Saimaa University of Applied Sciences
Technology, Lappeenranta
Double Degree Program in Civil and Construction Engineering

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THREE-PINNED GLUED-LAMINATED FRAME

Bachelor's Thesis 2017
ABSTRACT
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Three-pinned glued-laminated frame, 38 pp, 2 appendices
Saimaa University of Applied Sciences, Lappeenranta
Technology, DDCIV16
Bachelor’s Thesis, 2017
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The subject of the thesis was to design the Three-pinned glued-laminated frame for the sport centre with the use of modern software. All calculations are made according to European standards (EN 1990: 2002 + A1, EN 1991-1-1, EN 1991-1-3, EN 1991-1-4: 2005 + A1, EN 1995-1-1: 2004 + A1) and Finnish National Annex. Glulam characteristics are taken from EN 14080. Imposed loads are collected manually and applied in SCAD Office software. Cross section is chosen automatically in a special program created in Mathcad soft. According to the result of calculations, a 3D model of the structure is made with ARCHICAD software.

Thus, the main purpose to create a Mathcad program for choosing the cross section of three-pinned glued-laminated frame (including tapered sections) was achieved. This invention can significantly reduce the designing time.

Key words: construction design, structure, three-pinned frames, long-span, glued-laminated timber, Mathcad calculations, SCAD calculations, 3D ARCHICAD models.
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TERMINOLOGY

μi - the snow load shape coefficient
Ce - exposure coefficient,
Ct – thermal coefficient
sk - characteristic value of snow on the ground at the relevant site
Fs - the force exerted by a sliding mass of snow, in the direction of slide, per unit length of the building
s - the snow load on the roof relative to the most onerous undrifted load case appropriate for roof area from which snow could slide
b - the width on plan (horizontal) from the guard or obstacle to the next guard or to the ridge
α - a pitch of the roof, measured from the horizontal
Vb - the basic wind velocity
Vb,Q - the fundamental value of the basic wind velocity
Cdir - the directional factor
Cseason - the season factor
vm(z) - the mean wind velocity at a height z above the terrain
cr(z) - the roughness factor
c0(z) - the orography factor
zo - the roughness length
kr - terrain factor depending on the roughness length zo
zmin - the minimum height
σv - the standard deviation of the turbulence
kl - the turbulence factor
qp(z) - the peak velocity pressure at height z
ρ - the air density
ce(z) - the exposure factor
qb - the basic velocity pressure
we - the wind pressure acting on the external surfaces
ze - the reference height for the external pressure
**c_{pe}** - the pressure coefficient for the external pressure

**Q_{sp}** – maximum shear force in spans

**Q_{sup}** – maximum shear force on supports

**\sigma_{c,o,d}** – design compressive stress along the grain

**f_{c,o,d}** – design compressive strength along the grain

**f_{c,o,k}** – characteristic compressive strength along the grain

**f_{t,o,d}** – design tensile strength along the grain

**f_{t,o,k}** – characteristic tensile strength along the grain

**N_{r}** – lateral force in the ridge

**S** – length of the frame along the axis

**F** – section area

**I** – moment inertia

**W** – section modulus

**f_{m,k}** – characteristic bending strength

**E** – mean Young’s modulus

**E_{0.05}** – 5% value of mean Young’s modulus

**G** – shear modulus parallel to the grain

**G_{0.05}** – 5% value of shear modulus parallel to the grain

**I_{tor}** – torsional moment of inertia

**l** – span length

**l_{ef}** – effective length

**\sigma_{m,crit}** – critical bending stress

**\lambda_{rel,m}** – relative slenderness for bending

**r_{in}** – the inner radius of curved beam

**\lambda_{z}** – slenderness ratio corresponding to bending about z-axis

**\sigma_{m,d}** – design bending stress

**f_{m,k}** – characteristic bending strength

**f_{m,d}** – design bending strength

**h_{ap}** – depth of beam at the apex

**\tau_{d}** – design shear stress

**f_{v,d}** – design bending strength

**S’** – static momentum of the cross-section
1 GENERAL INFORMATION ABOUT GLUED-LAMINATED TIMBER

Glued-laminated timber is a wooden product, made up of wood lamellas (boards, laminates) glued together. (Handbook 1 – Timber structures 2008) Grain is directed longitudinally along the length of the product. It was invented in the beginning of the 20th century in Germany by Otto Hetzer. After some years, the inventor modified his creation and made curved elements from that product. For the first 50 years from Hetzer’s invention glulam was not very popular, but since then the manufacturing started to increase. Nowadays glue-laminated timber is one of the most competitive materials in wooden construction.

The manufacturing process is shown in Figure 1.

Figure 1. Glulam manufacturing process (reproduced from Handbook 1 – Timber structures 2008)
Spruce is commonly used as a material for laminates, but hardwoods are used, too. The lamella thickness is 40-50 mm and 1.5 – 5.0 m in length. For curved elements it is allowed to use thinner boards of 30-40 mm.

Glued-laminated timber has a range of advantages against solid wood:

- Greater quality
- Greater strength properties
- Greater stiffness properties
- Greater variety of shapes and forms
- More homogeneous
- Minimum drying effects
- Better chemical resistance
- Better fire-resistance
- Allows material economy (because of curved or tapered shapes)
- Uses less energy
- Greater variability of cross-sections: I, L, O, T, rectangular hollow section (Figure 2)
- Allows combination of laminates of different classes
- Visual appearance mostly appealing to people
- Can carry its full load immediately after erection
- Can be erected irrespectively to weather conditions

Figure 2. The variability of glue-laminated cross-sections (reproduced from Handbook 1 – Timber structures 2008)

Thus, it is better to use glulam for construction of large-spanned structures, unique buildings with complicated shapes.
Strength classes:

- GL30: homogeneous glulam, it is noted with the letter h, after the strength class designation (h = homogeneous glulam); the number 30 represents a characteristic flexible strength, in N / mm$^2$.
- GL30h: consists only of the slats in strength class T22; additionally, the letter s, is specified in the strength class designation (p = split, sawn glulam).

Figure 3. GL30h cross-section (reproduced from Nordic Glulam Handbook 2001)

- GL30c: combined glulam, the outer laminated zones can be at least 17% of the cross-sectional height; consists of laminated slats in strength class T22 in the outer zone and strength class T15 for the internal slats. (Nordic Glulam Handbook 2001)

Figure 4. GL30c cross-section (reproduced from Handbook 1 – Timber structures 2008)
Almost all modern portal frames with large spans are designed from glue-laminated timber. In Figure 5 there are some examples of glulam three-pinned frames.

Figure 5. Glulam portal frames (reproduced from Handbook 1 – Timber structures 2008)

2 INTRODUCTION

The main goal of this work is to create a Mathcad document for choosing the cross-section of a glue-laminated three-pinned curved frame with either constant or tapered section according to European Standards. This document will allow designers to reduce time of calculation and to avoid mathematical mistakes.

As an example for using the document, a glued-laminated three-pinned curved frame has to be calculated. Static linear calculation will be made in SCAD Office soft. However, only linear analyses will be used, which does not consider geometrical nonlinearity. The larger spans in the calculated structure, the bigger differences between the results of linear and nonlinear analyses appear. Nonlinear analyses is more exact, however, in frames with spans of 18 m the difference will
be insignificant. Besides, in this work joints are not designed. Only one specific load case is considered.

Additionally, a 3D model of the designed structure will be made in ARCHICAD.

3 DESIGNING METHODS

As a main designing tool, SCAD Office soft was used. SCAD Office is a new generation system developed by engineers for engineers and implemented by a team of experienced programmers. The system includes a high-performing computer complex SCAD, as well as a number of design and support programs that allow to solve complex problems of calculation and design of steel and reinforced concrete structures. The system is constantly developing, the user interface and computing capabilities are improving and new design components are included.

All calculation and designing methods are complied with SNIPs (Russian construction standards). However, it is possible to make calculations according to Eurocodes by entering user’s coefficients and combinations of actions.

SCAD Office includes the following programs:

- SCAD - a computational complex for strength analysis of structures by the finite element method.
- Kristall - calculation of elements of steel structures
- ARBAT - selection of fittings and expertise of elements of reinforced concrete structures
- COMEIN - calculation of stone and reinforced structures
- Decor - calculation of wooden structures
- Zapros - calculation of foundations and their elements
- SLOPE - analysis of the stability of slopes
- WeST - calculation of loads by SNiP "Loads and effects" and DBN (does not applicable for EN)
- Monolit - design of monolithic ribbed floors
- Comet - calculation and design of steel structures
- Cross - calculation of the bed coefficients of buildings and structures on an elastic foundation
- Section Designer - forming and calculation of geometrical characteristics of sections from rolling profiles and sheets
- Consul - construction of arbitrary sections and calculation of their geometrical characteristics on the basis of the theory of solid rods
- Tonus - construction of arbitrary sections and calculation of their geometric characteristics on the basis of the theory of thin-walled rods
- Sezam - search for equivalent sections
- CoCon - a guide to stress concentration coefficients and stress intensity factors
- KUST - the theoretical calculation guide of the designer

United graphical environment for the synthesis of the calculation scheme and analysis of the results provides unlimited possibilities for modeling the calculation schemes from the simplest to the most complex structures, satisfying the needs of experienced professionals and while still available to beginners.

A high-performance processor allows solving large-scale problems (hundreds of thousands of degrees of freedom under static and dynamic effects).

SCAD includes a developed finite element library for modeling rod, plate, solid and combined structures, stability analysis modules, forming designing combinations of forces, testing the stress state of structural elements in various strength theories, determining the interaction forces of the element with the rest of the structure, calculating the forces and displacements from load combinations. The complex includes programs for selecting reinforcement in elements of reinforced concrete structures and checking the cross-sections of elements of steel.

The program allows import of geometry from ArchiCAD, AutoCAD HyperSteel, and other soft producing data in DXF, DWG formats.

The calculation results are displayed both in graphical and tabular forms, which makes the understanding easier. For rod elements, deformed schemes can be obtained considering deflections and deflection diagrams for individual elements.
The forces in the rod elements are represented in the form of diagrams for the entire scheme or individual element, as well as the color indication of the maximum values of the selected force factor.

Forces and stresses in plate and volume elements are displayed in the form of isofields or isolines in the specified range of the color scale with the ability to display numerical values at the centers and nodes of the elements simultaneously.

The calculation results in a tabular form can be exported to the MS Word or MS Excel for easy editing and calculating.

The table presentation of the results can be supplemented with graphical materials selected in the process of creating the calculation scheme and analyzing the results.


4 CALCULATIONS

4.1 Actions on the structure

All actions are imposed according to European Standards. As permanent actions (G) is taken the self-weight of the main bearing structure and the self-weight of the glazed roof structure. Variable actions are: wind and snow actions in Lappeenranta. Accidental actions are not considered.

The variability of G is neglected, because it does not vary significantly. The self-weight of the structure is represented by a single characteristic value and it is calculated on the basis of the normal dimensions and mean unit masses.
For variable actions the characteristic value is taken as an upper value with an intended probability is not being exceeded. The combination values of variable actions \((\psi_0 Q_k)\) are used for the verification of ultimate limit states and irreversible limit states.

### 4.2 Partial factor method

Verifications are made by partial factor method. It checks that in all relevant design situations, no relevant limit state is exceeded when design values for actions or effects of actions and resistances are used in the design model. For these calculations and the relevant limit states, the individual actions for the critical load cases are combined with relevant coefficients.

Design values are obtained by using characteristic values multiplied by relevant factor. (According to EN 1990:2002+A1:2005)

In this work STR limit state is verified as following: internal failure or excessive deformation of the structure.

### 4.3 Initial data

Figure 6 shows the designing scheme of one frame of the structure. It is a 3-pinned curved frame with 18 m span. Frames’ layout in the structure is shown in Figure 5. The structure’s dimensions in axises on plan: 18 m x 36 m. It consists of 6 frames with steps of 6 m. Service class of the structure – 2. The building is located in Lappeenranta.
Figure 6. The designing scheme of the frame.

Figure 7. Plan of frames and frame sections (a – section of hinge, b – section of curved part).
Glulam characteristics (according to EN 14080):
Class of resistance - GL32h
Bending strength \( f_{m,k} \) - 35 MPa
Compressive strength \( f_{c,o,k} \) - 31 MPa
Tensile strength \( f_{t,o,k} \) - 24 MPa
Chipping strength - 4,8 MPa
Mean Young’s modulus \( E \) - 13500 MPa
Shear modulus parallel to grain \( G \) - 350 MPa

4.4 Collecting loads

4.4.1 Permanent actions (G)

This kind of actions are automatically created by SCAD program.

4.4.2 Variable actions (Q)

Snow load

Roof category: H - roofs not accessible except for normal maintenance and repair.

Snow loads \([\text{kN/m}^2]\) on roofs are determined as follows:

\[
s = \mu_i C_e C_t s_k
\]  \hspace{1cm} (1)

Where:

\( \mu_i \) - the snow load shape coefficient

\( C_e \) - exposure coefficient,

\( C_t \) – thermal coefficient

\( s_k \) - characteristic value of snow on the ground at the relevant site \([\text{kN/m}^2]\)

\( C_e \) - for normal topography (as in Lappeenranta) is 1,0.

\( s_k \) - for Lappeenranta is 2,75 kN/m²
\[ C_1 = 0,8 \]

\[ \mu_1 - \text{for pitched roof when snow is not prevented from sliding off the roof, angle of pitch of roof } 30^\circ \leq \alpha \leq 60^\circ \text{ and for cylindrical roof is } 0,8(60 - \alpha)/30. \ (\alpha = 33^\circ) \]

\[ \mu_1 = 0,8 \cdot (60 - 33) / 30 = 0,72 \quad (2) \]

The force \( F_s \) exerted by a sliding mass of snow, in the direction of slide, per unit length of the building should be taken as:

\[ F_s = s \cdot b \cdot \sin \alpha \quad (3) \]

Where:

- \( s \) - the snow load on the roof relative to the most onerous undrifted load case appropriate for roof area from which snow could slide
- \( b \) - the width on plan (horizontal) from the guard or obstacle to the next guard or to the ridge
- \( \alpha \) - a pitch of the roof, measured from the horizontal

\[ s = 0,72 \cdot 1 \cdot 0,8 \cdot 2,75 = 1,58 \text{ kN/m}^2 \]

\[ F_s = 1,58 \cdot 6 \cdot \sin 33^\circ = 5,21 \text{ kN} \]

**Wind load**

Terrarian cathegory - II

The basic wind velocity is calculated from Expression:

\[ V_b = C_{dir} \cdot C_{season} \cdot V_{b,0} \quad (4) \]

Where:

- \( V_b \) - the basic wind velocity, defined as a function of wind direction and time of year at 10 m above ground of terrain category II
- \( V_{b,0} \) - the fundamental value of the basic wind velocity
- \( C_{dir} \) - the directional factor
- \( C_{season} \) - the season factor.

For Finland \( v_{b,0} = 21 \text{ m/s} \). The recommended value of \( C_{dir} \) and \( C_{season} \) is 1,0.
\[ V_b = 1 \cdot 1 \cdot 21 = 21 \text{ m/s}. \]

The mean wind velocity \( v_m(z) \) at a height \( z \) above the terrain depends on the terrain roughness and orography and on the basic wind velocity, \( v_b \), and should be determined using Expression:

\[ v_m(z) = c_r(z) \cdot c_o(z) \cdot v_b \quad (5) \]

Where:

- \( c_r(z) \) - the roughness factor
- \( c_o(z) \) - the orography factor, taken as 1,0

\[ c_r(z) = k_r \cdot \ln \left( \frac{z}{z_0} \right) \quad (6) \]

Where:

- \( z_0 \) - the roughness length (= 0,05 m for terrain category II)
- \( k_r \) - terrain factor depending on the roughness length \( z_0 \) calculated using

\[ k_r = 0,19 \cdot \left( \frac{0,05}{0,05} \right)^{0.07} = 0,19 \quad (7) \]

Where:

- \( z_{0,\text{II}} = 0,05 \text{ m (terrain category II)} \)
- \( z_{\text{min}} \) - the minimum height defined (= 2 m for terrain category II)
- \( z_{\text{max}} = 200 \text{ m} \)

\[ k_r = 0,19 \cdot \left( \frac{0,05}{0,05} \right)^{0.07} = 0,19 \]

\[ c_r(z) = 0,19 \cdot \ln \left( \frac{11,34}{0,05} \right) = 1,03 \]

\[ v_m(z) = 1,03 \cdot 1 \cdot 21 = 21,63 \text{ m/s} \]

The standard deviation of the turbulence \( \sigma_v \) may be determined using Expression:

\[ \sigma_v = k_r \cdot v_b \cdot k_l \quad (8) \]

Where:

- \( k_l \) - the turbulence factor (the recommended value for \( k_l \) is 1,0)
\[ \sigma_v = 0,19 \cdot 21 \cdot 1 = 3,99 \text{ m/s} \]

\[ l_v(z) = \frac{\sigma_v}{v_m(z)} = \frac{3,99}{21,63} = 0,18 \quad (9) \]

The peak velocity pressure \( q_p(z) \) at height \( z \):

\[ q_p(z) = [1 + 7 \cdot l_v(z)] \cdot \frac{1}{2} \cdot \rho \cdot v_m^2(z) = c_e(z) \cdot q_b \quad (10) \]

Where:

\( \rho \) - the air density, which depends on the altitude, temperature and barometric pressure to be expected in the region during wind storms (1,25 kg/m\(^3\))

\( c_e(z) \) - the exposure factor given in Expression:

\[ c_e(z) = \frac{q_p(z)}{q_b} \quad (11) \]

\( q_b \) - the basic velocity pressure given in Expression:

\[ q_b = \frac{1}{2} \cdot \rho \cdot v_b^2 = \frac{1}{2} \cdot 1,25 \cdot 21^2 = 0,28 \text{ kN/m}^2 \]

\[ q_p(z) = [1 + 7 \cdot 0,18] \cdot \frac{1}{2} \cdot 1,25 \cdot 21,63^2 = 0,66 \text{ kN/m}^2 \]

\[ c_e(z) = \frac{0,66}{0,28} = 2,4 \]

**Wind loads on walls**

The wind pressure acting on the external surfaces, \( w_e \), should be obtained from Expression:

\[ w_e = q_p(z_e) \cdot c_{pe} \quad (12) \]

Where:

\( q_p(z_e) \) - the peak velocity pressure

\( z_e \) - the reference height for the external pressure

\( c_{pe} \) - the pressure coefficient for the external pressure
Figure 8. Key 1 for collecting wind loads on walls (reproduced from Eurocode 1: Actions on structures. Part 1-4: General actions. Wind actions 1991)

\[ \frac{h}{d} = \frac{11.34}{18} = 0.63 \Rightarrow \]

Table 1. \( c_{pe,10} \) and \( c_{pe,1} \) for walls.

<table>
<thead>
<tr>
<th>Zone</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{pe,10} )</td>
<td>-1.2</td>
<td>-0.8</td>
</tr>
<tr>
<td>( c_{pe,1} )</td>
<td>-1.4</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

\( c_{pe} = c_{pe,1} - (c_{pe,1} - c_{pe,10}) \log_{10} A \):

Table 2. \( c_{pe} \) for walls.

<table>
<thead>
<tr>
<th>Zone</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_{pe} )</td>
<td>-1.06</td>
<td>-0.24</td>
</tr>
</tbody>
</table>

\[ e = 2h = 22.69 \text{ m}, \quad e > d: \quad (13) \]
Figure 9. Key 2 for collecting wind loads on walls (reproduced from Eurocode 1: Actions on structures. Part 1-4: General actions. Wind actions 1991)

\[ w_e = 660.85 \cdot c_{pe}, \text{ Pa:} \]  

(14)

Table 3. \( w_e \) for walls.

<table>
<thead>
<tr>
<th>Zone</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_e )</td>
<td>-700.5</td>
<td>-158.6</td>
</tr>
</tbody>
</table>

Figure 10. Reference height, \( z_e \), depending on hand \( b \), and corresponding velocity pressure profile (reproduced from Eurocode 1: Actions on structures. Part 1-4: General actions. Wind actions 1991)

\[ h = 11.34 \text{ m} < b = 18 \text{ m} \]

**Wind loads on duopitch roof.**

Wind direction \( \theta = 0^\circ \) is considered because the value of wind pressure in this case is the highest.
Figure 11. Key for collecting wind loads on roof (reproduced from Eurocode 1: Actions on structures. Part 1-4: General actions. Wind actions 1991)

Table 4. External pressure coefficient $c_{pe,1}$ and $c_{pe,10}$ for roof.

<table>
<thead>
<tr>
<th>Pitch Angle $\alpha$</th>
<th>Zone for wind direction $\theta = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
</tr>
<tr>
<td>$c_{pe,10}$</td>
<td>$c_{pe,1}$</td>
</tr>
<tr>
<td>33°</td>
<td>-0.4</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

Table 5. External pressure coefficient $c_{pe}$ for roof.

<table>
<thead>
<tr>
<th>Pitch Angle $\alpha$</th>
<th>Zone for wind direction $\theta = 0^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F</td>
</tr>
<tr>
<td></td>
<td>$c_{pe}$</td>
</tr>
<tr>
<td>33°</td>
<td>-0.31</td>
</tr>
<tr>
<td></td>
<td>0.7</td>
</tr>
</tbody>
</table>

$$A_F = \frac{e}{4} \cdot \frac{e}{10} = \frac{22.69^2}{40} = 12.87 \text{ m}^2 \quad (15)$$

$$A_G = \left(b - \frac{e}{2}\right) \cdot \frac{e}{10} = \left(36 - \frac{22.69}{2}\right) \cdot \frac{22.69}{10} = 55.94 \text{ m}^2 \quad (16)$$

Table 6. $w_e$ for roof, kN/m².

<table>
<thead>
<tr>
<th>Zone</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_e$</td>
<td>-0.2</td>
<td>0.13</td>
<td>-0.11</td>
<td>-0.24</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>0.46</td>
<td>0.46</td>
<td>0.29</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 7. Determination of wind actions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>peak velocity pressure</strong> $q_p$</td>
<td>21 m/s</td>
</tr>
<tr>
<td>basic wind velocity $v_b$</td>
<td>11,34 m</td>
</tr>
<tr>
<td>reference height $z_e$</td>
<td>II</td>
</tr>
<tr>
<td>terrain category</td>
<td>0,66 kN/m²</td>
</tr>
<tr>
<td>characteristic peak velocity pressure $q_p$</td>
<td>0,18</td>
</tr>
<tr>
<td>turbulence intensity $I_v$</td>
<td>21,63 m/s</td>
</tr>
<tr>
<td>mean wind velocity $v_m$</td>
<td>1</td>
</tr>
<tr>
<td>orography coefficient $c_o(z)$</td>
<td>1,03</td>
</tr>
<tr>
<td>roughness coefficient $c_r(z)$</td>
<td></td>
</tr>
</tbody>
</table>

Wind pressures, e.g. for cladding, fixings and structural parts

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>external pressure coefficient $c_{pe}$</td>
<td>See Table 2,5</td>
</tr>
<tr>
<td>internal pressure coefficient $c_{pi}$</td>
<td>-0,28</td>
</tr>
<tr>
<td>net pressure coefficient $c_{p,net}$</td>
<td>-1</td>
</tr>
<tr>
<td>external wind pressure: $w_e=q_p c_{pe}$</td>
<td>See Table 3,6</td>
</tr>
<tr>
<td>internal wind pressure: $w_i=q_p c_{pi}$</td>
<td>-0,18 kN/m²</td>
</tr>
</tbody>
</table>

Figure 12. Pressure on surfaces (reproduced from Eurocode 1: Actions on structures. Part 1-4: General actions. Wind actions 1991)
4.4.3 Combination of actions

The following combinations of actions are considered in this case:

**Ultimate limit state**

\[ E_d = E (\gamma G + \gamma_{Q_s} Q_s + \psi_{0,W} \gamma_{Q,W} Q_W) \]  (17)

\[ \psi_{0,W} = 0,6, \ \gamma_G = 1,1, \ \gamma_{Q,S} = \gamma_{Q,W} = 1,15 \]

**Serviceability limit state**

\[ E_d = E (G + \psi_{1,S} Q_s + \psi_{2,W} \gamma_{Q,W} Q_W) \]  (18)

\[ \psi_{1,S} = 0,4, \ \psi_{2,W} = 0 \]

Where:

- \( E \) – effect of actions
- \( E_d \) – design value of effect of action
- \( G \) – permanent actions
- \( G_d \) – design value of a permanent action
- \( Q_s \) – variable snow action
- \( Q_w \) – variable wind action

**Calculations**

Further calculations were made in SCAD soft. Each element of the structure was divided on finite elements of 1 meter. Border conditions were set by bearings (Figure 13).
The following sections were set:

- for frames: 800x400 mm
- for pillars: 150x150 mm
- for bonds: steel cables 50 mm in diameter

Then self-weight, snow and wind loads were imposed. (Figures 14-16)
The following deflections were shown:

![Figure 17. Deflection values](image)

The maximum deflection is not higher than allowed: $58 \text{ mm} < 75 \text{ mm} \approx \frac{18000}{240}$.

Choosing cross section for the main bearing structure was made in Mathcad. (See Appendix 1). According to these calculations, section with alternating height along the frame is chosen. The minimum height of 450 mm is on hinge and on supports; the maximum height of 480 mm is in the curved part. The width of 240 mm is constant. The lamella thickness is 30 mm, the lamella width is 120 mm.
Table 8. The main results of calculations of tapered frame.

<table>
<thead>
<tr>
<th>Type of designed stress</th>
<th>Maximum stress value, MPa</th>
<th>Strength value, MPa</th>
<th>Durability reserve, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive $\sigma_{c,o,d}$</td>
<td>24,42</td>
<td>24,8</td>
<td>3,5</td>
</tr>
<tr>
<td>Bending $\sigma_{m,d}$</td>
<td>19</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>Shear $\tau_d$</td>
<td>3,4</td>
<td>4,8</td>
<td>30</td>
</tr>
</tbody>
</table>

5 MODELING

The 3-dimensional model is made in ARCHICAD 20 (Figures 18,19, Application 2). ARCHICAD is the leading Building Information Modeling (BIM) software application used by architects, designers, engineers and builders to professionally design, document and collaborate on building projects. Since its release over 30 years ago, ARCHICAD has been all about BIM. ARCHICAD is the obvious choice of many architecture, engineering and construction industry professionals worldwide. (GRAPHISOFT official web-site 2017)

Besides, that soft allows creating IFC models. The Industry Foundation Classes (IFC) specification is a neutral data format to describe, exchange and share information typically used within the building and facility management industry sector. IFC is the international standard for openBIM and with IFC4 an ISO standard (ISO 16739). (http://www.ifcwiki.org 2017). Today almost all models are released during the project in the IFC format. In addition, a native file format model may be required simultaneously. Methods of distribution are agreed upon for each project. At the end of the project all the designs and electronic documents including IFC and native format BIMs are delivered to the Client as described in the contracts. The Client is entitled to use the models according to the same terms as traditional project documents. The IFC files are compressed (e.g. zipped) when they are shared within the project. This operation reduces the file size by up to 80%. Even smaller file sizes can be achieved by using an IFC file optimization program in addition to the file compression utility (mostly for large projects). Native format model files may be compressed as well, but in most cases the effect is less pronounced. (COBIM. Common BIM Requirements 2012)

Unfortunately, in ARCHICAD it is not possible to make a curved beam as an element in vertical plane. There are two ways of creating it. The first one is to make it with the use of 3D shape, but it is not convenient because of incorrect representation in IFC model. Thus,
in this work the second way is chosen: the whole structure is made with redirection of planes (i.e. skates are created with the use of column tool, pillars and vertically oriented beams are created with the use of beam tool). The chosen way allows reading IFC model in the most correct way. (Figure 20, Appendix 2)

6 CONCLUSIONS

Nowadays timber structures are becoming more and more popular, because of lots of advantages. Modern timber buildings have great variability. Combinations of different materials, such as glass, timber and steel look appealing. Moreover, it is easy to erect these structures, because most of the assembling elements are prefabricated.

This study was released by using modern calculation and designing methods: all manual calculations were reduced to minimum. As the main calculation and designing programs the following softwares were chosen:

- SCAD software, which helped with determining stress values and deflections from different combinations of actions, scheme of deformed structure;
- Mathcad software, which significantly allowed to reduce designing time (choosing cross-section of the main bearing structure) and excluded the possibility of arithmetical mistakes.

Besides, 3D models of the designed structure were created, according to the results of calculations. This part of work was made in ARCHICAD soft. 3D modeling makes the understanding of the structure easier and allows to show all elements from any suitable angle.

The most significant result of this work is the Mathcad program for choosing cross section of the three-pinned glued-laminated frame, which can be used for further calculations of different structures with this kind of designed scheme.
REFERENCES

APPENDICES

Appendix 1. Mathcad calculations

Maximum shear force in spans $Q_{sp} = 22.04 \ \text{tonne}$

Maximum shear force on supports $Q_{sup} = 26.2 \ \text{tonne}$

Normal force in ridge is $N_r = N_2 = -6.07 \ \text{tonne}$

Figure 2. Curves of bending moments of 3-pinned frame (arch)
Curves of $M$, $N$ along the structure's axis

![Curves of $M$, $N$ along the structure's axis](image)

**Figure 3.** Curves of $M$, $N$ of 3-pinned frame (arch)

### 2. The calculation of designed value of compression, bending and tension resistances

#### 2.1. Geometric parameters of structure section

The section length is constant along the length. The lamella (layer) thickness is $\delta_{l} = 30 \text{ mm}$ and thier quantity $n_{l} > 17$

Section height is $h = n_{l} \cdot \delta_{l} = 510 \text{ mm}$

Section width, made of 2 elements with widths $b_{l} = 120 \text{ mm}$ and $b = 2 \cdot b_{l} = 240 \text{ mm}$

Class of resistance: GL 30

#### 2.2. Compression stress parallel to the grain

$\sigma_{c,d,e} \leq f_{c,d,e}$, where $\sigma_{c,d,e}$ - design compressive stress along the grain, $f_{c,d,e}$ - design compressive strength along the grain

Characteristic compressive strength along the grain is $f_{c,d,e} = 31 \text{ MPa}$

$$f_{c,d,e} = \frac{f_{c,d,e}}{1.25} = 24.80 \text{ MPa}$$

#### 2.3. Tension stress parallel to the grain

$\sigma_{t,d,e} \leq f_{t,d,e}$, where $\sigma_{t,d,e}$ - design tensile stress along the grain, $f_{t,d,e}$ - design tensile strength along the grain

Characteristic tension strength along the grain is $f_{t,d,e} = 24 \text{ MPa}$

$$f_{t,d,e} = \frac{f_{t,d,e}}{1.25} = 19.20 \text{ MPa}$$

Normal stress in the ridge is $N_{e} = -6.07 \text{ tonne}$

Length of the frame (arch) along the axis $S = \sum_{j=1}^{J-1} \sqrt{(X_{j+1} - X_{j})^2 + (Y_{j+1} - Y_{j})^2} = 30.76 \text{ m}$

Section area $A_{sec} = b \cdot h = 1224 \text{ cm}^2$

Moment inertia $I = \frac{b \cdot h^3}{12} = 265302 \text{ cm}^4$

Section modulus $W_{sec} = \frac{b \cdot h^2}{6} = 10404 \text{ cm}^3$
Critical bending stress for section 1: \[ \sigma_1 = \frac{N_1 F}{W} \]
Critical bending stress for section 2: \[ \sigma_2 = \frac{N_2 F}{W} \]
Critical bending stress for section 3: \[ \sigma_3 = \frac{N_3 F}{W} \]

Curves of critical bending stresses along the axis of structure

Characteristic bending strength is \( f_{m,k} = 35 \text{MPa} \)

**The results of section strength calculations**

- Section parameters: \( h = 51 \text{ cm} \), \( b = 24 \text{ cm} \)
- Elastic modulus \( E = 13600 \text{MPa} \)
- Fifth percentile elastic modulus \( E_{0.05} = E \cdot 5\% = 675 \cdot \text{MPa} \)
- Shear modulus parallel to grain \( G = 350 \text{MPa} \)
- Fifth percentile shear modulus parallel to grain \( G_{0.05} = G \cdot 5\% = 18 \cdot \text{MPa} \)
- Torsional moment of inertia \( I_{tor} = 0.0016 \text{m}^4 \)

**Section 1:**
- \( \max(\sigma_1) = 9.77 \cdot \text{MPa} \), \( \min(\sigma_1) = -21.30 \cdot \text{MPa} \)
- Durability reserve: \[ \frac{|f_{c,o,d}| - \max\{ |\max(\sigma_1)|, |\min(\sigma_1)| \}}{|f_{c,o,d}|} = 14.10 \cdot \% \]

**Section 2:**
- \( \max(\sigma_2) = 9.83 \cdot \text{MPa} \), \( \min(\sigma_2) = -20.59 \cdot \text{MPa} \)
- Durability reserve: \[ \frac{|f_{c,o,d}| - \max\{ |\max(\sigma_2)|, |\min(\sigma_2)| \}}{|f_{c,o,d}|} = 16.96 \cdot \% \]

**Section 3:**
- \( \max(\sigma_3) = 11.11 \cdot \text{MPa} \), \( \min(\sigma_3) = -21.31 \cdot \text{MPa} \)
- Durability reserve: \[ \frac{|f_{c,o,d}| - \max\{ |\max(\sigma_3)|, |\min(\sigma_3)| \}}{|f_{c,o,d}|} = 14.09 \cdot \% \]
2.4. Checking slenderness

Span length \( l = 9 \text{ m} \)

Effective length \( l_{ef} := 0.9 \cdot l = 8 \text{ m} \)

\[
\sigma_{m,cr} = \frac{0.78 \cdot b^2}{l_{ef} \cdot h} \cdot E_{0.05} = 7.34 \cdot \text{MPa}
\]

Relative slenderness for bending \( \lambda_{rel,m} = \left( \frac{f_{m,k}}{\sigma_{m,cr}} \right)^2 = 2 \)

The inner radius is \( r_{in} = 5.745 \text{ m} \)

Slenderness ratio corresponding to bending about the z-axis (deflection in the z-direction)

\[
\lambda_{z} = \frac{l_{ef}}{r_{m}} = 1.41
\]

\[
\lambda_{rel,z} = \frac{\lambda_{z}}{\pi} \cdot \sqrt{\frac{f_{c,0,k}}{E_{0.05}}} = 0.10
\]

\[
k_{z} := 0.5 \left[ 1 + 0.2 \cdot (\lambda_{rel,z} - 0.3) + \lambda_{rel,z}^2 \right] = 0.48
\]

\[
k_{z,s} := \frac{1}{k_{z} + \sqrt{k_{z}^2 - \lambda_{rel,z}^2}} = 1.04
\]

\[
k_{z,cr} := \begin{cases} 
1 & \text{if } \lambda_{rel,m} \leq 0.75 \\
1.56 - 0.75 \cdot \lambda_{rel,m} & \text{if } 0.75 < \lambda_{rel,m} < 1.4 \\
1 & \text{if } \lambda_{rel,m} > 1.4
\end{cases} = 0.21
\]

Section 1

\[
\left( \frac{\sigma_{1,m,d}}{k_{cr} \cdot f_{m,k}} \right)^2 + \frac{\sigma_{1,o,d}}{k_{c,z} \cdot f_{c,o,d}} \leq 1
\]

where:

\[
\sigma_{1,m,d} = \frac{\max \left( \left\{ \max (M_t) \right\}, \left\{ \min (M_t) \right\} \right)}{F} = 2.10 \cdot \text{MPa} \text{ is the design bending stress}
\]

\[
\sigma_{1,o,d} = \frac{\max \left( \left\{ \max (M_t) \right\}, \left\{ \min (M_t) \right\} \right)}{W} = 19.68 \cdot \text{MPa} \text{ is the design compressive stress parallel to grain}
\]
Section 2

\[
\left( \frac{\sigma^2_{m,d}}{k_{\text{crit}} \cdot f_{m,k}} \right)^2 + \frac{\sigma^2_{c,o,d}}{k_{c,z} \cdot f_{c,o,d}} \leq 1
\]

where:

\[
\sigma^2_{m,\text{max}} = \left| \frac{\max\left( \max(N2) \right)}{F} \cdot \frac{\max(N2)}{F} \right| = 2.04 \cdot \text{MPa} \text{ is the design bending stress}
\]

\[
\sigma^2_{c,\text{max}} = \left| \frac{\max\left( \max(M2) \right)}{W} \cdot \frac{\min(M2)}{W} \right| = 19.01 \cdot \text{MPa} \text{ is the design compressive stress parallel to grain}
\]

\[
\left( \frac{\sigma^3_{m,d}}{k_{\text{crit}} \cdot f_{m,k}} \right)^2 + \frac{\sigma^3_{c,o,d}}{k_{c,z} \cdot f_{c,o,d}} = 1
\]

Section 3

\[
\left( \frac{\sigma^3_{m,d}}{k_{\text{crit}} \cdot f_{m,k}} \right)^2 + \frac{\sigma^3_{c,o,d}}{k_{c,z} \cdot f_{c,o,d}} \leq 1
\]

where:

\[
\sigma^3_{m,\text{max}} = \left| \frac{\max\left( \max(N3) \right)}{F} \cdot \frac{\max(N3)}{F} \right| = 1.97 \cdot \text{MPa} \text{ is the design bending stress}
\]

\[
\sigma^3_{c,\text{max}} = \left| \frac{\max\left( \max(M3) \right)}{W} \cdot \frac{\min(M3)}{W} \right| = 19.68 \cdot \text{MPa} \text{ is the design compressive stress parallel to grain}
\]

\[
\left( \frac{\sigma^3_{m,d}}{k_{\text{crit}} \cdot f_{m,k}} \right)^2 + \frac{\sigma^3_{c,o,d}}{k_{c,z} \cdot f_{c,o,d}} = 1
\]
2.5. Calculation of bending stresses in curved (apex) area

Figure 5. Apex area

The depth of the beam at the apex is \( h_{ap} := h = 0.51 m \)

The width of the beam is \( b = 0.24 m \)

\[ \sigma_{m,d} \leq k_t f_{m,d} \]
where \( k_t \) takes into account the strength reduction due to bending laminates during production, \( f_{m,d} \) - design bending strength, \( \sigma_{m,d} \) - design bending stress

\[ f_{m,d} \geq \frac{f_{m,k}}{1.25} = 28.00 \cdot \text{MPa} \]

The designed moment at the apex is \( M_{ap,d} \geq M_2 = -20.17 \cdot \text{m \cdot tonnef} \)

\[ \sigma_{m,d} = \frac{6 |M_{ap,d}|}{b \cdot (h_{ap}^2)} = 19 \cdot \text{MPa} \]

\( r = r_m + 0.5 \cdot h_{ap} = 6.00 m \)
3. Calculations for tapered curved frames (arches)

Height of tapered curved section

\[ h_t = (42 \ 42 \ 45 \ 45 \ 45 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42 \ 42) \text{ cm} \]

\[ h_t \approx h_0 - \delta_t \]

Maximum section height is \( h_{t,\text{max}} = \max(h_t) = 48.00 \text{ cm} \)
Minimum section height is \( h_{t,\text{min}} = \min(h_t) = 45.00 \text{ cm} \)

Height distribution along the length

Curves of normal stresses along the structure length

Designed points along the structure axis

- Compression stresses in the 1st section
- Tensile stresses in the 1st section
- Compression stresses in the 2nd section
- Tensile stresses in the 2nd section
- Compression stresses in the 3rd section
- Tensile stresses in the 3rd section
- \( fc_{o,d} \) - design compressive strength along the grain
- \( ft_{o,d} \) - design tensile strength along the grain
Results of calculations by normal stresses of tapered frame along its length

Frame section height from \( h_{\text{max}} = \max(h_i) = 48 \text{ cm} \) (in curved part) to \( h_{\text{min}} = \min(h_i) = 45 \text{ cm} \)

**Maximum stresses (compression/tensile)**

For the 1st section in each element:
\[
\sigma_{1\text{max}} = \max(\sigma_{1\text{comp} taper}) = 129.06 \frac{\text{kgf}}{\text{cm}^2} \quad \sigma_{1\text{min}} = \min(\sigma_{1\text{comp} taper}) = -244.15 \frac{\text{kgf}}{\text{cm}^2}
\]

Durability reserve on the upper edge \( \frac{|f_{c,0,d} - |\sigma_{1\text{min}}|}{f_{c,0,d}} \) = 3.46 \%  

Durability reserve on the lower edge \( \frac{|f_{c,0,d} - |\sigma_{1\text{max}}|}{f_{c,0,d}} \) = 34.08 \%

For the 2nd section in each element:
\[
\sigma_{2\text{max}} = \max(\sigma_{2\text{comp} taper}) = 129.72 \frac{\text{kgf}}{\text{cm}^2} \quad \sigma_{2\text{min}} = \min(\sigma_{2\text{comp} taper}) = -235.92 \frac{\text{kgf}}{\text{cm}^2}
\]

Durability reserve on the upper edge \( \frac{|f_{c,0,d} - |\sigma_{2\text{min}}|}{f_{c,0,d}} \) = 6.71 \%  

Durability reserve on the lower edge \( \frac{|f_{c,0,d} - |\sigma_{2\text{max}}|}{f_{c,0,d}} \) = 32.74 \%

For the 3rd section in each element:
\[
\sigma_{3\text{max}} = \max(\sigma_{3\text{comp} taper}) = 148.17 \frac{\text{kgf}}{\text{cm}^2} \quad \sigma_{3\text{min}} = \min(\sigma_{3\text{comp} taper}) = -244.16 \frac{\text{kgf}}{\text{cm}^2}
\]

Durability reserve on the upper edge \( \frac{|f_{c,0,d} - |\sigma_{3\text{min}}|}{f_{c,0,d}} \) = 3.45 \%
4. Checking shear stresses

\[ \tau_d \leq f_{cd} \text{, where } \tau_d \text{ is design shear stress, } f_{cd} \text{ - design shear strength for the actual condition} \]

\[ \tau_d = \frac{Q_{sa}}{J \cdot b} + \Delta \tau = \frac{Q_{sa} \cdot S'}{J \cdot b} \leq f_{cd} = 4.6 \text{MPa} \]

4.1. Checking in span section in the designed point \( k = 29 \)

The maximum value of the shear force in span (taken from SCAD calculations) is \( Q_{sa} = 22.04 \text{ tonnel} \)

Static momentum of the designed section \( S' = \frac{b \cdot h^2}{6} = 7803.00 \text{ cm}^3 \)

Inertia moment of the designed section \( J = \frac{b \cdot h^2}{12} = 265302.00 \cdot \text{cm}^4 \)

Excentricity of the normal force in the designed section \( \Delta \tau = N_1 = -22.04 \cdot \text{tonnel} \) in the designed point \( k = 29 \) is \( \tau_d = 0 \cdot \text{mm} \)

\[ \Delta r = \frac{1.75 \cdot |N_1| \cdot e}{b \cdot h^2} = 0.00 \cdot \frac{\text{kgf}}{\text{cm}^2} \]

Thus,

\[ \tau_d = \frac{Q_{sa} \cdot S'}{J \cdot b} + \Delta r = 27.01 \cdot \frac{\text{kgf}}{\text{cm}^2} \]

The durability resource is \( \frac{f_{cd} - \tau_d}{f_{cd}} = 44.3 \cdot \% \)

4.2. Checking in supporting section in the designed point \( k = 1 \)

The maximum value of the shear force on supports (taken from SCAD calculations) is \( Q_{sup} = 26.20 \text{ tonnel} \)

Static momentum of the designed section \( S' = \frac{b \cdot h^2}{6} = 7803.00 \text{ cm}^3 \)

Inertia moment of the designed section \( J = \frac{b \cdot h^2}{12} = 265302.00 \cdot \text{cm}^4 \)

Excentricity of the normal force in the designed section \( N_1 = -24.74 \cdot \text{tonnel} \) in the designed point \( k = 1 \) is \( \Delta r = 30 \cdot \text{mm} \)

\[ \Delta r = \frac{1.75 \cdot |N_1| \cdot e}{b \cdot h^2} = 2.08 \cdot \frac{\text{kgf}}{\text{cm}^2} \]

Thus,

\[ \tau_d = \frac{Q_{sup} \cdot S'}{J \cdot b} + \Delta r = 34.19 \cdot \frac{\text{kgf}}{\text{cm}^2} \]

The durability resource is \( \frac{f_{cd} - \tau_d}{f_{cd}} = 30.2 \cdot \% \)
Appendix 2. 3D model of the building

Figure 18. Facade of the building

Figure 19. Bearing structure of the building
Figure 20. IFC model of the building