamma}{\varepsilon_f^2 H}WL \tag{26}$$

ε_f is strain failure during 1 cycle, H,W and L are same, γ is particle seize constant, η is distribution asperity of heights.(Wen and Huang 2012, 280)

4.5 Impact wear

Impact wear is produced either by erosion or percussion. Erosion is generated through kinetic energy of solid particles present in the air or liquid streams of interacting surface. This kinetic energy generates contact stress, then plastic deformation occurs until cutting of eroded material or intersection or cracks occur and hence wear of the material. The volume wear generated by erosion can be quantified as follows

$$V_e = \frac{kmv^2}{2H} \tag{27}$$

Where k is proportion of displaced material, m is mass of the body, v is velocity of the body and H is hardness of the material. Percussion is the cyclic collision repetition that machine elements incur. This mechanism results from combination of adhesion, abrasion, surface fatigue and fracture, the volume of wear resulting from this event is calculated as

$$V_p = N\frac{kmv^2}{2\mu H} \tag{28}$$

Where k, m, v are same as for erosion, μ is coefficient of friction and n is number of cycles (Bruce 2012, 137).

4.6 Wear due to chemical reactions

The reactivity of material surfaces and the interaction with the outer environment may lead to chemical reaction that can generate undesirable phenomena such as oxidation and corrosion. When these surfaces are subject to friction of rubbing, abrasion and cracks appear to lead to wear. The volume of chemical wear is generally estimated as

$$V_c = L\frac{kdW}{\xi^2\rho^2 VH} \tag{29}$$

Where d is diameter of asperity, L is the distance of constant, W is normal load, V is speed, k is factor of oxidation, ρ is thickness of reaction layer ξ critical thickness of layer and H is hardness.
There are also secondary types of wear like fretting and electrical induced wear. (Holmberg and Mattews 2009, 62)

4.7 Wear of lubricated contacts

The lubrication of contacting surfaces is characterized by the Lambda ratio which is the ratio of the film thickness over the equivalent roughness of the contacting solids. Generally if the ratio is greater than 3 then the probability of contact is neglected therefore mechanical wear would not occur. For values of Lambda between 1 and 3 the load would be shared by the lubricant and the asperities and contact can occur and hence adhesive wear between touching asperities is possible. Also if the value of ratio Lambda is less than 1 then the contact is serious and will lead to fatigue and adhesive wear. All contacts for different values of Lambda are either elastic or plastic depending on the plasticity index of the interacting solids. The introduction of lubricants changes the wear problem in contacting surfaces. In fact chemical wear can be avoided if adequate chemical composition of the lubricant is used, also abrasive wear is rare in lubricated contacts since the lubricant and the contacting surfaces in machine elements are mostly clean before use, then abrasion is not possible until accumulation of wear particles in the contacting interface. Hence the most common wear types in lubrication are fatigue and adhesive wear. Fatigue is difficult to estimate mathematically, so a different method is used to estimate the probable life for each machine element depending on different operating factors. Adhesive wear volume V can be calculated in lubricated metal elements using the following relation

$$V=L\ k_m(1+\mu^2)^{0.5}\beta(W/P_m) \tag{30}$$

Where L is the sliding distance, μ is friction coefficient, k_m is specific constant for rubbing materials, β is fractional film defect, W is the normal load and P_m is flow pressure under static load. P_m and β are calculated as following

$$P_m=(S/\mu)(1+3\mu^2)^{0.5} \tag{31}$$

$$\beta = \frac{A_m}{A_r} = 1-\exp[-(\frac{30.9\ 10^5 T_m^{0.5}}{vM^{0.5}})\exp(\frac{-E_c}{RT_s})] \tag{32}$$

Where S is shear strength, A_m is metal to metal area of contact, A_r is real area of contact, T_m is lubricant melting temperature, v is volume of lubricant, M is molecular weight of lubricant, E_c is absorption heat of the lubricant, R is the gas constant and T_s is absolute temperature of contact zone (Stolarski 2000, 37-41).

4.8 Wear of materials

Internal features of materials such as mechanical properties and chemical reactivity are fundamental in the resistance of wear, but external factors are also important like working environment, direct contact, presence of oxygen and temperature. The most common property that affects the behavior of a material to wear is hardness and generally the coefficient of wear is determined experimentally considering the effect of temperature and load. The following figure summarizes the range of wear coefficients for metals (Juvinall and Marsheck 2012, 390).

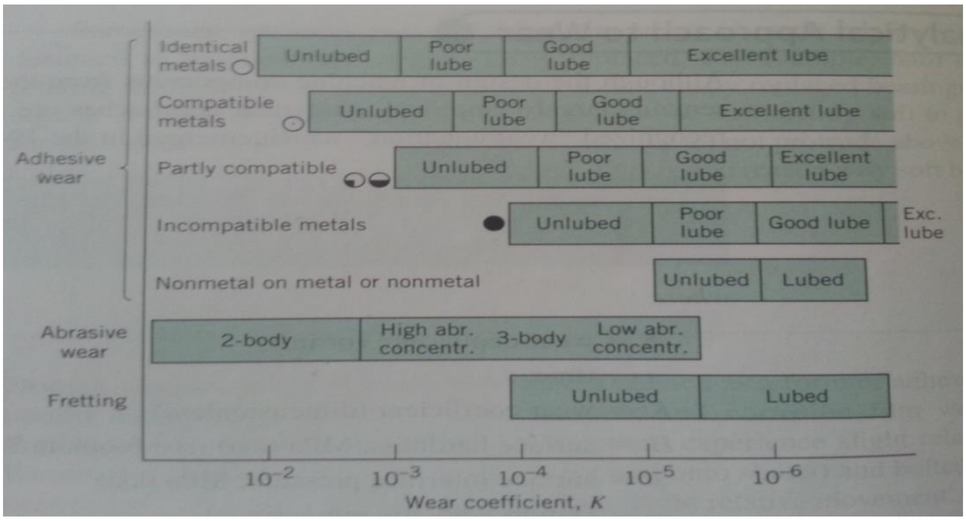

Figure 8 wear coefficient of metals (Juvinall and Marsheck 2012, 390)

5 LUBRICATION

5.1 Definition

As a third component of tribology science, Lubrication is a fundamental concept for the performance of machine elements. It is essential for elements experiencing direct solid contact or dynamic motion and it can be defined as the process of introducing a substance between two relatively close surfaces in relative motion, in order to reduce the volume of friction and wear. In absence of lubrication friction and wear will cause fast failures of the machine components and according to studies made by SKF more than 36 percent of bearings and gears are caused by poor lubrication. The formation of natural films on solid surfaces is not common so operational lubrication is compulsory for most machine elements and lubrication science and industry have been evolving since the creation of machinery. Based on the nature of the lubricant, lubrication can be solid or fluid. In fact, some machine elements operate under high temperatures and different pressures and fluid films are abortive, so the use of solid film lubricant is essential. Solid film is a substance such as powder or solid material used to reduce wear and friction effect. The most used dry lubricants are graphite and molybdenum disulfide that can reach up to 1100 Celcius degree and can be used for ball bearings and space industries. While fluid film can be liquid or gaseous, the thin film reduces contact and the thick one can prevent contact between lubricated surfaces. Air can be used as a lubricant while liquid lubricants are based on mineral oils (SKF, 2014).

5.2 Fluid film lubrication regimes

5.2.1 Hydrostatic lubrication

As the name implies, this regime is based on pressure. A relatively thick film is introduced into the contact area at a continuous pressure high enough to separate the contacting surfaces. It does not require motion of the elements. This kind of lubrication leads to absolute minimum friction and it is used in applications where velocities are low or when rubbing or interaction contact is not desired. Usually, corrosion results from hydrostatic lubrication due to interaction between the lubricant and the material of the lubricated surfaces.

5.2.2 Hydrodynamic lubrication

In this regime the pressure to separate contacting surfaces is built up when the sliding velocities of contacting machine elements pull the introduced lubricant into the cavity of the contacting interface. It provides a load carrying capacity that is explained by the viscosity of the lubricant that prevents instantaneous squeezing out of the contact interface. It is the most applied regime in modern industries and it is considered as ideal because the films are many times thicker than the heights of the contacting surfaces asperities but contact can still occur at stars and stops because of low speeds. These start-stop conditions can cause adhesive wear. High speeds can increase the friction and induce chemical corrosive wear .It is also called full film lubrication or fluid lubrication. The following figure shows the lubrication for a hydrodynamic bearing

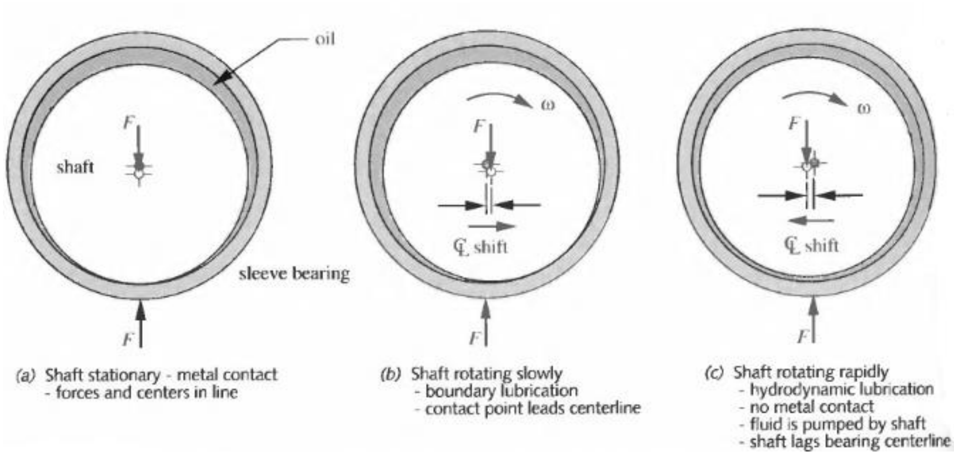

Figure 9 Boundary and hydrodynamic lubrication for a sleeve bearing (Norton 2011, 584)

5.2.3 Elastohydrodynamic lubrication

This regime is the combination of the hydrodynamic lubrication and the elastic deformation of asperities in the contacting interface. Velocity and load result in flat surfaces through contact that allows a full hydrodynamic film to form. It is used for rolling of non-conforming surfaces such as rolling element bearing or gears and the lubricant film is thinner than the hydrodynamic regime, it is used with high loads. These high loads affect several aspects such as geometry, viscosity and deformation that alter important properties such as thermal and shear rates. Fatigue wear is most common for this regime while corrosive wear can happen and starts-stops situations induce adhesive wear.

5.2.4 Boundary lubrication

At extreme conditions like high pressure, temperature or load, velocity and fluid viscosity may decrease blocking the formation of full film, actually surfaces come too close together and the contact is dominated by interaction between surface asperities and the monomolecular or multimolecular films. Some asperities of the contacting surfaces come in contact while the rest are separated by the thin film. The phenomena of formation of boundary films, is induced by physical or chemical absorption and chemical reactions. This kind of lubrication is frequent in machine elements operating at high loads and low speeds. Usually, in boundary lubrication viscosity does not affect wear and friction behavior, though adhesive and chemical wear are likely to appear.

5.2.5 Mixed lubrication

Mixed lubrication is a state when two lubrication regimes can be present. It can be defined as intermediate lubrication between hydrodynamic or elastohydrodynamic and boundary lubrication. It is characterized by multimolecular lubricant films. Contact can occur as parts of the interface are covered by a partial film. In absence of films absorption adhesion and metal interaction can occur which eventually lead to adhesive and chemical wear (Budynas and Nisbett 2008, 298-599).

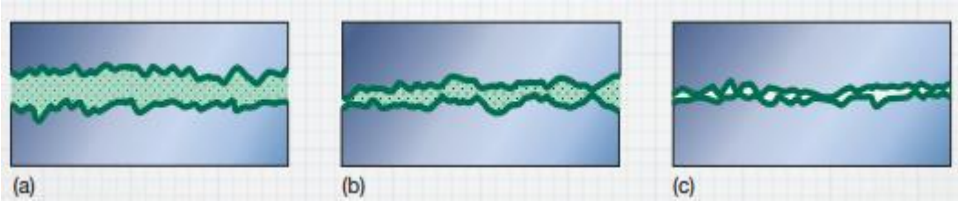

Figure 10 Film conditions of lubrication regimes. (a) Fluid film lubrication - surfaces separated by a bulk lubricant film; (b) partial lubrication - both bulk lubricant and boundary film play a role; (c) boundary lubrication - performance depends entirely on boundary film.(Hamrock, Scmid and Jacobson 2004, 8)

5.3 Favorable conditions

Generally the failure of film lubrication is due to the melting of films at extreme temperatures and the degradation of contacting surface multilayers as a result of critical load and velocity. Besides, there are many conditions that can improve the lubrications. In fact the lubricated material itself can improve lubrication if the surface has high energy which facilitates absorption of molecules such as in the case of metals. Also the reactivity of the surfaces enhances the formation of stable chemical films. From another side, the nature of the lubricant determines its performance, for example the increase in the interaction of molecules improves boundary lubrication and the monomolecular and multimolecular layers diminish friction rates. Besides the length of the carbon chain in the lubricant is important since the friction decreases as length increases. Also chemical films that can form from fatty acids or mineral oils block wear and friction of reactive surfaces. Moreover, absorbed gases like air, which is most common in boundary lubrication, are important because it was proven that oxygen presence decrease the friction rate (Bhushan 2013, 504-510).

5.4 Liquid lubricants

Liquid lubricants are formed by natural or synthetic oils and the most used oils in industry are a mixture of both to improve the oil performance. Natural organic oils are based on animal fat and vegetable and considered as good boundary lubricants but they are thermally and oxidatively unstable and the maximum operating temperature is 130 degrees. While mineral oils are superb boundary lubricants because of the presence of paraffins that increase the working temperature up to 200 degrees. Synthetic oils are used because of critical conditions of pressure and humidity and operating temperature can reach up to 450 degrees. A synthetic hydrocarbon is based on replacement of weak bonds fraction in the carbon chain by hydrocarbon chain while chloroflorocarbons replace hydrogen by chlorine and fluorine to protect carbon bonds. Esters exhibit good temperature and viscosity properties because they contain strong bonds. Acids and alcohol can produce many esters. Silicones constitute excellent hydrodynamic lubricants because of their thermal stability, their low surface tension and because they are chemically inert but they are the most expensive. Also silanes have supreme temperature stability. Polyphenyl ethers have good resistance to oxidation and can reach over 500 degrees. Equally perfluoropolyethers are a good class of synthetic oils because of their stability at high temperature and a polar group can be easily attached to them.

To enhance the performance of these oils, different additives are applied. Additives control the function of the lubricants and hence the classification as nonreactive, low-friction, anti-wear or extreme pressure lubricants. For example anti-wear additives are used for the surface of bearings to form a coating that resist forces responsible of wear, while extreme powerful anti-wear and pressure additives are used in eloashydrodynamic bearings because of high loads. For good boundary lubricants up to 0.5 % additives are applied. The most used additives are ZDDP and TCP.
In some applications where liquids cannot be contained, greases are applied for spatial and cost considerations. Greases can function under maximum temperatures of 175 degrees (Bhushan 2013, 511-520).

5.5 Viscosity

Viscosity can be defined as the resistance of the oil to shear or the resistance the resistance to flow between individual layers of the oil which caused by intermolecular forces. Despite compressibility and stability, viscosity is the most important characteristic of lubricants since it has a direct effect on the temperature, friction, power and heat dissipation of lubricated contact. It is measured using viscometers and there are three main types based on their geometry, falling, capillary and rotational viscometers. Numerically viscosity is defined as following

$$\eta = \frac{\tau}{\dot{\gamma}} = \frac{F}{\dot{\gamma} A} \qquad (33)$$

Where η is absolute viscosity, τ is shear stress $\gamma^{\cdot}$ is shear strain rate, F is the force and A is the area.
In absence of slip and if velocity is continuous absolute viscosity is then

$$\eta = \frac{\tau h}{V} \qquad (34)$$

Where h is film thickness and V is relative linear velocity. It can also be measured as kinematic viscosity through the relation

$$\nu = \frac{\eta}{\rho} \qquad (35)$$

Where ρ is density of the lubricant.
Viscosity ranges and grades are used to classify different lubricant oils. The main systems are those of the SAE (Society of Automotive Engineers), the AGMA (American Gear Manufacturers Association), the ISO (International Standards Organization), and the ASTM (American Society for Testing and Materials). Other systems are applied in special cases. There useful charts for conversion of different system grading such as ASTM D 2161 for viscosities conversion.

ISO Grade	Equivalent SAE Grade	Viscosity				Density	
		Centistokes		10^{-6} reyns (lb s/in^2)		kg/m^3	lb/in^3
		40 °C	100 °C	104 °F	212 °F		
32	10W	32	5.4	4	0.6	857	0.0310
46	20	46	6.8	5.7	0.8	861	0.0311
68	20W	68	8.7	8.5	1.1	865	0.0313
100	30	100	11.4	12.6	1.4	869	0.0314
150	40	150	15	19	1.8	872	0.0315
220	50	220	19.4	27.7	2.4	875	0.0316

Figure 11 examples of ISO oil grades

In order to define a relation between temperature change and viscosity, an arbitrary numbering scale called viscosity index VI was set. Viscosity index is described as low when it is below 35, medium between 35 and 80, high between 80 and 110 and very high above 110. Slight temperature changes occur with high index and big changes occur with low index. Temperature has a direct effect on viscosity. In fact the oil expands and the intermolecular forces weaken as the temperature rise what results in a decrease of the viscosity. The simplest relation to describe this dependence is

$$\eta = \eta_o \exp[\beta(\frac{1}{t} - \frac{1}{t_0})] \qquad (36)$$

Where η and tare viscosity at temperature t and η_o is is viscosity at time t_0 while β is temperature- viscosity coefficient.

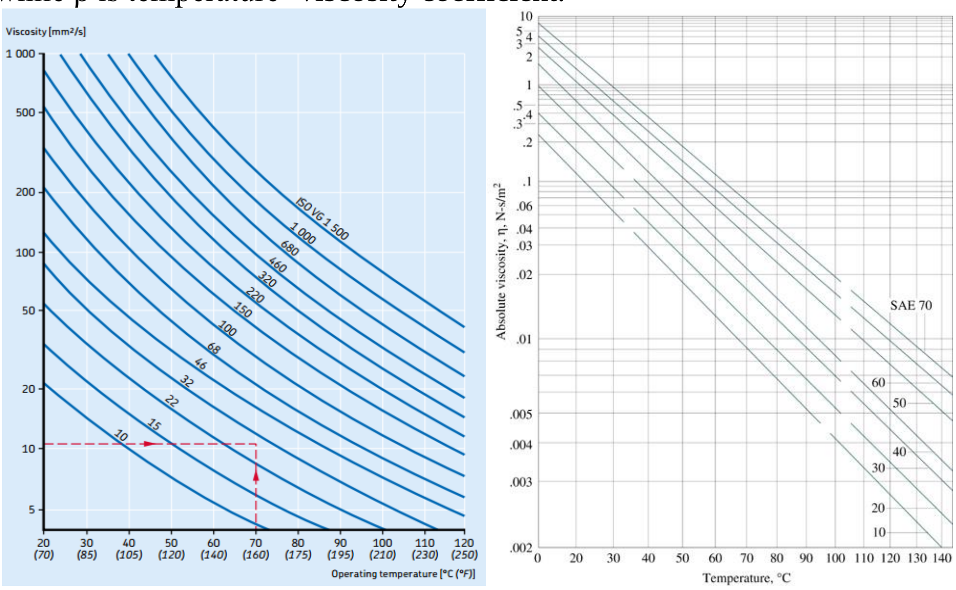

Figure 12 viscosity-temperature changes (SKF,2014)

High pressure causes closeness of molecules that strengthen intermolecular forces and hence increase viscosity, this can be described with the relation

$$\eta = \eta o \exp[\alpha p] \qquad (37)$$

Where p is the pressure difference and α is the viscosity-pressure coefficient (Stachowiak and Batchelor 2005, 11-22).

6 SLIDING ELEMENT BEARINGS

6.1 Hydrodynamic bearings

6.1.1 Journal bearing

6.1.1.1 Unloaded journal bearing

Journal bearing belongs to the plain bearings type and they are used to carry radial load. Both of their wear surfaces are cylindrical, they can be sleeve bearings or a segmented journal. They are characterized by high-load capacity, low friction rate, resistance to shock and vibration and low sensitivity to lubricant contaminations. The investigation of journal bearings showed that friction is generated as result of the lubricant shearing, that the bearing is totally supported by the lubricant film and that the viscosity is the main factor governing the behavior of friction and load carrying capacity. In order to estimate friction of unloaded or lightly loaded concentric journal bearing the following equation was established by Petroff

$$F = \eta_0 \frac{u_a}{h} A \tag{38}$$

Where *A* is the surface area of the bearing interface, *h* is the film thickness, μ_a is the relative velocity, and η_o is the viscosity at ambient pressure and constant temperature

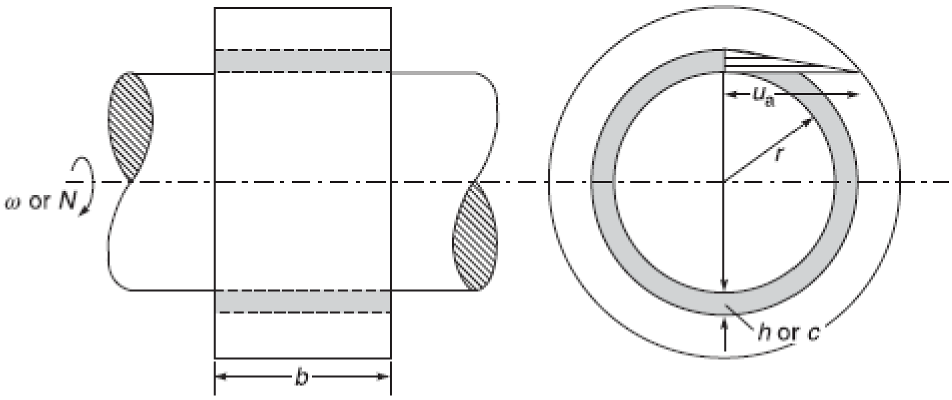

Figure 13 concentric journal bearing (Bhushan 2013, 410)

If the radius is r, b is the width and the angular velocity is ω as shown in the figure above then

$$F = \frac{2\pi\eta_0 r^2 b\omega}{h} \tag{39}$$

If W is the normal load then coefficient of friction μ is

$$\mu = \frac{F}{W} = \frac{2\pi\eta_0 r^2 b\omega}{Wh} \tag{40}$$

The heat loss can be calculated using friction torque T as follows

$$H_v = Fu_a = T\omega = \frac{2\pi\eta_0 r^3 b\omega^2}{h} \tag{41}$$

6.1.1.2 Loaded journal bearing

Based on the equilibrium conditions of a loaded journal bearing, the total load W carried by the shaft is equated by the net contact force F_r. The contact force F_r is the resultant of the normal force F_n and the friction force F_f. The quotient of the friction component by the normal component gives the friction coefficient μ at the contact point. The rotation of the shaft and the bearing at an angular velocity ω creates a radial clearance c between the center of journal and the bearing. If rotation occurs when a viscous fluid is applied, this fluid is inserted into the gap by the convergent channel of this gap. Reaching sufficient velocity, a hydrodynamic pressure can be built exceeding the applied load. Hence the separation of the contacting surfaces and the journal rotates in the same direction of the bearing until equilibrium is reached at a distance separating the two centers, this distance is called eccentricity and it is assumed to be nil at light loads. If eccentricity and clearance are equal then there is contact. The ratio of eccentricity to clearance is known as eccentricity ratio ε.

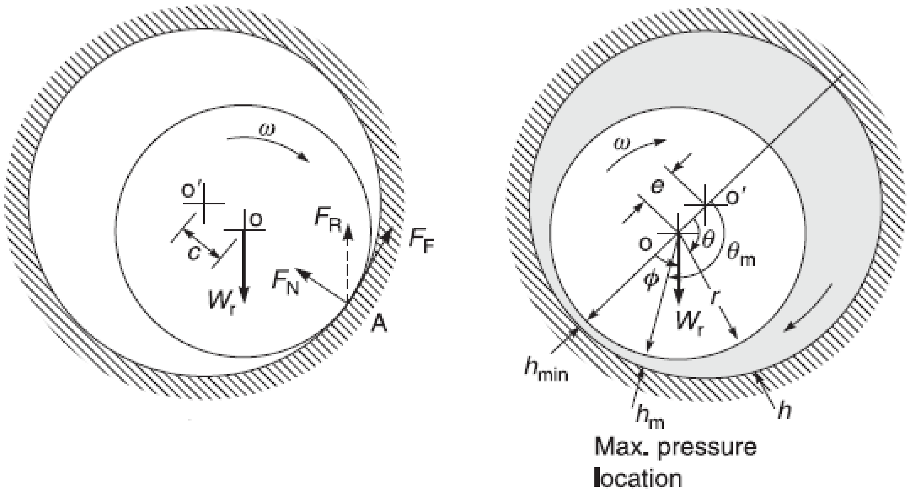

Figure 14 Unlubricated and lubricated rotating journal bearing (Bhushan 2013, 444)

Further investigation of the behavior of film thickness related to rotational angle θ shows that maximum pressure and minimum film thickness occur when θ is equal to π while minimum pressure and maximum film thickness appear at nil θ as follows

$$h_{min} = c - e = c\,(1 - \varepsilon) \tag{42}$$

$$h_{max} = c\,(1 + \varepsilon) \tag{43}$$

Between these 2 limits the film thickness is assumed to be

$$h = c\,(1 + \varepsilon \cos\theta) \tag{44}$$

From another side it has been suggested that the width can affect the bearing features and analytical solutions for film thickness lubrication can be evaluated for short and long bearings (Norton 2011, 589-590).

6.1.1.3 Infinitely-wide bearings

Assuming that the fluid flow is one-dimensional and if the bearing width is largely greater than the diameter, then the flow between surfaces is assumed to be in the circumferential direction. Using the Sommerfeld substitution for the Reynolds integral of pressure over the angular position of the bearing, the following results for an extremely wide bearing were deducted

$$p = p_0 + 6\eta_0\omega(\frac{r}{c})^2 \frac{6\varepsilon\sin\theta(2+\varepsilon\cos\theta)}{(2+\varepsilon^2)(1+\varepsilon\cos\theta)^2} \quad (45)$$

$$h_m = \frac{2\,c(1+\varepsilon^2)}{2+\varepsilon^2} \quad (46)$$

$$\theta_m = \cos^{-1}(-\frac{3\varepsilon}{2+\varepsilon^2}) \quad (47)$$

$$p_m = 6\eta_0\omega(\tfrac{r}{c})^2 \frac{3\varepsilon(4-5\varepsilon^2+\varepsilon^4)^{1/2}(4-\varepsilon^2)}{2(2+\varepsilon^2)(1+\varepsilon^2)^2} \quad (48)$$

$$\phi = \tan^{-1}[\tfrac{\pi}{2\varepsilon}(1-\varepsilon^2)]^{1/2} \quad (49)$$

Where p is pressure at position θ, p_0 is pressure at θ equal 0, r is the journal radius, h_m is maximum film thickness, θ_m is position of maximum pressure p_m and ϕ is the attitude angle separating the minimum film location and the resultant load axis.

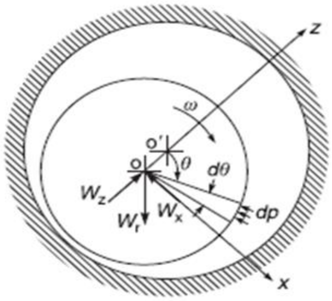

Figure 15 Force system action on the journal(Bhushan 2013, 448)

Using the geometry above and the Sommerfeld substitution, the load capacities per width are

$$W_x = 6\eta_0\omega r(\tfrac{r}{c})^2 \frac{\pi\varepsilon}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} \quad (50)$$

$$W_z = 6\eta_0\omega r(\tfrac{r}{c})^2 \frac{\varepsilon^2}{(2+\varepsilon^2)(1-\varepsilon^2)} \quad (51)$$

$$W_r = \eta_0\omega r(\frac{r}{c})^2 \frac{6\varepsilon[\pi^2-\varepsilon^2(\pi^2-4)]^{1/2}}{(2+\varepsilon^2)(1-\varepsilon^2)} \quad (52)$$

While the shear forces on the journal and the bearing are

$$f_j = 4\pi\eta_0\omega(\tfrac{r^2}{c}) \frac{1+2\varepsilon^2}{(2+\varepsilon^2)(1-\varepsilon^2)^{1/2}} \quad (53)$$

$$f_b = 4\pi\eta_0\omega(\tfrac{r^2}{c})\frac{(1-\varepsilon^2)^{1/2}}{(2+\varepsilon^2)} \tag{54}$$

The journal loads are balanced as follows

$$rf_j = rf_b + W_r \tag{55}$$

Hence the friction coefficient and heat dissipation power are

$$\mu = \frac{f_j}{W_r} = (\frac{r}{c})\frac{4\pi(1+2\varepsilon^2)(1-\varepsilon^2)^{1/2}}{6\varepsilon[\pi^2-\varepsilon^2(\pi^2-4)]^{1/2}} \tag{56}$$

$$H_v = f_j r\omega \tag{57}$$

The lubricant is normally supplied through a hole or a groove. The fluid pressure inserted to the clearance of the journal bearing is the supply pressure and it can be different from ambient pressure. The above results allowed the presentation of the Sommerfeld number known also as bearing characteristic number

$$S = \frac{\eta_0 N}{P}\left(\frac{r}{c}\right)^2 = \frac{(2+\varepsilon^2)(1-\varepsilon^2)}{6\pi\varepsilon[\pi^2-\varepsilon^2(\pi^2-4)]^{1/2}} \tag{58}$$

Where P is load per unit projected bearing area ($P = W_r/2r\, b$) and N is the angular speed in revolution per seconds (Williams 2005).

6.1.1.4 Infinitely-short bearings

In the case of short bearings (d/b>2) the pressure flow is considered in the axial direction and pressure variation is found to be parabolic in function of the angle θ and the width direction as

$$p = 3\eta_0\omega\frac{\varepsilon}{c^2}(\frac{b^2}{4}-y^2)\frac{\sin\theta}{(1+\varepsilon\cos\theta)^3} \tag{59}$$

While maximum pressure and position are

$$p_m = \frac{3\eta_0\omega\varepsilon b^2\sin\theta_m}{4c^2(1+\varepsilon\cos\theta_m)^3} \tag{60}$$

$$\theta_m = \cos^{-1}[1-\frac{(1+24\varepsilon^2)^{1/2}}{4\varepsilon}] \tag{61}$$

The loads are

$$W_x = \eta_0\omega rb(\frac{r}{c})(\frac{b}{d})^2\frac{\pi\varepsilon}{(1-\varepsilon^2)^{3/2}} \tag{62}$$

$$W_z = \eta_0\omega rb(\frac{r}{c})(\frac{b}{d})^2\frac{4\varepsilon^2}{(1-\varepsilon^2)^{1/2}} \tag{63}$$

$$W_r = \eta_0\omega rb(\frac{r}{c})(\frac{b}{d})^2\frac{\varepsilon}{(1-\varepsilon^2)^2}[16\varepsilon^2+\pi^2(1-\varepsilon^2)]^{1/2} \tag{64}$$

The attitude angle and the shear forces acting on the journal are

$$\phi = \tan^{-1}[\frac{\pi(1-\varepsilon^2)^{1/2}}{4\varepsilon}] \tag{65}$$

$$f_j = f_b = \eta_0\omega rb(\frac{r}{c})\frac{2\pi}{(1-\varepsilon^2)^{1/2}} \tag{66}$$

While friction coefficient and the heat loss are

$$\mu = \frac{f_j}{W_r} \qquad (67)$$

$$H_v = f_j r\omega \qquad (68)$$

$$\Delta t = \frac{\mu W_r \omega r}{Q\rho c_p} = \frac{4\pi P(\frac{r}{c})\mu}{\rho c_p Q_d\,(1-0.5\frac{Q_s}{Q})} \qquad (69)$$

Where Q, Q_s and Q_d are respectively volumetric, side and dimensionless flow, c_p is specific heat of the fluid and ρ is the density (Hamrock, Scmidt and Jacobson 2004, 270-275 280-285). Also, the mean temperature of the oil can be calculated as

$$t_m = t_i + \frac{\Delta t}{2} \qquad (70)$$

6.1.1.5 Slenderness ratio

Slenderness is the ratio of the length of the bearing over the diameter. For various values of this ratio Raimondi and Boyd developed different charts of characteristic numbers in functions of different variables such as film thickness, attitude angle, flow pressure and friction coefficient. These charts allowed the determination of other variables such as the change in the oil film temperature defined as following (Bubynas and Nisbett 2008).

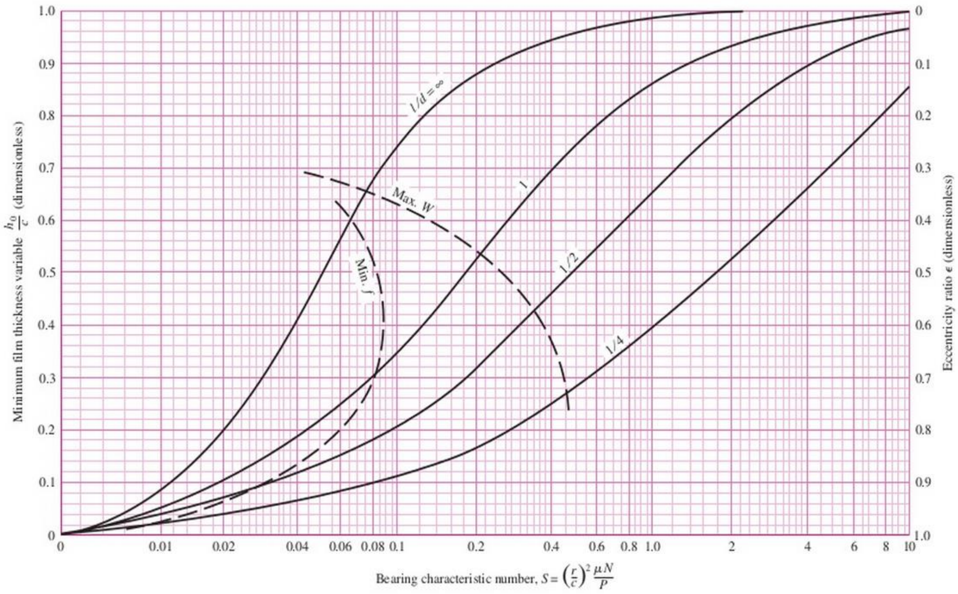

Figure 16 Chart example for hydrodynamic bearing (Bubynas and Nisbett 2008,616)

6.1.2 Thrust bearings

6.1.2.1-Fixed-pad thrust bearing

Thrust bearings are designed to support thrSust loads and their surfaces are perpendicular to the axis of rotation, they contain different pads in order to ensure satisfactory lubrication. These pads can be fixed-inclined or pivot pads. The following figure shows a fixed-inclined-pad thrust bearing where h_i and h_0 are maximum and minimum film thickness

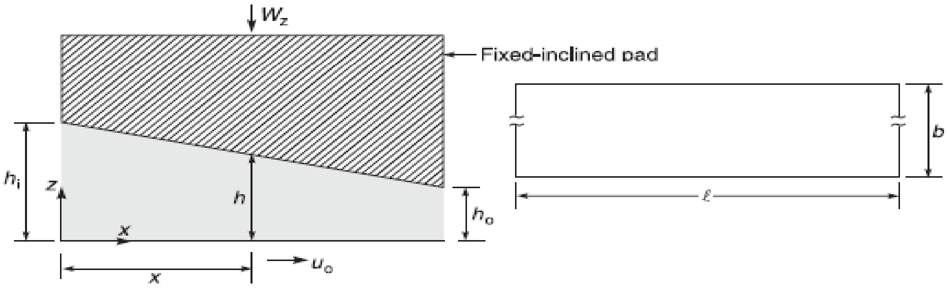

Figure 17 Figure16:Fixed-inclined thrust bearing lubrication (Bhushan 2013,431)

For the case of a single pad, assuming that the lubricant flow is in the direction of the sliding, and using the Reynolds equation the film thickness h can be expressed through the following relation

$$\frac{h}{h_0} = 1 + m(1 - \frac{x}{l}) \tag{71}$$

With further integrations the following results were found

$$h_m = 2h_0(\frac{1+m}{1+m}) \tag{72}$$

$$p = \frac{6\,\mathrm{lu}_0\eta_0}{h_0^2}[\frac{m\frac{x}{l}(1-\frac{x}{l})}{(2+m)(1+m-m\frac{x}{l})^2}] \tag{73}$$

$$p_m = \frac{\mathrm{lu}_0\eta_0}{h_0^2}[\frac{3m}{2(2+m)(1+m)}] \tag{74}$$

$$x_m = l(\frac{1+m}{2+m}) \tag{75}$$

Where m = (h_i /h_0)-1, p is pressure flow at position x, p_m is maximum pressure at position x_m and h_m is the film thickness at maximum pressure. Also the following result s were developed

$$q = \frac{u_0 h_m}{2} \tag{76}$$

$$w = \frac{\mathrm{l}^2\mathrm{u}_0\eta_0}{h_0^2}[\frac{6\ln(1+m)}{m^2} - \frac{12}{m(2+m)}] \tag{77}$$

$$F = \frac{\mathrm{lu}_0\eta_0}{h_0}[\frac{4}{m}\ln(1+m) - \frac{6}{(2+m)}] \tag{78}$$

$$\mu = \frac{F}{W} \tag{79}$$

$$H_v = Fu_0 \tag{80}$$

Where q is shear induced flow and U_0 is the sliding velocity, W is normal load and F is shear force. For Multiple pads bearing the load capacity, the heat loss and the volumetric flow rate are to be multiplied by number of the pads (Bhushan 2013, 430-436).

6.1.2.2 Pivot-pad and Rayleigh thrust bearings

For the pivot-pad thrust bearings it is imperative that pressure center x_c is at same position of the pivot to locate the resultants forces at this same position as follows

$$x_c = \frac{(1+m)(3+m\ln)(1+m)-3m-2.5m^2}{m(2+m)\ln(1+m)-2m^2} \quad (81)$$

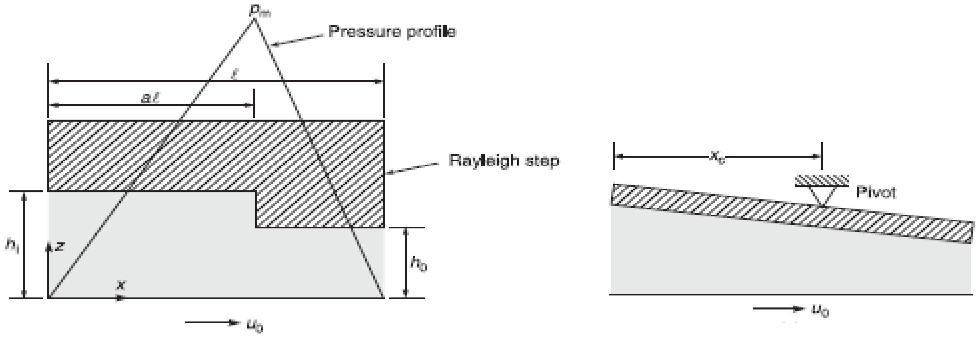

Figure 18 Rayleigh and pivot-pad thrust bearings (Bhushan 2013, 439)

Even it is difficult to manufacture the step of Rayleigh step bearing, it is characterized by great load capacity and the following equations can be used

$$p_m = \frac{6\mathrm{a}\ \mathrm{l}u_0\eta_0(1-a)m}{h_0^2[a+(1-a)(1+m)^3]} \quad (82)$$

$$W = \frac{lP_m}{2} \quad (83)$$

$$q = \frac{(1+m)^3 p_m}{12\eta_0 la} + \frac{(1+m)h_0}{u_0} \quad (84)$$

Where a is the step length, l is bearing length, u_0 is sliding velocity and q is volumetric flow (Bhushan 2013, 438-440).

6.1.2.3 Slenderness ratio approach

Identically to journal bearings Raimondi and Boyd presented diagrams to define hydrodynamic thrust bearings in function of different variables governing lubrication.

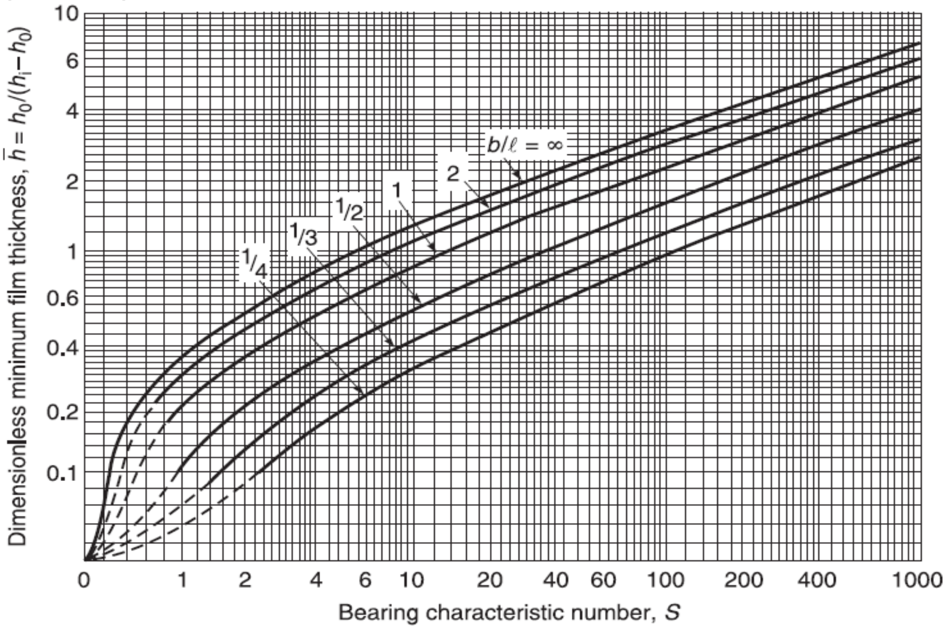

Figure 19 chart example for a fixed-inclines-past thrust bearing(Bhushan 2013, 443)

6.1.3 Squeeze film bearings

This type is generated by high frequencies oscillation of bearings, in fact bearings are under a cushioning effect resulting in that the lubricant cannot be immediately squeezed out of the contacting area. Hence a load-carrying capacity is developed. The investigation of two parallel-surface squeeze film bearings using Reynolds equation gave the following relations

$$p = \frac{3\eta_0 v}{2h_0^3}(l^2 - 4x^2) \quad (85)$$

$$p_m = \frac{3\eta_0 v l^2}{2h_0^3} \quad (86)$$

$$W = \eta_0 v (\frac{l}{h_0})^3 \quad (87)$$

$$q = vx \quad (88)$$

$$\Delta t = \frac{\eta_0 l^3}{2W}[\frac{1}{h_2^2} - \frac{1}{h_1^2}] \quad (89)$$

Where x is the position from center bearing axis, v is the approaching velocity, h_2 and h_1 are film thicknesses before and after time change Δt (Szeri 2010, 120-130).

6.2 Hydrostatic bearings

Hydrostatic bearings operate with an outer pressurized fluid source that generates load-carrying capacity. Besides increased damping and stiffness, they provide high load capacity at low speeds and excellent positioning at high speeds. They function with high-pressure pumps supplying the lubricant to recess area at the bearing interface. The following figure shows the geometry of hydrostatic thrust bearing and the presence of a supply pressure p_s and a recess pressure p_r. the difference between these pressures is changing depending on applied load.

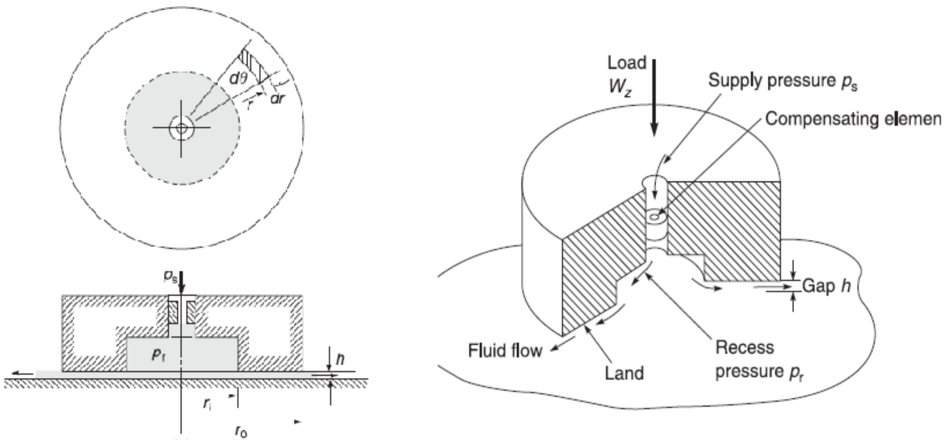

Figure 20 hydrostatic thrust bearing geometry (Williams 2005, 206)

Considering the outer radius of the bearing r_0 and the recess radius r_i the evaluation of Reynolds equation on this geometry at a radius r gives the following relations

$$p_r = \frac{2W \ln(r_0/r_i)}{\pi(r_0^2 - r_i^2)} \tag{90}$$

$$Q = \frac{\pi h^3 p_r}{6\eta_0 \ln(r_0/r_i)} \tag{91}$$

$$p = \frac{6\eta_0 Q}{\pi h^3} \ln(r_0/r) \tag{92}$$

Where Q is the total volumetric flow and p is pressure at distance r from bearing center. For friction torque T, generated by shear force, and the normal load W the following is true

$$W = \frac{3\eta_0 Q}{h^3}(r_0^2 - r_i^2) \tag{93}$$

$$T = \frac{\pi\eta_0\omega}{2h}(r_0^4 - r_i^4) \tag{94}$$

For hydrostatic bearing the power loss H_t consist of Viscous dissipation H_v generated by friction and the pumping loss H_p defined as

$$H_t = H_v + H_p = T\omega + p_r Q \tag{95}$$

The temperature change Δt of the lubricant is given as

$$\Delta t = \frac{H_t}{Q\rho c_p} \tag{96}$$

The previous relations are true for a single circular recess, while hydrostatic bearings can contain multiple recesses with circular, annular or rectangular shapes. For the case of rectangular recesses the following analytical relations are found using Laplace equation

$$p_r = \frac{6\eta_0 c Q}{h^3 l} \tag{97}$$

$$p = p_r(1 - \frac{y}{c}) \tag{98}$$

$$W = \frac{6\eta_0 c}{h^3} Q(b + c) \tag{99}$$

Where l is bearing length, b is bearing width, c is the distance where zero pressure occurs and y is distance from recess center (Williams 2005, 225-229).

6.3 Wear

Sliding bearings are used where motion is not required and also where contact is to be avoided especially during start-stop operations so chemical wear is the most common type of wear as a consequence of lubricant and neighboring materials. While for hydrodynamic bearings beside fatigue, adhesive wear is common for start-stop processes and chemical wear results from lubricant effects on the lubricated surfaces. This chemical wear is best avoided by formation of inactive film on the bearing surface when additives are used like phosphate or organo-metals salts. However modern design of bearings is trying to include all factors responsible of wear in the calculation of their estimated working life.

Vibration is also responsible for wear of all kinds of bearings, in fact, when bearings are static, the lubricant cannot reach all the surfaces and thus with the presence of vibrations in close machinery of working area solid contact occurs and causes the separation of particles which lead to depressions of the raceways (Khonsari and Boozer 2008, 460-465).

7 ROLLING ELEMENTS

7.1 Contact forms

The rolling motion of machine elements under considerable loads can cause effects on viscosity of the lubricants and deformation in the contact zones. Hence, elastohydrodynamic lubrication is the predominant regime for this type of motion. This phenomena occurs widely with nonconforming contacts and it is crucial for highly-loaded contacts such as line contacts of a roller bearing, point contact for a ball bearing, also for point and line contact in the case of gear teeth. In this lubrication regime, factors such as temperature and shear rate have big influence on the behavior of the lubricant. In order to understand the tribological behavior of these machine elements, types of contacts has to be evaluated regarding loads and motion. Line and point contact are labeled as footprints contacts, line contacts occurs when 2 cylinders are in contacts, and as the load increases this lines can form a rectangular footprint. While the contact of a sphere and a flat surface causes the formation of circular points contacts. The contact of a ball and a raceway of a bearing under considerable loads, initiates the formation of elliptical footprints. The analysis of nonconforming contacts can be reduced to the case of a nonconforming surface and a plane surface. The following example shows the case of two cylinders with respective velocities u_a and u_b transformed to an equivalent cylinder having equivalent radius R (Williams 2005,410-420). It is also important to evaluate the relative sliding velocity as the difference $|u_a - u_b|$ and also the entraining velocity of interest known also as rolling velocity and defined as

$$u = (u_a + u_b)/2 \qquad (100)$$

it is also useful to define the slide to roll ratio S as

$$S = \frac{sli.velocity}{2rol.velocity} = \frac{u_a - u_b}{u_a + u_b} \qquad (101)$$

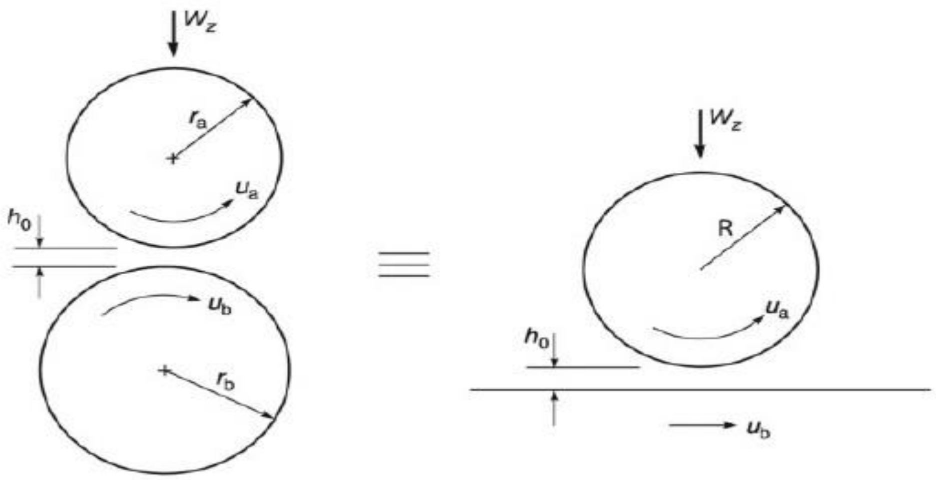

Figure 21 Equivalent 2 cylinders system (Bhushan 2013, 483)

7.1.1 Line contact

7.1.1.1 Rigid cylinder contact

For a constant viscosity, the integration of Reynolds equation over the area of a cylinder moving in the x direction on a flat surface, yields that the film thickness has a parabolic function as

$$h = h_0 + \frac{x^2}{2R} \quad \text{when } x \ll R \tag{102}$$

Where R is the equivalent radius (effective radius) and h_0 is the minimum film thickness.
Considering the inlet and the outlet pressures to be ambient, the maximum pressure and the load capacity can be expressed as

$$p_{\max} = 2.15\eta_0 u(\frac{R}{h_0^3})^{1/2} \tag{103}$$

$$W = 4.9\eta_0 u \frac{R}{h} \tag{104}$$

For a variable viscosity and using Barus relation, it is useful to define a new variable called reduced pressure as

$$p_r = \frac{1-\mathrm{e}^{-\alpha p}}{\alpha} \tag{105}$$

Then the maximum pressure and corresponding film thickness can be simply defined as

$$(p_r)_{\max} = 2.15\eta_0 u(\frac{R}{h_0^3})^{1/2} \tag{106}$$

$$h_0 = 1.66(\alpha\eta_0 u)^{2/3} R^{1/3} \tag{107}$$

The load capacity is found after integrating the area under the pressure curve (Bhushan 2013, 483-485)

7.1.1.2 Elastic cylinder contact

Elastic deformation occurs in nonconforming surfaces under any loads. The analyze of the lubricant pressure distribution shows that it is elliptical and near Hertzian pressure and that it is constant through the contact region. The evaluation of the film thickness for elastic contacts is done using elasticity and Reynolds equation as follows

$$h = h_0 + \frac{x^2}{2r} + w \tag{108}$$

Where x is the position, r is the equivalent radius of curvature for elliptical contact and w is combined deformation of the solids in contact. The minimum film thickness can be determined as

$$\frac{h_0}{R} = 2.08(\frac{\eta_0 u\alpha}{R})^{8/11}(\frac{E'R}{W})^{1/11} = 2.08(UG)^{8/11}(W')^{-1/11} \tag{109}$$

Where U is a speed parameter $(\frac{\eta_0 u}{E'R})$, G is a materials parameter (αE'), W' is a load parameter $(\frac{W}{E'R})$ while E' is the composite modulus. Based on different investigations of elastohydrodynamic behavior, the following results for the minimum and the center film thickness have been made

$$\frac{h_{\min}}{R} = 1.714 U^{0.694} G^{0.568} (W')^{-0.128} \qquad (110)$$

$$\frac{h_c}{R} = 2.922 U^{0.692} G^{0.47} (W')^{-0.166} \qquad (111)$$

The Thickness of the film for the viscous case is greater than the isoviscous case and generally for a successful lubrication, the film thickness must be high compared to the surface roughness of the bearing (Bhushan 2013, 485-488).

7.1.2 Point contact

The circular contact points occur as a result of two spheres contact while elliptical contact points manifest when a sphere comes into contact with a raceway. This high pressure spots induce the deformation at the contact interface and elevate the lubricant oil viscosity. Similarly tribilogical features for this type of contact can be solved using elasticity and Reynolds equation. The elastohydrodynamic behavior of point contacts for the case of high elastic modulus materials permitted the development of the relations below

$$\frac{h_{\min}}{R_x} = 3.63 U^{0.68} G^{0.49} W'^{-0.073} [1 - \exp(-0.68k)] \qquad (112)$$

$$\frac{h_c}{R_x} = 2.69 U^{0.67} G^{0.53} W'^{-0.067} [1 - 0.61 \exp(-0.73k)] \qquad (113)$$

For U is a speed parameter $(\frac{\eta_0 u}{E'R_x})$, G is a materials parameter (α E'), W' is a load parameter $(\frac{W}{E'R_x})$ while E' is the composite modulus, K is the ellipticity parameter ($k=R_y/R_x$) and R_y is the effective radii in y direction while R_x is the effective radii in the sliding direction x.

For materials with low elastic modulus the minimum film thickness and the center film thickness can be evaluated as following

$$\frac{h_{\min}}{R_x} = 7.43 U^{0.64} W'^{-0.21} [1 - 0.85 \exp(-0.31k)] \qquad (114)$$

$$\frac{h_c}{R_x} = 7.32 U^{0.64} W'^{-0.22} [1 - 0.72 \exp(-0.28k)] \qquad (115)$$

The following pictures shows examples of point and line contacts

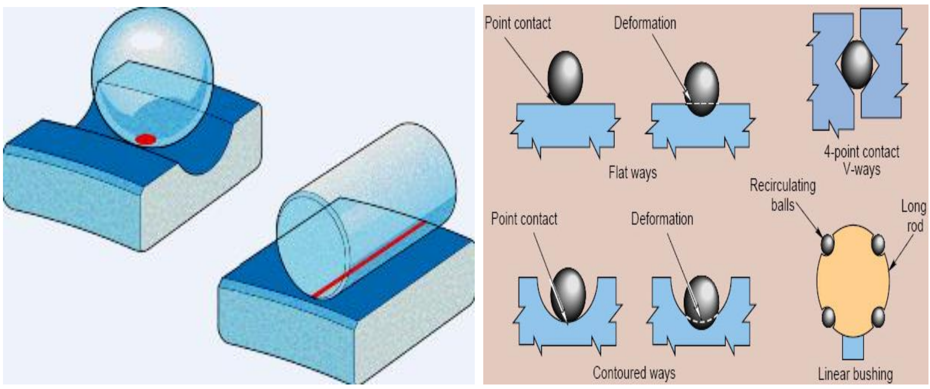

Figure 22 Contacts examples for sphere, cylinder and ball bearing (machine design, 2014)

Further studies showed that in presence of lubricant, the pressure distribution is almost Hertzian, hence the maximum pressure for a circular point contact can be evaluated as

$$p_{\max} = \frac{3W}{2\pi a^2} \qquad (116)$$

$$a = \sqrt[1/3]{\frac{3RW}{4E'}} \qquad (117)$$

Where W is the applied load, R is the equivalent radius, E’ is the composite modulus and a is contact radius. Further formulas are set in figure (Hamrock,Jacobson and Scmid 2004,1490-495).

7.1.3 Thermal correction

The line and point contact theory presented before was in the case of isothermal conditions. However in most of the applications, the temperature increases at the contact interface what changes the liquid viscosity. Generally it is better to determine the temperature at the contact zone and to include it while setting the absolute viscosity of the lubricant. Moreover, engineering methods were developed to approximate temperature correction. In fact for point contact studies are not yet advanced while for line contact the following equation for thermal reduction factor is used

$$T_r = 0{,}857 - 0.0234T_l + 0.000168(T_l)^2 \qquad (118)$$

$$T_l = \frac{\eta_0 \beta u^2}{\rho} \qquad (119)$$

Where u is the entraining velocity, η_0 is the absolute viscosity at atmospheric pressure and surface temperature, ρ is the lubricant thermal conductivity and β is the viscosity-temperature coefficient. This thermal reduction factor is multiplied by the film thickness to estimate its true value. Different viscosity coefficients and values are to be measured at different pressures and temperatures to estimate true values of the film thickness, and the most used method for research is the rolling disk-type apparatus (Stolarski 2000, 244-245).

7.2 Rolling bearings

7.2.1 Friction

Rolling bearings contain a number of balls or rollers that roll in the annular space. The contact between rolling elements and rings or races causes resistance to the relative motion of these elements. Actually, the motion of rolling elements over yielding surfaces is characterized by the generation of friction and unsymmetrical load distribution over the rollers. Friction is not only due to rolling or slip sliding but also to different factors. The total friction torque is the sum of the individual torques caused by mechanical and dynamic factors.

7.2.1.1 Differential sliding

The rolling of rollers or balls along curved rings develops pure rolling, along 2 lines of an ellipse, and sliding because of difference in distances between contact points to the axis of rotation. This is called differential sliding that origins a friction torque what can be estimated as

$$M_{ds} = \frac{\mu_s W D_0 Z}{4 d_b}\left(1 - \frac{d_b^2}{D_0^2}\cos^2\alpha\right)(r_0 B_0 - r_i B_i) \qquad (120)$$

Where μ_s is the sliding coefficient of friction, W is the load,D_0 is the mean diameter, d_b is the ball diameter, Z is is the number of balls, α is the contact angle, r_i and r_0 are the radii of the outer and inner raceways while B_0 and B_i are factors based on geometric deformation.

7.2.1.2 Gyroscopic spin

The contact angle α causes a friction torque that can be expressed as following

$$M_{gr} = I\omega_b Z\sin\alpha = \frac{\pi}{60} d_b^2 \frac{\gamma}{g}\omega_b Z\sin\alpha \qquad (121)$$

Where I is the ball moment of inertia, ω_b is the angular velocity of the ball, γ is the density of the ball and g is the acceleration of gravity.

7.2.1.3 Elastic hysteresis

There are energy losses due to elastic hysteresis of the material and the friction torque can be estimated as

$$M_{hs} = 0.000125 D_0 d_b^{-2/3}\Sigma p_i^{4/3} \qquad (122)$$

Where p_i is the load on the i^{th} ball.

7.2.1.4 Raceway torque

The weight of the inner ring causes contact with the roller in a focused point and the friction torque can be defined as following

$$M_c = \frac{\mu_c G_c D_0}{4}\left(1 - \frac{d_b^2}{D_0^2}\cos^2\alpha\right)\sin\left[\alpha + \tan^{-1}\left(\frac{d_b}{2R_r}\sin\alpha\right)\right] \qquad (123)$$

Where μ_c is the friction coefficient between the rolling element and the raceway housing, R_r is the raceway radius and G_c is the raceway housing mass.

7.2.1.5-Lubricant shear

The presence of the lubricant induces an additional energy loss that is relative to the lubricant features, the friction torque generated by the shear of the lubricant for a cylindrical roller is

$$M_{ls} = \frac{13Z\pi v^2\eta}{\sqrt{\lambda h_0}} \tag{124}$$

And for rolling ball is

$$M_{ls} = \frac{12Z\pi v^2\eta}{(2\beta/3\lambda)\sqrt{\beta}}\ln(\frac{4m^2\beta}{h_0}) \tag{125}$$

$$\lambda = 0.5(\frac{1}{R_x} \pm \frac{1}{R_1}) \tag{126}$$

$$\beta = 0.5(\frac{1}{R_y} \pm \frac{1}{R_2}) \tag{127}$$

Where v is the longitudinal velocity of the contact point in rolling, m is half of the film thickness, R_x is the curvature radius of the roller, R_1 is the raceway radius of curvature While R_y R_2 are the radii perpendicular the motion direction.

There are also other factors contributing in the total torque friction such as geometric errors that causes deviation from the theoretical line of action, also friction can be generated by working medium environment and by temperature increase (Stolarski 2000, 249-254).

7.2.1.6-SKF friction model

SKF established models to estimate the exact torque of friction in rolling element bearings. The simplest model can be used when the bearing dynamic load P is 10 times smaller than the thrust load, then the total friction torque can be estimated as

$$M_t = 0.5\mu P\ d \tag{128}$$

Where μ is the bearing coefficient of friction and d is the bearing bore diameter (F inner ring raceway diameter for needle roller bearings).

Generally SKF established a model to calculate friction on all rolling element types suggesting that the total torque is the sum of the rolling frictional moment, sliding friction moment, seals frictional moment and the friction moment caused by drag losses.

Rolling frictional moment M_r can be calculated using the following relation

$$M_r = \Phi_{ish}\ \Phi_{rs} G_{rr}\ (n\eta)^{0.6} \tag{129}$$

Where Φ_{ish} is the inlet shear heating reduction factor, Φ_{rs} is the kinematic replenishment/starvation reduction factor and G_{rr} is a variable (depending on the bearing type the bearing mean diameter d_m [mm]= 0,5(d + D),the radial load F_r [N] the axial load F_a [N]) while n is the rotational speed [r/min] and η is the actual operating viscosity of the oil or the base oil of the grease [mm2/s].

The shear heating reduction can be calculated as

$$\Phi_{ish} =[1+1.84\ 10^{-9}(n\ d_m)^{1.28}\ \eta^{0.64}]^{-1} \qquad (130)$$

$$\phi_{rs} = \frac{1}{e^{[k_{rs} n\eta (d+D)\sqrt{\frac{k_z}{2(D-d)}}]}} \qquad (131)$$

Where K_{rs} is a replenishment/starvation constant (3 10^{-8} for low level oil bath and oil jet lubrication, 6 10^{-8} for grease and oil-air lubrication), K_Z is a geometric constant related to the bearing type, d is the bearing bore diameter [mm] and D is the bearing outside diameter [mm].

Sliding frictional moment M_s can be calculated as following

$$M_s=G_{sl}\ \mu_{sl} \qquad (132)$$

Where G_{sl} is a variable depending on loads, type and diameter of the bearing and μ_{sl} is the sliding friction coefficient that is defined as

$$\mu_{sl}= \Phi_{bl}\ \mu_{bl} +(1- \Phi_{bl})\ \mu_{EHL} \qquad (133)$$

Where μ_{bl} is a coefficient depending on the additive package in the lubricant, generally ≈ 0.15, μ_{EHL} is the sliding friction coefficient in full-film conditions (0,02 for cylindrical roller bearings 0,002 for tapered roller bearings and 0,05 for lubrication with mineral oils 0,04 for lubrication with synthetic oils 0,1 for lubrication with transmission fluids) and Φ_{bl} is a weighting factor for the sliding friction coefficient defined as

$$\phi_{bl} = \frac{1}{e^{2.6(n\eta)^{1.4} 10^{-8} d_m}} \qquad (134)$$

The frictional moment of seals M_{seal} is calculated as

$$M_{seal}=k_{s1} d_s^{\beta}+k_{s2} \qquad (135)$$

Where K_{S1} is a constant depending on the seal type and the bearing type and size, d_s is the seal counter-face diameter [mm], β is an exponent depending on the seal type and the bearing type while K_{S2} is constant depending on the seal type and the bearing type and size.

The rotation of the bearing in oil bath causes drag losses and the friction torque M_{drag} for a roller bearing is

$$M_{drag} = 0.4 V_m k_{roll} C_w B d_m^4 n^2 + 1.093\times 10^{-7} n^2 d_m^3 (\frac{n d_m^2 f_t}{\eta})^{-1.379} R_s \qquad (136)$$

For a ball bearing

$$M_{drag} = 0.4 V_m k_{ball} d_m^5 n^2 + 1.093\times 10^{-7} n^2 d_m^3 (\frac{n d_m^2 f_t}{\eta})^{-1.379} R_s \qquad (137)$$

All the constants and the factors needed for the friction torques calculation are set in SKF catalogues for rolling elements bearings. The starting torque for this type of bearing can be estimated as the sum of the sliding frictional moment and seals frictional moment. Consequently the power loss generated from friction N_R and the temperature increase Δt are calculated as

$$N_R = 1.05\ 10^{-4}\ M_t\ n \tag{138}$$

$$\Delta t = N_R/W_s \tag{139}$$

Where W_s is the cooling factor (SKF, 2014)

7.2.2 Deflection

In order to estimate the tribological features of rolling bearings, it is crucial to approximate their elastic deformation under applied load. Palmgren established the relations in the following figure to predict deflection of rolling bearings where P is the applied load, d is the rolling element diameter, l is the length of the contact and α is the bearing contact angle (the angle between the line drawn through rolling element and the radial plane of the bearing).

	Radial deflection	Axial deflection
Self-align ball bearing	$\delta_r = \frac{6.858 \times 10^{-8}}{\cos\alpha}\left(\frac{P^2}{d}\right)^{\frac{1}{3}}.$	$\delta_a = \frac{6.86 \times 10^{-8}}{\sin\alpha}\left(\frac{P^2}{d}\right)^{\frac{1}{3}}$
Deep-groovr and anugular bearing	$\delta_r = \frac{4.3 \times 10^{-8}}{\cos\alpha}\left(\frac{P^2}{d}\right)^{\frac{1}{3}}.$	$\delta_a = \frac{4.3 \times 10^{-8}}{\sin\alpha}\left(\frac{P^2}{d}\right)^{\frac{1}{3}},$
roller- bearing for point-line contact	$\delta_r = \frac{5.61 \times 10^{-9}}{\cos\alpha}\frac{(P^3)^{\frac{1}{4}}}{l^{\frac{1}{2}}},$	$\delta_a = \frac{5.61 \times 10^{-9}}{\sin\alpha}\frac{(P^3)^{\frac{1}{4}}}{l^{\frac{1}{2}}},$
Roller-bearing for line contact	$\delta_r = \frac{3.0 \times 10^{-10}}{\cos\alpha}\frac{P^{0.9}}{l^{0.8}}.$	$\delta_a = \frac{3.0 \times 10^{-10}}{\sin\alpha}\frac{P^{0.9}}{l^{0.8}}.$

Figure 23 Palmgren deflections (Stolarski 2000, 256)

7.2.3 Speeds

The interaction between the rolling element and the races in this type of bearing has a straight effect on performance. The angular velocity of the center, the rolling velocity at the point contact and centrifugal force for a rolling element respectively are

$$\omega_c = \frac{R_{in}}{R_{in} + R_{out}}\omega \tag{140}$$

$$U_r = \omega_c R_{out} \tag{141}$$

$$F_c = m\omega_c^{\ 2}(R_{in} + r) \tag{142}$$

Where ω is angular velocity of the bearing, r is the element radius and m is the mass while R_{in} and R_{out} are inner and outer ring radiuses (Harnoy 2002, 240-242).

Limiting temperatures are reached at different speed depending on the heat generated by friction loss and other factor such as applied loads, lubrication and bearing features. Hence it is crucial to estimate reference speed and operating temperature in order to predict bearing performance. The ISO 15312 standards present thermal speed ratings for rolling bearings under some specific conditions. When loads and oil viscosity values exceed the standards references the frictional heat loss increase what affects the reference speed of the bearings. Based on these standards, SKF established relations in order to adjust the reference speeds influenced by high viscosity or loads. In fact the adjusted speed for oil lubrication can be calculated as following

$$n_{ar}= n_r f_p f_v \tag{143}$$

Where n_{ar} is adjusted reference speed [r/min], n_r = nominal reference speed [r/min] f_P is adjustment factor for bearing load P and P_v is adjustment factor for oil viscosity. While for grease lubrication the equation is

$$n_{ar} = n_r f_p \frac{f_{v\ actual\ \text{base oil viscosity}}}{f_{v\ \text{ISO VG150}}} \tag{144}$$

It is also important to set limiting speed for bearings because even under favorable conditions limiting values should not be exceeded. Generally product data specifies the reference and limiting speeds and provide diagrams for speed adjustment factors (SKF 2014).

7.2.4 Failure and wear

In addition to excessive thrust load, rolling element bearings can fail due interruption or inappropriate supply of the lubricant since in some applications this can start immediate damage. Besides thermal imbalance is harmful because inadequate thermal gradient between the outer and the inner races can result in additional radial preloading. This imbalance may also be the result of excessive sliding what may cause contact if film thickness is not redefined. Outer elements can also generate higher temperature than the bearing what can affect the lubricant and the bearing element so the design should take in consideration the dissipation of this heat away from the bearing. Also if radial clearance between the two raceways is lost or the bearing is misaligned, the load may increase and fracture occurs at the cage. In applications where bearings are exposed to electrical current, tiny holes will eventually form in the bearing surfaces. Moreover, adhesive wear is common in rolling element bearings especially by start-stop operations while chemical corrosive wear could occur as result of interaction with the outer environment or the lubricant. Impact wear is probable with applications where sudden impacts are endured. Moreover fatigue wear is the most common for these bearings especially for heavy-loaded ones. (Harris and Kotzalas 2007, 274-280)

7.2.5 Lubrication

Generally for efficient lubrication the minimum film thickness must exceed the equivalent surface roughness at the contact. Lubrication is defined by ratio Λ of film thickness to equivalent roughness of the contacting surfaces. For rolling bearings this ratio is usually between 3 and 5. In fact rolling bearing lubrication operates under elastohydrodynamic regime. Minimum thickness for ball bearing can be calculated from the equation of point contact, while roller bearing is defined by line contact minimum thickness equation. Attention should be kept while determining the equivalent radius of curvatures between the rolling elements and races. The following figure presents a summary of sign convention for rolling bearings radii.

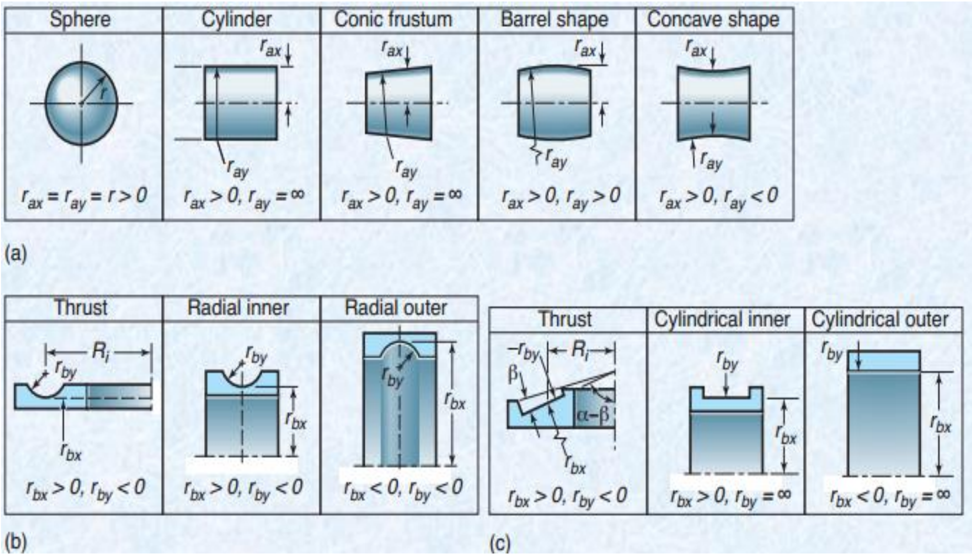

Figure 24 Sign designations for radii of curvature (a) Rolling elements; (b) ball bearing races; (c) rolling bearing races(Schmid, Hamrock and Jacobson 2013, 300)

Usually the range of elastohydrodynamic film thickness lubrication is between 0.1 and few micrometers with bearings operating under normal conditions. Since the geometry of the rolling elements is not perfect then contact can occur, thus additives are used to enhance the lubricant performance. There are different lubrication methods used for rolling elements bearings depending on the application. The most used mode is grease lubrication because it is the less expensive and the simplest. The grease is transferred away from the rolling elements path and a portion of the fluid flow into the raceway ensuring the lubrication function. Grease is held using seals or shields. However grease does not react to operating speed change and the selection of the grease type is based on the working conditions. The grease operating life depends on leakage rate of the oil out of the grease and by degradation or evaporation. The period of relubrication is estimated using previous experience or similar model of bearings. Generally grease is widely used because of the light weight, the simple design and the facile maintenance. Solid film lubrication is used for extreme conditions like very low or very high temperatures and also low pressure. The film can be powder, bonded or transfer films. In most of bearing using this lubrication, the cage is made from PTFE. Rolling elements pick a PTFE film while contacting the cage pocket and the bearing is cooled. It is mostly used for aerospace industry. From another side jet lubrication is mostly used with extreme bearing speeds to immediately remove the heat generated. The performance of this technique depends on the supplying nozzles and the flow rate of lubricant. The centrifugal forces participate in the heat decrease by moving the lubricant through the bearing. The cage design is also important by inserting the jets into the cage and the ring gaps so outer ring persist more and not fail fast as usual and the larger is the gaps the more efficient is the lubrication. The best design is that allows the use of the natural pumping ability of the bearing. Another technique is used where the lubricant is directed under and through the holes of the inner race. This lubrication using under-race passages is more successful in operation where cooling the inner race is needed since jet lubrication failed because of lubricant outwards conduction by centrifugal forces. This technique is difficult to apply with small bores because of the limited space but it is very efficient with large bores such used in aircraft turbine engines. The difficulty of this technique has limited the use of this lubrication to special applications since it requires holes through the inner race of the bearing what may lead to fast fracture and failure. Besides, mist lubrication is common with these bearings. Actually, the technique suggests the suspension of oil droplets in air as a fog to be carried into the bearing. The condensation of these particles will result to lubrication. Nozzles are used to control the speed of the mist. However the oil mist passes only once through the bearing and then discarded. This lubrication mode is very effective with high speed and precision tools in machinery for example in helicopter turbine engines. The following figures shows the operating principles of some lubrication techniques (SKF 2014;Stolarski 2000,260-265).

7.2.6 Bearing life

7.2.6.1 Lundberg-Palmgren bearing life theory

Weibull distribution is a probability model that estimates the distribution of a bearing life based on statistical data. The theory established by Weibull was developed by Lundberg and Palmgren based on the influence of the rolling element-raceway contact normal stress, on the fatigue life of raceways. Based on Weibull probability distribution of metal fatigue the concluded the basic theory of the stochastic dispersion. They assumed that the logarithm of the reciprocal of the probability of survival S could be expressed as a power function of orthogonal shear stress x_o, life N, depth to the maximum orthogonal shear stress Z_o, and stressed volume V. That is

$$\ln\frac{1}{S}=\frac{\tau_0^c N^e}{z_0^h}V \tag{145}$$

Including a as the semi-major axis of the Hertzian contact ellipse, and l as the length of the running track of the race where V= alz_0 then

$$\ln\frac{1}{S}=\frac{\tau_0^c N^e al}{z_0^{h-1}} \tag{146}$$

By substituting the Hertzian contact parameters (in terms of the applied load and contact geometry), they obtained the load-life relationship for rolling bearings. This can be expressed in its final form, called basic rating life, in a very simple way:

$$L_{10} = (\frac{C_r}{P_{eq}})^p \qquad (147)$$

Where *C* is the bearing basic dynamic load rating, p is factor that depends on the bearing geometry (3 for ball bearings and 10/3 for roller bearings), and *P* is the equivalent bearing load and L_{10} is the rated life at 1 million inner-race revolutions with 90% probability.

$$C_r=f_{cm}(i\cos\alpha)^{0.7}Z^{2/3}d^{1,8} \qquad (148)$$

Where Cr is the basic dynamic load rating, f_{cm}, is the bearing geometry and material coefficient per ANSI/ABMA standards, i is the number of rolling-element rows, α is the bearing contact angle, Z is the number of elements per row, and d is the ball diameter, while

$$P_{eq}=XF_r+YF_a \qquad (149)$$

Where F_r and F_a are, respectively, the radial and axial loads applied to the bearing and X and Y are factors calculated by Lundberg and Palmgren to provide the proper equivalent load based on the ratio F_a/F_r . For pure radial loads, P_{eq} equals F_r.

The Palmgren theory can be proved starting with assumption that any material has a certain fatigue limit that can be infinite under limited load level. So the curve life is asymptotic, also the material must yield to a finite load. With the further assumption that the curve has the profile of an exponential function the following function can be established

$$k= C(an+e)^{-x}+u \qquad (150)$$

Where k is the specific load or Stribeck's constant, C is the material constant, a is the number of load cycles during one revolution at the point with the maximum load exposure, n is the number of revolutions in millions, e is the material constant that is dependent on the value of the elasticity or fracture limit, u is the fatigue limit, and x is an exponent (always close to 1/3) (NASA, 1997).

The previous equation can be written as

$$Life(millions\ of\ stress\ cycles) = (\frac{C}{k-u})^3 - e\frac{n!}{r!(n-r)!} \qquad (151)$$

if e is zero and eliminating the fatigue limit the relation becomes

$$Life(millions\ of\ race\ rev) = (\frac{CZd^2/5}{Q})^3 \qquad (152)$$

Recalling the equation of basic load C_r , using f_{cm}=C/5 while P_{eq}=Q and α=0 finally the result is

$$L = (\frac{C_r}{P_{eq}})^3 \qquad (153)$$

7.2.6.2 ISO life

The DIN ISO 281(1990) standards state the use of the basic life rating for life determination. And also if additional information known about the material and the lubrication more accurate raring can be used called adjusted life rating L_{na} as follows

$$L_{na}=a_1a_2a_3L_{10} \qquad (154)$$

Where a_1 is life adjustment factor for requisite reliability other that 90%, a_2 is adjustment factor for materials characteristics and a_3 is factor for operating conditions. However in 2007 ISO improved this to new rating called expanded adjusted rating life defines as

$$L_{nm}=a_1 a_{iso} L_{10} \text{ (in } 10^6 \text{ revolutions)} \qquad (155)$$

Where a_{iso} is life adjustment factor for operating conditions, a_1 was redefined. The following table presents the new standards values

Requisite reliability	Expanded adjusted rating life	Life adjustment factor	Requisite reliability	Expanded adjusted rating life	Life adjustment factor
%	L_{nm}	a_1	%	L_{nm}	a_1
90	L_{10m}	1	99,4	$L_{0,6m}$	0,19
95	L_{5m}	0,64	99,6	$L_{0,4m}$	0,16
96	L_{4m}	0,55	99,8	$L_{0,2m}$	0,12
97	L_{3m}	0,47	99,9	$L_{0,1m}$	0,093
98	L_{2m}	0,37	99,92	$L_{0,08m}$	0,087
99	L_{1m}	0,25	99,94	$L_{0,06m}$	0,08
99,2	$L_{0,8m}$	0,22	99,95	$L_{0,05m}$	0,077

Figure 25 2007 ISO a_1 adjustment factor

The a_{iso} factor takes into consideration loads, lubrication and lubricant features, residual stress and fatigue limit of the material and operating conditions. This factor can be calculated as

$$a_{iso} = f\left(\frac{e_c C_u}{P} \text{k}\right) \qquad (156)$$

Where e_c is a life adjustment factor for contamination, C_u is the fatigue limit load, P is the equivalent dynamic bearing load, k is the viscosity ratio (For k $>$ 4, calculation should be carried out using k = 4.For k $<$ 0,1, this calculation method cannot be used)

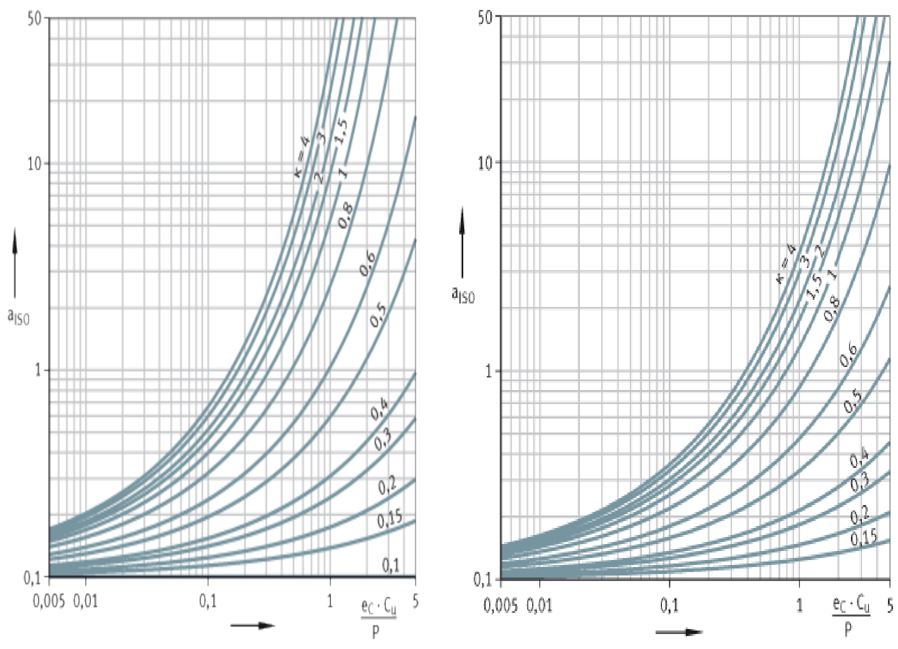

Figure 26 a_{iso} values for radial and axial roller bearing (ISO, 2007)

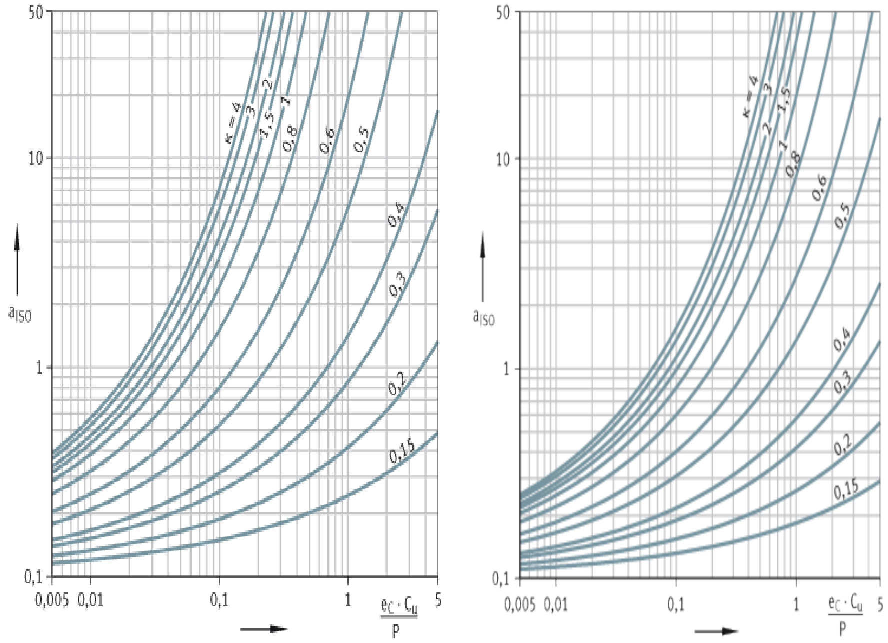

Figure 27 a_{iso} values for radial and axial ball bearing(ISO,2007)

7.3 Involute gears

7.3.1 Gears contact and stress

The study of tribological behavior of involute gears is based on the understanding of the contact generated between the mating gears. In order to establish a concluding analysis some assumptions should be respected. In fact involute teeth should be perfectly shaped and equally spaced, also having equal share of load while contacting and constant pressure distribution. Analysis of mating gears as cylinder is possible but not precise since the radii of curvature are variable because of the teeth profile. Furthermore, the deformation of teeth is hard to predict especially with the introduction of lubricants. As shown in the following figures, the mesh of 2 involute teeth generates different successive line of contact and the succession of these lines forms the line of action in the transversal direction.

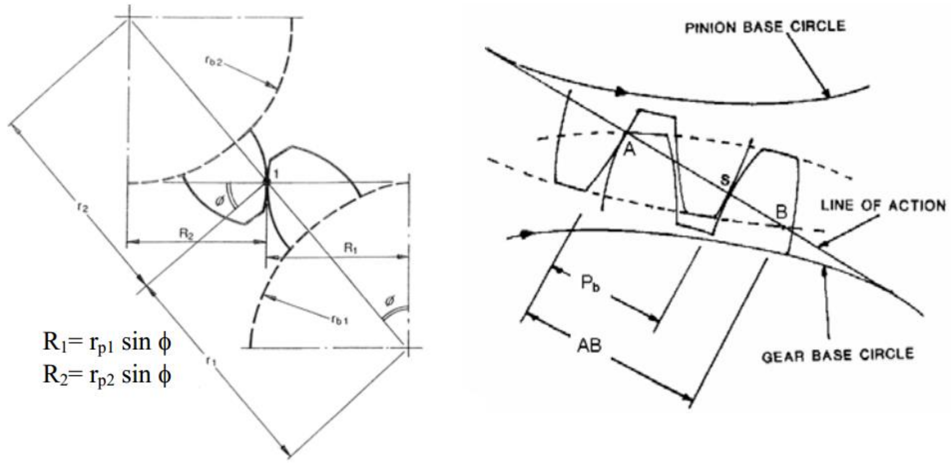

Figure 28 Involute teeth and line of action (wastet, 2014)

The maximum Hertzian contact for involute teeth is then

$$\sigma_0 = \sqrt{\frac{W(1+r_{p1}/r_{p2})}{f\pi r_{p1}[(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2]\sin\phi}} \qquad (157)$$

Where W is the normal load, E_1 and E_2 are modulus, ν_1 ν_2 are Poisson ratio, r_{p1} r_{p2} are gears diameters, f is gear width and φ is the pressure angle. The contact is quantified using the contact ratio to define the number of contacting teeth for spur gear at any time as follows

$$C_r = \frac{q_t}{p_b} = \frac{q_t p_d}{\pi\cos\varphi} \qquad (158)$$

Where p_b is base pitch and p_d is diameter pitch and q_t is length of arc action. Generally for efficient gears this ratio should be less than 1,2. For other types of gears the additional parameters should be included to evaluate this ratio such as working pressure angle, center distance and corrected radii values, and standards should be followed depending on the gear type. Furthermore the design of gears is based on the evaluation of 2 important parameters which are contact stress number S_c and bending stress number S_b defining the maximum stress values for contacting teeth. For different types of gears, the calculation of these parameters is set by different standards such ISO 6336 and SFS 4790. The AGMA standards2001-C95 for spur and helical gears use the following equation

$$S_c = \frac{f_t p_d}{fJ} k_0 k_s k_m k_b k_v \qquad (159)$$

$$S_b = C_p \sqrt{\frac{f_t k_0 k_s k_m k_b k_v}{fID_p}} \qquad (160)$$

Where f_t is tangential force on gear teeth, p_d diametral pitch of the gear, f face width of the gear, I and J are geometry factors for contact stress and bending stress, k_0 overload factor, k_s seize factor, k_b rim thickness factor, k_m load distribution factor, k_v dynamic factor and C_p is elastic coefficient of the gear material and D_p pitch diameter of the pinion. Standards should be followed for different factors and for calculations depending of the gear types (WASET, 2014).

7.3.2 Gears friction

7.3.2.1-Friction components

The friction in gears exists between contacting teeth as result of relative sliding. The analysis of friction in this case should be taken into 2 stages, start point-pitch point and pitch point-end point. There is no friction at the pitch point location. As shown in the following figure, the friction force component direction changes at this point due to change in sliding direction.

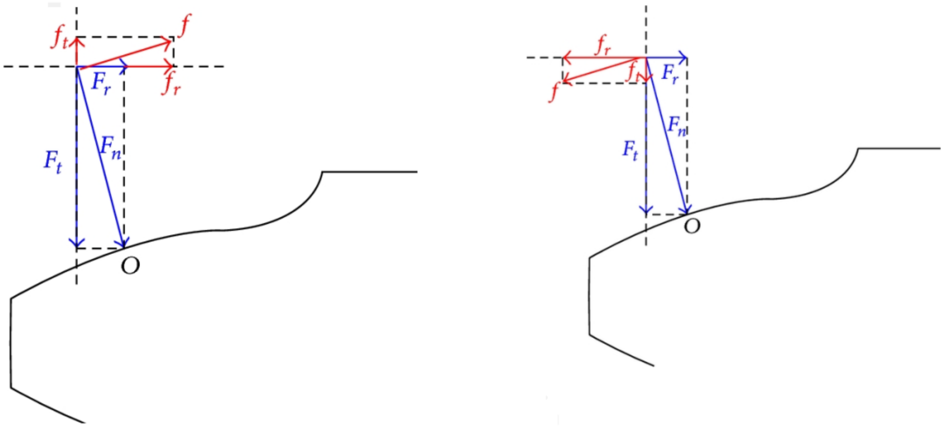

Figure 29 force system during approach and recess (Hindawi Publishing Corporation, 2013)

For both driven and driving gears, the friction force is always defined as the product of the net normal force by coefficient of sliding friction μ. However the vertical input force F_v and the horizontal input force F_h are defined as following

During approach

$$F_v = F_t - f_t \quad (161)$$

$$F_h = F_r + f_r \quad (162)$$

During recess

$$F_v = F_t + f_t \quad (163)$$

$$F_h = F_r - f_r \quad (164)$$

This phenomena shows the rolling/friction duality and has great effect on bending stress behavior, In fact bending stress is amplified during approach and weaken during recess by friction effect. Hohn et al developed the following expression for friction coefficient in gears

$$\mu = 0.048\left(\frac{F_t}{\rho_c b v_{\Sigma c}}\right)^{0.2} \nu_0^{-0.05} R_a^{0.25} X_L X_{SC} \quad (165)$$

Where F_t is the normal force in transverse direction, b is the face width, $v_{\Sigma c}$ is the sum velocity at pitch point, ρ_c is the equivalent curvature radius at the same point, ν_0 is the kinetic viscosity, R_a is the composite surface roughness, X_L is a correction factor for additives and X_{SC} is a correction factor for coating (Waset, 2014).

7.3.2.2 Friction loss

Power losses in gears are mainly due friction and churning and generally for different types of gears the friction losses are about 1 % form total input power. For gears it is common to define the friction losses as efficiency percentage. There are different methods to estimate these fiction losses. Actually, if the work done by pinion and gear is to be considered and if the friction coefficient is constant the following relation are found

$$f_{/\min} = \frac{F_n \omega R_b}{F_t + F_r}[\frac{\mu'}{2}(\beta_t^2 + \beta_r^2)(1+\frac{1}{i})] \quad (166)$$

$$efficiency = 1 - [\frac{1+(1/i)}{\beta_t + \beta_r}]\frac{\mu'}{2}(\beta_t^2 + \beta_r^2) \quad (167)$$

Where $f_{/min}$ is the friction loss per minute, i s the gear ratio, ω is the angular velocity of driving gear, μ' is average friction coefficient, R_b is base radius of the driver,β_t and β_r is the arc of approach and recess of the driver. Different speeds, loads, lubricants can change the friction coefficient from constant to variable then the relations shift to

$$f_{/\min} = \frac{F_n \omega R_b}{F_t + F_r}[(1+\frac{1}{i})(\frac{\mu_a}{2}\beta_t^2 + \frac{\mu_r}{2}\beta_r^2)] \quad (168)$$

$$efficiency = 1 - [\frac{1+(1/i)}{\beta_t + \beta_r}(\frac{\mu_a}{2}\beta_t^2 + \frac{\mu_r}{2}\beta_r^2)] \quad (169)$$

Where μ_a and μ_r are average friction coefficient during approach and recess (Stolarski 2000,289-293). If the losses are to be calculated using energy calculation approach the results are

$$Effecicncy = 100 - P = 1 - \frac{50\mu}{\phi}\frac{(H_s^2 + H_s^2)}{(H_s + H_t)} \quad (170)$$

For spur gear and helical gears

$$H_t = (\frac{R_g \pm 1}{R_g})[\sqrt{((\frac{r_0}{r_p})^2 - \cos^2\alpha) - \sin\alpha}] \quad (171)$$

$$H_s = (R_g \pm 1)[\sqrt{((\frac{R_0}{R_p})^2 - \cos^2\alpha) - \sin\alpha}] \quad (172)$$

Gear Type	Φ	sign of R_g term
External spur	cos α	(R_g+1)
Internal Spur	α	(R_g-1)

Single Helical	$\cos \varphi_n$ / $\cos^2 \beta$ $\tan \varphi_n$ = $\tan \alpha.\cos\beta$ β = helix angle	(R_g+1)

Figure 30 Φ parameter calculation (Roymech ,2014)

For worm gear the following formula is used

$$effecicncy-worm=\frac{\cos\alpha_n-\mu\tan\gamma}{\cos\alpha_n+\mu\cot\gamma} \quad (173)$$

Where R_g is Gear ratio, Ro is Outside Diameter of gear (m), r_o is Outside Diameter of pinion(m), R_p is Pitch Diameter of gear (m), r_p is Pitch Diameter of pinion(m), E is efficiency of gear pair (%), P is Power loss of gear pair as a percentage of input power (%), α_n is normal pressure angle, v_s is gear surface sliding velocity (m/s), α is pressure angle, β is helix angle (deg), γ is worm lead angle (deg) and μ is coefficient of friction.

7.3.3 Wear and failure modes for gears

ISO 10825:1995(reviewed 2000) and ANSI/AGMA 1010E95 defines failure and wear modes for different types of gears. There are many gears failures modes that can manifest as wear, deformation, fracture, cracking or fatigues. Usually these modes are classified under lubrication related or nonlubrication related. Gear teeth flanks are subject to failure caused by wear, pitting and scuffing while gear root fillets brake after fatigue. Bending fatigue is the most common type of fatigue failure caused usually by imperfections on the teeth surface or exceeding of the bending contact number. It is initiated by cracks that propagate under loading until fracture. Contact fatigue is also important; it is usually triggered when rolling or sliding loadings cause friction that changes the distribution of the contact interface stress. When contact stress number is exceeded cracks are formed at the gear teeth until propagation to the surface and possible pitting. Thermal fatigue can occur at high friction or high temperature working environment. Moreover, impact should be also considered, actually it can be tooth-bending impact by fast removing of gear tooth, tooth-shear when high impact is endured at short contacting time or tooth chipping when an external force alter the stress features of the teeth. Pitting is very common phenomena in gears and it exists even with lubrication and without contact. In fact lubricant may enhance pitting under high pressures by forcing lubricant into cracks what lead to their spread. Smooth surfaces usually present less amount of pitting. For through-hardened gears, pitting occurs at the surface of the teeth, it starts at the pitch line and progress towards dedendum which is usually the first to manifest pitting. Usually this surface originated pitting risk is evaluated by comparing the predicted stress in teeth to the contact stress number. Surface-hardened or case carburized teeth usually develop subsurface originated pitting. Cracks are initiated it the case/core interface and after coalescence lager volume in teeth is harmed. Subsurface pitting is frequent in slow motion final drives using spur, helical bevel or hypoid gears. Deepening the case interface and decreasing loads are efficient ways to limit this kind of pitting. Generally, subsurface pitting should be considered while designing by evaluation of the half width of Hertzian contact and the shear stress below surface to secure the case-depth. Abrasive wear can occur with gears especially if hard abrasive particles are inserted into contact area. These particles can be generated by lubricant or by neighboring elements such seals or bearing. Also chemical wear can be caused by chemical elements in operating area. However adhesive wear is not common and only manifest when particles from other failed components are impinged into the gear teeth (Davis 2005). The following figure summarizes failure modes for gears

Category	Failure modes
Wear	Adhesion. Mild Moderate Severe Abrasion. Mild Moderate Severe Polishing. Corrosion Fretting corrosion Scaling. Cavitation. Erosion. Electrical discharge. Rippling
Scuffing	Mild scuffing Moderate scuffing Severe scuffing.
Plasitic deformation	Indentation. Cold flow. Hot flow Rolling. Tooth hammer. Rippling Ridging. Burr. Root fillet yielding. Tip-to-root interference.
Contact fatigue	Pitting (macro-pitting). Initial Progressive Flake. Spall. Micro-pitting Subcase fatigue.
Cracking	Hardening cracks. Grinding cracks. Rim and web cracks. Case/core separation. Fatigue cracks
	Hardening cracks. Grinding cracks. Rim and web cracks. Case/core separation.
Fracture	Brittle fracture. Ductile fracture. Mixed mode fracture. Tooth shear Fracture after plastic deformation
Bending fatigue	Low-cycle fatigue. High-cycle fatigue. Root fillet cracks. Profile cracks. Tooth end cracks.

Figure 31 Gear failure modes (ANSI/AGMA 1010E95)

Scuffing is most frequent excessive damage of gears wear characterized by local weld between surfaces and it is mainly due to lubrication failure at high temperature. Scuffing exhibit torn markings in sliding direction and it is hard to be controlled. In fact, with sliding friction generates additional stress that can start plastic deformation in the surface layer and hence welding is possible. So it is the result of friction coefficient increase and it is usually localized away from the pitch line. Beside natural ductile films, scuffing can be reduced by appropriate material selection by using chemically similar material with different hardness to keep at least one intact and also it is know that material having low solid solubility are less sensitive to weld. However, the best way to limit the scuffing risk is to ensure adequate lubrication. In addition to film thickness, critical temperature at the contact zone is very important factor affecting scuffing. This explained by the failure of lubricating oil at high temperature and therefore eventual gear failure (Davis 2005, 258-265). Therefore a limit temperature called flash temperature T_f is evaluated and then compared to the maximum lubricant temperature. It is calculated as follows

$$T_f = T_b + (\frac{1.25}{1.25 - R_a})G_c(\frac{P_e}{b})^{0.75}(\omega_1^{0.5}\frac{m^{0.25}}{1.094}) \quad (174)$$

Where T_b is gear bulk temperature, b is face width, m is module, R_a is surface finish, P_e is to be taken as full load on teeth (60% only for perfect geometry) and G_c is geometry constant as shown in the following figure

G_c (at pinion tip)	pinion (number of teeth)	gear (number of teeth)	G_c (at gear tip)
0.0184	18	25	−0.0278
0.0139	18	35	−0.0281
0.0092	18	85	−0.0307
0.0200	25	25	−0.0200
0.0144	25	35	−0.0187
0.0088	25	85	−0.0167
0.0161	12	35	−0.0402
0.0101	35	85	−0.0087

Figure 32 G_c values for 20º pressure angle(Stolarski 2000,280)

7.3.4 Gears Lubrication

7.3.4.1 Lubrication regimes of gears

Gears are generally operating under one of three regimes depending mainly on the operating speed. In fact, at low speeds boundary lubrication is the dominant lubrication regime. Eleastohydrodynamic films do not exist and gear protection from wear and failures is done through absorption of very thin surface film built by lubricant and additives, the friction coefficient in this regime is about 0.1 to 0.2. At moderate velocities, mixed lubrication is the dominant regime when the speed allows the formation of film that does not separate totally the contacting teeth, the development of this film reduces friction and wear rates, friction values are between a range of 0,05 to 0.1. However at sufficient speeds elastohydrodynamic regime is the ruling regime after formation of an elastohydrodynamic thick film that ensure total separation of the contacting teeth and hence the friction is reduced only the share of this film having a range between 0.01 and 0.04, while the wear rate is neglected and only persists if abrasive particles in the lubricant initiate some holes into lubricated surfaces. Also, the average load intensity on surface known as factor Q contributes at the range of different gears lubrication regimes (Stolarski 2000, 275-278). The following figure shows the regime limits in function of factor at different velocities.

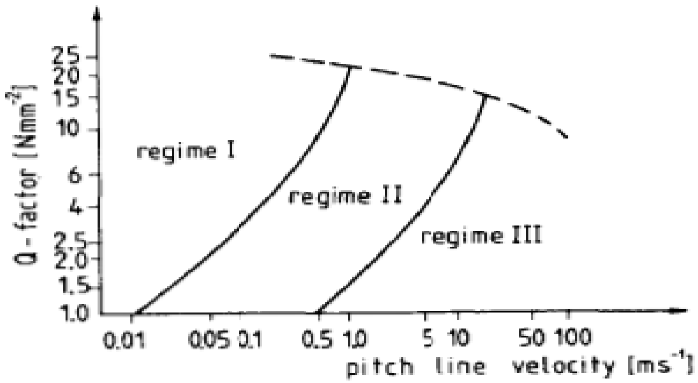

Figure 33 regimes range for different loads intensity and pitch line velocities.(Stolarski 2000,277)

7.3.4.2 Minimum film thickness

For fast estimation, the lubrication ratio Lambda can be used for film thickness. For gears usually values greater than 1.2 are sufficient. Also, theory of line contacts and replacement of gear teeth by cylinders can be used but not very efficient. However modern design of gars use a standard formula for spur and helical gear as following

$$h = 44.6 C_f \frac{L_f V_f}{p_f} \tag{175}$$

$$C_f = \text{curvature factor} = \frac{C \sin\phi}{\cos^2\psi} \frac{m}{(m+1)^2}$$

L_f = lubricant factor= $(\alpha E')^{0.54}$

$$P_f = \text{loading factor} = \left(\frac{P_t}{bE'A}\right)^{0.13}$$

$$V_f = \text{Velocity factor} = \left[\frac{\mu\eta_0}{AE'}\right]^{0.7}$$

$$u = \text{rolling velocity} = \frac{\pi n_p d_p \sin\phi}{60}$$

$$E' = \text{effective modulus} = \frac{\pi}{2} \frac{E}{(1-\nu)^2}$$

Where c is center distance, ψ is helix angle, ϕ is pressure angle, m is module,α is pressure –viscosity coefficient,η_0 is viscosity at operating temperature, d_p is pitch diameter, n_p is pinion velocity in rpm, P_t is tangential load on tooth , a is loaded area and b is face width.

7.3.4.3 Volume and lubrication methods

In order to limit power loss, low viscosity lubricant is used at high speeds while viscosity is more required at high loads. Most of the gears manufacturers suggest recommended viscosity values at different speeds and loads. Furthermore, it is empirical to set the volume of lubricant in gearbox since high quantity increase power loss due liquid shear and low quantity would cause friction and wear. The following formula was set to estimate appropriate lubricant volume in a gearbox

$$V = (0.0035 - 0.011)P\left(\frac{0.1}{Z_p \cos\psi} + \frac{0.03}{2+v}\right) \quad (176)$$

Where P is power transmitted (KW), Z_p is teeth number in the pinion, ψ is helix angle (0 for spur gear) and v is pitch line velocity (m/s). Low volume is used with single gear train while high values are needed for multi-stage train.

There are different methods used for lubrication but grease, splash and forced oil are the most common. Grease lubrication is suitable for low speed applications but not with high loads or continuous operations because of the cooling effect if grease. The quantity of grease should be medium to avoid power loss and it can be used in an open or closed environment. Splash lubrication is done by splashing lubricant into the gear system by mean of rotating gears. The oil level must be considered carefully for this method. Usually bath level is between one and three times the tooth depth for horizontal shaft spur and helical gears and about one-third to one times of tooth depth with vertical shafts. For bevel gears one –third to one times the face width. While for above worm gear the level is about one-third the reference diameter of worm and about one-fourth to half of worm reference diameter if worm is below. Splash lubrication is affected by centrifugal force and pitch line velocity. The limit speed V_l at which this lubrication is still effective can be estimates as

$$V_l = \frac{0.4\nu m}{\alpha h_0^2} \quad (177)$$

Where m is the gear module, ν is kinematic viscosity, α is the angle between the immersion point and the engagement point and h_0 is the film thickness. Besides, forced oil circulation is done through different application such as drop method when the lubricant is delivered to the contact by the mean pipe connected to a pump, also spray method is used when pump sprays directly the lubricant into the mesh area while with high speed applications, mist method or jet method are the most common. For jet, one of the following formulas can be used to approximate the flow rate of oil Q needed

$$Q=0.6+2\ 10^{-6}\ mv \text{ or } Q=0.015\ 10^{-3}\ P \quad (178)$$

Where m is gear module, v is the pitch line velocity and P is the transmitted power.

Generally spur, helical and bevel gears have the same lubrication requirements, they usually need low viscosity with rust and oxidation inhibited oils. Hypoid gears need high anti-scuffing additives because they operated at high loads and velocities. While worm gears are characterized by significant frictional losses at high velocities so fatty and low acid oils are used (Stolarski 2000, 286-288; Davis 2005, 19-20).

7.3.5 Gears life

7.3.5.1 S_N curves

In order to predict life service of gears, different velocities and loads were evaluated. Without surprise, it was found that lubrication extends the limit life of gears since it affects directly pitting and scuffing. S-N curves were established to show gear fatigue strength for different lubrication regimes. The S-N curves for gears shows that Fatigue occur faster in boundary lubrication regime and that service life is longer in mixed lubrication and more extended in elastohydrodynamic lubrication. These results were expected because of contact stress behavior for these regimes. This was also proved by the establishment of the following relations

Boundary lubrication regime

$$N_a / N_b = (\frac{P_a}{P_b})^{3.2} \tag{179}$$

Mixed lubrication regime

$$N_a / N_b = (\frac{P_a}{P_b})^{5.3} \tag{180}$$

Full film lubrication regime

$$N_a / N_b = (\frac{P_a}{P_b})^{8.4} \tag{181}$$

Where P_a is the tooth load for N_a cycles before pitting and P_b is the tooth load for N_b cycles before pitting. The exponent in the previous equations can vary depending on the load, lubrication and the material properties (Stolarski 2000, 277). The following figures shows an example of S-N curve for gears

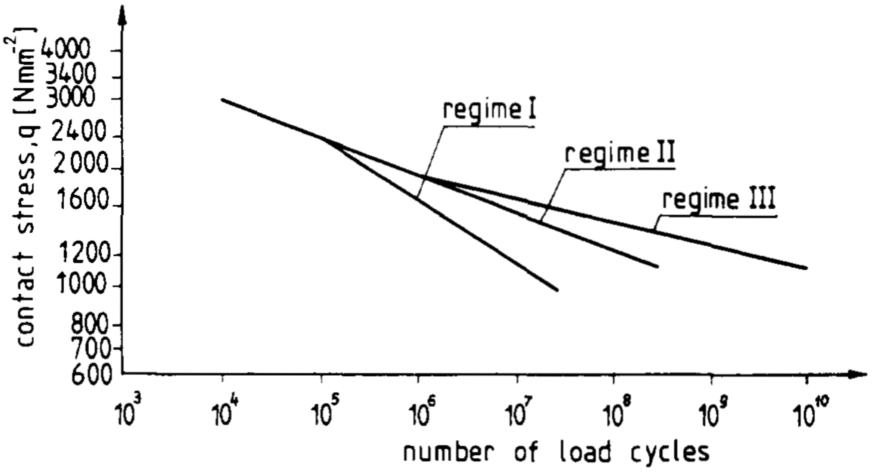

Figure 34 S-N curve example.(Stolarski 2000, 277)

7.3.5.2 Fatigue failure

Service life for gears is based on the study of their fatigue failure. Gears behavior under different loads revealed 2 principal stresses responsible for failure, contact stress at the mesh of the contacting teeth and bending stress at the tooth root. The following figure shows loads and stress distribution at teeth contact and tooth root.

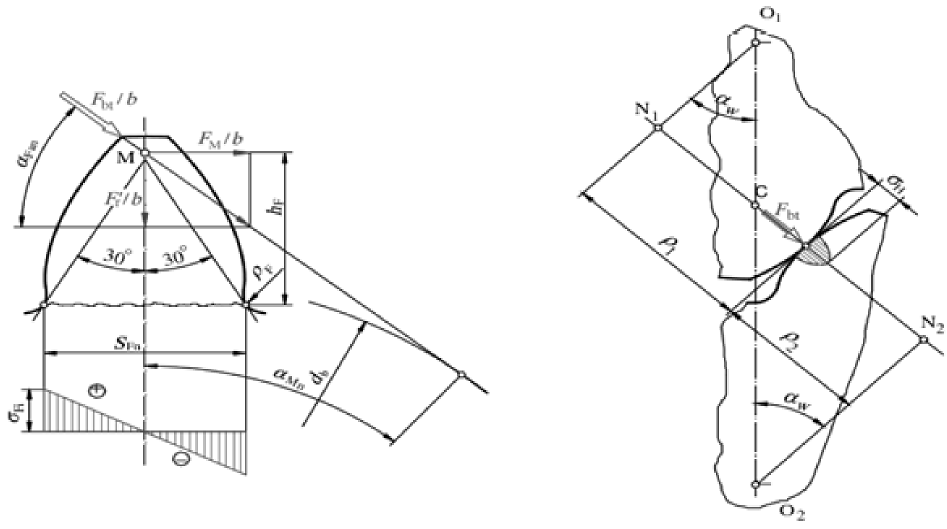

Figure 35 loads and stress at tooth root and gear mesh (Jelaska 2012,172)

Based on Hertzian contact, ISO 6336-6 established the following relations

$$\sigma_{H0} = Z_H Z_E Z_\varepsilon Z_\beta \sqrt{\frac{F_t}{d_1 b}\frac{u+1}{u}} \quad (182)$$

$$\sigma_H = \sigma_{H0} Z_B \sqrt{K_A K_V K_{HB} K_{H\alpha}} \leq \sigma_{HP} \quad (183)$$

$$\sigma_{HP} = \frac{\sigma_{H\lim} Z_{NT}}{S_{H\min}} Z_L Z_V Z_R Z_W Z_X \quad (184)$$

$$\sigma_{H\lim} = A_H x + B_H \quad (185)$$

Where σ_{H0} is the nominal contact stress, σ_H is the calculated contact stress, σ_{HP} is the permissible contact stress, and σ_{Hlim} is the allowance contact stress number obtained from ISO 6336-5(derived from the contact pressure that may be sustained for 2 million load cycles without the occurrence of progressive pitting for a 1% probability of damage) and S_{Hmin} is the minimum safety factor, F_t is tangential force on gear tooth, u is gear ratio, d_1 is pinion diameter and b is face width. The zone factor Z_H, the single pair tooth contact factor (pinion) Z_B, the contact ratio factor Z_ε the helix angle factor Z_β, and the size factor Z_X are all determined by gear geometry. Work hardening factor Z_W, the life factor Z_{NT} for standards reference test gears and the elasticity factor Z_E are material factors. The lubricant factor Z_L, the velocity (pitch line velocity) factor Z_V, and the surface roughness factor Z_R are film thickness factors, all the previous factor are determined using ISO 6336-2.A_H and B_H are contact stress constants and x is hardness While The application factor K_A, internal dynamic factor K_V, face load factor $K_{H\beta}$ and transverse load factor $K_{H\alpha}$ are set by ISO 6336-1.
The service life of gears can be determined using S-N curves or Woehler curves using the following relation

$$Z_N = (\frac{N_H}{N})^{1/m_H} = \frac{\sigma_H}{\sigma_{H\lim}} \qquad (186)$$

Where Z_N is life factor, N_H is load cycles number at the curve knee, N is number of cycles and m_H is curve slope. Moreover, based on Lewis bending equation the following relations were established

$$\sigma_{F0} = \frac{F_t}{m_n b} Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta \qquad (187)$$

$$\sigma_F = \frac{F_t}{m_n b} Y_{Fa} Y_{Sa} Y_\varepsilon Y_\beta K_A K_v K_{F\alpha} K_{F\beta} \qquad (188)$$

$$\sigma_{FP} = \frac{\sigma_{F\lim} Y_{ST} Y_N}{S_{F\min}} Y_\delta Y_R Y_X \qquad (189)$$

$$\sigma_{F\lim} = A_F x + B_F \qquad (190)$$

Where σ_{F0} is the nominal bending stress, σ_F is the calculated bending stress, σ_{FP} is the permissible bending stress, and $\sigma_{H\lim}$ is the allowance bending stress number obtained from ISO 6336 and $S_{F\min}$ is the minimum safety factor, F_t is tangential force on gear tooth, b is face width and m_n is normal module. Y_{Fa} is tooth form factor, Y_{Sa} is stress concentration factor, Y_ε is contact ratio factor, Y_β is helix angle factor, Y_δ notch sensitivity factor, Y_R is surface factor for root stress, Y_X size factor. K_A application factor, K_v is internal dynamic factor, $K_{F\alpha}$ transverse load factor and $K_{F\beta}$ face load factor. A_F and B_F are constants for bending and x is hardness. Methods and charts for all factors and constant are listed in ISO 3663. Similarly the life can be calculated using the following relation based on Woehler curves

$$Y_N = (\frac{N_F}{N})^{1/m_F} = \frac{\sigma_F}{\sigma_{F\lim}} \qquad (191)$$

Where Y_N is life factor for bending, N_F is load cycles number at the curve knee, N is number of cycles and m_F is curve slope. For AGMA service calculation, the same idea is used based on surface fatigue strength and bending fatigue strength and different constants, curves and factors are set for American system.

8 CONCLUSION

The understanding of tribology is based on the analysis of solid surfaces, and deviations are always present on these surfaces. The distribution of the height deviations has a direct effect on wear and friction. However, solid surfaces can form natural lubricating films depending on the nature of the material and the working environment. Surface roughness is he most used aspect to characterize surface texture asperities and it is used to predict wear and friction since they have a direct proportionality relation. The analysis of interacting solid surfaces required the application of Hertzian contact formulas .The analysis of this contact is based on the evaluations of the elasticity index and contact area which are important to estimate wear and friction resulting from surface interactions.

Friction is a system response resulting from the interaction of surfaces asperities. Machine elements exhibit sliding and rolling frictions. Sliding is proportional to the applied load while rolling also depends on the geometry of the contacting elements. Friction mainly results from adhesion, ploughing and deformation of contacting asperities. In addition to the material nature the determination of friction coefficients depends on the asperities dimensions, hardness, applied loads and especially shear strength since friction is actually the force needed to shear the contacts resulting from adhesive and ploughing bonds. Lubrication presence affects friction amplitude and coefficient by meniscus forces. Static and dynamic friction coefficients are different depending on contacting materials.

Wear is also a system response leading to potential failure of machine elements. The main types of wear are adhesive, abrasive, chemical impact and fatigue wear. The estimation of the volume of different types of wear depends on the working environment and the mechanical features of contacting materials such as hardness, surface roughness and fracture toughness. Also geometry of asperities, applied load, and number of cycles are necessary for wear estimation. The introduction of lubricants limits the contact between machine elements that can reduce wear only to fatigue and adhesive wear. The lubricant characteristics are also included in wear calculations for lubricated contact. Materials present different wear coefficient depending on external conditions and generally wear decreases as hardness gets higher.

Machine elements operate under different lubrication regimes depending on the lubricant volume and the elements speed. Hydrodynamic regime is based on speed movement while hydrostatic lubrication is based on pressure. Elastohydrodynamic regime includes deformation while boundary and mixed are intermediate regimes. Synthetic and naturals lubricants are classified depending on temperature, length of carbon chain and amount of additives. The most important lubrication factor is viscosity which changes depending on pressure and temperature.

Sliding elements bearings operates under hydrodynamic or hydrostatic regimes. Friction and minimum film calculations of different types of bearings depend on applied load, velocity, viscosity and bearing eccentricity. Bearing length and diameter are crucial for friction losses and film thickness choice. Slenderness ratio charts can be used for fast estimation of tribology functions. Fatigue wear is the main type of wear affecting sliding bearings.

Rolling element tribological behavior is set by evaluation of lines and contact points. Rolling machine elements operates under an elastohydrodynamic regime. Friction in rolling bearings is built by different factors and lubrication film thickness is calculated depending on contact theory. Wear and fatigue failure are common and bearing life is calculated using standards. The frictional behavior of gears is different at recess and approach, and losses can be evaluated using work or energy considerations. Film thickness and oil volume for gears depends on geometry, speed load and material properties. Scuffing and pitting are the most common types of wear and gear’s life is set depending on surface and bending failure.

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