Saimaa University of Applied Sciences
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Double Degree Program in Civil and Construction Engineering

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COMPARISON OF REINFORCED AND FIBER SLAB

Bachelor`s Thesis 2018

ABSTRACT
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Saimaa University of Applied Sciences
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This study was commissioned by «Gradient Ltd.». The purpose of the study was to analyze and calculate different variants of reinforcement of concrete slabs. The main objective of the research was to compare the total costs, strength properties, and material consumptions.

The data for this study was collected from Russian norms and European literature for designers. The calculations are based on official Russian norms SP 52-104-2006 «Steel fibre reinforced concrete structures», SP 63.13330.2017 «Concrete structures», SP 20.13330.2017 «Loads and actions».

The results of the calculations show the possibility of replacement of ordinary reinforcement with combined. Combined reinforcement is a combination of bar and fiber reinforcement, with the preservation of the longitudinal bars as working.

Keywords: Fiber reinforcement, ribbed slab, combined reinforcement

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TERMINOLOGY

Fiber-reinforced concrete - is concrete containing fibers that are uniformly distributed and randomly oriented.

Fibers - short longitudinal pieces of different materials, that are used for concrete reinforcement.

R_b - design compression strength of concrete

R_{bt} - design tensile strength of concrete

R_{fb} - design compression strength of fiber-reinforced concrete

R_{fbt} - design tensile strength of fiber-reinforced concrete

R_s - design strength of steel

R_f - design strength of fiber

E_b - concrete elasticity modulus without creep

E_s - steel elasticity modulus

E_f - fiber elasticity modulus

μ_{fm} - mass reinforcement factor

μ_{fv} - volume reinforcement factor

μ_{fa} - area reinforcement factor

d_f - diameter of a fibre

d - diameter of a reinforcement bar

l_f - length of a fibre

$l_{f,an}$ - minimal anchoring length, that provides fiber rupture instead of pulling out

k_{or} - fiber orientation factor, in relation to main tensile stress distribution lines

k_n - fiber orientation factor, that takes into account section size

φ_f - efficiency factor of indirect fiber reinforcement

η - adhesion characteristic of fibers to concrete on surface area

$J_{f,red}$ - moment of inertia of section

$W_{f,red}$ - resistance moment of section

h_0 - working height of section

α_r - verifying coefficient, is used in calculation to make sure that concrete in compressed zone is more durable, than reinforcement in tensioned zone

1 INTRODUCTION

1.1 General information

At the current time, fibre-reinforced concrete is given more attention in construction field. This is because with all the advantages of concrete and reinforced concrete, they have a number of drawbacks. One of the most serious is low crack resistance, the reason of the brittle failure of structures under load. This and other disadvantages can be eliminated by using fiber reinforcement, which gives a possibility to obtain fibre-reinforced concrete - a composite material consisting of a cement matrix with a uniform or predetermined distribution throughout the volume of oriented or chaotically arranged discrete fibers of different sizes (2).

The first information about fiber-reinforced concrete appeared at the beginning of the 20th century when a strength increasing research was conducted. The main goal of this research was to increase the resistance of concrete structures in compressed and stretched layers by adding thin iron wires to concrete. At the first stage of the study, this approach was considered only as assistance related to bar reinforcement, but the expediency of using dispersed concrete reinforcement at that time was obvious (4).

At present, the nomenclature of the used fibers is very extensive, and according to the accepted classification fibers are divided by (4): modulus of elasticity of fiber to high-modulus (steel, carbon, glass, etc.) and to low-modulus (polypropylene, viscose, etc.); natural origin (asbestine, basalt, wool, etc.) and artificial (viscose, polyamide, etc.); the main material to metal (most often steel) and non-metallic (synthetic, mineral).

Dispersed reinforcement is a loose material in the form of a set of discrete fibers of different origin, type, and size, which is intended for disperse reinforcement of concrete, as a reinforcer and modifier of the structure of the composite. In the case of dispersed reinforcement, the strengthening of concrete with fiber is based on the hypothesis that the composite matrix transmits the applied load to

the fibers uniformly distributed in it due to tangential forces that act on the phase interface.

In the case where the elastic modulus of the fiber exceeds the modulus of elasticity of the concrete matrix, the main part of the stresses is taken up by the fibers, and the overall strength of the composite is directly proportional to their volume content (5).

For effective use, different fibers must meet the following requirements: the modulus of elasticity of the fibers must be higher than the modulus of elasticity of the composite matrix; the fibers must be chemically stable and work normally in alkaline concrete environments; the cost of fibers should be minimal.

The production of steel fiber is based on cutting low-carbon wire, sheet steel or foil; molding from a melt, milling of slabs and strips, as well as intermittent vibrational cutting during turning of the workpiece. Nonmetallic fibers are pieces of monofilaments, fibrillated films, and complex filaments, which, among other things, can be made from the industrial waste of the appropriate production. The main task in the development of the compositions and technology of fiber-reinforced concrete is the optimization of the geometric parameters and dimensions of the fibers that ensure reliable adhesion of the fibers to the concrete matrix under the permissible loads, improving the processability, lowering the labor and energy intensity of operations for the production. The adhesion of the fiber to the concrete and the increase in the processability of operations, first of all, depends on the ratio of the length of the fiber to its diameter (l / d), which can vary widely and affect the degree of fiber anchoring in the concrete matrix, as well as the technological properties of fiber-reinforced concrete mixtures.

Taking into account the above, when using steel fiber, the ratio of its length to diameter is taken equal to l / d = 80 ... 100 (4).

To improve the design capabilities of fiber, the cross-sectional shape changes from round to angular, thus increasing the lateral surface area of the fiber. The cross-sectional shape provides considerable rigidity of such fiber in the longitudinal direction, which greatly facilitates the processes of its transportation

and dosing, as well as facilitates the uniform distribution of fibers at an increased content in the concrete mix, which leads to a serious improvement in the strength, deformation and performance characteristics of fiber-reinforced concrete.

1.2 Task description

The purpose of the thesis is to make a technical and economic comparison of the characteristics of fiber-reinforced concrete and reinforced concrete on the example of typical structural elements.

1.3 Methodology

The work carried out a large number of comparisons, the main ones are the comparison of the strength and economic profitability of the use of fiber reinforced concrete. In addition, other important properties of the material, such as crack resistance, fracture toughness and etc. did not remain without attention.

2 THEORETICAL BACKGROUND

2.1 Composition of fiber-reinforced concrete

Mineral binders are in the composition of concrete (5). The properties of mineral binders have been thoroughly studied in the relevant literature. Therefore, it is advisable to consider in the work only those features that are most important for ensuring the necessary structural qualities of dispersed reinforced concrete. Portland cement environment is an active alkaline environment (Ph = 13 or more). This determines the effect on reinforcing fibers. Hydration products reliably protect metal reinforcement from corrosion but are very aggressive to any mineral reinforcing fibers (fiberglass, basalt). If it is necessary to use mineral reinforcing fibers, it is possible to reduce alkalinity by changing the proportions of the minerals in the cement clinker.

Then, there are reinforcement fibers in fiber-reinforced concrete (5). Not all fibers are suitable for reinforcing concrete. First of all, it is necessary to take into account such properties as strength, deformability (modulus of elasticity),

adhesion to concrete, the coefficient of thermal expansion, etc. Also, important issue is the cost of fiber, as well as the volume of its production.

Currently, three types of reinforcing fibers are used (4): thin steel wire, glass fiber, and various polymer fibers, for example, polypropylene. Types of fibers are very different from each other, so each of them needs to be approached separately. For example, in the constructive way, the steel fiber reinforcement is more suitable, because its modulus of elasticity is higher than the modulus of elasticity of concrete. Glass fibers of small diameter (8-10 microns) in strength correspond to steel, but it is 3.5 times lighter. The modulus of elasticity of glass fibers is lower than that of steel but higher than that of concrete, which makes it possible to use it as a supporting reinforcement. Polymer fiber is characterized by increased deformation, so it can not act as a supporting reinforcement. However, with the help of polymeric fibers, it is possible to implement new methods of constructive reinforcement (an increase of impact strength, improvement of abrasion resistance, prevention of fractures during transportation).

Table 1. Types and properties of reinforcement fibers.

Fiber	Density, kg/m^3	Tensile strength, N/mm^2	Elasticity modulus, N/mm^2	Deformation before rapture, %
Polypropylene	900	40-70	3.5-8	10-25
Polyethylene	950	30-70	1.4-4.2	10
Nylon	1100	77-84	4.2	16-20
Acrylic	1100	21-42	2.1	25-45
Polyester	1400	73-78	8.4	11-13
Cotton	1500	42-70	4.9	3-10
Asbestos	2600	91-3000	68	0.6
Glass	2600	105-385	70-80	1.5-3.5
Basalt	2600	160-360	80-110	1.4-3.6
Steel	7800	600-1500	210	3-4

Carbon	1550	>2000	245	1

In the production of fiber-reinforced concrete steel fibers (4) are most commonly used, because its modulus of elasticity is higher than the modulus of elasticity of concrete matrix, which makes it possible to obtain advantages in bearing capacity and in maintenance, especially in the tension zone of an element during bending. More, steel fiber is used when it is necessary to obtain high explosion and burglary resistance (for example bank vault). According to previous researches with «Dramix» fibre, in case of reinforcing with 0.3 mm in diameter fibers, at a saturation of 3% by volume, tensile strength increases by factor of 5 in comparison with unreinforced concrete, and in the case of saturation of 2% by volume, bending strength increases by 2 times.

Steel fiber differs in the manner of production and in form. The following are the different types of fibers.

1. Anchoring fiber

Anchoring steel fiber - fibers from low-carbon wire with anchors (couplers) at both ends. The bent ends of the fiber provide better adhesion to the concrete in the mix and upon drying.

Figure 1. «Dramix» 80/60 anchoring fiber (Rabinovich 2003)

2. Anchoring fiber from sheet metal

Fiber sheet is produced by cutting a sheet of high-strength steel. The fiber has a triangular or trapezoidal cross-section with a rough texture and bends at the ends.

Figure 2. «Fibraprom» milled anchoring fiber («Fibraprom» official website)

3. Wave fiber

Wave fiber is made of low-carbon and high-carbon wire. A special structure made it possible to facilitate the production of fiber without loss of quality.

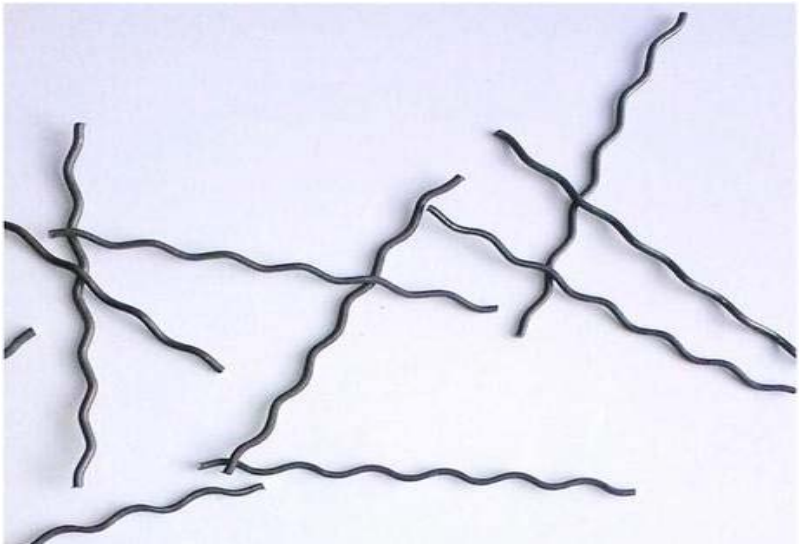

Figure 3. «Rosfiber» wave fiber (GOST 14613-83)

4. Milled from the steel melt fiber

The milled fiber from the melt is produced by cutting a melt with a milling cutter. Steel billets are milled using the patented technology of the company "Vulkan Harex" (Germany). The technology involves the use of special tools to give the fiber a triangular cross-section. The key feature of a fiber of this type is that this fiber never sticks to each other.

Figure 4. «Rosfiber» milled from the steel melt fiber (Rabinovich 2003)

Polymer and mineral fiber. According to the "rule of mixtures" the strength of concrete increases directly in proportion to the volume content of the fibers, if their modulus of elasticity is higher than the modulus of elasticity of the matrix, hence low-modulus fibers in heavy concrete are not able to fulfill the role of the reinforcing agent in concrete. However, according to the work of the researchers, with the introduction of low-modulus fibers, the strength of concrete not only decreases, which would correspond to the rule of mixtures, but increases. For example, according to Russian scientist Puharenko Y.V., when 1-2% of polypropylene fiber (l = 20mm) is introduced, the strength increase reaches 15-18%. The same effect is confirmed in the work of Zotov A.N., who comes to the conclusion that the use of polypropylene fiber contributes to the improvement of the strength properties of concrete, so an

increase in the consumption of fibers leads to a decrease in the compressive strength, but to an increase in the tensile strength in bending.

Basically, polymer and mineral fibers (except fiberglass and carbon fiber) are introduced to concrete to increase abradability resistance and to decrease cracks appearance. Also, polymer and mineral fiber differs in the manner of production and in form. The following are the different types of fibers.

1. Basalt fiber is a natural fiber derived from a natural mineral. It is a chemically inert fiber with an ideal adhesion to concrete, not reacting with salts, alkalis or dyes. Concretes with the addition of basalt fiber are successfully used: for the construction of offshore structures; road surfaces; in decorative concretes. The advantage of basalt fiber is that it is obtained from raw material in one stage, while the preparation of glass fiber requires many stages.

Figure 5. «Fibraprom» basalt fiber («Fibraprom» official website)

2. Nylon fiber. Introduction of nylon fiber in concrete adds to the mortar tenacity, elasticity, and strength, improves waterproofing properties, frost and chemical resistance.

Figure 6. «Fibraprom» nylon fiber («Fibraprom» official website)

3. Glass fiber is a chopped glued fiberglass. When the mortar is mixed, the additive breaks down into fibers and becomes almost invisible. Glass fiber for concrete helps to reduce shrinkage and cracking of concrete. It is resistant to the effects of aggressive environment and corrosion. It has good strength and elasticity. The biggest drawback of glass fiber is the destruction in the environment of hydratable cement as a result of alkaline corrosion.

Figure 7. «Fibraprom» glass fiber («Fibraprom» official website)

4. Polypropylene fiber is one of the most actively used additives. Polypropylene fiber for concrete is produced by extrusion. As a result, thin fibers of white or yellowish color, 6-18 mm in length, are obtained.

Figure 8. «Fibraprom» polypropylene fiber («Fibraprom» official website)

In conclusion, it can be said that properties of fiber-reinforced concrete mainly depend on the properties and the volume of fiber introduced into it and that fiber-reinforced concrete is a material with a wide range of defined properties, starting from the requirements of the increased modulus of elasticity of the concrete matrix and improved resistance to cracking, and ending with the requirements for water tightness and explosion resistance.

Aggregates.

As a fine aggregates quartz sands are used. As a coarse aggregate, both dense and porous minerals can be used. The maximum diameter of the used fraction should be less than 0.25 of fiber length and less than «s» factor

$$s = \frac{d_{red}}{\sqrt{\mu_{fv} * K_{or}}} \; ; \; d_{red} = \sqrt{\frac{4A_f}{\pi}}$$

Where μ_{fv} is volume reinforcement factor, K_{or} is fiber orientation factor and d_{red} is a diameter of round fibers, A_f is a cross-section area of fibers (5).

3 CALCULATION OF NORMAL REINFORCED SLAB

The ribbed slab is a monolithic structure containing a slab, longitudinal and transverse ribs, working on bending in different directions.

Figure 9. Example of concrete ribbed slab

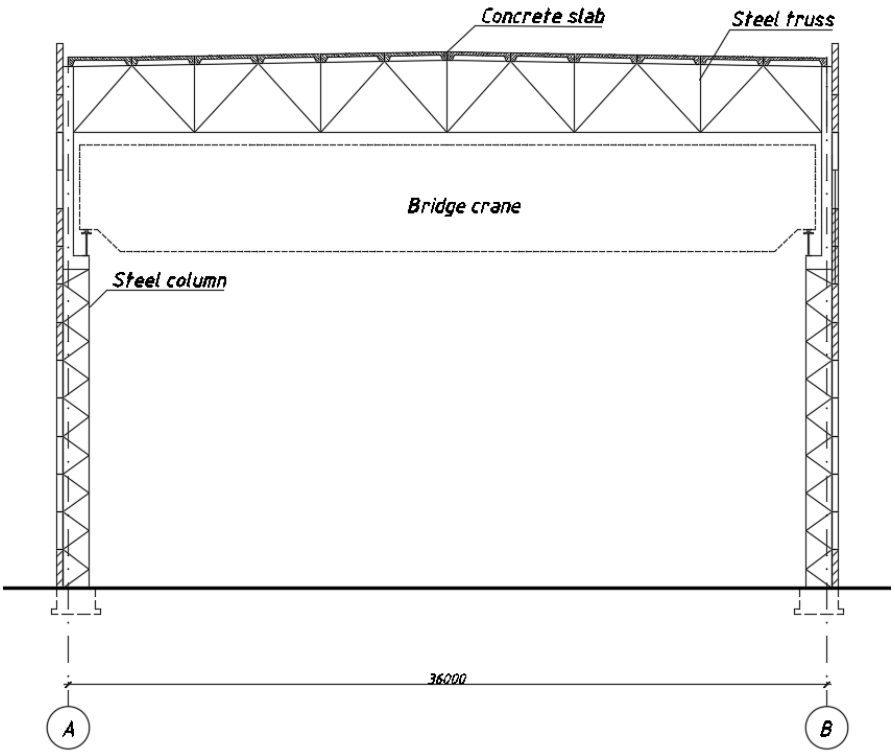

Figure 10. Section view of manufactory building

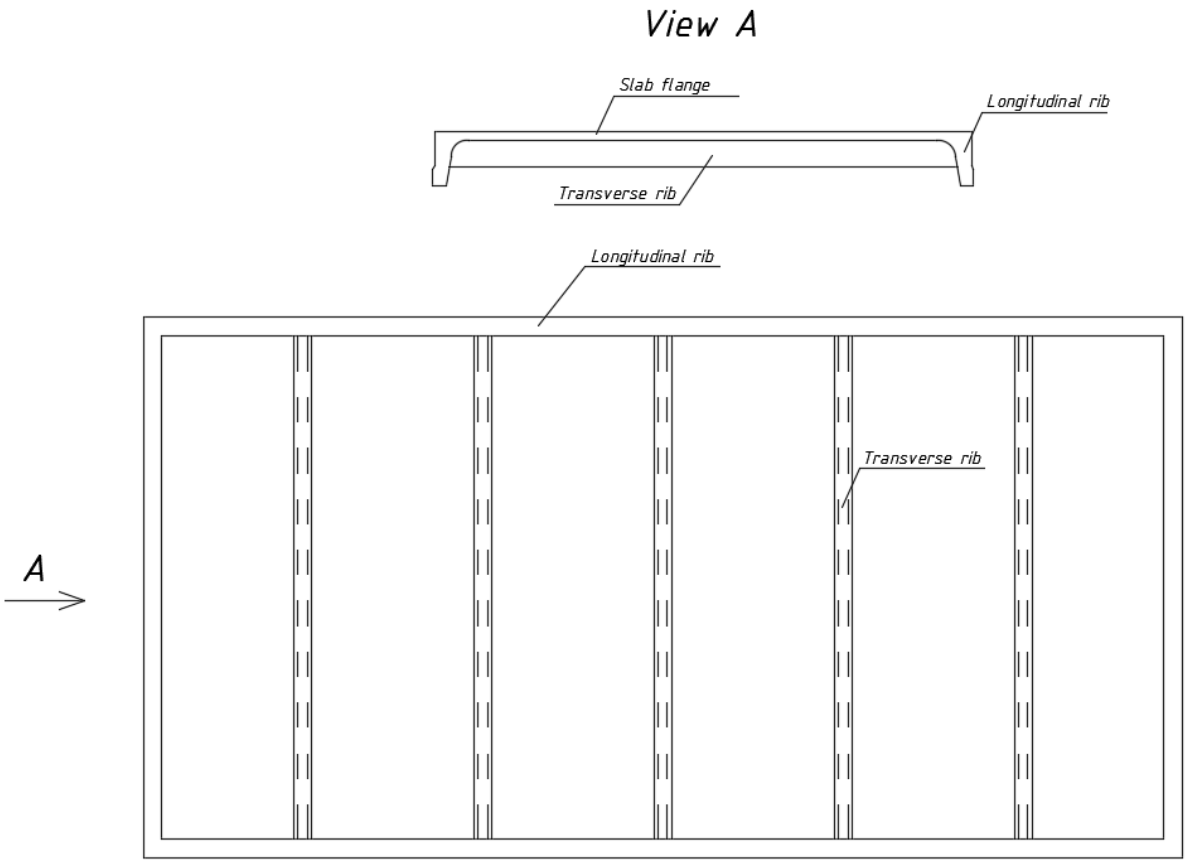

Figure 11.General elements of concrete ribbed slab

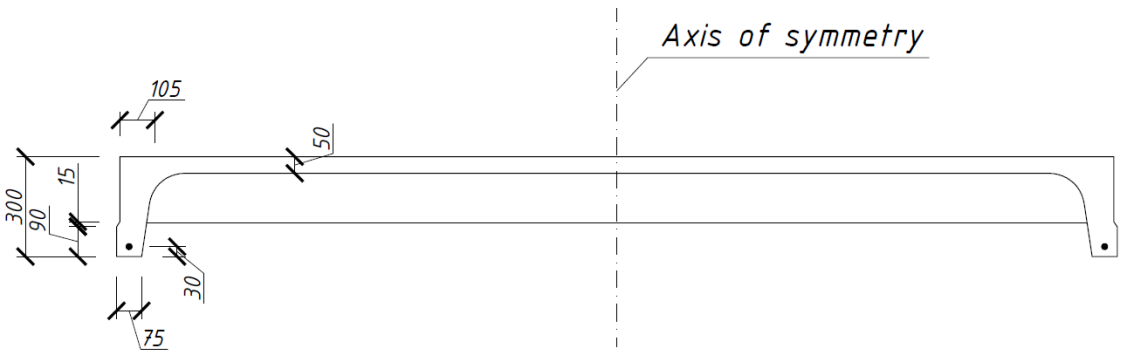

Figure 12. Cross-section of slab

Given : Slab dimensions 5760·2980, evenly distributed load q = 7,3 kN/m^2, Concrete B25 (Rb = 13,2 MPa, Rbt = 0,95 MPa), Rs = 680 MPa - for prestressed reinforcement. Tension type - electrothermal tension. Rs = 355 MPa for non-prestressed reinforcement, for slab reinforcing mesh B500 with Rs = 415 MPa, Es = 2.1·10^5 MPa, Eb = 2.75·10^4 MPa.

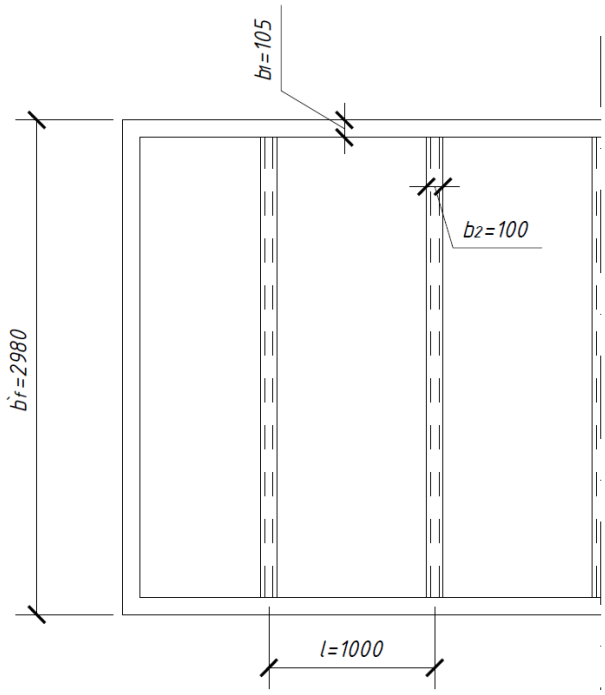

Figure 13. Calculated span determination

3.1 Slab flange calculation

For a calculated span are accepted:

- in the transverse direction $l_1 = b'_f - 2b_1 = 2980 - 105 \cdot 2 = 2770$ mm;
- in the longitudinal direction $l_2 = l - b_2 = 1000 - 100 = 900$ mm, where b_1 и b_2 – width at the top of the longitudinal and transverse ribs, respectively.

$$\frac{l_2}{l_1} = \frac{2770}{900} = 3{,}07.$$

According to span ratio = 3,07 slab flange is calculated as a beam (7). For calculation it is necessary to cut a conditional strip of 1 meter wide.

<u>Static calculation.</u>

The load on the strip of a slab with a nominal width of 1 meter and a slab flange thickness of 50 mm is given in Table 2.

Table 2. Load determination for slab flange.

№	Type of load	Normative load, kN/m	Load safety factor	Designed load, kN/m
Dead load				
1	Cement floor weight δ=20 mm, ρ = 2000 kg/m^3	0,4	1,3	0,52
2	Weight of concrete slab (self weight) δ=50mm, ρ =2500 kg/m^3	1,25	1,1	1,375
Total		1,65		1,895
Temporary load				
3	Evenly distributed load	7,3	1,2	10,4
Total		8,95		12,3

Since the flange is working in two directions, it is necessary to take into account the moments in the transverse directions.

$$M_1 = \frac{ql^2}{48} = \frac{12,3 \cdot 2,77^2}{48} = 2.15 kN \cdot m$$

$$M_1 = \frac{ql^2}{48} = \frac{12,3 \cdot 0,9^2}{48} = 0.26 kN \cdot m$$

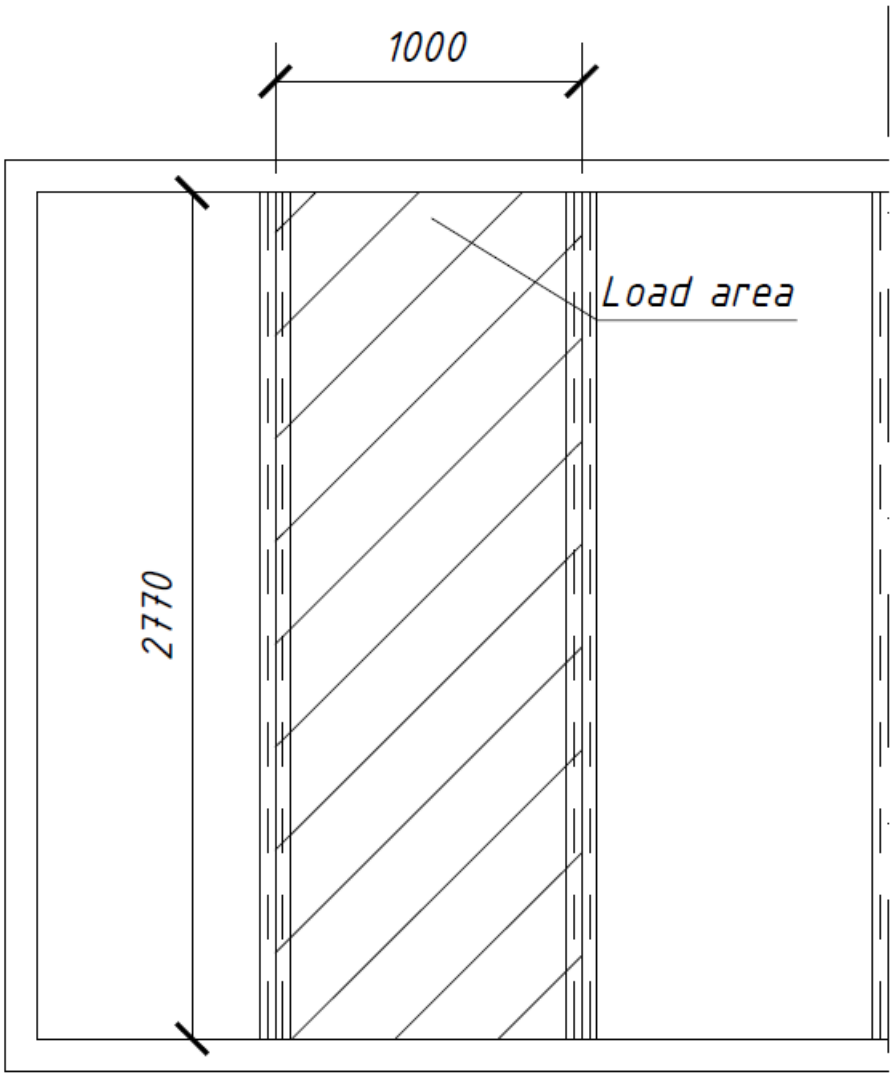

Figure 14. Design scheme of slab flange

<u>Reinforcement calculation.</u>

The flange is designed from concrete of class B25 with the following characteristics: R_b = 13,2 MPa; R_{bt} =0,95 MPa; $R_{b,ser}$ = 18,5 MPa; $R_{bt,ser}$ =1,35 MPa; E_b = 27 500 MPa.

A wire of class B500 is used as a working reinforcement with a calculated resistance Rs = 415 MPa, in the form of welded roll mesh with longitudinal and transverse working reinforcement, and in longitudinal and transverse ribs - rod reinforcement of class A400 in the form of flat welded frames with Rs = 355 MPa. A240 reinforcement is taken as a transverse reinforcement in the ribs of the slab.

Refinement of the thickness of the plate, taking into account reinforcement factor μ_s = 0,006:

$$\xi = \mu_s \cdot \frac{R_s}{R_b} = 0,006 \cdot \frac{415}{13,2} = 0,19$$

$\alpha_m = \xi(1 – 0,5\xi) = 0,19(1 – 0,5\cdot 0,19) = 0,18$

$$h_0 = \sqrt{\frac{M}{R_b \cdot b \cdot \alpha_m}} = \sqrt{\frac{2150000}{13,2 \cdot 1000 \cdot 0,18}} = 30,2$$

$h = h_0 + a = 30,2 + 15 = 45,2$ mm.

In order to protect against corrosion, the slab is taken 50 mm thick with h_0 = 50 – 15 = 35 mm.

Determination of reinforcement area on 1 meter of slab (7)

$\alpha_m = \frac{M}{R_b \cdot b \cdot h^2_0} = \frac{2,15}{13,2 \cdot 1000 \cdot 0,035^2} = 0,132 < \alpha_r = 0,376$ (α_r is taken according to reinforcement class)

Compressed reinforcement is not required by calculation.

$$A_s = \frac{R_b \cdot b \cdot h_0 \cdot (1 - \sqrt{1 - 2\alpha_m})}{R_s} = \frac{13,2 \cdot 1000 \cdot 35 \cdot (1 - \sqrt{1 - 2 \cdot 0.132})}{415} = 155,6\, mm^2$$

As a reinforcement in slab flange will be used reinforcement mesh $\frac{5B500-125}{5B500-125}$ (in numerator - specification of longitudinal reinforcement, in denominator - specification of transverse reinforcement; 5 - diameter of rods, 125 - distance between rods) with longitudinal and transverse working reinforcement with A_s =157,1 mm^2.

3.2 Calculation of the intermediate transverse rib

The transverse ribs of the panel are monolithically connected with the longitudinal ribs, however, considering the possibility of turning them under the action of an external load, the beam is taken with free support for the design scheme of the transverse rib in the safety margin. The designed span of the transverse rib is calculated as the distance between the axes of the longitudinal ribs: l_0 = 2980 − 2 · 105 · 0,5 = 2875 mm. The height of the transverse ribs is taken 200 mm, the width at the bottom – 50 mm, at the top – 100 mm.

Static calculation.

The maximum load on the transverse edge is transferred from the trapezoidal load areas.

$q^1 = q(l_2+b_r) = 12,3(0,9+0,075) = 12,0$ kN/m

where $b_r = (100+50)/2 = 75$ мм – medium width of transverse rib;

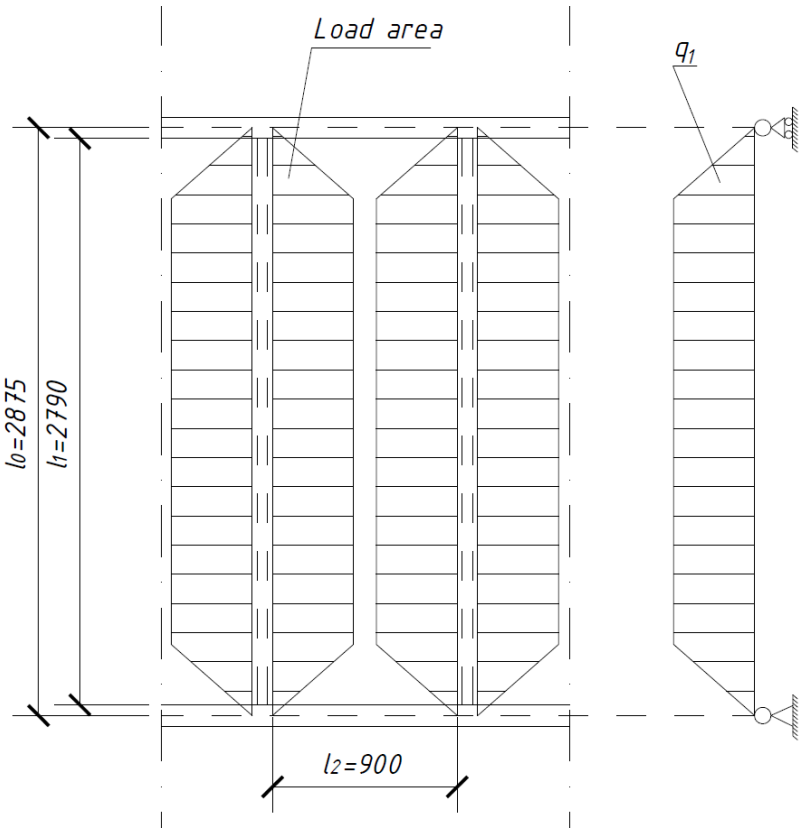

Figure 15. Design scheme of the intermediate rib.

The self weight of transverse rib:

$$q_c = b_r\left(h_r - h_f\right)\ \rho\cdot\gamma_f = 0,075\left(0,2-0,05\right)\cdot 25\cdot 1,1 \approx 0,31 \text{ kN/m.}$$

Design efforts:

$M = (q^1+q_c)\cdot(l_0^2/8) - (0.5\cdot q_1\cdot l_2^2)/6 = (12+0,31)\cdot(2,875^2/8) - (0.5\cdot 12,0\cdot 0,9^2)/6 =$ $12,1$ kN$\cdot$m;

$Q = (q^1+q_c)\cdot(l_0/2) - (q_1\cdot l_2)/4 = (12+0,31)\cdot(2,875/2) - (12,0\cdot 0,9)/4 = 15,1$ kN

Reinforcement calculation.

The ratio of the thickness of the plate to the height of the rib $\frac{h_f}{h} = \frac{5}{20} = 0{,}25 > 0{,}1$

therefore for the calculated cross-section of the rib T-section is selected with flange width in compressed zone equal to:

$$b`_f = 2 \cdot \frac{l_0}{6} + b`_r = 2 \cdot \frac{2875}{6} + 100 = 1065mm < \frac{1}{2} l_0 + b`_r = 550mm$$

From these conditions it is necessary to choose the smallest. The width of flange is taken equal to 550mm.

The necessary amount of reinforcement A400 at $h_0 = 200 - 25 = 175$ mm:

$\alpha_m = \gamma_n \frac{M}{R_b \cdot b \cdot h_0^2} = \frac{12{,}1 \cdot 10^6}{13{,}2 \cdot 550 \cdot 175^2} = 0{,}0544$. For the A400 reinforcement $\alpha_r = 0{,}39$ (7).

If $\alpha_m > \alpha_r$, then in necessary to calculate reinforcement in compressed zone.

Since $\alpha_m < \alpha_r$, a compressed reinforcement is not required by calculation (7).

$$A_s = 13{,}2 \cdot 550 \cdot \frac{175\left(1 - \sqrt{1 - 2 \cdot 0{,}0544}\right)}{355} = 200{,}29 \text{ mm}^2$$

In the transverse ribs flat welded frames with longitudinal reinforcement of rods with a diameter of 16 mm with A_s = 201,1 mm^2 are chosen.

Calculation of the strength of inclined sections.

The strength of the inclined strip between the inclined sections is needed to be checked.

Q = 15,1 kN $< 0{,}3 R_b \cdot b \cdot h_0 = 0{,}3 \cdot 13{,}2 \cdot 0{,}05 \cdot 0{,}175 \cdot 1000$ = 34,65 kN, it means that the strength of the inclined strip between the inclined sections is enough.

With a rib height of 20 cm and a longitudinal reinforcement of ∅16 mm, transverse rods in frames of A240 reinforcement class with rod ∅ 6 mm with A_s=28 mm^2 are taken. In accordance with the recommendations distance between transverse rods should be no more than s = $0{,}5 h_0 = 0{,}5 \cdot 175 =$ 87,5 mm and no more than 300 mm. Therefore s_w=75мм.

Strength of inclined sections under the action of transverse force. If $q_{sw} = \frac{R_{sw} \cdot A_{sw}}{s_w} = \frac{170 \cdot 28 \cdot 1}{75} = 63{,}47$ N/mm > $0{,}25 R_{bt} b = 0{,}25 \cdot 0{,}95 \cdot 50 = 11{,}88$ N/mm, then stirrups must be taken fully into account in calculation. Value of M_b is determined with formula:

$M_b = 1{,}5 R_{bt} b h_0^2 = 1{,}5 \cdot 0{,}95 \cdot 50 \cdot 175^2 = 2{,}2 \cdot 10^6$ kN*m.

Determination of the projection length of the unfavorable incline section.

$q_1 = 12{,}31$ kN/m.

Since

$$\sqrt{\frac{M_b}{q_1}} = \sqrt{\frac{2{,}2 \cdot 10^6}{12{,}31}} = 432{,}25 \text{ mm} < \frac{2h_0}{1 - 0{,}5 \frac{q_{sw}}{R_{bt} b}} = \frac{2 \cdot 175}{1 - 0{,}5 \frac{63{,}47}{0{,}95 \cdot 50}} = \frac{350}{0{,}347} = 1054{,}6 \text{ mm},$$

length of projection should be taken equal to

$$c = \sqrt{\frac{M_b}{0{,}75 \cdot q_{sw} + q_1}} = \sqrt{\frac{2{,}2 \cdot 10^6}{0{,}75 \cdot 63{,}47 + 12{,}31}} = 195{,}9 mm < 2h_0 = 350 mm$$

Then $c_0 = c$ = 196 мм.

$Q_{sw} = 0{,}75 q_{sw} \cdot c_0 = 0{,}75 \cdot 63{,}47 \cdot 196 = 9330{,}1$ N;

$$Q_b = \frac{M_b}{c} = \frac{2{,}2 \cdot 10^6}{196} = 11734{,}7 N$$

$Q_b + Q_{sw} = 9330{,}1 + 11734{,}7 = 21064{,}8$ N

$Q = Q_{max} - q_1 \cdot c = 15{,}1 - 12{,}31 \cdot 0{,}196 = 12{,}69$ kN,

$Q_b + Q_{sw} = 21{,}06$ kN > Q = 12,69 kN, hence the strength of inclined sections is ensured.

The final check of distance between stirrups.

$$S_{w,\max} = \frac{R_{bt} \cdot b \cdot h^2}{Q_{\max}} = \frac{0{,}95 \cdot 50 \cdot 175^2}{15100} = 96{,}33 mm > S_w = 75 mm$$

Hence requirement is fulfilled.

3.3 Longitudinal rib calculation

For the unification reasons height of section is increased to 300 mm (1).

Static calculation.

Table 3. Load determination for slab longitudinal rib.

№	Type of load	Normative load, kN/m^2	Load safety factor	Designed load, kN/m^2
Dead load				
1	Cement floor weight δ=20 mm, ρ = 2000 kg/m^3	0,4	1,3	0,52
2	Weight of concrete slab (self weight) δ=50mm, ρ =2500 kg/m^3	1,25	1,1	1,38
3	Transverse ribs* (5 pieces) b= $\frac{100+50}{2}$ = 75,0 mm h = 200-50 = 150 mm ; ρ = 2500 kg/m^3	0,23	1,1	0,253
4	Longitudinal ribs** (2 pieces) b= 90 mm; h = 300-50 = 250mm ρ = 2500 kg/m^3	0,33	1,1	0,36
Total		2,21		2,513
Temporary load				
5	Evenly distiributed load	7.3	1,2	8,76
Total		**9,51**		**11,27**

Annotation: * q =5 $\frac{\rho b_p(h_p-0.05)l_1}{b_n l_n}$ =5 $\frac{25\cdot 0{,}075\cdot 0{,}15\cdot 2{,}79}{3{,}0\cdot 5{,}76}$ = 0,23 kN/m^2

** q =2 $\frac{\rho * 0.08 * h_p * l_n}{b_n l_n}$ =2 $\frac{25\cdot 0{,}08\cdot 0{,}25\cdot 5{,}76}{3{,}0\cdot 5{,}76}$ = 0,33 kN/m^2

Total load on longitudinal rib:

Total designed load:

$$q\cdot b_n = 13{,}01 \cdot 3{,}0 = 33{,}81 \text{ kN/m},$$

Total normative load (for SLS):

$$(g+v)b_n = 9{,}51 \cdot 3{,}0 = 28{,}53 \text{ kN/m},$$

For the design scheme for longitudinal ribs a single-span beam with one free support of the ends is chosen (Fig.16), the calculated span is defined as the distance between the centers of the support surfaces of the panel edges.

$$l_0 = 5760 - 2 \cdot 0{,}5 \cdot 100 = 5660 \text{ мм} = 5{,}66 \text{ m.}$$

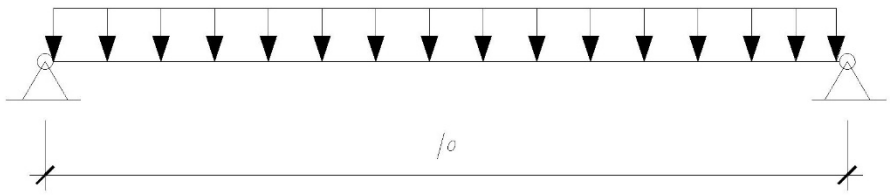

Figure 16. Design scheme of longitudinal rib

Efforts in two longitudinal ribs:

of designed load

$M = 0{,}125 q l_0^2 \cdot \gamma_n = 0{,}125 \cdot 33{,}81 \cdot 5{,}66^2 = 135{,}4$ kN·m=135,4 · 10^6 N·mm;

$Q = 0{,}5 q l_0 \cdot \gamma_n = 0{,}5 \cdot 33{,}81 \cdot 5{,}66 = 95{,}68$ kN;

of normative load

$M^n = 0{,}125 q^n l_0^2 \cdot \gamma_n = 0{,}125 \cdot 28{,}53 \cdot 5{,}66^2 = 114{,}3$ kN·m;

$Q^n = 0{,}5 q^n l_0 \cdot \gamma_n = 0{,}5 \cdot 28{,}53 \cdot 5{,}66 = 80{,}73$ kN;

Calculated cross-section – T-section with flange in compressed zone.

Flange width at h'_f : h = 50 : 250 = 0,125 > 0,1

$b'_f \leq 2 \cdot l_0 / 6 = 2 \cdot 5660 / 6 \leq 1887$ mm.

$b'_f \leq 1490$ mm.

The effective height of section: $h_0 = h - a = 300 - 30 = 270$ mm. With the width of the longitudinal ribs at the top of 105 mm and at the bottom of 75 mm, the total thickness of the two ribs in the level of the center of gravity of the reinforcement, excluding the monolithical seams, will be 180 mm.

Calculation of the strength of normal cross-sections.

The work of concrete in the monolithical seams into the safety margin is not taken into account, assuming that, under unfavorable conditions, reliable joint work of the concrete with the longitudinal ribs due to their adhesion may not be ensured. Then the calculated width of the flange:

$$b'_f = 1490 - 30 = 1460 \text{ mm.}$$

The calculation is made on the assumption that the compressed reinforcement is not required by calculation, A'_s = 0:

$R_b b'_f h'_f \left(h_0 - 0{,}5h'_f\right) = 13{,}2 \cdot 1460 \cdot 50\left(270 - 0{,}5 \cdot 50\right) = 236{,}1 \cdot 10^6$ N·mm = 236,1 kN·m > *M* = 135,4 kN·m

This means that the neutral axis passes within the flange ($x<h'_f$) and the element is calculated as a rectangular with a width b'_f = 1460 мм.

The required amount of prestressed reinforcement of A600 class at

$\alpha_m = \dfrac{135{,}4 \cdot 10^6}{13{,}2 \cdot 1460 \cdot 270^2} = 0{,}1$ <α_R = 0,34,

$\alpha_R = \xi_R \cdot (1 - 0{,}.5 \cdot \xi_R)$, where ξ_R is accepted at the ratio $\dfrac{\sigma_{sp}}{R_s} = 0{,}6$.

$\xi_R = 0{,}43$ (for this calculation), $\alpha_R = 0{,}43(1 - 0{,}5 \cdot 0{,}43) = 0{,}34$

Hence compressed reinforcement is not required by calculation.

$$A_{sp} = R_b b'_f h'_0 (1 - \sqrt{1 - 2\alpha_m}) / \gamma_{s3} R_s \text{ mm}^2,$$

$$\gamma_{s3} = 1{,}25 - 0{,}25\frac{\xi}{\xi_R} = 1 - 0{,}25 \cdot \frac{1 - \sqrt{1 - 2\alpha_m}}{0{,}43} = 1{,}25 - 0{,}25 \cdot \frac{1 - \sqrt{1 - 2 \cdot 0{,}1}}{0{,}43} = 1{,}19$$

Condition: $\gamma_{s3} \le 1{,}1$

$$A_{sp} = 13{,}2 \cdot 1460 \cdot 270 \cdot (1 - \sqrt{1 - 2 \cdot 0{,}1}) / (1{,}1 \cdot 680) = 806{,}55$$

Bar reinforcement of 4 roads is accepted 4∅18 A600 c A_s^f = 1018 mm^2 > 806,55 mm^2.

A240 reinforcement with a diameter of 10 mm is chosen for mounting reinforcement of longitudinal ribs in form of flat welded frames with

A'_s = 78,5 · 2=157 mm^2 = 0,00016 m^2.

Calculation of the strength of inclined sections.

For structural reasons, we adopt a transverse reinforcement of class A240 with R_{sw} = 170 МПа. In two flat welded frames, the transverse rods according to the conditions of the welding technology are taken equal to 6 mm in diameter.

The maximum permissible distance between rods of the transverse reinforcement at the supports at h_0 = 300–30 mm = 270 mm: $s \leq 0{,}5h_0 = 0{,}5 \cdot 270 = 135$ mm; $s \leq 300$ mm.

The distance between the rods is taken equal to s = 100 mm on the length section near the support (1/4 span distance) и 200 mm ($0{,}75\ h_0 = 0{,}75 \cdot 0{,}27 = 202{,}5$ mm) in the middle.

Calculation of the strength of the strip between inclined sections.

The width of the 2 ribs in the middle of cross-section $b = 0{,}\cdot 2 = 0{,}18$ m.

$Q \leq 0{,}3R_b bh_0$, where Q is taken at the distance from the support not less than h_0; $0{,}3\ R_b bh_0 = 0{,}3 \cdot 13{,}2 \cdot 10^3 \cdot 0{,}18 \cdot 0{,}27 = 192{,}5$ kN >

> $Q = Q - qh_0 = 95{,}68 - 33{,}81 \cdot 0{,}27 = 86{,}55$ kN, hence strength of strip between inclined sections is guaranteed.

Calculation of the inclined section under a shear force effect (7).

The calculation of the bent elements in an in is made from the condition:

$$Q \leq Q_b + Q_{sw}$$

where Q is the shear force from the vertical external load applied to the upper edge of the element.

The transverse force carried by concrete is determined by the formula

$$Q_b = \frac{M_b}{c}$$

but, no more than $Q_{b,\max} = 2{,}5R_{bt}bh_0$ and not less than $Q_{b,\min} = 0{,}5\varphi_n R_{bt}bh_0$

$M_b = 1{,}5\varphi_n R_{bt}bh_0^2$, c - projection of the unfavorable inclined section.

$$Q_{sw} = 0{,}75q_{sw}c_0$$

To take into account the positive effect of prestress on the load-bearing capacity of concrete in transverse force (coefficicient φ_n) it is necessary to determine the value of the pre-compression force P taking into account the loss of prestressing in the reinforcement.

<u>Determination of geometric characteristics of the cross-section.</u>

A_s and A_s' in the calculations of the characteristics of the reduced section, we do not take into account conditionally.

Coefficient of reduction $\alpha = \frac{E_s}{E_b} = \frac{2{,}1 \cdot 10^5}{2{,}75 \cdot 10^4} = 7{,}63$

Concrete area:

$A = b'_f \cdot h'_f + 2 \cdot A_r = 1460 \cdot 50 + 2 \cdot 300 \cdot 90 = 127000$ mm^2

Reduced section area:

$A_{red} = A + \alpha A_{sp} = 127000 + 7{,}63 \cdot 1018 = 134767{,}3$ mm^2.

The static moment relative to the bottom edge of the longitudinal ribs:

$$S_{red} = \sum A_i \cdot y_{bi} + \alpha S_{sp} = 1460 \cdot 50 \cdot (300 - 0{,}5 \cdot 50) + 180 \cdot 250 \cdot 250 / 2 +$$
$$+ 7{,}63 \cdot 1018 \cdot 30 = 25933020\ mm^3$$

The distance from the most stretched concrete fiber to the center of gravity of the reduced section:

$y = S_{red} / A_{red} = 25933020 / 134767{,}3 = 192{,}4$ mm.

The moment of inertia of reduced section relative to the center of gravity:

$$J_{red} = \frac{b'_f \cdot h'^3_f}{12} + b'_f \cdot h'_f \left(h - y - 0{,}5 \cdot h'_f\right)^2 + \frac{b\left(h - h'_f\right)^3}{12} + b\left(h - h'_f\right) \cdot \left[y - 0{,}5\left(h - h'_f\right)\right]^2 +$$
$$+ \alpha \cdot A_{sp}\left(y - a\right)^2 = \frac{1460 \cdot 50^3}{12} + 1460 \cdot 50\left(300 - 192{,}4 - 0{,}5 \cdot 50\right)^2 + \frac{180 \cdot \left(300 - 50\right)^3}{12} +$$
$$+ 180 \cdot \left(300 - 50\right) \cdot \left[192{,}4 - 0{,}5\left(300 - 50\right)\right]^2 + 7{,}63 \cdot 1018 \cdot \left(192{,}4 - 30\right)^2 = 1156922974\ mm^4 =$$
$$= 1{,}16 \cdot 10^9\ mm^4$$

The moment of resistance of the reduced section:

$$W = J_{red}/y = 1{,}16 \cdot 10^9 / 192{,}4 = 6{,}02 \cdot 10^6 \text{ mm}^3.$$

We take into account the inelastic deformations of the stretched concrete by multiplying W by the coefficient γ = 1.3,

$$W_{pl} = 6{,}02 \cdot 10^6 \cdot 1{,}3 = 7{,}84 \cdot 10^6 \text{ mm}^3.$$

Determination of prestressing value taking into account losses σ_{sp}.

The initial value of prestressing of the reinforcement is:

$$\sigma_{sp} \le R_{sn} - p = 785 - 93,6 = 586,4\,MPa \;\; \Delta\sigma_{sp2} = 0;$$

The initial value of prestressing is taken equal to 450 MPa

First loses:

a) From stress relaxation in the reinforcement under the electrothermal method of tension

$$\Delta\sigma_{sp1} = 0,03\sigma_{sp} = 0,03 \cdot 450,0 = 13,5 \text{ MPa};$$

b) From temperature drop

$$\Delta\sigma_{sp2} = 0;$$

c) From form deformation

$$\Delta\sigma_{sp3} = 0;$$

d) From anchors deformation

$$\Delta\sigma_{sp4} = 0;$$

The sum of the first losses

$$\Delta\sigma_{sp(1)} = 13,5 \text{ MPa}.$$

Compression force taking into account the first losses:

$$P_{(1)} = A_{sp}\left(\sigma_{sp} - \Delta\sigma_{sp(1)}\right) = 1018\left(450,0 - 13,5\right) = 444,35 \text{ kN}.$$

Since in the upper zone there is no tensional reinforcement ($A'_{sp} = 0$), then

$$e_{0p1} = y_{sp} = y - a_{sp} = 192,4 - 30 = 162,4 \text{ mm}.$$

The maximum compressive stress in the concrete ae the lower edge of the rib from the action $P_{(1)}$ at the moment of its own weight $M = 0$, $y_s = y$, is equal to

$$\sigma_{bp} = \frac{P_{(1)}}{A_{red}} + \frac{P_{(1)} e_{0p1} \cdot y}{J_{red}} = \frac{0,444 \cdot 10^6}{0,134 \cdot 10^6} + \frac{0,444 \cdot 10^6 \cdot 162,4 \cdot 192,4}{1,16 \cdot 10^9} = 3,13 + 11,96 =$$
$$= 15,27\,MPa$$

15,27 MPa $< 0,9R_{bp} = 0,9 \cdot 18,5 = 16,65$ MPa.

The strength of concrete is sufficient to perceive the compression force.

Second losses:

a) From shrinkage of concrete B25

$$\Delta\sigma_{sp5} = \varepsilon_{b,sh} \cdot E_s = 0{,}0002 \cdot 2{,}1 \cdot 10^5 = 42 \text{ MPa};$$

where $\varepsilon_{b,sh}$ - is the shrinkage deformation of concrete, taken equal to: 0,0002 - for concrete of classes B35 and below;

b) From concrete creep

$$\Delta\sigma_{sp6} = \frac{0{,}8\varphi_{b,cr} \cdot \alpha \cdot \sigma_{bp}}{1 + \alpha\mu_{sp}\left(1 \pm \frac{e_{0p1} y_{sp} A_{red}}{J_{red}}\right)\left(1 + 0{,}8\varphi_{b,cr}\right)}.$$

For concrete of class B25

$\varphi_{b,cr}$ = 2,5; E_b = 27500 MPa; $\alpha = \frac{E_s}{E_b} = \frac{2.1 \cdot 10^5}{27{,}5 \cdot 10^3} = 7{,}63.$

Calculation of the bending moment in the middle of the span of the slab from its own weight:

$$M_b = \frac{g_2 \gamma_n l_0^2}{8} = \frac{5{,}46 \cdot 5{,}66^2}{8} = 23{,}06 \text{ kN·m}.$$

The stress in the concrete in the level of the center of gravity of the prestressed reinforcement from the compression force taking into account the bending moment from the plate's own weight:

$$\sigma_{bp} = \frac{P_{(1)}}{A_{red}} + \frac{P_{(1)} e_{0p1} \cdot y_{sp}}{J_{red}} - \frac{M \cdot y_{sp}}{J_{red}} = \frac{0{,}444 \cdot 10^6}{0{,}134 \cdot 10^6} + \frac{0{,}444 \cdot 10^6 \cdot 162{,}4 \cdot 192{,}4}{1{,}16 \cdot 10^9} -$$

$$- \frac{23{,}06 \cdot 10^6 \cdot 192{,}4}{1{,}16 \cdot 10^9} = 3{,}13 + 11{,}96 - 3{,}82 = 11{,}27\, MPa$$

$$11{,}27 \text{ MPa} < 0{,}9 R_{bp} = 0{,}9 \cdot 18{,}5 = 16{,}65 \text{ MPa}.$$

Stress in concrete in the level of the center of gravity of the upper non-tensioning reinforcement

$$y_s^{'} = h - a_s' - y = 300 - (30 + 192{,}4) = 77{,}6 \text{ mm};$$

$$\sigma'_{bp} = \frac{P_{(1)}}{A_{red}} - \frac{P_1 e_{0p1} - M_b}{J_{red}} y'_s = \frac{0,444 \cdot 10^6}{0,134 \cdot 10^6} - \frac{0,444 \cdot 10^6 \cdot 162,4 - 23,06 \cdot 10^6}{1,16 \cdot 10^9} \cdot 77,6 =$$
$$3,31 - 3,28 = 0,03 MPa$$

The distance between the centers of gravity of the prestressed reinforcement and the reduced cross section

$$y_{sp} = e_{0p1} = 162,4 \text{ mm.}$$

Reinforcement ratio

$$\mu_{sp} = \frac{A_{sp}}{A} = \frac{1018}{127000} = 0,008, \ \mu'_s = \frac{A'_s}{A} = \frac{157}{127000} = 0,001.$$

$$\Delta\sigma_{sp6} = \frac{0,8 \cdot 2,5 \cdot 7,63 \cdot 15,27}{1 + 7,63 \cdot 0,008\left(1 + \frac{162,4 \cdot 192,4 \cdot 0,134 \cdot 10^6}{1,16 \cdot 10^9}\right)(1 + 0,8 \cdot 2.5)} = 126,3 MPa$$

$$\Delta\sigma'_{sp6} = \frac{0,8 \cdot 2,5 \cdot 7,63 \cdot 0,03}{1 + 7,63 \cdot 0,001\left(1 + \frac{239,04 \cdot 274,04 \cdot 0,157 \cdot 10^6}{3,17 \cdot 10^9}\right)(1 + 0,8 \cdot 2.8)} = 0,4 MPa$$

$$\Delta\sigma'_{sp6} \cong 0 MPa$$

Total second losses:

$$\Delta\sigma_{sp(2)} = \Delta\sigma_{sp5} + \Delta\sigma_{sp6} = 42 + 126,3 = 168,3 \text{ MPa.}$$

Total loss of prestression:

$$\Delta\sigma_{sp(1)} + \Delta\sigma_{sp(2)} = 13,5 + 168,3 = 181,8 \text{ MPa} > 100 \text{ MPa.}$$

Stress σ_{sp2} with all losses:

$$\sigma_{sp2} = 450 - 181,8 = 268,2 \text{ MPa.}$$

The compression force taking into account all stress losses P is determined when the stresses in the non-tensioning reinforcement are equal to σ_s. σ_s conditionally assumed to be equal to the second losses, i.e.

$$\sigma_s = \Delta\sigma_{sp(2)} = 168,3 \text{ MPa}$$

$$P = \sigma_{sp2} A_{sp} - \sigma_s A_s = 268,2 \cdot 1018 - 168,3 \cdot 157 = 246,6 \text{ kN.}$$

$Q_{b,\min} = 0,5 \varphi_n R_{bt} b h_0 = 0,5 \cdot 1,41 \cdot 0,95 \cdot 10^3 \cdot 0,18 \cdot 0,27 = 32,5$ kN

$Q_{b,\max} = 2,5 R_{bt} b h_0 = 2,5 \cdot 0,95 \cdot 10^3 \cdot 0,18 \cdot 0,27 = 115,42$ kN.

Value of the φ_n:

$$\varphi_n = 1 + 1{,}6\frac{P}{R_b A_1} - 1{,}16\left(\frac{P}{R_b A_1}\right)^2 = 1 + 1{,}6 \cdot \frac{246{,}6 \cdot 10^3}{13{,}5 \cdot 300 \cdot 180} - 1{,}16 \cdot \left(\frac{246{,}6 \cdot 10^3}{13{,}5 \cdot 300 \cdot 180}\right)^2 = 1{,}41;$$

Since the value $q_{sw1} = \frac{R_{sw} A_{sw}}{S_w} = \frac{170 \cdot 10^3 \cdot 000057}{0{,}1} = 96{,}9$ >

$$> 0{,}25\varphi_n R_{bt} b = 0{,}25 \cdot 1{,}41 \cdot 0{,}95 \cdot 10^3 \cdot 0{,}18 = 60{,}2 \text{ N/mm},$$

it is necessary to calculate stirrups fully.

Determination of the length of the projection of the unfavorable incline section *c*.

$c = \sqrt{\frac{M_b}{q}} = \sqrt{\frac{1{,}5\varphi_n R_{bt} b h_0^2}{q}} = \sqrt{\frac{1{,}5 \cdot 1{,}41 \cdot 0{,}95 \cdot 10^3 \cdot 0{,}18 \cdot 0{,}27^2}{39{,}03}} = 0{,}82$ m, but no more than $3h_0 = 3 \cdot 0{,}27 = 0{,}81$ m.

The projection length of the unfavorable incline section is taken equal to c = 0,81 m.

The projection length of the inclined crack c_0 is taken equal to *c*, but no more than $2h_0 = 2 \cdot 0{,}27 = 0{,}54$ m.

The length of the inclined crack is c_0 = 0,54 m.

Then

$$Q_{sw} = 0{,}75 q_{sw} c_0 = 0{,}75 \cdot 96{,}9 \cdot 0{,}54 = 39{,}25 \text{ kN}.$$

$$Q = Q - qc = 95{,}68 - 33{,}81 \cdot 0{,}81 = 68{,}3 \text{ kN}.$$

With the sum of the shear capacities $Q_{sw} + Q_{b,min}$ = 39,25 + 32,5 = 71,75 kN > Q = 68,3 kN, strength is ensured.

3.4 Longitudinal ribs calculation according to SLS

Determination of the crack width, normal to the longitudinal axis of the element.

The reduced moment of resistance along the stretched edge of the longitudinal ribs:

$$W_{red} = \frac{J_{red}}{y} = \frac{1{,}16 \cdot 10^9}{192{,}4} = 6{,}03 \cdot 10^6 \text{ mm}^3.$$

Distance to the core point:

$$r = \frac{W_{red}}{A_{red}} = \frac{6{,}03 \cdot 10^6}{134767} = 44{,}74 \text{ mm}.$$

The moment of crack formation:

$$M_{crc} = \gamma W_{red} R_{bt,ser} + P(e_{0p} + r) = 1{,}3 \cdot 6{,}03 \cdot 10^6 \cdot 1{,}85 +$$
$$+ 0{,}246 \cdot 10^6 (162{,}4 + 44{,}74) = 190{,}3 \cdot 10^6 \text{ N·mm.}$$

The values of P, J_{red}, y, A_{red}, e_{0p} are presented above

Coefficient $\gamma = 1{,}3$,

Bending moment from normative load:

$$M^n = 114{,}3 \text{ kN·m;}$$

Consequently, cracks are formed.

$$a_{crc} = \varphi_1 \varphi_2 \varphi_3 \psi_s \frac{\sigma_s}{E_s} l_s \le a_{crc,ult}$$

φ_1 = *1,4* with a continuous action of the load

φ_2 = *0,5* for a reinforcement of the periodic profile

φ_3 = *1* for bent elements

$$\sigma_s = \frac{M - P \cdot z}{A_{sp} \cdot z}$$

z = 0,7h₀

lₛ - base length between cracks

$$l_s = 0{,}5 \cdot \frac{A_{bt}}{A_{sp}} \cdot d_s \;;\; A_{bt} = h_f \cdot b_f + (y_t - h_f) \cdot b \;;\; y_t = k \cdot y_0$$

$$y_0 = \frac{S_{red}}{A_{red} + \frac{P}{R_{bt,ser}}} \;;\; \text{k =0,95;}$$

$$y_0 = \frac{25933}{1347 + \frac{444}{0{,}155}} = 6{,}15\, cm$$

$y_t = 0.95 \cdot 6{,}15 = 5{,}84\, cm < 2a = 6\, cm$, y_t = 6 cm

$$A_{bt} = 5 \cdot 149 + (6-5) \cdot 18 = 763\, cm^2$$

$$l_s = 0{,}5 \cdot \frac{763}{10{,}18} \cdot 1{,}8 = 67{,}5\, cm$$

ls need to be no more than 40 cm.

$$\sigma_s = \frac{11430 - 440 \cdot 18{,}9}{10{,}18 \cdot 18{,}9} = 16{,}18 kN/cm^2 = 161{,}8 MPa$$

$$a_{crc} = 1{,}4 \cdot 0{,}5 \cdot 1{,}0 \cdot 1{,}0 \frac{161{,}8}{21 \cdot 10^4} \cdot 400 = 0{,}22\, mm < a_{crc,ult} = 0{,}4\, mm$$

Hence, the crack width is smaller than the ultimate width.

Deformation calculation.

Determination of curvature in the middle of the span from the action of permanent and long-term loads, since the deflection is limited to aesthetic requirements.

$$E_{b,red} = \frac{R_{b,ser}}{\varepsilon_{b,red}} = \frac{18,5 \cdot 10^3}{28 \cdot 10^{-4}} = 0,66 \cdot 10^7 \ kN/m^2$$

For reinforcement in tensioned zone:

$$\alpha_{s2} = \frac{E_s}{\psi_s \cdot E_{b,red}} = \frac{21 \cdot 10^7}{1,0 \cdot 0,66 \cdot 10^7} = 31,8$$

$$\mu\alpha_{s2} = \frac{A_{sp}}{bh_0}\alpha_{s2} = \frac{1018}{180 \cdot 270} 31,8 = 0,66$$

$$\varphi_f = \frac{(b`_f - b)h`_f}{bh_0} = \frac{(1490 - 180) \cdot 50}{180 \cdot 270} = 1,34$$

$$e_{sp} = y_в - e_0 - a = 107,6 + 162,4 - 30 = 240 \, mm;$$

where $y_в$ - distance from the center of gravity to the upper edge.

$$\frac{e_{sp}}{h_0} = \frac{240}{270} = 0,88$$

$$\phi_c = 0,88$$

Curvature of element:

$$\frac{1}{r} = \left(\frac{1}{r}\right)_3 = \frac{M}{\phi_c \cdot b \cdot h_0^3 \cdot E_{b,red}} = \frac{114,3}{0,88 \cdot 0,18 \cdot 0,27^3 \cdot 0,66 \cdot 10^7} = 0,0055 \ \text{m}^{-1}$$

Curvature from prestressing:

$$\sigma_{sb} = \Delta\sigma_{sp5} + \Delta\sigma_{sp6}$$

$$\sigma`_{sb} = \Delta\sigma_{sp5} + \Delta\sigma`_{sp6}$$

$$\left(\frac{1}{r}\right)_4 = \frac{\sigma_{sb} - \sigma'_{sb}}{h_0 \cdot E_s} = \frac{168,3 - 0,4}{0,270 \cdot 21 \cdot 10^7} = 0,000003 \ \text{m}^{-1}$$

$$\left(\frac{1}{r}\right)_{\max} = \left(\frac{1}{r}\right)_3 - \left(\frac{1}{r}\right)_4 = 0,0055 - 0,000003 = 0,0055 \, m^{-1}$$

$$f = S \cdot l^2 \left(\frac{1}{r}\right)_{\max} = \frac{5}{48} \cdot 5,660^2 \cdot 0,0055 = 0,018 \ m.$$

f_{ult} = l/200 = 28 *mm* > 18,0 *mm*, calculation for the SLS is completed.

4 CALCULATION OF FIBER REINFORCED CONCRETE SLAB

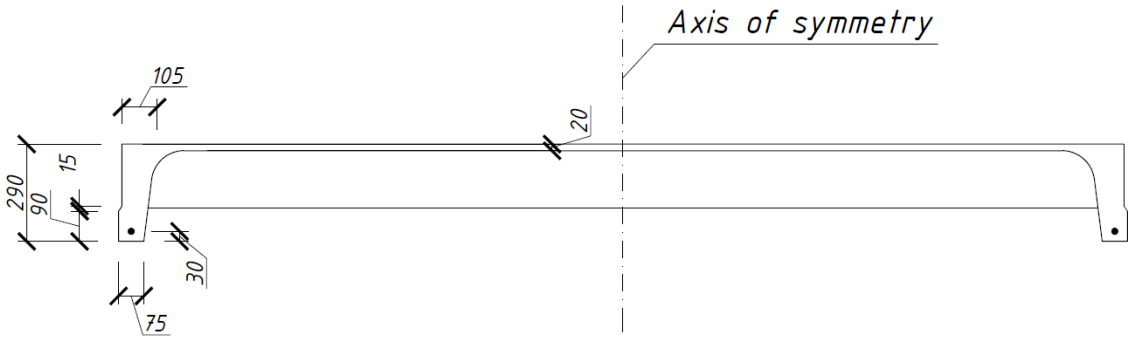

Figure 17. Cross-section of slab.

Given: Slab dimensions 5760·2980, evenly distributed load q = 7,3 kN/m^2, Concrete B25 (R_b = 13,2 MPa, R_{bt} = 0,95 MPa), R_s = 680 MPa - for prestressed reinforcement. Tension type - electrothermal tension. The diameter of steel fiber = 0,8 mm, length of steel fiber = 80 mm, R_f = 500 MPa, $R_{f,n}$ = 600 MPa, μ_{fv} = 0,008, A_s = 254,5 mm^2 (for 1 longitudinal rib).
$E_s = 2.1 \cdot 10^5$ MPa, $E_b = 2.75 \cdot 10^4$ MPa.

<u>Static calculation.</u>

Table 4. Load determination for fiber reinforced slab longitudinal rib.

№	Type of load	Normative load, kN/m^2	Load safety factor	Designed load, kN/m^2
Dead load				
1	Cement floor weight δ=20 mm, ρ = 2000 kg/m^3	0,4	1,3	0,52
2	Weight of concrete slab (self weight) δ=20mm, ρ =2500 kg/m^3	0,5	1,1	0,55
3	Transverse ribs* (5 pieces) b= $\frac{100+50}{2}$ = 75,0 mm h = 200-20 = 170 mm ; ρ = 2500 kg/m^3	0,25	1,1	0,28
4	Longitudinal ribs** (2 pieces) b= 90 mm; h = 300-20 = 270mm ρ = 2500 kg/m^3	0,36	1,1	0,39
Total		1,51		1,74
Temporary load				
5	Evenly distiributed load	7.3	1,2	8,76

Total	**8,81**		**10,5**

Total load on longitudinal rib:

Total designed load:

$$q \cdot b_n = 10{,}5 \cdot 1{,}49 = 19{,}4 \text{ kN/m},$$

Total normative load (for SLS):

$$(g+v)b_n = 8{,}81 \cdot 1{,}49 = 13{,}13 \text{ kN/m},$$

4.1 Calculation of the reduced cross section of the fiber reinforced concrete

In view of the symmetry of the cross section, the calculation is carried out for the half of the cross section of the slab.

Determination of the geometric characteristics of the reduced cross section:

Coefficients of reduction: $\alpha_{s,f} = \frac{E_s}{E_b} = \frac{2{,}1 \cdot 10^5}{2{,}75 \cdot 10^4} = 7{,}63$

Reduced cross section area: $A_{red} = A + A\mu_{fa}\alpha_f + A_s\alpha_s$

μ_{fa} - coefficient of the fiber reinforcement.

$\mu_{fa} = k^2{}_{or}\mu_{fv}$, where $k^2{}_{or}$ is coefficient taken from “Guidlines for designing fiber reinforced structures table 4”

Coefficient for the web: $\mu_{faw} = 0{,}676^2 \cdot 0{,}008 = 0{,}0036$

Coefficient for the flange: $\mu_{faf} = 0{,}634^2 \cdot 0{,}008 = 0{,}0032$

Since the values differ slightly, the average meaning is applied $\mu_{fa} = 0{,}0034$

$$A_{red} = (1490 \cdot 20 + 80 \cdot 270 + 15 \cdot 90 + 15^2 \cdot 0{,}5) \cdot$$
$$\cdot (1 + 7{,}63 \cdot 0{,}0034) + (255 \cdot 7{,}63) = 56560\, mm^2$$

$$S_{red} = S(1 + \alpha_f \mu_{fa}) + \alpha S_{sp} = (1490 \cdot 20 \cdot 280 + 60 \cdot 270^2 \cdot 0{,}5 +$$
$$+ 45 \cdot 270 \cdot 0{,}5 \cdot \frac{2}{3} \cdot 270 + 15 \cdot 90 \cdot 90 \cdot 0{,}5 + 15^2 \cdot 0{,}5 \cdot 95) \cdot$$
$$\cdot (1 + 7{,}63 \cdot 0{,}0034) + 7{,}63 \cdot 255{,}5 =$$
$$= 12031000\, mm^3$$

Distance from the center of gravity to the bottom edge:

$$y_{cw} = \frac{S_{red}}{A_{red}} = \frac{12031 \cdot 10^3}{56560} = 213\,mm$$

$$y_w = y_{cw} - a = 213 - 30 = 183\ mm$$

<u>Calculation of the moment of the inertia of the reduced cross section.</u>

$$J_{red} = J(1+\alpha_f \mu_{fa}) + \alpha_s J_s = (\frac{1490 \cdot 20^3}{12} + 1490 \cdot 20 \cdot (280-213)^2 +$$
$$+ \frac{60 \cdot 270^3}{12} + 60 \cdot 270 \cdot (213 - \frac{270}{2})^2 + \frac{45 \cdot 270^3}{36} + \frac{45 \cdot 270}{2} \cdot$$
$$\cdot (213 - \frac{2}{3} \cdot 270)^2 + \frac{15 \cdot 15^3}{36} \cdot \frac{15^2}{2} \cdot (213 - \frac{1}{3} \cdot 15 - 90^2) \cdot$$
$$\cdot (1 + 7{,}63 \cdot 0{,}0034) + 7{,}63 \cdot 255 \cdot 183^3 = 46666 \cdot 10^4\ mm^4$$

<u>Determination of strength properties of fiber reinforced concrete.</u>

Fibreconcrete destruction can occur in two cases: all fibers are pulled out of concrete, or part of the fiber is torn and the other part is pulled out of the concrete matrix.

$l_{f,an} \geq \frac{l_f}{2}$ - first case of destruction,

$l_{f,an} < \frac{l_f}{2}$ - second case of destruction.

$l_{f,an} = \frac{\eta \cdot d_f \cdot R_{f,n}}{R_b} = \frac{0{,}6 \cdot 0{,}8 \cdot 600}{13{,}2} = 21{,}8\,mm$, where η - is a coefficient that takes into account profile of fiber (in case of fiber of periodic profile $\eta = 0{,}6$). Consequently, the destruction occurs in the second case.

Strength of fiber reinforced concrete in the second case:

$R_{fbt} = m_1(k^2_{or} \mu_{fv} R_f (1 - \frac{l_{f,an}}{l_f}) + R_b \cdot (0{,}08 - 5{,}5\mu_{fv}))$, where m_1 - is a coefficient that takes into account the presence of the anchors (in this case there are no anchors m_1 = 1).

Design tensile strength of the fiber reinforced concrete:

in rib: $R_{fbt} = 0,676^2 \cdot 0,008 \cdot 500 \cdot (1 - \frac{21,8}{80}) + 0,48 = 2,1\,MPa$,

in flange: $R_{fbt} = 0,634^2 \cdot 0,008 \cdot 500 \cdot (1 - \frac{21,8}{80}) + 0,48 = 1,9\,MPa$

Design compressive strength of the fiber reinforced concrete:

$R_{fb} = R_b + (k^2{}_n \cdot \varphi_f \cdot \mu_{fv} \cdot R_f)$, where k_n - is a coefficient that takes into account the work of the fibers in a section that perpendicular to the direction of the external compressive force. The coefficient depends on the ratio of the length and width of the section. In this case for rib $k_n = 0,513$, for flange $k_n = 0,634$.

φ_f - is a coefficient of efficiency of indirect reinforcement. $\varphi_f = \frac{5+L}{1+4,5L}$,

where $L = \frac{k_n \cdot \mu_{fv} \cdot R_f}{R_b}$.

for rib: $L = \frac{0,513^2 \cdot 0,008 \cdot 500}{13,2} = 0,079$; $\varphi_f = \frac{5+0,079}{1+4,5 \cdot 0,079} = 3,748$;

for flange: $L = \frac{0,634^2 \cdot 0,008 \cdot 500}{13,2} = 0,113$; $\varphi_f = \frac{5+0,113}{1+4,5 \cdot 113} = 3,39$.

$R_{fbw} = 13,2 + (0,513^2 \cdot 3,748 \cdot 0,008 \cdot 500) = 17,1\,MPa$;

$R_{fbf} = 13,2 + (0,634^2 \cdot 3,39 \cdot 0,008 \cdot 500) = 18,7\,MPa$.

Determintation of prestressing value taking into account prestression loses σ_{sp}.

The initial value of prestressing of the reinforcement is:

$$0,3R_{s,ser} + p < \sigma_{sp} \le R_{sn} - p = 225,5 + 93,6 < 450 < 785 - 93,6;$$

Initial value of prestressing is taken equal to 450 *MPa*

First loses:

From stress relaxation in the reinforcement under the electrothermal method of tension

$$\Delta\sigma_{sp1} = 0,03\sigma_{sp} = 0,03 \cdot 450,0 = 13,5 \text{ MPa};$$

Compression force taking into account the first losses:

$P_{(1)} = A_{sp}\left(\sigma_{sp} - \Delta\sigma_{sp(1)}\right) = 255\left(450,0 - 13,5\right) = 111,31\,kN$

$e_{0p1} = y_{sp} = y_{cw} - a = 213 - 30 = 183$ mm.

The maximum compressive stress in the concrete ae the lower edge of the rib from the action $P_{(1)}$ at the moment of its own weight $M = 0$, $y_s = y$, is equal to

$$\sigma_{bp} = \frac{P_{(1)}}{A_{red}} + \frac{P_{(1)} e_{0p1} \cdot y}{J_{red}} = \frac{0,111 \cdot 10^6}{0,056 \cdot 10^6} + \frac{0,111 \cdot 10^6 \cdot 183 \cdot 213}{0,466 \cdot 10^9} = 11,65\, MPa$$

11,65 MPa $< 0,9 R_{bp} = 0,9 \cdot 18,5 = 16,65$ MPa.

The strength of concrete is sufficient to perceive the compression force.

Calculation of the bending moment in the middle of the span of the slab from its own weight:

$$M_b = \frac{g_2 \gamma_n l_0^2}{8} = \frac{1,74 \cdot 5,66^2}{8} = 11,42 \text{ kN·m.}$$

The stress in the concrete in the level of the center of gravity of the prestressed reinforcement from the compression force taking into account the bending moment from the plate's own weight:

$$\sigma`_{bp} = \frac{P_{(1)}}{A_{red}} + \frac{P_{(1)} e_{0p1} \cdot y_{sp}}{J_{red}} - \frac{M \cdot y_{sp}}{J_{red}} = \frac{0,111 \cdot 10^6}{0,056 \cdot 10^6} + \frac{0,11 \cdot 10^6 \cdot 183 \cdot 213}{0,46 \cdot 10^9} -$$

$$- \frac{11,42 \cdot 10^6 \cdot 213}{0,46 \cdot 10^9} = 10,66\, MPa$$

10,66 MPa $< 0,9 R_{bp} = 0,9 \cdot 18,5 = 16,65$ *MPa.*

Stress in concrete, taking into account the first losses:

$\sigma_{sp1} = 450 - 13,5 - 10,66 = 425,84\, MPa$ **.**

Compressing force, taking into account the first losses:

$$P = \sigma_{sp1} A_{sp} = 425,84 \cdot 255 = 108,65\, kN$$

Second losses:

a) From shrinkage of concrete B25

$\Delta\sigma_{sp5} = \varepsilon_{b,sh} \cdot E_s = 0,0002 \cdot 2,1 \cdot 10^5 = 42$ MPa;

where $\varepsilon_{b,sh}$ - is the shrinkage deformation of concrete, taken equal to: 0,0002 - for concrete of classes B35 and below;

b) From concrete creep:

$$\Delta\sigma_{sp6} = \frac{0{,}8\varphi_{b,cr} \cdot \alpha \cdot \sigma_{bp}}{1 + \alpha\mu_{sp}\left(1 \pm \frac{e_{0p1} y_{sp} A_{red}}{J_{red}}\right)\left(1 + 0{,}8\varphi_{b,cr}\right)}.$$

For concrete of class B25

$\varphi_{b,cr}$ = 2,5; E_b = 27500 MPa; $\alpha = \frac{E_s}{E_b} = \frac{2.1 \cdot 10^5}{27{,}5 \cdot 10^3} = 7{,}63.$

The distance between the centers of gravity of the prestressed reinforcement and the reduced cross section

$$y_{sp} = e_{0p1} = 183 \text{ mm.}$$

Reinforcement ratio

$$\mu_{sp} = \frac{A_{sp}}{A} = \frac{255}{56560} = 0{,}005,$$

$$\Delta\sigma_{sp6} = \frac{0{,}8 \cdot 2{,}5 \cdot 7{,}63 \cdot 11{,}65}{1 + 7{,}63 \cdot 0{,}005\left(1 + \frac{183 \cdot 213 \cdot 0{,}056 \cdot 10^6}{0{,}46 \cdot 10^9}\right)\left(1 + 0{,}8 \cdot 2.5\right)} = 107{,}2\, MPa$$

Total second losses:

$\Delta\sigma_{sp(2)} = \Delta\sigma_{sp5} + \Delta\sigma_{sp6}$ = 42 + 107,2 = 149,8 MPa.

Total loss of prestression:

$\Delta\sigma_{sp(1)} + \Delta\sigma_{sp(2)} =$ 13,5 + 10,66 + 149,8 = 173,9 MPa > 100 MPa.

Stress σ_{sp2} with all losses:

σ_{sp2} = 450 – 173,9 = 276,1 MPa.

Compressing force, taking into account all losses:

$$P_{02} = \sigma_{sp2} A_{sp} - \sigma_s A_s = 276{,}1 \cdot 255 = 70{,}405\, kN.$$

Calculation of strength of normal cross-sections.

Determination of the position of the boundary of the compressed zone. $R_s \cdot A_s + R_{f,btw} \cdot A_{wt} \le R_{f,bf} \cdot A_f$;

$680 \cdot 255 + 2.1 \cdot 23740 \le 18{,}7 \cdot 29800$; $223254 < 557260$ - The boundary of the compressed zone passes through the flange, since the condition is satisfied.

Determination of the relative comrpession height:

$$\xi = \frac{x}{h} = \frac{R_s \cdot A_s + R_{f,btw} \cdot A_{wt}}{R_{fb} \cdot b`_f \cdot h} = \frac{680 \cdot 255 + 2.1 \cdot 23740}{18.7 \cdot 1490 \cdot 20} = 0,400$$

Maximum value of the relative compression height:

$\xi_r = \dfrac{\omega}{1 + \dfrac{\sigma_{sr}}{\sigma_{sc,u}} \cdot (1 - \dfrac{\omega}{1,1})}$, where $\omega = 0,8 - 0,008 \cdot R_b = 0,8 - 0,008 \cdot 13,2 = 0,695$,

and $\sigma_{sr} = R_s + 400 - \sigma_{sp2} \cdot \gamma_{sp} = 680 + 400 - 276,1 \cdot 0,9 = 831,51$,

and $\sigma_{sc,u}$ - maximum stress in compressed reinforcement equal to 400 MPa

$\xi_r = \dfrac{0,695}{1 + \dfrac{831,31}{400} \cdot (1 - \dfrac{0,695}{1,1})} = 0,613$; $\xi < \xi_r$ - the compressed zone is within the flange.

Strength condition: $M \le R_{f,btw} \cdot A_w \cdot \dfrac{h_w + t_f}{2} + R_s \cdot 2A_s \cdot (h - \dfrac{t_f}{2} - a)$.

$M = 0,125 q l_0^2 \cdot \gamma_n = 0,125 \cdot 19,4 \cdot 5,66^2 = 77,6$ kN·m = 77,6 · 10^6 N·mm;

$$R_{f,btw} \cdot A_w \cdot \frac{h_w + t_f}{2} + R_s \cdot 2A_s \cdot (h - \frac{t_f}{2} - a) =$$

$$= 2,1 \cdot 23700 \cdot \frac{270 + 20}{2} + 680 \cdot 2 \cdot 255 \cdot (290 - 10 - 30) = 93,916\, kN \cdot m$$

$77,6 < 93,9$ - strength is ensured.

Calculation of the strength of the concrete strip between inclined sections.

Strength condition: $Q = 0,3 \cdot \varphi_{w1} \cdot \varphi_{b1} \cdot R_b \cdot h$, where $\varphi_{w1} = 1 + 5 \cdot \dfrac{E_f}{E_b} \cdot \mu_{fa}$, where

$\mu_{fa} = \mu_{fv} \cdot k_n^2 = 0,008 \cdot 0,513^2 = 2,1 \cdot 10^{-3}$; $\varphi_{w1} = 1 + 5 \cdot \dfrac{2,1 \cdot 10^4}{0,27 \cdot 10^4} \cdot 2,1 \cdot 10^{-3} = 1,08$;

$\varphi_{b1} = 1 - 0,01 \cdot R_b = 1 - 0,01 \cdot 13,2 = 0,868$.

$Q = 0,3 \cdot 1,08 \cdot 0,868 \cdot 13,2 \cdot 270 = 60,3\, kN$

$Q = 0,5 \cdot q l_0 \cdot \gamma_n = 0,5 \cdot 19,4 \cdot 5,66 = 54,9\, kN$

$54,9 < 60,3$ - strength is ensured.

Calculation of the strength of inclined sections.

Strength condition: $Q \le Q_{fb} + Q_b$, where $Q_{fb} = \dfrac{R_{f,btw} t_w \cdot h_w}{\sin(90^0 - \beta)}$;

For determination of tensile strength of it is necessary to change k_{or} to $k_n = 0,513$.

$R_{f,btw} = 0,513^2 \cdot 0,008 \cdot 500 \cdot (1 - \dfrac{21,8}{80}) + 0,48 = 1,5\ MPa$. Since application of load is perpendicular to flange surface, angle $\beta = 0$.

$$Q_{fb} = \frac{R_{f,btw} t_w \cdot h_w}{\sin(90^0 - \beta)} = \frac{1,5 \cdot 90 \cdot 270}{1} = 36450\,N;$$

$$Q_b = \frac{0,75 \cdot R_{bt} \cdot t_w \cdot h^2}{h_w \cdot \sin(90^0 - \beta)} = \frac{0,75 \cdot 0,95 \cdot 90 \cdot 290^2}{270} = 19973\,N$$

Condition check: $Q \le Q_{fb} + Q_b$; $54,9 \le 36,45 + 19,97 = 56,4\,kN$, hence strength is ensured.

4.2 Longitudinal ribs calculation according to SLS

Determination of the crack width, normal to the longitudinal axis of the element.

Service limit state condition: $M \le M_{crc}$, where M_{crc} - is a crack appearance moment;

$$M_{crc} = R_{bt,ser} \cdot W_{pl} + M_{rp};$$

moment of resistance of the fiber reinforced reduced cross section:

$$W_{pl} = \frac{2 \cdot (J_{bc} + \alpha_f \cdot J_{fc1} + \alpha_f \cdot J_{ft1} + 1)}{h - x} + S_{bt};$$

where J_{bc}, J_{fc1} and J_{ft1} - are moments of inertia of compressed concrete zone, of fiber reinforcement area located at the compressed and tensioned zones respectively.

Determination of the position of the neutral line (border between compressed and tensioned areas):

$S_{bc} + \alpha_f \cdot S_{fc1} - \alpha_f \cdot S_{ft1} = \frac{(h-x)\cdot A_{bt}}{2}$, where *x* - is a coordinate of neutral line in Y axis. Firstly it is necessary to determine coefficient of fibre reinforcement by area $k_{an} = 1 - 0,5\frac{l_{f,an}}{l_f} = 1 - 0,5\frac{21,8}{80} = 0,864$.

$\mu_{faf} = \mu_{fvf} \cdot k_{or,f}^2 \cdot k_{an} = 0,008 \cdot 0,634^2 \cdot 0,864 = 0,0028$,

$\mu_{faw} = \mu_{fvw} \cdot k_{or,w}^2 \cdot k_{an} = 0,008 \cdot 0,676^2 \cdot 0,864 = 0,0031$.

Value of *x* - is determined by equation:

$$b_f \cdot t_f \cdot (h - 0,5 \cdot t_f) \cdot (1 + \alpha_f \cdot \mu_{faf}) + b_w \cdot (x - t_f) \cdot (h - \frac{x + t_f}{2}) \cdot (1 + \alpha_f \cdot \mu_{faw}) -$$

$$-\alpha_f \cdot b_w \cdot \frac{(h-x)^2}{2} = \frac{(h-x)^2}{4} \cdot b_w$$

$x = 150\,mm$.

$$W_{pl} = \frac{2}{140} \cdot (\frac{1490 \cdot 20^3}{12} + 1490 \cdot 20 \cdot 140^2) \cdot (1 + 7,63 \cdot 0,0031) + (\frac{80 \cdot 130^3}{12} + 80 \cdot 130 \cdot 65^2) \cdot$$

$$\cdot (1 + 7,63 \cdot 0,0028) + (7,63 \cdot 0,0028) \cdot (\frac{80 \cdot 140^3}{12} + 80 \cdot 140 \cdot 70^2) + 80 \cdot 140 \cdot 70 = 1706,44 \cdot 10^4\ mm^3$$

$M_n = 0,125 q_n l_0^2 \cdot \gamma_n = 0,125 \cdot 13,3 \cdot 5,66^2 = 53,26\,kN \cdot m$

$M_{rp} = P_{02} \cdot (e_{op} \cdot r)$; where $r = \varphi \cdot \frac{W_{red}}{A_{red}} = \varphi \cdot \frac{J_{red}}{y_{cw} \cdot A_{red}}$;

where $\varphi = 1,6 - \frac{\sigma_b}{R_{b,ser}} = 1,6 - 0,75 = 0,85$

$r = \frac{0,85 \cdot 46666 \cdot 10^4}{213 \cdot 56560} = 33\,mm$, condition checking: $M \le M_{crc}$;

$53,26 > 1,85 \cdot 1706,44 \cdot 10^4 + 70405 \cdot (183 + 33) = 46,77\,kN$, condition is not satisfied, hence cracks appears.

$a_{crc} = \delta \cdot \varphi_1 \cdot \eta_{f1} \cdot \eta_{red} \cdot \frac{\sigma_f}{E_f} \cdot 20 \cdot (3,5 - \mu_{red}) \cdot \sqrt[3]{d_{red}} \le a_{crc,ult}$; where $\delta = 1$, $\varphi_1 = 1,75$;

$$\eta_{f1} = \frac{0,5}{0,5 + m};\ m = \frac{1}{\frac{40 \cdot d_f^2 \cdot (\mu_{fa} + 5 \cdot \mu_s)}{\mu_{fa}^2 \cdot A} + 1} = \frac{1}{\frac{40 \cdot 0,8^2 \cdot (0,0034 + 5 \cdot 0,006)}{0,0034^2 \cdot 42062} + 1} = 0,36;$$

where *A is* an area of cross section.

$$\eta_{red} = \frac{\eta_{f2} \cdot \mu_{fa} + \eta_s \cdot \mu_s}{\mu_{fa} + \mu_s} = \frac{1 \cdot 0,0034 + 1 \cdot 0,006}{0,0034 + 0,006} = 1;\ \mu_{red} = \mu_{fa} + \mu_s = 0,0034 + 0,006 = 0,0094$$

$$d_{red} = \frac{d^2 \cdot \mu_{fa} + d^2 \cdot \mu_s}{d \cdot \mu_{fa} + d \cdot \mu_s} = \frac{0,8^2 \cdot 0,0034 + 0,8^2 \cdot 0,006}{0,8 \cdot 0,0034 + 0,8 \cdot 0,006} = 17,5\,mm$$

Stress σ_f is determined with accordance to SP 96.13330.2016 by formula 6.60

$\sigma_f = \frac{M - P \cdot (e_{op} + r)}{W_{f1}}$; where $W_{f1} = \frac{J_f}{y_c}$; J_f - is a moment of inertia of cross section reduced to steel, including in it steel fibers area, area of bar reinforcement, and areas of tensioned and compressed zones of concrete.

$$J_f = \frac{b_f \cdot h_f^3 \cdot (\alpha + \mu_{faf})}{12} + b_f \cdot h_f \cdot (\alpha + \mu_{faf}) \cdot (x - \frac{h_f}{2})^2 + \frac{b_w \cdot (\alpha + \mu_{faw}) \cdot (x - h_f)}{12} +$$
$$+ \frac{b_w \cdot (x - h_f)^3}{2} \cdot (\alpha + \mu_{faw}) + \frac{b_w \mu_{faw} \cdot (h - x)^3}{12} + \frac{b_w \cdot \mu_{faw} \cdot (h - x)^3}{2} + A_s \cdot \alpha \cdot (h - x - a)^2 =$$
$$= 8647,4 \cdot 10^4\ mm^4;$$

$$\sigma_f = \frac{M - P_{02} \cdot (e_{op} + r) \cdot y_c}{J_f} = \frac{(53260000 - 70405 \cdot (183 + 33)) \cdot 140}{8647,4 \cdot 10^4} = 61,6\,MPa$$

$$a_{crc} = 1 \cdot 1,75 \cdot 0,36 \cdot 1 \cdot \frac{61,6}{2,1 \cdot 10^5} \cdot 20 \cdot (3,5 - 0,0094) \cdot \sqrt[3]{17,5} = 0,034\,mm;\ a_{crc,ult} = 0,05\,mm$$

Deformation calculation.

$\left(\frac{1}{r}\right)_{\max} = \left(\frac{1}{r}\right)_5 - \left(\frac{1}{r}\right)_6 + \left(\frac{1}{r}\right)_7 - \left(\frac{1}{r}\right)_4$, where $\left(\frac{1}{r}\right)_5 = \frac{M_{crc}}{B_{f1}} + \frac{M - M_{crc}}{B_{f3}}$; B_{f1} - short-time rigidity. $B_{f1} = 0,85 \cdot E_b \cdot J_b$, where J_b - moment of inertia of cross section reduced to concrete.

$$x = h - \frac{S_b}{A_{red} + A} = 290 - \frac{12031 \cdot 10^3}{56484} = 77\,mm;$$

$$J_b = \frac{1490 \cdot 20^3}{12} + 1490 \cdot 20 \cdot (77 - 10) \cdot (1 + 7,63 \cdot 0,0031) + \frac{60 \cdot 270^3}{12} + 60 \cdot 270 \cdot$$
$$\cdot (213 - 0,5 \cdot 270)^2 + \frac{45 \cdot 270^3}{36} + \frac{45 \cdot 270 \cdot (213 - \frac{2}{3})^2}{2} + \frac{15 \cdot 15^3}{36} + \frac{15}{2} \cdot (213 - \frac{1}{3} \cdot 15 - 90)^2 \cdot$$
$$\cdot (1 + 7,63 \cdot 0,0028) + 7,63 \cdot 255 \cdot 183^2 = 46626 \cdot 10^4\ mm^4;$$

$$B_{f1} = 0,85 \cdot 0,275 \cdot 46626 \cdot 10^4 = 108988 \cdot 10^8 \; N \cdot mm^2;$$

$$B_{f3} = 0,9 \cdot E_b \cdot J_b = 0,9 \cdot 0,275 \cdot 46626 \cdot 10^4 = 115390 \cdot 10^8 \; N \cdot mm^2;$$

$$B`_{f3} = 0,5 \cdot E_b \cdot J_b = 0,5 \cdot 0,275 \cdot 46626 \cdot 10^4 = 64105 \cdot 10^8 \; N \cdot mm^2;$$

$$\left(\frac{1}{r}\right)_5 = \frac{4,66 \cdot 10^7}{108988 \cdot 10^8} + \frac{(5,32 - 4,66) \cdot 10^7}{115390 \cdot 10^8} = 0,48 \cdot 10^{-5} \; mm^{-1};$$

$$\left(\frac{1}{r}\right)_6 = \frac{M_n}{B_{f3}} = \frac{5,33 \cdot 10^7}{115390 \cdot 10^8} = 0,46 \cdot 10^{-5} mm^{-1};$$

$$\left(\frac{1}{r}\right)_7 = \frac{M_n}{B`_{f3}} = \frac{5,33 \cdot 10^7}{64105 \cdot 10^8} = 0,83 \cdot 10^{-5} mm^{-1};$$

$$\left(\frac{1}{r}\right)_4 = \frac{\sigma_b}{E_s \cdot h} = \frac{149,8}{2,1 \cdot 10 \cdot 290} = 0,245 \cdot 10^{-5} mm^{-1};$$

$$\left(\frac{1}{r}\right)_{\max} = \left(\frac{1}{r}\right)_5 - \left(\frac{1}{r}\right)_6 + \left(\frac{1}{r}\right)_7 - \left(\frac{1}{r}\right)_4 = 0,605 \cdot 10^{-5} mm^{-1}$$

$f = S \cdot l^2 \left(\frac{1}{r}\right)_{\max} = \frac{5}{48} \cdot 5660^2 \cdot 0,605 \cdot 10^{-5} = 20 \, mm < f_{ult} = 28 \, mm$. SLS calculation completed.

5 COMPARISON OF TWO TYPES OF SLAB

Table 5. Material consumption comparison.

	Concrete, m³		Steel, kg	
Floor slab	Steel reinforced sample	Fiber reinforced sample	Steel reinforced sample	Fiber reinforced sample
	1,37 m³	0,916 m³	136,1	110,67

Table 6. Material cost and labor cost comparison.

	Material cost, eur		Labor cost, man-day	
Floor slab	Steel reinforced sample	Fiber reinforced sample	Steel reinforced sample	Fiber reinforced sample
	112	108	10,39	7,79

Fiber consumption per m^3 of concrete = 57 kg. For these samples the combined method of reinforcement was used. In fiber reinforced slab, longitudinal and transverse reinforcement was left, vertical stirrups and reinforcement mesh in flange were replaced by evenly distributed steel fibers. It has been proven by experience that despite of the absence of a reinforcing mesh in the flange of the plate, the monolithic connection of the shelf and ribs is saved. The total weight of a normal reinforced slab is equal to 3560 kg, the weight of fiber reinforced is equal to 2400 kg (weight reduction was achieved by reducing the thickness of the flange of the slab).

According to the researches of N.I. Vatin (9), other properties of reinforced concrete in comparison with conventional reinforced concrete are presented below. Economic comparison of slabs made according to F.N. Rabinovich (2003, p-p. 338-350).

Table 7. Increasing of properties of fiber reinforced sample.

Property	Increasing of property,%
Compressive strength	10-15
Tensile strength	100-150
Impact strength	500-900
Fire resistance	100-300
Abradability	30-40
Watertightness	50-100
Frost resistance	50-100
Durability	50-100

Increasing of strength properties is possible to reach if fiber reinforcement is used with normal reinforcement jointly. It is possible to see on an example of load-carrying structures such as slabs, beams, and etc. Increasing of service properties (such as fire abradability, watertightness and etc.) is possible to reach without normal reinforcement. For example as a top layer of coating of industrial floors, coating of airfields andother abrasion-resistant surfaces.

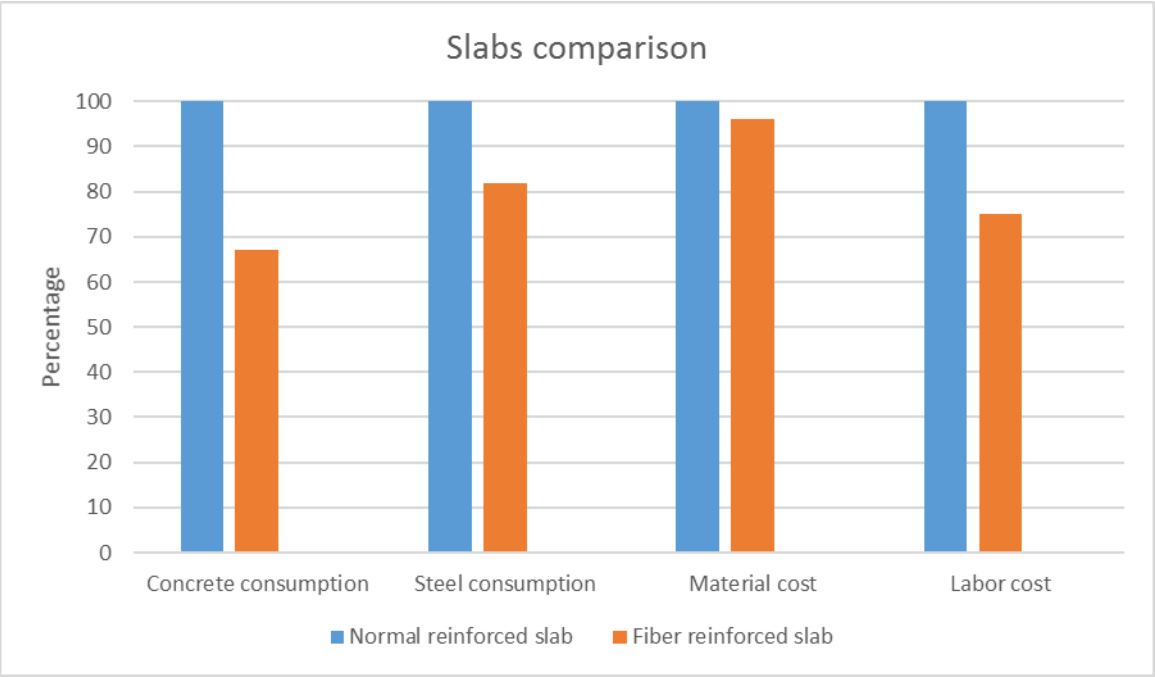

Figure 18. Diagram of slabs comparison.

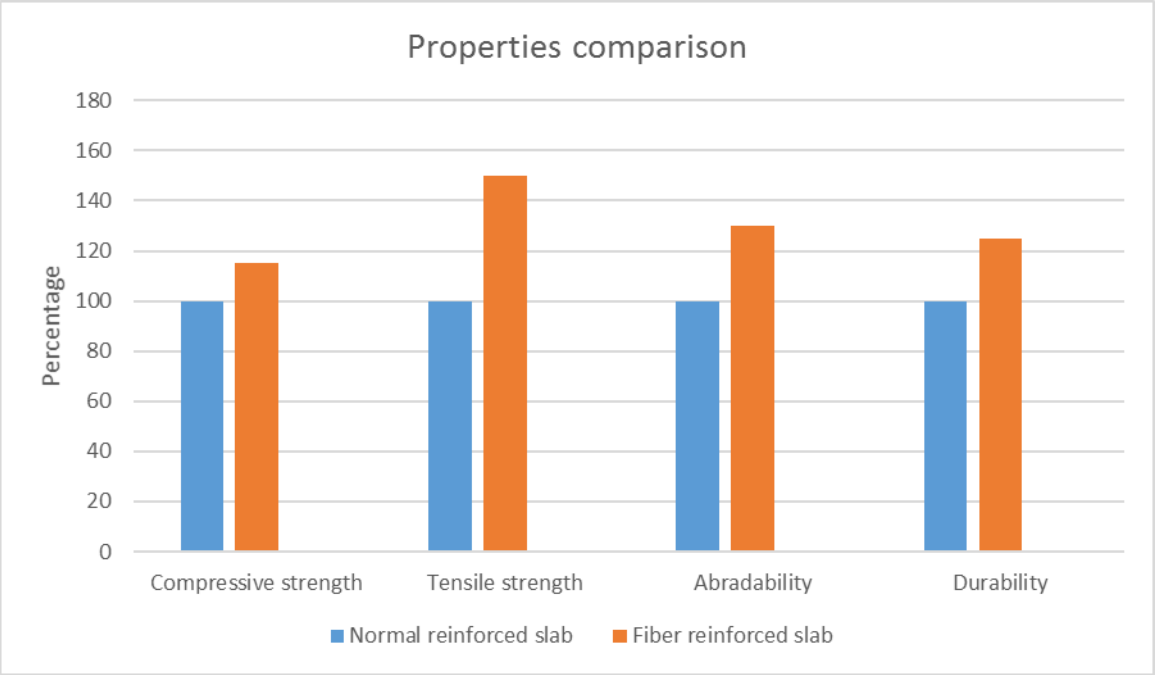

Figure 19. Diagram of slabs properties comparison.

6 CONCLUSION

Nowadays, there are many variations of use of fiber reinforcement in the construction industry. The required properties of concrete are possible to reach with addition of different types of fibers. Fiber-reinforced concrete is a general name of the structural material, because fibers differ strongly. One type of fiber reinforced concrete (with addition of mineral fibers) can be used instead of normal reinforced concrete for increasing specific properties of material. Another one (with addition of steel fibers) can be used to increase the main properties of an element (such as strength, deformations and crack formation).

On an example of different reinforced concrete slabs, the effectiveness evaluation of fiber reinforcement application was carried out. The results of this study show that it is possible to save about 10 percent of the total cost of the element, which may become a huge figure in the construction of large objects. Despite of the high efficiency of the application, the use of fiber reinforced concrete is constrained by the value of applicated load, because fiber reinforcement is chaotically oriented and can not fully replace rod reinforcement.

The most effective application of fiber reinforced concrete is structures that carry dynamic loads, that are exposed to frost and water.

Figures

Tables

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