VAASAN AMMATTIKORKEAKOULU
UNIVERSITY OF APPLIED SCIENCES

Hanna Syrjälä

SINGULARITY VALUE MANAGEMENT FOR OPTIMIZED CONCRETE DESIGN

Technology and Communication
2019

VAASAN AMMATTIKORKEAKOULU
Rakennustekniikka

TIIVISTELMÄ

Tekijä	Hanna Syrjälä
Opinnäytetyön nimi	Singulariteettiarvojen hallinta optimoidulle betonisuunnittelulle
Vuosi	2019
Kieli	englanti
Sivumäärä	40 + 1 liitettä
Ohjaaja	Marja Naaranoja

Opinnäytetyö tehtiin insinööritoimiston toimeksiannosta. Työn tilaajalla oli tiedostanut tarpeen saada suunnittelijoilleen ohjeistus, joka ottaisi huomioon singulariteettiarvojen hallinnan mallinnettaessa betonilaattarakenteita FEM-mitoitusohjelmalla.

Opinnäytetyön tarkoitus oli selvittää teräsbetonilaatan mallinnukseen sekä sen mitoitukseen vaikuttavia erinäisiä tekijöitä sekä ratkaista singulariteettiongelmat, jotka vaikuttavat teräsbetonilaatan mitoitukseen. Opinnäytetyön tavoite oli luoda selkeä ja helppolukuinen ohjeistus teräsbetonilaatan mallinnukseen singulariteettiarvojen hallinta huomioiden.

Työ koostuu teoriaosuudesta, jossa selvitetään hyväksyttäviä tapoja käsitellä singulariteettiarvojen hallintaa sekä laskentaosuudesta, jossa eri lähestymistapoja ongelmaan testataan FEM-mallin sekä Excel-taulukkolaskennan avulla sekä vertaillaan niistä saatuja tuloksia.

Opinnäytetyön tuloksena syntynyt singulariteettiarvojen hallinnan huomioiva optimoitu betonisuunnittelun ohjeistus on salassa pidettävä liite. Työn julkinen osuus ottaa kantaa teräsbetonilaatan oikeanlaiseen FEM-mallintamiseen sekä antaa näkemyksiä siihen, miten singulariteettiarvot kannattaa ottaa teräsbetonilaatan suunnittelussa huomioon.

Avainsanat FEM-laskenta, elementtiverkko, singulariteetti arvot, optimoitu betonilaatan raudoitus

VAASAN AMMATTIKORKEAKOULU
UNIVERSITY OF APPLIED SCIENCES
Rakennustekniikka

ABSTRACT

Author	Hanna Syrjälä
Title	Singularity Value Management for Optimized Concrete Design
Year	2019
Language	English
Pages	40 + 1 Appendices
Name of Supervisor	Marja Naaranoja

The thesis was commissioned by an engineering company, to develop alternatives and best practice for peak value management when analyzing concrete slab structures with FEM design software.

The purpose of the thesis was to find out various factors affecting the modelling of the reinforced concrete slab and its dimensioning, and to solve the singularity problems that affect the dimensioning of the reinforced concrete slab. The aim of the thesis was to create a clear and easy-to-read guideline for the modelling of reinforced concrete slabs, focusing on the management of singularity values.

The thesis consists of a theoretical part, which explores acceptable ways to deal with singularity values management, and a calculation part, where different approaches to the problem are tested using the FEM model and Excel calculations, and compare the results obtained between these two.

The result of the thesis, the guideline for optimized concrete design taking into account the management of singularity values, is a confidential annex. The public part of the thesis takes a stand on the correct FEM modelling of the reinforced concrete slab and gives an insight on how to analyze singularity values when designing a reinforced concrete slab.

Keywords Finite Element Method, mesh, singularity values and optimized reinforced concrete slab

CONTENTS

LIST OF FIGURES

LIST OF APPENDICES

1. INTRODUCTION

1.1 Background

The Client for this thesis is a technical engineering company, which provides professional multi-discipline engineering and design services for the global energy and power industry. The company consists of several different services; energy, oil & gas, process, civil, manufacturing and vehicles. The Client's Concrete group has invested in improved optimization in their concrete design.

The Concrete team uses FEM software to model and calculate structures. Correctly used FEM models provides data that can be used for cost-optimizing the structures.

This thesis studies how to manage best the singularity values in FEM when analyzing concrete structures. The thesis focuses on finding the most cost–efficient solutions, to provide optimized concrete design for the Client's end–customers.

1.2 Contents

The thesis consist of three main chapters: The literary part studies how to find out what the accepted methods for FEA singularity value management are through, university books and the RFEM software manual. The second part consists of an FEM model and calculations. This part will concentrate on solving the correct ways to model the structure. The third part is Excel calculations and a comparison between the FEM and Excel results. Through these findings and results, the thesis studies what the optimal average area around peak values is that gives the most realistic moment for reinforcement design.

1.3 Aims and Goals of the Thesis

The aim of this thesis is through comparisons, examinations and studying literature to find the most optimized mesh size, giving the most cost-effective design, without compromising the structural safety. The aim is construction cost savings from an optimized FEM analysis of reinforced concrete slabs. This leads to more satisfied customers for the Client through better quality and cost.

The goal is to find proper methods for the FEM– analysis of concrete slabs, a topic that has limited guidance in both the building code, our educational system and at designer levels.

The result of the thesis is a confidential guide for the RFEM implementation. The public thesis is an explanation of methods and resources for FEM in general.

2. LITERATURE REVIEW

2.1 General

The finite element method is commonly used in the civil sector for designing of the reinforcement in concrete slabs. Due to necessary simplifications in the model, unrealistic concentrations of moments and shear forces will occur. To obtain an economical design it is important to put deeper attention to these concentrations. The study focusses on solving the width, which is distributed over a certain length that gives realistic design moment. /1/

2.2 The Finite Element Method

In the Finite Element Method, a complex structure is subdivided into a finite number of individual components. The components are referred to as “elements”. /4/

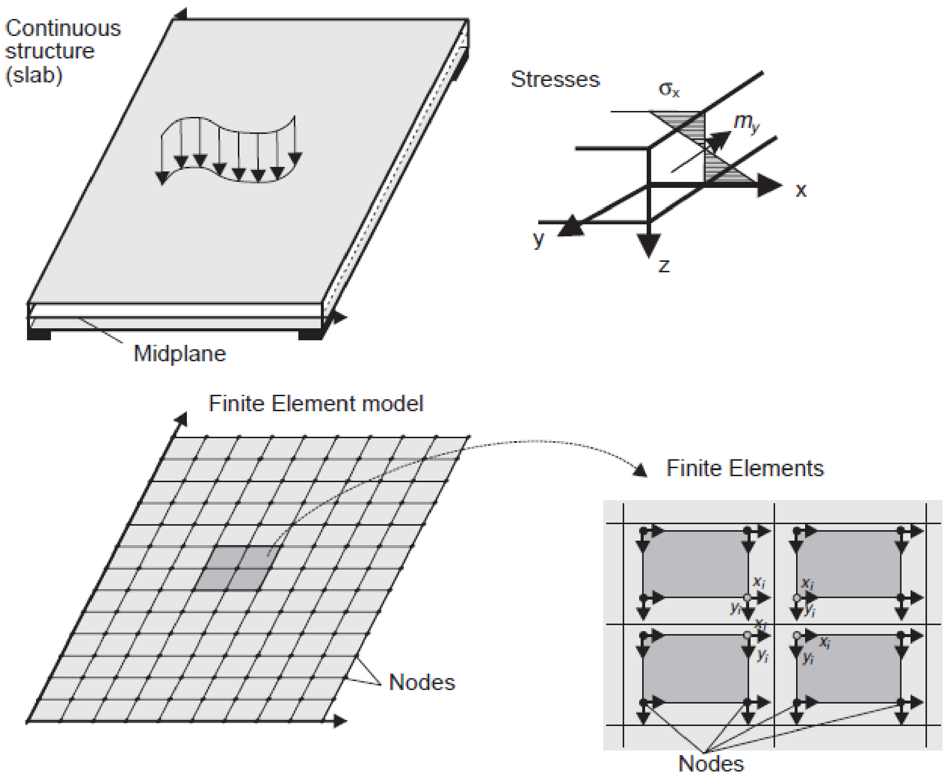


Figure 1. Numerical analysis, Finite Element Method /redrawn 4/

It is possible to specify the relation between the nodal displacements of the elements and their nodal reactions. In the FE method, all displacements, strains and stresses within an element, as well as the resulting nodal forces, are treated as unknown variables to be solved by a series of algebraic equations. The result of the complete system comes from the assembly of all of these calculated elements. The main task in the Finite Element Method is to discover form functions that can best approximate the behavior of a special structural element and also satisfy the compatibility condition. /4/

2.3 RFEM

RFEM is a powerful 3D FEA program for the analysis and design of plates, walls, shells, solids, and frame structures. The program forms the basis of a modular software system: RFEM is used to define internal forces, deformations, and support reactions for planar and spatial structural systems with or without member and solid elements. /6/

FEA is a computer method of analysis that can be used by engineers to perform solutions to a wide range of one–, two– and three– dimensional structural problems. /2/

2.4 Model Creation

2.4.1 Design Process

It is very important to remember that the designer has a strong impact on the FE analysis results due the modelling of the structure and choices made during the modelling.

The correctly done model creation of reinforced concrete slab follows a certain path (see **Figure 2**) and there are numerous issues to consider during the model creation.

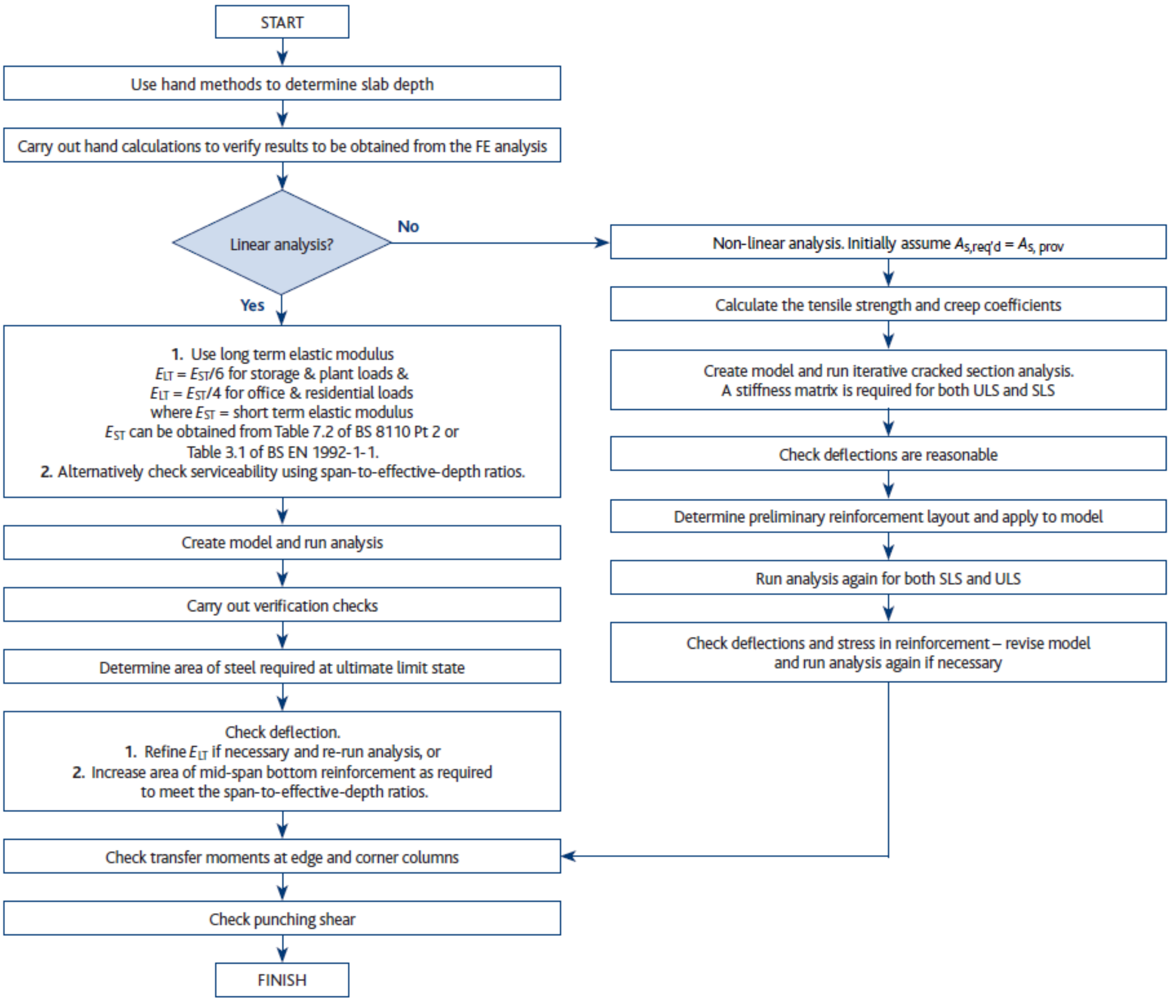


Figure 2. Design process when using FEA /redrawn 2/

When modelling the reinforced concrete structure, the designer needs to determine at least:

- The properties of used concrete
- Type of the model
- Element size and shape
- Loading
- Support conditions
- Result interpretation /2/

When modelling the reinforced slab, standards determine regulations for plate structures regarding size and direction of the reinforcement to be used.

The following **Figure 3** demonstrates the relation between the user–defined Type of Model, the model for the design, and the element of structures according to the standard EN 1992-1-1. **Figure 3** shows the different elements of structures, which are used to determine the size and direction of the minimum or maximum reinforcement in the standard in question. /5/

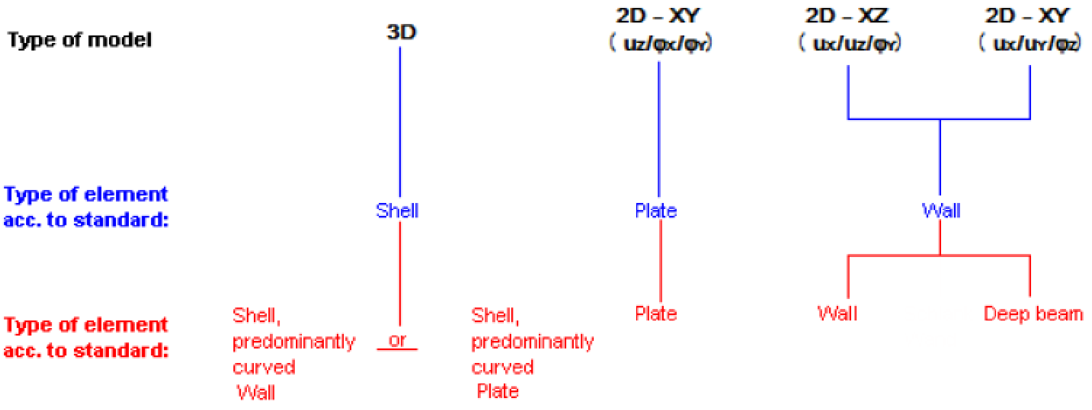


Figure 3. Choice of the model type /redrawn 5/

2.4.2 Meshing

The term 'mesh' describes the sub– division of surface members into elements as shown in **Figure 4**. /2/

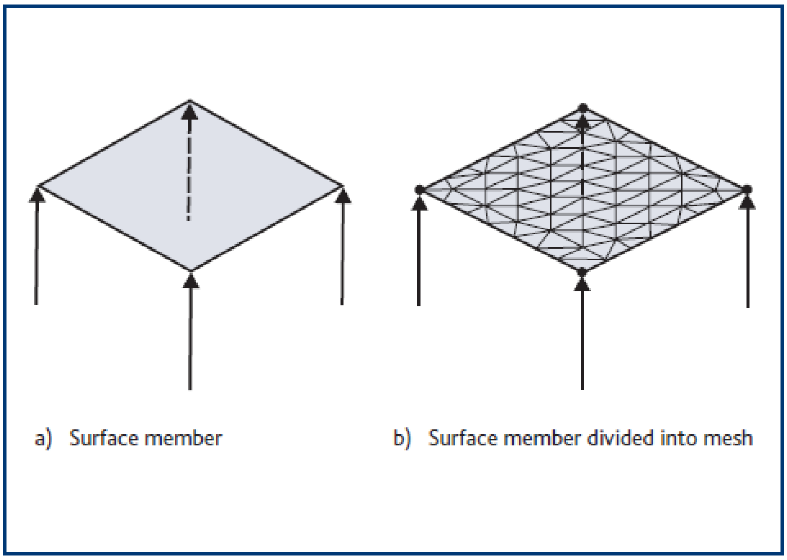


Figure 4. Surface divided into mesh /redrawn 2/

Several studies show that there is a dependency of chosen mesh size – the finer the mesh, the higher the peak values of moments will arise. A finer mesh can give more accurate results but the designer must have the knowledge to estimate, which results are usable for the design and which are not. A finer mesh takes more time to analyse and it is subject to the law of diminishing returns. It should be noted that the finer mesh gives higher peak moments and can lead to over–dimensioning in concrete design, if the peak values are not considered and understand correctly. /2/

It is also important to acknowledge that a coarse mesh does not always give a correct demonstration of the forces. This is due to locations where the stresses change quickly in a short space. These kind of locations are for example at certain type of supports, near openings or under concentrated loads. /2/

The designer has to assess how fine or coarse the mesh should be. For sufficiently accurate results, it is important to select the correct mesh size or at least, understand the results and the influence of the mesh size on the peak values. There are no definite answer for the ideal mesh size. A good starting point for elements can be considered a span/10 or 1000mm, depending on which gives a smallest value. /2 /

When assessing the right mesh size, it is good to remember that in an FE analysis of a concrete slab, moments and cross–sectional forces tend to go towards the infinity upon mesh refinement. In order to obtain sufficiently accurate results at these critical sections, the mesh density around the point support node in a slab should be chosen so that there is at least one shell element regardless of order, between the support node and the critical cross-section. This mesh refinement is illustrated in **Figure 5**. /1/

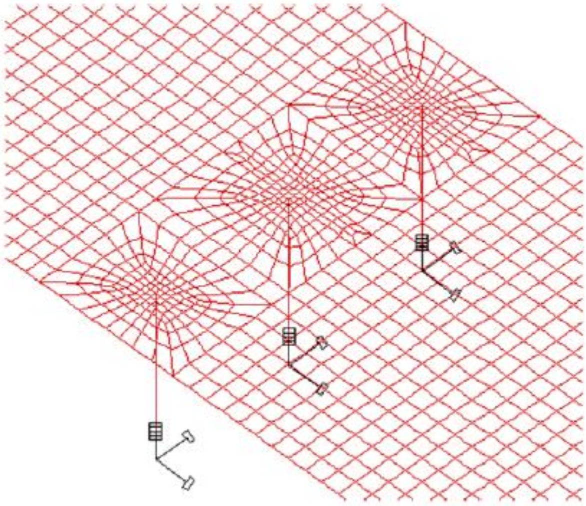


Figure 5. Mesh refinements around column support /redrawn 1/

2.4.3 Boundary Conditions

In reality, a structure does not show any singularities. The singularity problem is caused by the inaccuracy of the boundary conditions of the model. Therefore, it is not always important to calculate with the maximum value of the stresses. The support from a foundation or other structural parts provides stiffness with respect to both translation and rotation. It is easy to make a model error when modelling the support conditions. For example, the line support to which the rotation is released, allows the outer edge of the slab to rotate freely versus a slab with a rigid support, the results of the calculations differ significantly. In reality, the support of the slab is somewhat in between a rigid and a hinged support. /1, 4/

It is of high importance that the modelling of the support conditions is done correctly, to ensure realistic bending moments at the supports and in the mid-span. In this way, the column moments can be derived and the punching shear stress can be evaluated realistically. The model should also have a sufficient amount of mesh nodes in order to obtain correct results. However, it is possible that the model contains too many nodes, especially at supports, where the peak moments tend to be highlighted. /2/

It is good to know that singularity issues do not arise in cases where a slab is supported by a line support. The element mesh still needs to be fine enough in order to give sufficient results in the adjacent critical sections. In addition, it is recommended that at least one element length is provided between the line support node and the critical cross section, no matter the order of the shell element. As an alternative, one can use the maximum moment and shear force at the line support as a sufficient approximation. /1/

The support conditions for slabs supported by bearings or columns are often modelled as concentrated at single nodes. By doing so, the singularity is introduced in the solution. The moments and sectional forces tend to go towards infinity upon the mesh refinement. There are in two different ways to deal with this kind of problem: either the modeling of the support can be improved to avoid the singularity, or the results in the failure-critical sections adjacent to the supports can be evaluated. /1/

2.4.4 Problems with Model Creation

When using the RFEM software, the designer must pay attention to several important points when modelling the concrete structure.

First, there are potential program errors. There are problems that may occur in a numerical model due the necessary simplifications in it. Problems also can occur due the false assumptions of the element behavior.

These are also the model errors to consider. FE analysis results are highly dependent on the user and the chosen FE code. The designer must have a good knowledge of FEM and software behavior to design the structure correctly enough to get sufficient results.

The designer must pay attention to material choices in the model. The calculation in the concrete slab model are based on a linear elastic behavior. In reality, it is known that concrete is a nonlinear material.

When modelling the structure, the load distributions are also a point of interest. The designer must consider for example how to evaluate the concentrated load results properly.

The modelling of support conditions is when the model errors are very easily made. The designer needs to model the support conditions with great care to ensure that resulting bending moments in the mid– span and at the supports are realistic. /4, 2, 1/

2.5 Stress Analysis

2.5.1 Moment Distribution

Moment distributions occur on top of columns and at receding walls. The problem is also the dependency on the mesh fineness; the finer the mesh, the higher the peak.

One should not concentrate on the peak value itself, but on the area of the moment diagram over the section. For this area, it is sufficient to consider a part of the section, which extends to the left and right of the column. This is an average area, which gives the design moment for the reinforcement calculations. /1, 3/

As **Figure 6** illustrates: The area of the bending moment diagram should be the area of interest. The designer should not pay too much attention to the peak bending moments.

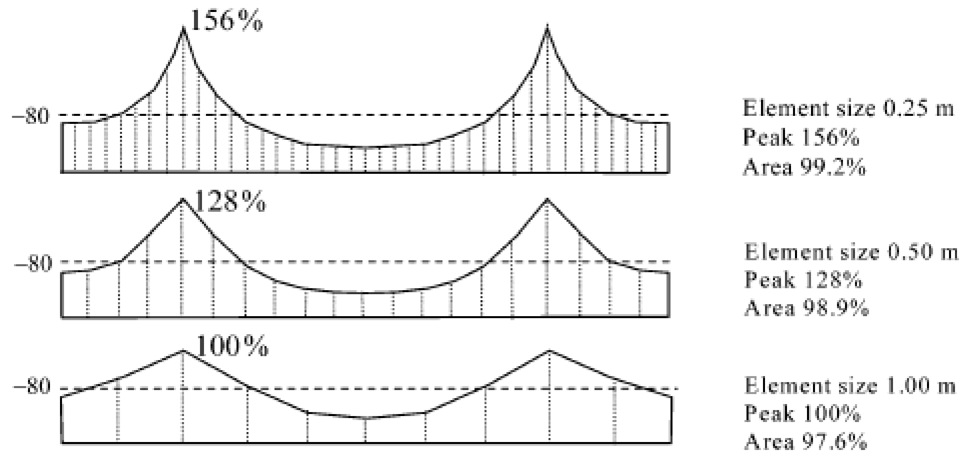


Figure 6. The relation between mesh size and moment distribution /redrawn 3/

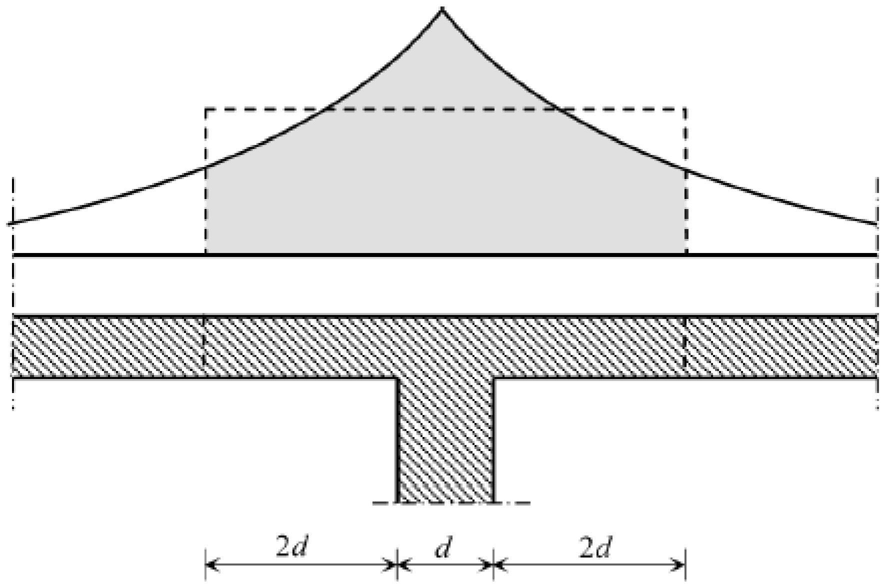


Figure 7. Moment peak value management /redrawn 3/

There are a couple ways to try to smooth out the peak moments. One example is introduced in **Figure 7**: The plate length two times the column width at each side of the column, in total five times the column width. The integral over this section part gives a good estimate for the reinforcement which is needed in this section part. /1/

It needs to be noted that when modelling the supports in a simplified way, in single points or for example along discrete lines, the maximum bending moments obtained from the FE analysis will over–estimate the real moments. This does not mean that the modelling of the supports should be done in a different way, it just has to be noted that this way of modelling the supports gives peak moments that can be neglegted and smoothed out. When studyin the bending moments in a slab over a support with a certain width, it will be discovered that the bending moment has its maximum above the support. /1/

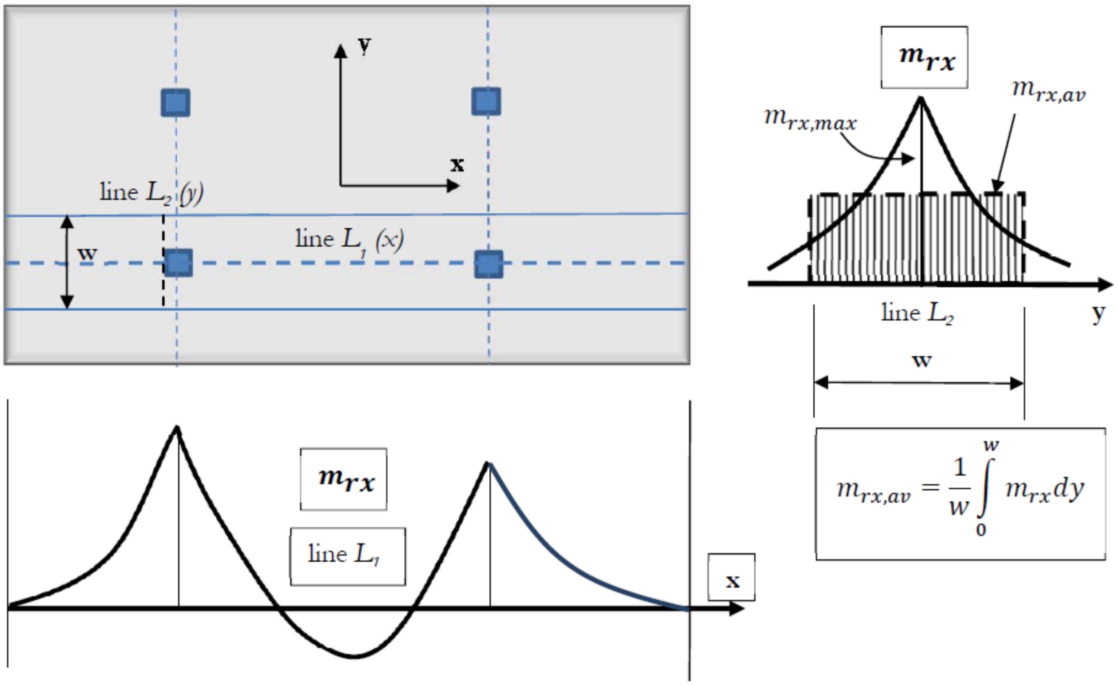


Figure 8. Reinforcement moment's redistribution /redrawn 3/

In **Figure 8** ***w*** means a certain width, over which the reinforcement moments can be redistributed. The distribution widths at a support can be chosen from the recommendations in **Figure 9**.

$$w = \min(3h, \frac{L_c}{10}) \quad \text{for } \frac{x_u}{d} = 0.45 \text{ (0.35 for concrete strength classes} \geq \text{C55/67)}$$

$$w = \min\left(5h, \frac{L_c}{5}\right) \quad \text{for } \frac{x_u}{d} = 0.30 \text{ (0.23 for concrete strength classes} \geq \text{C55/67)}$$

$$w = \frac{L_c}{4} \quad \text{for } \frac{x_u}{d} = 0.25 \text{ (0.15 for concrete strength classes} \geq \text{C55/67)}$$

$$w = \frac{L_c}{2} \quad \text{for } \frac{x_u}{d} = 0.15 \text{ (0.10 for concrete strength classes} \geq \text{C55/67)}$$

$$w = \min\left(5h, \frac{L_c}{5}\right) \quad \text{for } \frac{x_u}{d} = 0.0$$

Figure 9. ULS distribution width recommendations for a support /1/

Another way of reducing the peak moment values is introduced in the following **Figure 10.**

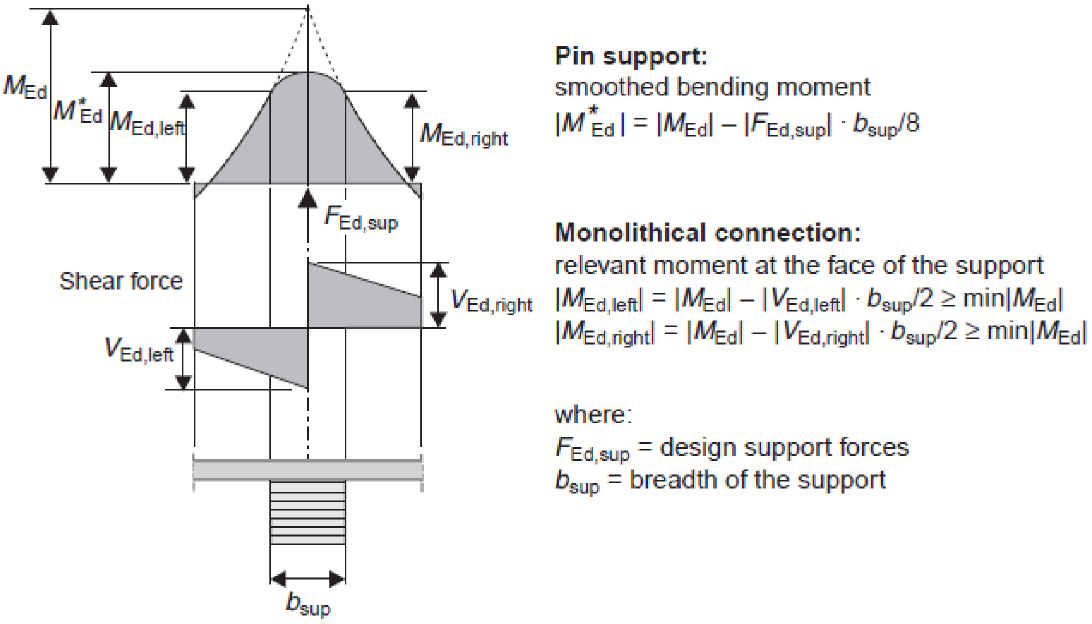


Figure 10. Smoothing the bending moment /redrawn 4/

2.5.2 Smoothing of the Shear

There are no specific guidelines given for redistribution of shear forces from a linear FE analysis in Eurocode 2. Additionally, very little scientifically based information has been found in literature on the subject. /1/

There are some recommendations, which are based on the assumption that the reinforcement concrete structures in ULS have good capabilities for plastic redistributions. It is also assumed that the same method can be used for choosing the distribution widths for shear forces that is used for the reinforcement moments. /1/

When starting to re– evaluate the shear force distribution, it has to be taken into account that the redistribution is done in a direction, which is orthogonal to the direction of the resultant shear force. **Figure 11** demonstrates this.

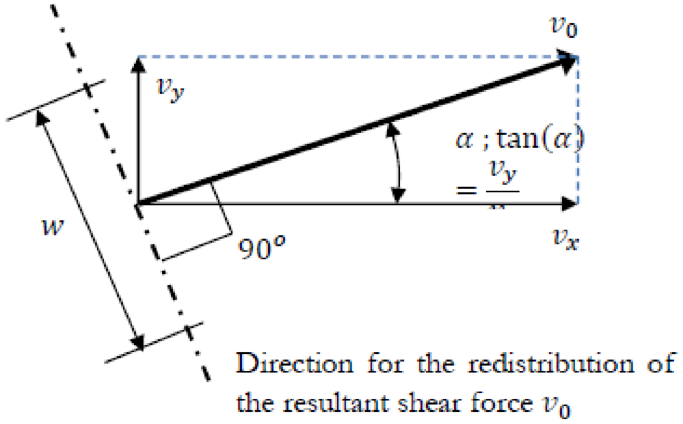
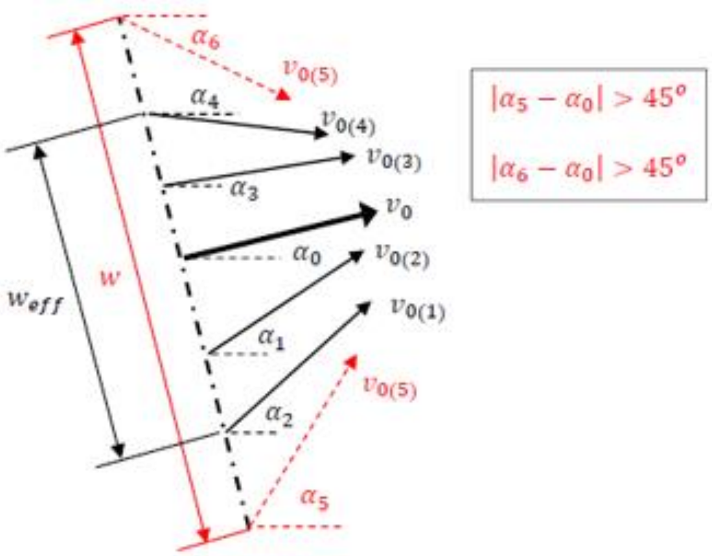


Figure 11. Redistribution of the shear forces resultant /redrawn 1/

The distribution width for shear forces can, as earlier mentioned, be chosen as equal to that is used for the reinforcement moments.

There are some limitations to this rule:

- The distribution width should not exceed 5h. The thickness of the slab is *h*.
- The same rules apply that are given in **Figure 9.**
- The distribution width should be restricted to a variation of the angle α of less than 45°. See **Figure 12**. /1/



Case	$\alpha_{1,i}$	$\alpha_{2,i}$	$v_{dx,i}$	$v_{dy,i}$
(1)	$\leq 45°$	$\leq 45°$	$\max(v_{01,i}, v_{02,i})$	0
(2)	$\leq 45°$	$> 45°$	$v_{01,i}$	$v_{02,i}$
(3)	$> 45°$	$\leq 45°$	$v_{02,i}$	$v_{01,i}$
(4)	$> 45°$	$> 45°$	0	$\max(v_{01,i}, v_{02,i})$

Figure 12. Distribution width recommendations for shear forces /redrawn 1/

The direction of the resultant shear force can be determined as shown in **Figure 13.**

Its angle α_i to the x axis is determined as:

$$\alpha_i = \arctan\left(\frac{v_{y,i}}{v_{x,i}}\right)$$

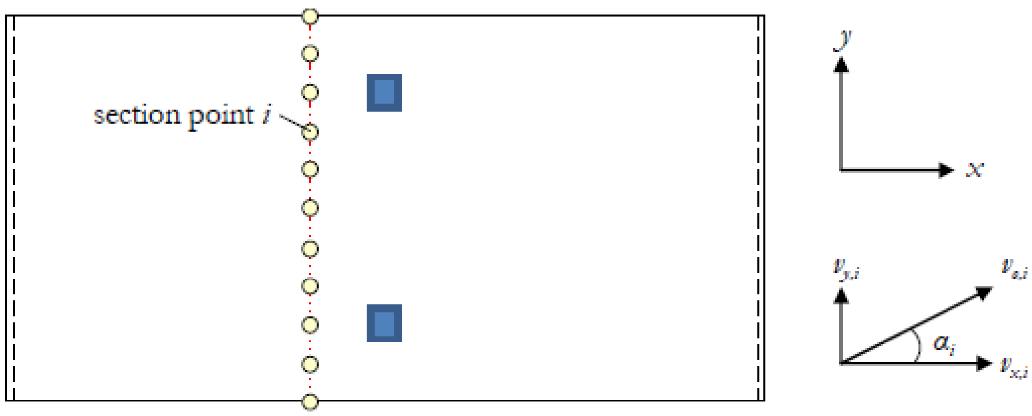


Figure 13. Direction of the resultant shear force /redrawn 1/

The maximum values and the distribution of the shear forces over the supports are usually irrelevant for the design of concrete slabs. In case of a stiff monolithic connection between the slab and the supported columns or walls, the design in bending is carried out for the forces at the face of the support. If the slab is restrained in vertical direction only, the maximum design bending moment is calculated from the values at the face of the supports and then smoothed out. /4/

One way to check the punching shear is to take the reactions from the FEM model, then carry out the calculation, for example using a spreadsheet for the design of reinforced concrete. If there is a need to model the area of the column, then shear stresses can be taken from the model. It still necessary to have knowledge of the structural behavior of the model to make a sufficient decision how to take into account the fact that there will be peaks which are greater than the design limits in the codes. /2/

The slab should be designed for punching in regions of concentrated single loads or pin supports by columns and not for the high resultant shear forces. /4/

The critical result section of the shear force in the slab are not placed closer to the support edge than d. This is illustrated in **Figure 14**. This helps the designer to estimate the correct location for the critical result section in the concrete slab. However, it has to be remembered that this is dependent on the stiffness of the slab-support connection and the design itself. /1/

To summarize, the maximum values and the distribution of the bending moments and shear forces over the supports are generally not required for the design of concrete slabs.

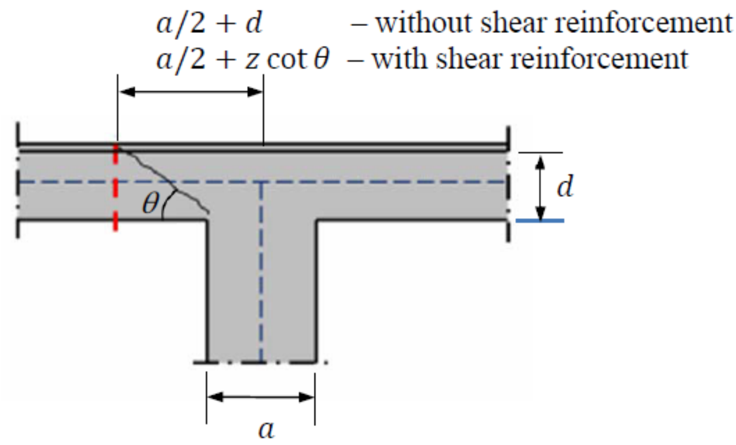


Figure 14. The critical result section of the shear force /redrawn 1/

2.5.3 Punching and Deflection Control

The slab should be designed for punching in regions of concentrated single loads or pin supports by columns and not for the high resultant shear forces. The punching shear can determine the thickness of the concrete slab. If the maximum shear force of the model overcomes the maximum shear capacity of the structure, the concrete will crack. /4/

Punching shear and deflection control are usually the main criteria for flat slabs. Punching shear should be checked using code rules. /2/

Deflection in concrete is dependent on:

- Concrete tensile and compressive strength
- Elastic modulus
- Creep
- Shrinkage
- Cracking of the member
- Ambient conditions
- Restraint
- Magnitude, time and duration of loading
- Stiffening by other elements

Deflection prediction is based on various different assumptions and therefore can only be an estimate at its best. /2/

3. CALCULATIONS

3.1 FEM Model

The model is an example from a foundation for a transmission tower. The FEM model is a concrete slab:

- Concrete Grade: C30/37
- Slab Dimensions: 5.6 m x 7.0m x 0.6m
- Pedestals 4 pcs 700x700mm, 2.90 m high.

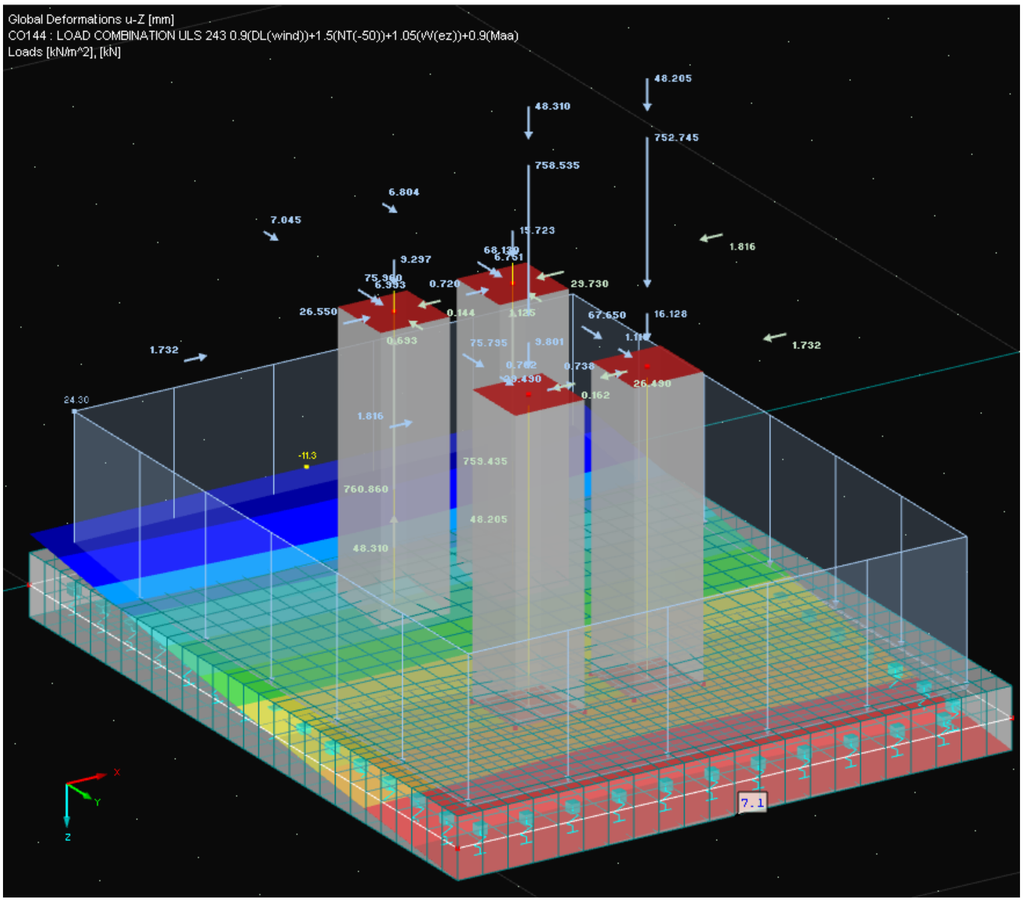


Figure 15. The example model of the tower foundation

Loads were imported from Project data for steel structure, and critical load case for bottom reinforcement and for the calculation was determined to be thermal minimum (- 50 degrees). In this temperature the electrical cables shrink and add max load to the foundation. The mesh size in the calculated model was 250 mm.

The minimum reinforcement for bending was 746 mm² (0,13%), and the maximum reinforcement was 24000 mm² (4%).

Slab shear capacity without shear reinforcement was 338 kN/m (338 kN/m -> 203 kN), as illustrated in **Figure 16**.

	A_s [mm^2]	d [mm]	ρ_l	σ_{cp} [N/mm^2]	k	v_{min} [N/mm^2]	$V_{Rd.c}$ [kN]	V_{Ed} [kN]	$V_{Ed}/V_{Rd.c}$
Btm.main	1 608	542.0	0.00297	0.0	1.61	0.36	203.9	**203.0**	**1.00**
Top.main	905	559.0	0.00162	0.0	1.60	0.35	197.6	**10.0**	**0.05**
Btm.trans	1 028	528.0	0.00195	0.0	1.62	0.36	189.7	**10.0**	**0.05**
Top.trans	754	547.0	0.00138	0.0	1.60	0.36	194.6	**10.0**	**0.05**

Figure 16. Slab shear capacity without shear reinforcement

3.2 FEM Model – without Singularity Management

The design moment without singularity value management in the model is 774 kNm/m as **Figure 17** below shows.

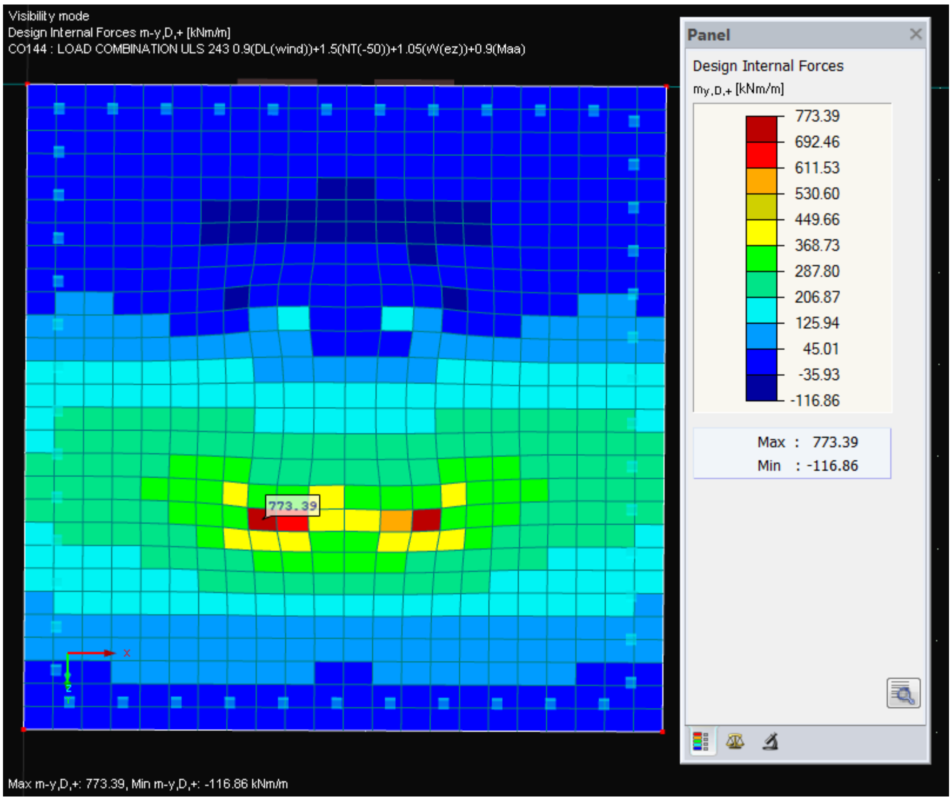


Figure 17. The design moment without singularity value management

The required reinforcement in the slab are presented in **Figure 18** below.

CALCULATIONS IN THE MAIN DIRECTION

Materials

Concrete =	C30/37		Design situation =	Persistent & Transient		
f_{ck} =	30	N/mm²	γ_c =	1.5		
$f_{cd} = \alpha_{cc} * f_{ck} / \gamma_c$ =	17.00	N/mm²	α_{cc} =	0.85		
f_{ctm} =	2.896	N/mm²	λ =	0.8		(3.1.7(3))
E_{cm} =	32837	N/mm²	η =	1.0		(3.1.7(3))
ε_{cu} =	3.5	‰				
Steel =	S500					
f_{yk} =	500	N/mm²	γ_s =	1.15		
$f_{yd} = f_{yk}/\gamma_s$ =	434.8	N/mm²				
E_s =	200000	N/mm²				

Creep and development of tensile strength EC2 3.1.2(9) & Annex B

Creep factor, φ =	0.00		s =	0.25	(3.1.2(6))
Cement type =	N		α =	0.667	(3.1.2(9))
Age at cracking = t =	28	days	$\beta_{cc(t)}$ =	1.000	(3.2)
$f_{ctm}(t)$ =	2.896	N/mm²	$f_{ctm}(t) = \beta_{cc}(t)^{\alpha} f_{ctm}$		(3.4)

Slab geometry

b =	1000	mm	h =	600	mm
c_{bottom} =	50	mm	c_{top} =	35	mm

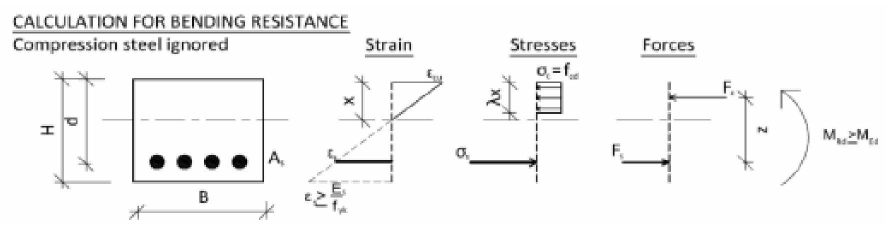
Which reinforcement is to be outermost (<M>ain or <T>ransversal)? M

Effective heigths

$c_{btm.main}$ =	50	mm	$c_{top.main}$ =	35	mm
$d_{btm,main}$ =	540	mm	$d_{top,main}$ =	559	mm
$d'_{btm,main}$ =	60	mm	$d'_{top,main}$ =	41	mm

Minimum reinforcement for bending EC2 9.3.1.1

$A_{s.min.btm.main}$ =	813.3	mm²	$A_{s.min.btm.main} = b * d_{btm.main} * \max(0.26 * f_{ctm}/f_{yk}; 0.0013)$	(9.2.1.1(1))
$A_{s.min.top.main}$ =	841.9	mm²	$A_{s.min.btm.top} = b * d_{top.main} * \max(0.26 * f_{ctm}/f_{yk}; 0.0013)$	
$A_{s.max}$ =	24000	mm²	$A_{s.max} = b * h * 0.04$	(9.2.1.1(3))
$s_{max.slabs}$ =	400	mm	$s_{max.slabs} = \max(3 * h; 400)$	(9.3.1.1(3))
w_{max} =	0.3	mm		(Table 7.1N)



Page : 2(6)

BOTTOM reinforcement in MAIN direction

d =	540	mm	A_s =	3 703	mm²	c =	50	mm
d' =	41	mm	A_{sc} =	905	mm²			

Check for cross section resistance EC2 6.1

$M_{Ed.btm.m}$ =	774.0	kNm/m	Ok, no need for compression reinforcement	
β_{req} =	0.171		β_{bal} = 0.467	$\omega_{bal} = \lambda \varepsilon_{cu} / (\varepsilon_{cu} + f_{yk}/E_s)$
$A_{s.req}$ =	3604.3	mm²		$A_{s.req} = \omega_{req} * b * d * f_{cd} * \eta / f_{yd}$

Provided bending reinforcement

8pcs/m	ø =	16	mm, c/c	125	mm	=>	A_s =	1608	mm²
6.67pcs/m	ø =	20	mm, c/c	150	mm	=>	A_s =	2094	mm²
	ø =		mm, c/c		mm	=>	A_s =		mm²
	$ø_{max}$ =	20	mm, c/c	68.1818182			$A_{s.btm.m}$ =	3 703	mm²
				c/c OK!			As > As.min, OK!		

λx =	94.7	mm	$\lambda x = A_{s.btm.m} * f_{yd} / (b * f_{cd} * \eta)$	
ε_s =	1.247%		$\varepsilon_s = (d - x) * \varepsilon_{cu} / x$	εs>fyk/Es OK!
$M_{Rd.btm.m}$ =	793.1	kNm/m	$M_{Rd.btm.m} = A_s * f_{yd} * (d - 0.5 * \lambda x)$	MRd OK!

Figure 18. Calculation results for the reinforcement Client EC2 Slab Excel tool

Figure 19 shows the Excel calculation of the reinforcement that is required for the design moment 774 kNm/m. The total amount of the reinforcement provided in the bottom slab in main direction is 3703 mm².

Which reinforcement is to be outermost (<M>ain or <T>ransversal) | M

	Loads acting on cross section				Reinforcements		
	M_{Ed} kNm	**M_{Ek} kNm**	**$\eta = M_{Eqp}/M_{Ek}$**	**V_{Ed} kN**	**ø mm**	**c / c mm**	
Btm.main	774.0	0.0	1.00	10.0	16	125	As > As.min, OK!
					20	150	As.min.c OK!
							c/c OK!

Figure 19. The provided bending reinforcement

The maximum shear in the design without singularity value management is 943 kN/m. This is presented in **the Figure 20**. As presented earlier in Figure 16, this exceeds capacity by 240%, and thus needs thicker slab or better analysis.

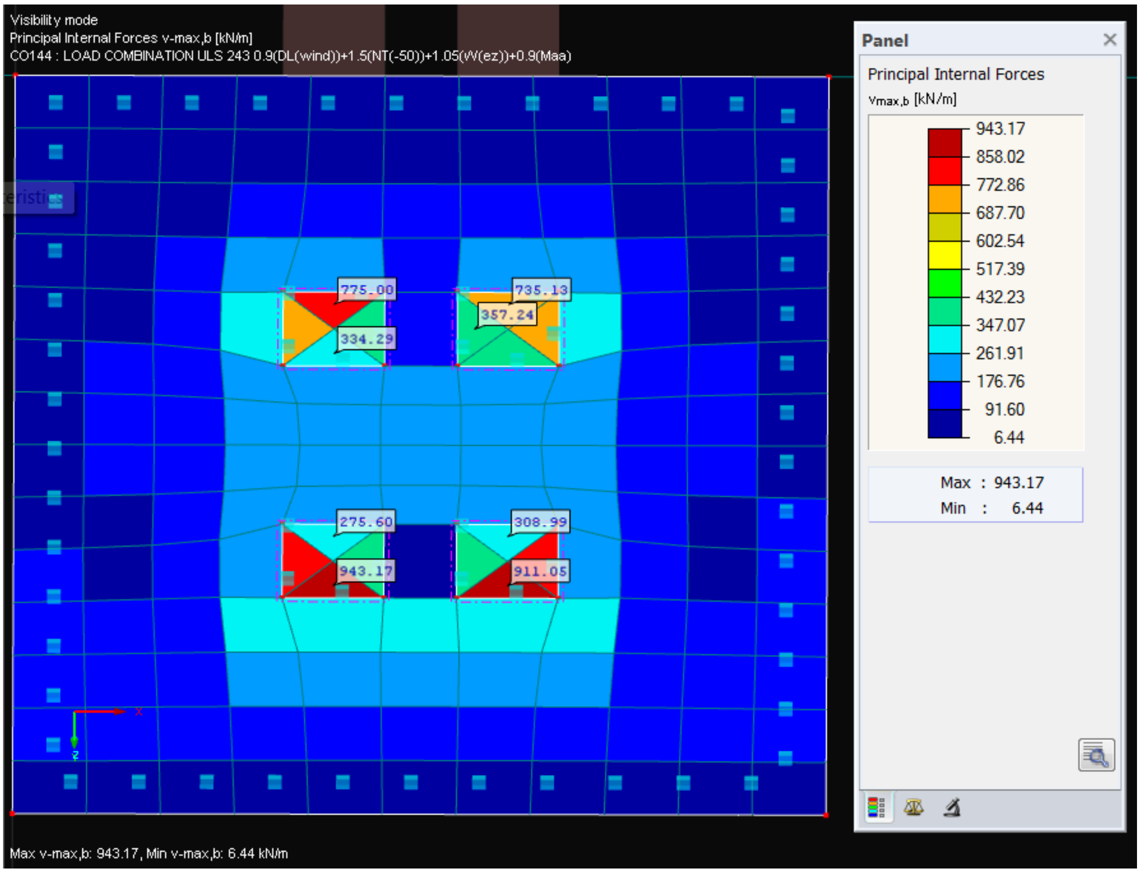


Figure 20. The maximum shear force in the design.

3.3 FEM Model with Singularity Value Management

3.3.1 Dependency of Mesh Size

The following **Figures 21, 22** and **23** demonstrate the fact that the peak forces of the structure depend on the given mesh size. The structure, its supports and its loads are identical in the examples. The only thing changed in the model is the mesh size. As it can be seen in the Figures: The finer the mesh is – the higher the moment peak values become.

The different mesh sizes are chosen according to the following:

- The recommendation 10/span (500 mm)
- 250 mm, finer than the recommendation for comparison
- 750 mm, coarser than the recommendation for comparison

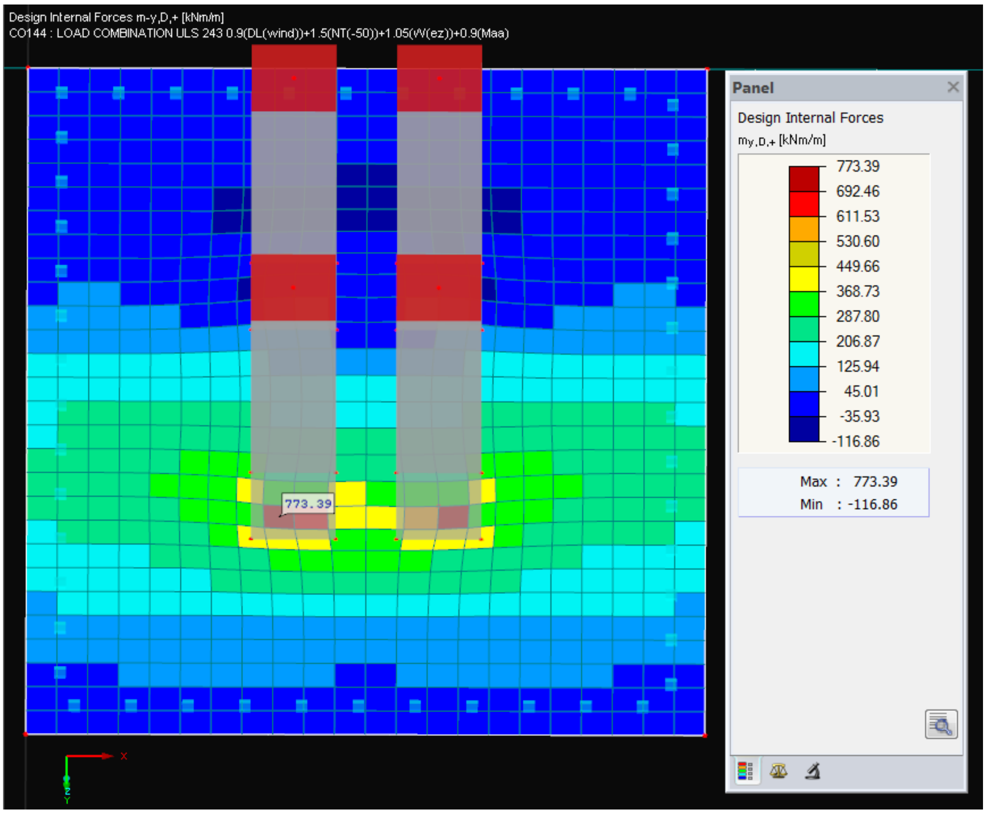


Figure 21. Moment distribution, mesh size 250mm

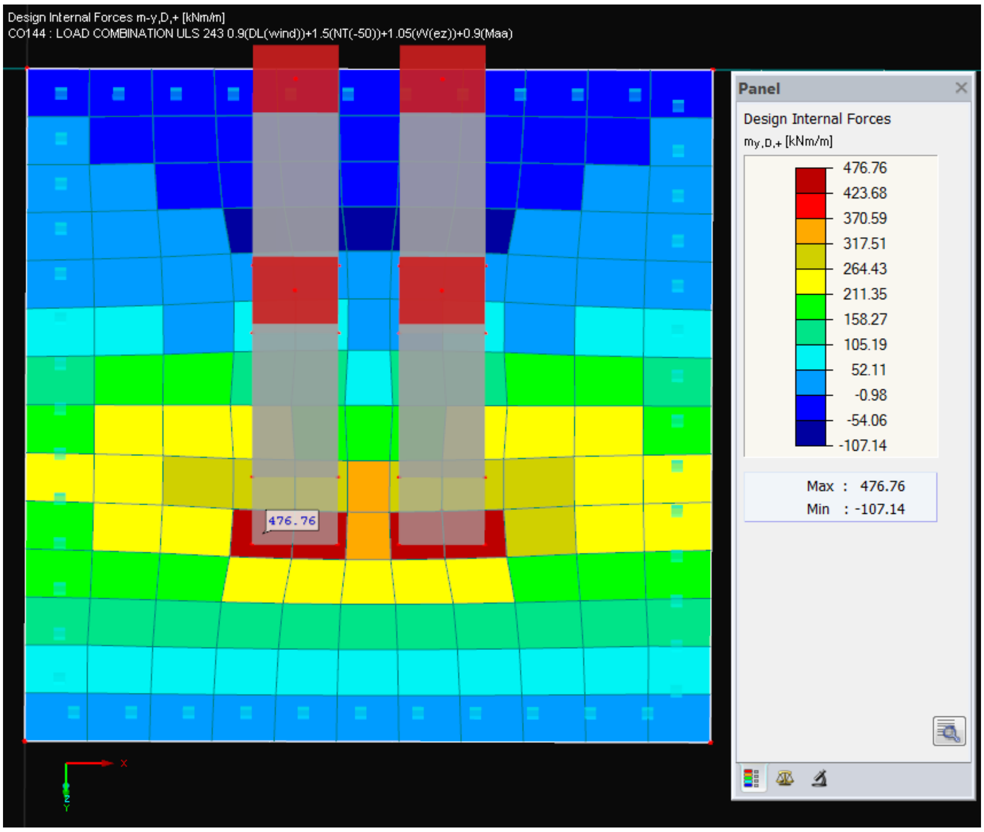


Figure 22. Moment distribution, mesh size 500mm

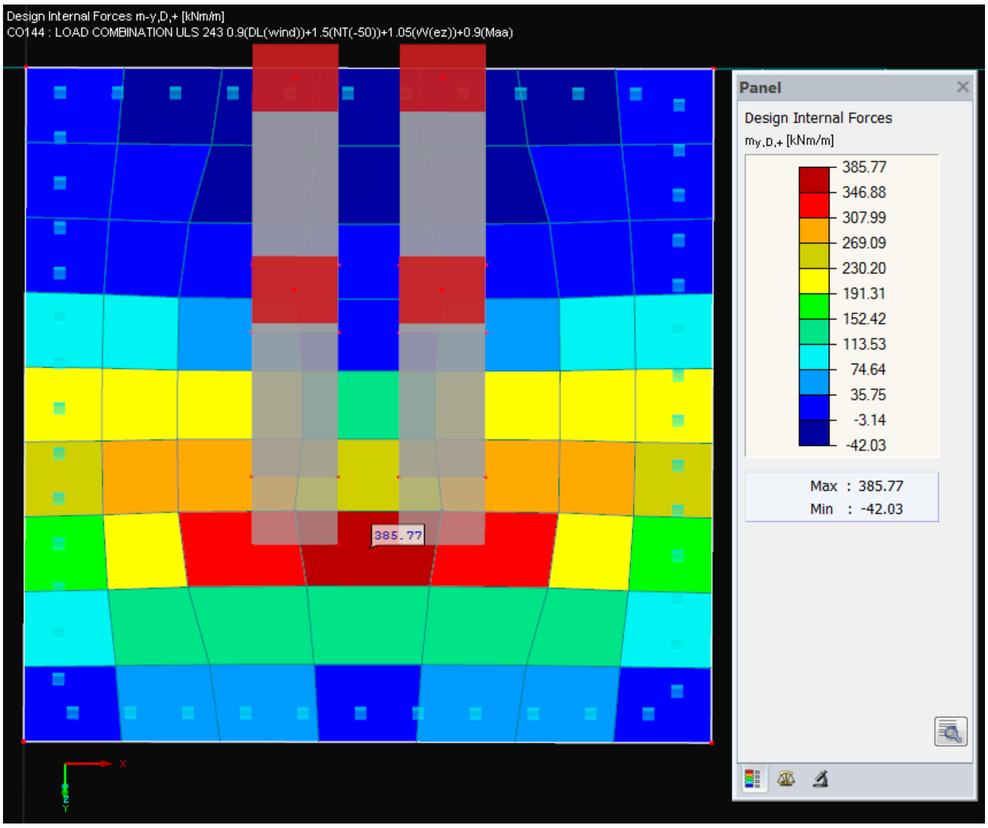


Figure 23. Moment distribution, mesh size 750mm

As the calculation results in **Figures 21, 22** and **23** demonstrate, there is a 50 % difference in the design moment between the mesh size 250 mm and the mesh size 750 mm. For the reinforcement in the concrete this means that As. req = 1722 mm² increases to As. req = 3624 mm², a difference of 52 %.

3.3.2 Smoothing of the Peak Values: Moment

The determination of the allowable width for the moment peak values starts by testing of the different recommendations. One has the moment distribution showing in the FEM–model and then the right size of the currently studied average width is measured in the model. When the width of the critical section is known, then the average of the moments of this width is calculated. In **Figure 24** the basic idea is presented.

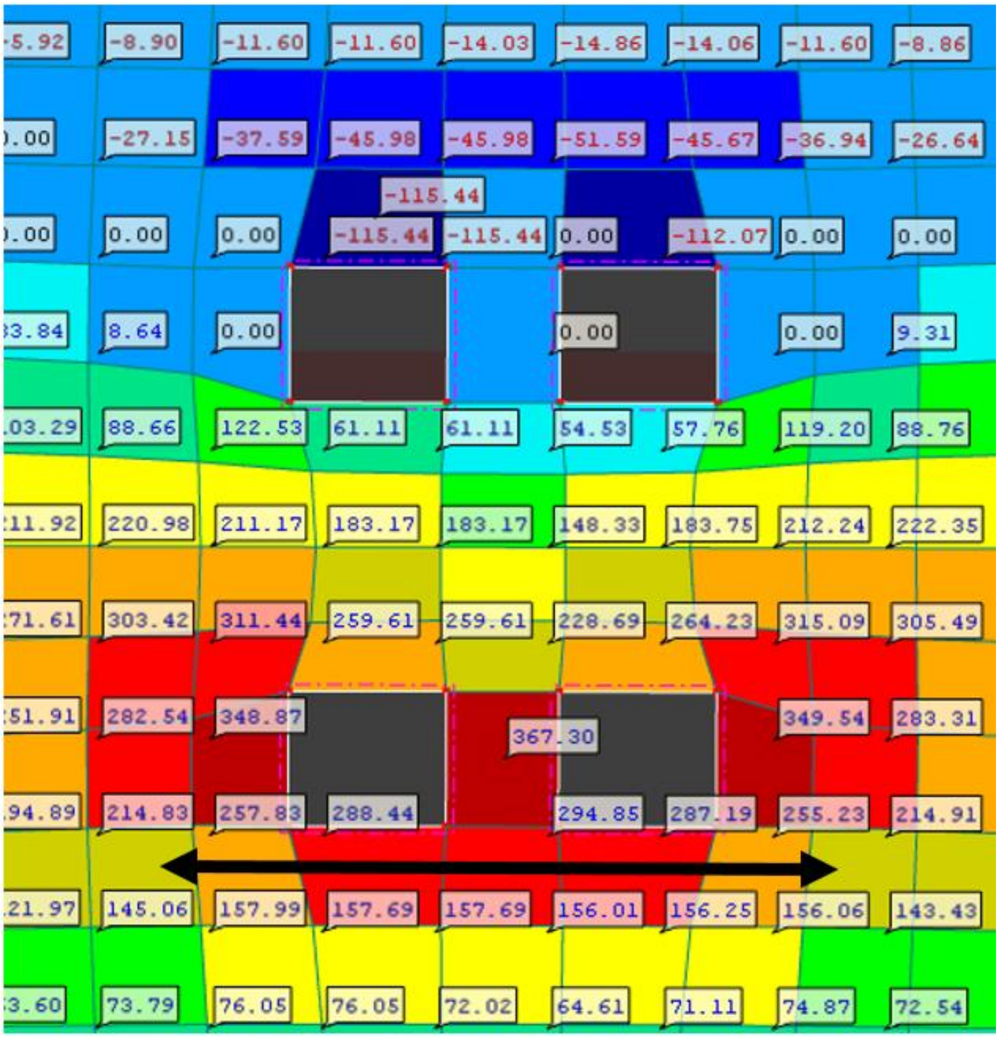


Figure 24. Max bottom moments with width of critical section marked.

The Excel calculation (presented in appendix) gives the following distribution widths:

- 5 x d – Method (illustrated in the **Figure 7**) gives a width 3.0 m
- Lc/4 – ULS Distribution Method (illustrated in the **Figure 9**) gives a width 0.9 m

From the calculation of the most suitable recommendations found for the optimized design moment, the results are the following:

- 5 x d – Method gives a design moment: 256 kNm/m
- Lc/4 – ULS Distribution Method gives a design moment: 256 kNm/m

The reinforcement provided for the slab is recalculated with the new given design moment: 256 kNm/m, and a new result for the reinforcement is shown in the **Figure 25.**

Which reinforcement is to be outermost (<M>ain or <T>ransversal) | M |

	Loads acting on cross section				Reinforcements		
	M_{Ed}	M_{Ek}	$\eta =$	V_{Ed}	ø	c / c	
	kNm	kNm	M_{Eqp}/M_{Ek}	kN	mm	mm	
Btm.main	256.0	150.0	1.00	10.0	16	175	As > As.min, OK!
							As.min.c OK!
							c/c OK!

Figure 25. The recalculated bending reinforcement

The total amount of the reinforcement provided in bottom slab is 1116 mm².

3.3.3 Smoothing of the Peak Values: Shear Force

When determining the allowed average width for the shear forces, the same distribution widths can be tested when determining the average width for the moments.

The Excel calculations gives the following distribution widths:

- 5 x h – gives a width: 3.0 m
- Lc/4 – ULS Distribution gives a width: 0.9 m
- a/2 + d – Method (illustrated in the **Figure 14**) gives a width: 1.05 m

The calculation of the most suitable recommendations found for the optimized design shear force, the results are the following:

- 5 x h – gives a design shear force: 268 kN/m
- Lc/4 – ULS Distribution gives a design shear force: 290 kN/m
- a/2 + d – Method gives a design shear force: 210 kN/m

The new shear force calculated is now: 290 kN/m. The maximum shear capacity of the slab was calculated in the Client EC2 Slab Excel Tool to be 338 kN/m. With the shear force being 290 kN/m, the concrete structure will not crack.

3.4 Excel Comparison

Loads for the excel comparison was taken by adding a support at top of column to get the result forces from the respective load combination.

Figure 26 shows the FEM model and the ULS forces:

- N_d= 818 kN, Vd_y=76kN, Vd_x= 33kN.

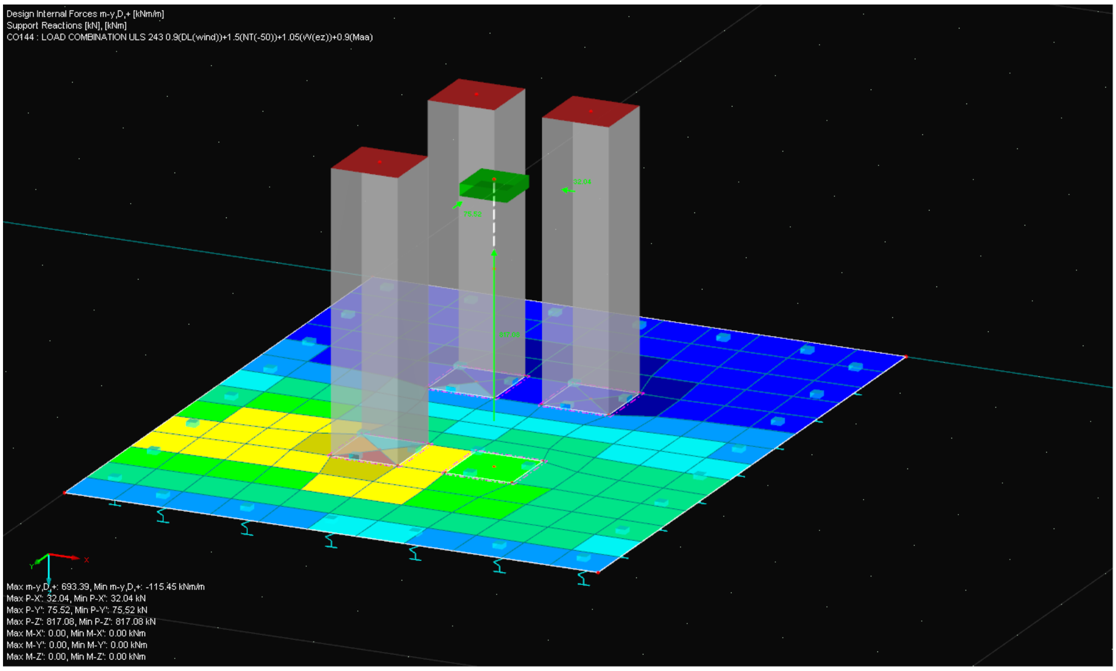


Figure 26. The reaction forces in the FEM model

On the next page, **Figure 27** presents the Excel calculation of the reinforcement. Dimensions were modified to have matching edge distances, as the excel tool is built for a single pedestal. The redefined dimensions for the structure in the Excel calculation are 4.4m x 4.8m.

CHECK N_d, V_d, M_d and γ_g FOR WORST CASE

FORCES (ULS and SLS)		ULS,STR	ULS,EQU			SLS	
Vertical load (Z)	$N_{d(Z)}$ =	818	0	kN	$N_{k(Z)}$ =	0	kN
Horizontal load (X)	$V_{d.X}$ =	33	0	kN	$V_{k.X}$ =	0	kN
Horizontal load (Y)	$V_{d.Y}$ =	76	0	kN	$V_{k.Y}$ =	0	kN
Overturning moment around Y	$M_{d1.Y}$ =	0	0	kNm	$M_{k1.Y}$ =	0	kNm
Overturning moment around X	$M_{d1.X}$ =	0	0	kNm	$M_{k1.X}$ =	0	kNm

PAD FOUNDATION DIMENSIONS				COLUMN DIMENSIONS		
Pad length (dimension in Y-direction)	L =	4.40	m	Column length C_L =	0.70	m
Pad width (dimension in X-direction)	B =	4.80	m	Column width C_W =	0.70	m
Pad thickness	H =	0.60	m	Column height C_H =	2.90	m
Initial depth	H_p =	1.50	m			
Concrete density	γ_{conc} =	25.0	kN/m³			

CHECK FOR GROUND BEARING RESISTANCE AND OVERTURNING
ACCORDING TO EN 1997-1, section 2.4.7.3.4.3 (Design Approach 2)
RECTANGULAR BASEPRESSURE DISTRIBUTION ASSUMED IN ULS CALCULATIONS

Bearing capacity (short term)	**R/A' = q_m =**	**200**	**kN/m²**	γ_g = 1.35
Soil density	γ_{soil} =	18	kN/m³	γ_{g2} = 0.9
Weight of concrete	$N_{k.conc}$ =	352.3	kN	$N_{k.conc} = \gamma_{conc}*(H*B*L+C_L*C_W*C_H)$
Weight of soil on top of found.	$N_{k.soil}$ =	334.2	kN	$N_{k.soil} = \gamma_{soil}*C_H*(B*L-C_L*C_W)$
Total normal force	$N_{d.tot}$ =	1744.8	kN	$N_{d.tot} = N_d+\gamma_g*(N_{k.conc}+N_{k.soil})$
Total overturning moment around Y	$M_{d.tot.Y}$ =	115.5	kNm	$M_{d.tot.Y} = M_{d1.Y} + (H+C_H)*V_{d.X}$
Total overturning moment around X	$M_{d.tot.X}$ =	266.0	kNm	$M_{d.tot.X} = M_{d1.X} + (H+C_H)*V_{d.Y}$
Eccentricity, ULS (STR)	**$e_{d.X} = M_{d.tot.Y}/N_{d.tot}$ =**	**0.066**	**m**	**ed<B/3 OVERTURNING OK.**
Eccentricity, ULS (STR)	**$e_{d.Y} = M_{d.tot.X}/N_{d.tot}$ =**	**0.152**	**m**	**ed<L/3 OVERTURNING OK.**
Eccentricity, ULS (EQU)	**$e_{d2.X} = M_{d.tot.Y}/N_{d.tot}$ =**	**0.000**	**m**	**ed<B/3 OVERTURNING OK.**
Eccentricity, ULS (EQU)	**$e_{d2.Y} = M_{d.tot.X}/N_{d.tot}$ =**	**0.000**	**m**	**ed<L/3 OVERTURNING OK.**
Effective width	B' =	4.67	m	$B' = B-2*e_{d.X}$
Effective length	L' =	4.10	m	$L' = L-2*e_{d.Y}$
Ground pressure	**$q_d = N_{d.tot}/(L'B')$ =**	**91**	**kN/m²**	**qd<qm BEARING RESISTANCE OK.**

REINFORCEMENT DESIGN ACCORDING EN-1992-1-1:2004
Dimensioning for bending moment. Sect. 6.1 & 9.3.1.1

Concrete grade	f_{ck} =	C30	MPa	γ_c =	1.5	
Steel yield strength	f_{yk} =	500	MPa	α_{cc} =	0.85	
Dimensioning bandwidht	L_{mit} =	1	m	f_{cd} =	17.00	MPa
	$p_{d.tot} = q_d - (p_{d.conc} + p_{d.soil})$ =	49.2	kN/m²	f_{ctm} =	2.90	MPa
Concrete cover to bottom rebars	c_{nom} =	75	mm	γ_s =	1.15	
	Outer bars:	X-direction		f_{yd} =	434.8	MPa

Dimensioning around Y-axis (bars in X-direction)			Dimensioning around X-axis (bars in Y-direction)			
$M_{Ed.Y}$ =	**103.3**	**kNm**	**$M_{Ed.X}$ =**	**84.1**	**kNm**	M_{Ed} = Bending moment at column face
$Ø_X$ =	16	mm	$Ø_Y$ =	16	mm	Ø = Main bar diameter
c/c_X =	250	mm	c/c_Y =	250	mm	c/c = Center to center distance
d_X =	517	mm	d_Y =	501	mm	d = H - c - Ø/2
$A_{s,prov.x}$ =	804	mm²	$A_{s,prov.Y}$ =	804	mm²	
$A_{s,req.x}$ =	465	mm²	$A_{s,req.Y}$ =	390	mm	$A_{s,req} = [-d+(d^2-2M_{Ed}/(L_{mit}f_{cd}))2]/[f_{yd}/(L_{mit}f_{cd})]$
λx_x =	20.6	mm	λx_Y =	20.6	mm	$\lambda x = A_s*f_{yd}/(L_{mit}*f_{cd})$
ε_{sx} =	6.7	%	ε_{sY} =	6.5	%	$\varepsilon_s = (d-x)*\varepsilon_{cu}/x$
$M_{Rd.X}$ =	**177.2**	**kNm**	**$M_{Rd.Y}$ =**	**171.6**	**kNm**	**$M_{Rd} = A_s*f_{yd}*(d-0.5*\lambda x)$**
εs>εyk STEEL STRAIN OK.			**εs>εyk STEEL STRAIN OK.**			
MRd>MEd OK.			**MRd>MEd OK.**			

$A_{s,min} = Max(0.0013;0.26*f_{ctm}/f_{yk})*L_{mit}*Max(d_x,d_y)$ = 779 mm² **As.min(9.1) OK.**

Figure 27. Calculation results for the reinforcement, Client EC2 pedestal Excel tool

The required reinforcement in the main direction of the slab are presented in **Figure 28**. The total amount of the reinforcement provided is 804 mm².

SUMMARY OF RESULTS				
Soil bearing utility ratio:	q_d/q_m =	45.6%		
Bending reinforcement ratio	M_{Ed}/M_{Rd} =	58.3%		
Punching shear ratio	$v_{Ed}/v_{Rd.c}$ =	55.0%		
Flexural shear ratio	$V_{Ed}/V_{Rd.c}$ =	36.8%		
Cracking moment ratio (SLS)	$M_{Ek.Y}$ / $M_{Cr.Y}$ =	1.6%	kNm	Mcr > MEk: no cracking in Theory.
	$M_{Ek.X}$ / $M_{Cr.X}$ =	1.3%	kNm	Mcr > MEk: no cracking in Theory.
Max Crack width (SLS)	w_k =	0.02	mm	OK, wk<wk.max
Main reinforcement X-direction:	Provide ø16 at c/c 250 mm			As.min(9.1) OK. & As.min(7.1) OK.
Main reinforcement Y-direction:	Provide ø16 at c/c 250 mm			
Shear reinforcement	vEd/vRd.c ≤ 100% => No need to provide shear reinforcement			

Figure 28. The provided reinforcement

4. EVALUATION

4.1 Comparison

It is clear that the singularity value management approach gives more optimized design moments. The difference between reinforcement with singularity value management and without the singularity value management is significant:

- **With SVM**: The total amount of the reinforcement provided in bottom slab in main direction is 1116 mm^2.
- **Without SVM**: The total amount of the reinforcement provided in bottom slab in main direction is 3703 mm^2.

The difference between the reinforcements provided is: 70 %

It can be noted that the Excel calculations give the reinforcement amount provided 804mm^2. This is in line with the calculations done with singularity value management, but not fully comparable as it was for a single pedestal. It might be safe to say that there is some truth in the assumption that the moment peak values are not the point of the interest, when designing a reinforced concrete slab.

Other observation is the mesh size comparison and the different result in the design moments. The calculation results show that there is a 50 % difference in the design moment between the mesh size 250 mm and the mesh size 750 mm. It should be taken into account, when modelling the structure that is not necessary to calculate the model with too fine a mesh.

For the reinforcement in the concrete the difference in the design moments means that As. req = 1722 mm^2 increases to As. req = 3624 mm^2, when the mesh size changes from 250 mm to 750 mm. The difference in reinforcement required is then 52 %. The change between the design moments with the different mesh sizes is substantial and cannot be neglected.

When smoothing out the shear force, the results also point out the fact that there is a huge difference with the maximum shear force and the smoothed one. The maximum shear force is 70 % larger. The higher shear force will lead to increased thickness of the concrete slab and higher costs. The difference between the shear force results are in line with the differences in calculations between the different design moments as well.

4.2 Risks of Singularity Value Management

The structure cannot be modelled to be an exact copy of the real structure. It should always be kept in mind that even at its best a numerical model is only a simplification of reality. It is only as accurate as its basic assumptions. There is a lot that depends on the designer itself. Different designers will get different results from the same structure.

Most design engineers believe that a detailed model and numerical calculation saves reinforcement. Because of the complexity of the model, that is not very often the case. /4/

If the singularity problems are neglected without any singularity value management, there is a risk for over–dimensioning of the structures. Singularity problems usually may occur in the regions of:

- Walls that end within a slab
- Discontinuous line supports
- Pin supports
- Openings
- Re–entrant corner and obtuse corners
- Concentrated loads /4/

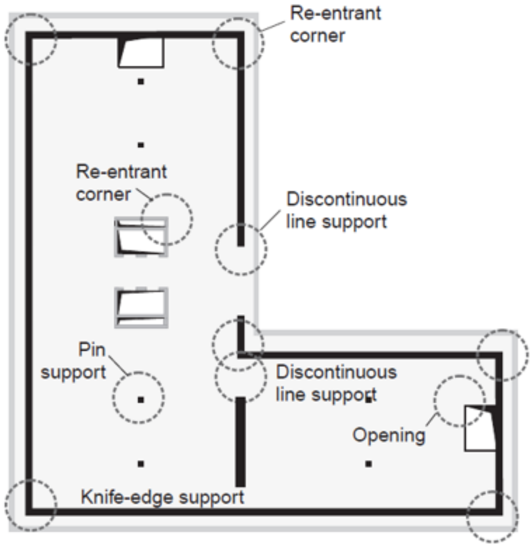


Figure 29. Singularity problem in the regions /redrawn 4/

4.3 Open Points

There is a limited guidance and literature about the subject. That makes the assumptions and recommendations about singularity value management a difficult issue to investigate. At its best, the findings and comparison results can only be a good estimate.

It would be useful to do further research on the results of several real projects, so there can be even more accurate results to present.

The thesis does not take any stand on singularity value management on other structures than a concrete slab. The subject is quite challenging to limit and there would be different type of structures that can be studied about the topic, if the study was expanded to other concrete structures as well.

5. CONCLUSION

There was a real need for a guide that uses singularity value management when optimizing concrete structures. The differences between the calculations with or without the singularity value management revealed that the peak value issues of the moments and shear forces are important and need to be taken into account in order to prevent over–dimensioning the structure and to improve the cost–efficiency in the concrete design.

The understanding of the Finite Element Method and singularity value management are the keys in understanding how optimally design the concrete structure.

Through the study I have learned a great deal regarding the Finite Element Method, the behavior of concrete structures, the singularity value management and how to model the structures in FEM properly. The aim of the thesis was achieved by studying the literature about the subject and by implementing the learned things into to FEM-model and calculations.

The challenge during the thesis was the fact that there was limited information about the subject area of the thesis. This made the study very time consuming, but on the other hand, very interesting to explore.

The goal of the thesis was to provide a short and easy-to-read summery for designers to use, when modelling the reinforcement slabs. The goal was achieved and the summary will be shared to all designers in the concrete team.

6. REFERENCES

/1/ Pacoste, C., Plos, M. & Johansson, M. 2012. Recommendations for finite element analysis for the design of reinforced concrete slabs. Stockholm.

/2/ The Concrete Center. 2006. How to design reinforced concrete flat slabs using FEA. Surrey.

/3/ Blauuwendraad, J. 2010. Plates and FEM. Surprises and Pitfalls. London: Springer Publishing.

/4/ Rombach G.A. 2011. Finite-element Design of Concrete Structures. Practical problems and their solutions. 2. Edition. London: ICE Publishing.

/5/ Dlubal 2017. RFEM Add-on Module RF-Concrete Surfaces. Program description. Accessed 29.4.2019. https://www.dlubal.com/en/downloads-and-information/documents/online-manuals/rf-concrete-surfaces/01/01.

/6/ Dlubal 2016. RFEM 5 Spatial Models Calculated According to Finite Element Method. Program Description. Accessed 03.5.2019. https://www.dlubal.com/en/downloads-and-information/documents/online-manuals/rfem-5/01/0.

APPENDIX 1 Design guide for FEM implementation (CONFIDENTIAL)