Investigating the Creep Behaviour of Polypropylene

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Abstract:

A viscoelasticity consisting viscos and elastic part, is a model defining mechanical property of both elastic and viscous behaviour of a material. A polymer or a material can have both a linear and non-linear viscoelastic behaviour. Linear viscoelasticity of a thermoplastic polymer is a theory explaining a relationship between linear stress and corresponding strain rate at any particular time period. There are several mathematical models of linear viscoelasticity based on the Boltzmann superposition principle, some of a significant principle and theories, that portrays creep deformation are Maxwell, Kelvin-Voigt, Standard linear solid, Burgers model. Creep is a gradual deformation of a material. Factors like time, temperature, material properties, load etc, affects the creep deformation. But only the time dependent deformation or a creep of a polypropylene and stress relaxation under room temperature is analysed using Standard Linear Solid (SLS) model of viscoelasticity. Using a Testometric machine, step tensile test of a polypropylene as a thermoplastic material was conducted. A sample piece (dog-bone) of a polypropylene was used as a specimen, different parameters were set for each test and a test lasted for approximately 2 hours each. Basically, test for creep and stress relaxation were conducted using a step tensile method. As a creep test, initial stress was applied, and the same stress was held over time to get a creep curve and the rate of strain over time. For the stress relaxation, initial strain was applied, and the same strain was kept throughout a test to get a relaxation curve and a rate of change of stress. Under SLS model a stiffness or young's modules of a linear spring and a coefficient of viscosity of a linear viscous dashpot were calculated from the result. A time dependent stress and strain equation were developed based the constitutive equation and resulted parameters from the experiment. The result compromised that creep or deformation varies linearly over time for PP as a thermoplastic material and also verified that a constitutive model of linear viscoelasticity based on Boltzmann Superposition is very much applicable to analyse a time dependent creep or deformation under a room temperature. As expected, the experimental work is very much adequate to the past experiments and theory of viscoelasticity.

Keywords:	Viscoelasticity, Creep, Linear Viscoelasticity, Stress Relaxation, Models of Linear Viscoelasticity, Polypropylene
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Forewords

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1 INTRODUCTION

1.1 Background

During the 19th century Maxwell, Boltzmann, Kelvin and other physicists started the research about viscoelasticity, creep and recovery of glasses, metals and rubbers. In 20th century, following this research, further studies with synthetic polymers were made to examine the viscoelastic behaviour of different polymers by providing different mathematical models of viscoelasticity. Today, most of the experiments are based on Maxwell and Kelvin-Voigt model combined with the Boltzmann superposition principle, to describe a creep behaviour of different materials. Depending on these models, various research (specially time-temperature based) has been completed in the past, regarding creep behaviour also described as viscoelastic behaviour of a material. [1]

Creep, being a slow and gradual or continuous deformation of any material under constant stress (significantly lower than ultimate tensile strength) would have a great impact on the reliability of a material structure, dimension and failure of a material, over a certain period of time [2].

Polypropylene is a rigid, semi-crystalline thermoplastic polymer made by combining propylene monomers in 1951 by a Phillips Petroleum scientist named Paul Hogan and Robert Banks. It was commercialized after 3 years, in 1954 by Italian chemist Professor Giulio Natta. Polypropylene being highly chemical resistance, tough (ability to deform without breaking), fatigue resistance (retain shape after bending), low density, highly insulative over electricity, relatively slippery surface, inexpensive, made it most commonly produced and used plastic in today's world. Despite of its shortcomings like highly flammable, susceptible to UV degradation and oxidation, high thermal expansion, it is an ideal choice over many other materials. [3] Considering its advantages, it is widely used in plastics furniture, gears in a machinery, packaging and containers, living hinges, casing electrical products, marine sector, medical application [4].

So, it is clear that understanding a creep behaviour of a polypropylene is a very important aspect during the designing and manufacturing process. Various factors influence the creep of thermoplastic polymer or polypropylene example; time, temperature, alignments and geometry, fibre volume fraction, molecular chain and bonding etc. [2] However, this experimental work depends only upon the continuous stress over a period of time under a room temperature, neglecting other factors that affect creep strain.

1.2 Objectives

Polypropylene, being a largest and widely used plastic material, it is necessary to know the durability and the factors affecting it. Time-dependent behaviour under certain load is one of the most common factors affecting the life of any thermoplastic polymer. So, this experiment is mainly focused on analysing a creep response of polypropylene as a thermoplastic material. The aim of this work also is to define a stress-strain model as a function of time, which can be used to predict a time-dependent deformation of polypropylene. The resulted mathematical expression and model can also be used in a finite element analysis.

The main objectives of this thesis work are pointed as:

- Analysing creep and relaxation of a polypropylene using Standard Linear Solid models of viscoelasticity.
- Developing a stress and strain model as a function of time, to predict a possible creep behaviour of a polypropylene and be able to assume a deformation over any period of time.

2 LITERATURE REVIEW

2.1 Viscoelasticity

Ludwig Boltzmann, an Austrian physicist was the first person to coin the term viscoelasticity in the 19th century. Afterward, several researches have been conducted and various models have been developed to define the viscoelastic behaviour of a material. [5]

The term viscoelasticity consists of viscous and elastic part meaning, a mechanical property of a material showing both viscous and elastic characteristics under a certain load and temperature on the process of deformation [5]. A viscous material exhibits a time dependent behaviour, when a load is applied, the material starts to deform at a constant rate and after removing the load, the material forgets its original state and stays in a deformed state. Whereas an elastic material deforms under a certain stress and after removing a load or a stress, it remembers its original shape and returns back to its original state. A material showing both of these properties is viscoelastic material and it presents a time-dependent strain showing a fading memory. It can also be defined as the time and temperature dependent properties under a certain load and can be classified into three forms as linear, non-linear and plastic depending upon the response of a material undergoing deformation. [1]

2.2 Creep

A slow and a gradual deformation of a material under a constant stress at any temperature is defined as a creep. Generally, it occurs due to long-term influence of high temperature (may also occur at a room temperature) or a constant stress over time or with aging. It can be divided into three stages: primary, secondary and tertiary stage as it goes for continuous deformation at a decreasing, constant and increasing rate with respect to these stages and finally at some point it may end with a rupture. [2]

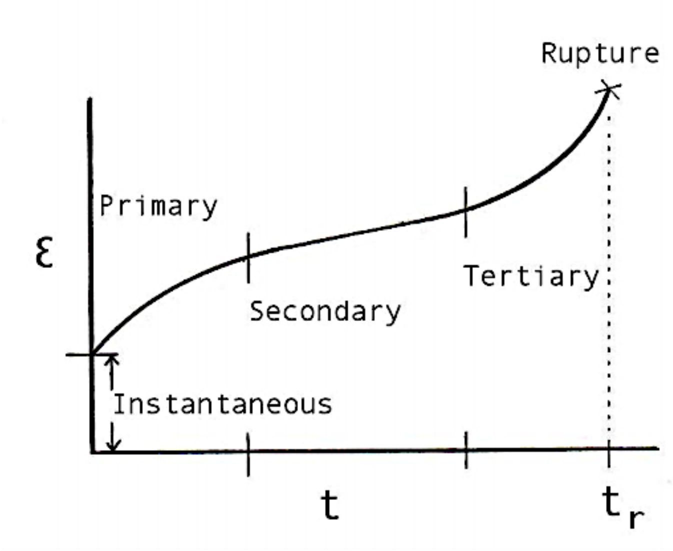


Figure 1. Classical creep stages curve (creep strain over time) [1]

2.2.1 Creep-Recovery

The phenomenon of loading a material at a constant stress and holding that stress for a period of time and removing the stress is a creep-recovery test [6]. As shown in the figure below, an increasing strain over time, after an instantaneous strain is creep strain, and the test focusing only until this point or a loading period is a creep test [7].

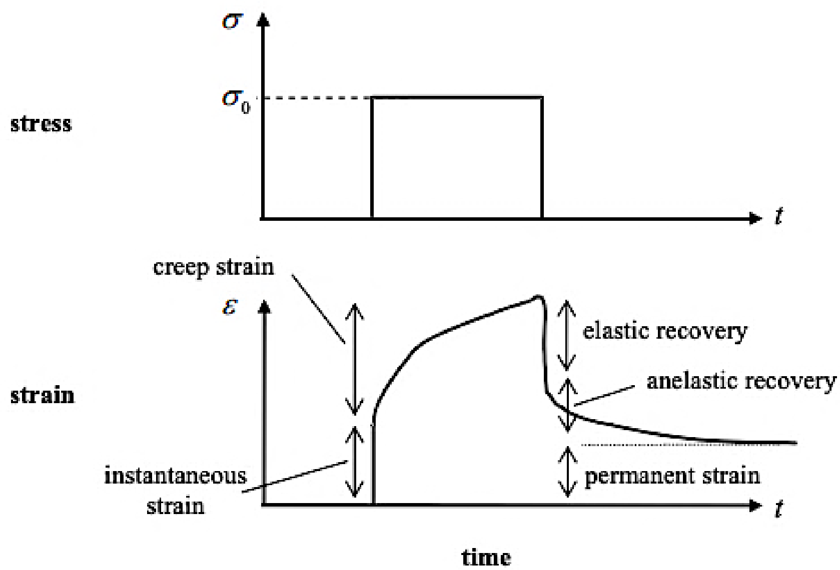


Figure 2. Response to creep-recovery test [7]

As per figure 2, at first there is an instantaneous elastic strain, followed by an increasing strain over time is creep strain, before the recovery. The ratio of total strain to an applied constant stress is creep compliance. [8]

$$J(t) = \frac{\varepsilon(t)}{\sigma_0} \qquad \text{Eq. 1}$$

Where,

$J(t)$ *is creep compliance*

$\varepsilon(t)$ *is total strain*

σ_0 *is a constant stress applied*

When a load is removed, a reverse elastic strain or a recovery of the creep strain appears at a decreasing rate, this stage is defined as a recovery stage. The recovery process is also called a delayed elasticity as it is a part of the elastic region. The creep recovery starts with an immediate elastic recovery, followed by anelastic recovery (strain recovered over time). Finally, may be left with a permanent strain or deformation, depending on the type of a material; it can be seen above in figure 2. [7]

2.2.2 Stress Relaxation

The phenomenon of viscoelastic material to return to their original shape upon removal of loading, where stress varies with time under a constant strain is stress relaxation. It is a time-dependent decrease in stress under a constant strain. It describes how a material relieves a state of stress under a constant strain whereas creep test describes how a material strain under a constant stress or load. [9] Basically, both creep and stress relaxation are long-term deformation over time, that help to determine dimensional stability and viscoelastic properties of a polymer or any material [10].

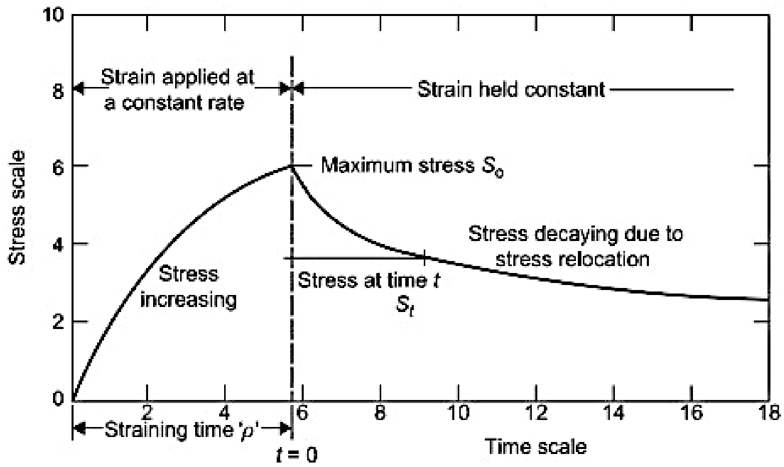


Figure 3. General stress relaxation curve over time [10]

The above figure 3 describes a general experience of the stress relaxation curve over time. The basis of this test is measuring the amount of load required to maintain a certain strain over a different time interval. It takes some amount of load to stretch a test material to a certain level and after some time period, a gradual decrease in amount of load required to maintain that strain will be observed. This process is also known as stress decay, as the stress is decreasing with time. It is a difficult task to perform accurately, and the calculation may differ from material to material. [10]

2.3 Linear viscoelasticity

Linear viscoelastic materials are those materials having a linear relationship between strain rate and stress applied over time. The theory to define such a relationship with an extension of linear elasticity and hyper elasticity in a certain time is linear viscoelasticity. [11] All models of linear viscoelasticity, simple or complex, are formed by combining a Hookean elastic deformation (represented by a linear elastic spring) and a Newtonian flow (represented by a linear viscous dashpot). Complex models with more components are likely to get more accurate results [6]. Some mechanical models of linear viscoelasticity to describe the behaviour of a system or a material are discussed below.

2.3.1 Linear elastic spring model

This model has a general linear-elastic spring with a stiffness “E”. When a stress is applied, there will be an instantaneous strain into the system and an instantaneous de-strain of a material will be seen upon the removal of load or a stress. There will be no viscosity involved in this model. [12] It can be defined by a simple relationship between stress and strain,

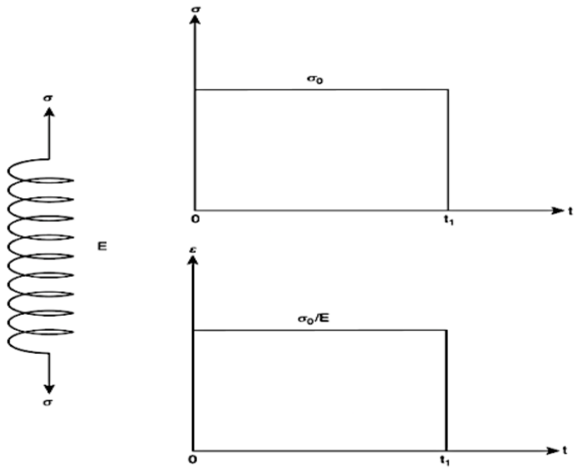


Figure 4. Linear spring response to a constant stress [2]

Mathematically [7],

$$\varepsilon = \frac{\sigma}{E} = S\sigma$$

Rearranging,

$$\boldsymbol{\sigma = E\varepsilon} \qquad \text{Eq. 2}$$

Where,

E *is stiffness or a young´s modulus*

S *is a compliance modules (or inverse of stiffness)*

σ *is stress applied*

ε *is strain*

2.3.2 Linear viscous dashpot model

A dashpot is a mechanical device or a damper with piston-cylindrical arrangement, filled with a viscous liquid. By pulling the piston through the fluid, a certain response in strain is realized. During a test, a stress will be applied and kept fixed for a period of time, before removing the stress completely. [12]

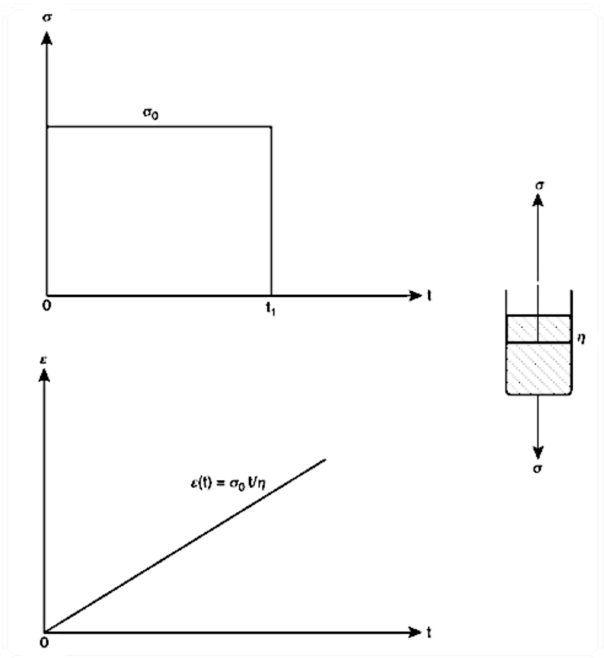


Figure 5. Linear viscous dashpot response to a constant stress [2]

At the beginning, when a stress is applied, the strain initially will be zero as shown in the above figure 5, as it takes some time for the molecules to rearrange. Then the strain will be seen in an increasing order reaching its maximum level, with a constant stress. At this period the rate of strain is constant. Beyond this point, even if the stress is removed the deformation is going to stay at the same place (permanent deformation) as there will be no elasticity or no stress to move the piston back in the system. [12] This phenomenon can be defined as,

$$\dot{\varepsilon} = \frac{\sigma}{\eta}$$

Rearranging,

$$\boldsymbol{\sigma = \eta\dot{\varepsilon} = \frac{d\varepsilon}{dt}\eta}$$ Eq. 3

where,

$\dot{\varepsilon}$ *is rate of change of strain with time*

η *is coefficient of viscosity*

Actual strain can be achieved by integrating the above equation and assuming no strain at time $t = 0$.

$$\varepsilon = \frac{\sigma_0}{\eta}t$$

Where,

σ_0 *is initial stress*

t *is a consecutive time*

2.3.3 Maxwell model

James Clerk Maxwell defined this model in 1867 and named the model by his name [2]. The model consisting a linear spring and a linear dashpot in a series or a straight line is represented by a Maxwell model of linear viscoelasticity, as shown in figure 6 below,

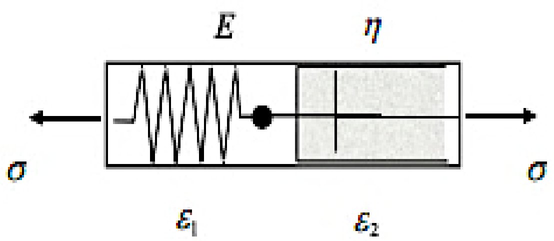


Figure 6. Schematic of Maxwell model [1]

According to the model, if a load or stress is applied, the stress for both the spring and dashpot will be the same, but the total strain will be divided into strain on spring and strain on dashpot. [7] Which gives,

$$\varepsilon = \varepsilon_1 + \varepsilon_2$$
$$\sigma = \sigma_1 = \sigma_2$$

Where,

$\sigma_1 = E\varepsilon_1$(spring) and $\sigma_2 = \eta \frac{d\varepsilon_2}{dt}$ (dashpot)

Taking derivatives of strain equation with respect to time,

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt}$$

Replacing value of $\varepsilon_1 = \frac{\sigma_1}{E}, \frac{d\varepsilon_2}{dt} = \frac{\sigma_2}{\eta}$ and $\sigma = \sigma_1 = \sigma_2$

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}$$

Gives, the basic constitutive equation of Maxwell Model [7],

$$\boldsymbol{\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\dot{\varepsilon}} \qquad \text{Eq. 4}$$

2.3.3.1 Creep-Recovery

When a load or an initial stress (σ_0) is applied to the model, the spring will stretch immediately but the dashpot will take some time to react. So, the initial strain $\varepsilon(0) = \frac{\sigma_0}{E}$ will be realized [8].

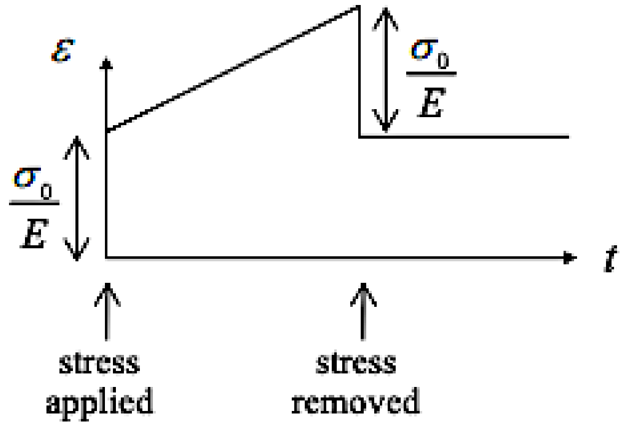


Figure 7. General creep-recovery response of Maxwell model [7]

During a test, there will not be any change in the stress rate i.e., stress will be kept constant throughout the test at σ_0, and $\dot{\sigma} = 0$. Then the equation will be,

$$\dot{\varepsilon} = \frac{\sigma_0}{\eta}$$

Integrating both sides, gives,

$$\varepsilon(t) = \frac{\sigma_0}{\eta} t + C$$

To get the values of integrating constant, apply initial condition at t=0, $\varepsilon(0) = \frac{\sigma_0}{E}$

$$C = \frac{\sigma_0}{E}$$

Replacing value of C,

$$\varepsilon(t) = \frac{\sigma_0}{\eta} t + \frac{\sigma_0}{E}$$

Rearranging,

$$\boldsymbol{\varepsilon(t) = \sigma_0 \left(\frac{1}{\eta} t + \frac{1}{E}\right)} \quad \text{Eq. 5}$$

is function of creep response. [8]

Can also be expressed as a creep compliance function as [7],

$$\varepsilon(t) = \sigma_0 J(t)$$

Where,

$$J(t) = \left(\frac{1}{\eta} t + \frac{1}{E}\right)$$

Now, on removing the load, the spring will react immediately and the elastic part $\frac{\sigma_0}{E}$ will be recovered instantly, but the strain on dashpot will not be recovered at all and the creep strain will remain unrecovered. So, there will be some elastic response, permanent strain but no anelastic recovery. As per creep, this model assumes strain to increase at an increasing rate with time. It is totally the inverse of polymeric behaviour as most parts of polymer illustrate the rate of strain to increase at a decreasing rate with time. Thus, Maxwell model is considered less accurate on analysing the creep of a polymer or any other material. [7]

2.3.3.2 Stress Relaxation

The stress relaxation begins with a constant strain of ε_0 at time t=0. The strain will be held constant i.e., $\dot{\varepsilon} = 0$, and the amount of stress over time will be recorded. [7]

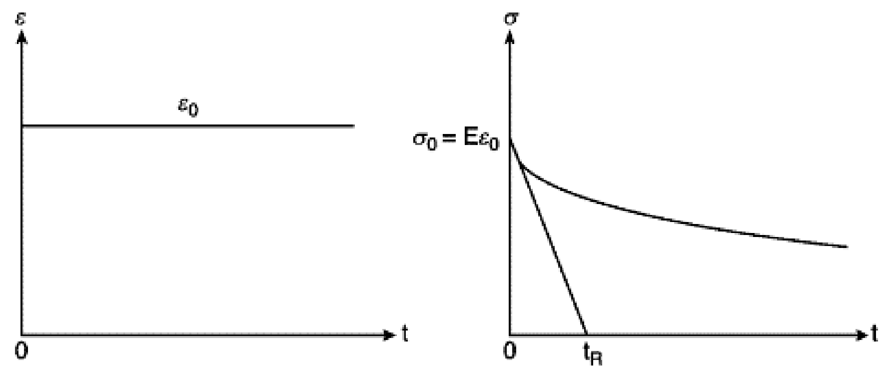


Figure 8. General relaxation response of Maxwell model [2]

The stress function is given by integrating constitutive equation of Maxwell model ($\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\dot{\varepsilon}$) and applying above initial condition [2],

$$\boldsymbol{\sigma(t) = \sigma_0 e^{-\frac{Et}{\eta}} = \varepsilon_0 E e^{-\frac{Et}{\eta}}} \qquad \text{Eq. 6}$$

Can be written as [12],

$$\sigma(t) = \varepsilon_0 E(t)$$

Where,

$E(t) is\ a\ relaxation\ function = \ Ee^{-\frac{t}{t_R}}$ and,

$t_R\ is\ a\ relaxation\ time = \frac{\eta}{E}$

So, the Maxwell model states that the stress decays exponentially with time, which is genuine for most of the polymers. So, this model is very much accurate in predicting stress relaxation of a material. [12]

2.3.4 Kelvin-Voigt model

A British physicist, Lord Kelvin and a German physicist, Woldemar Voigt defined this model and named it after their name. It is a combination of linear spring and linear dashpot kept in a parallel circuit as shown below [13],

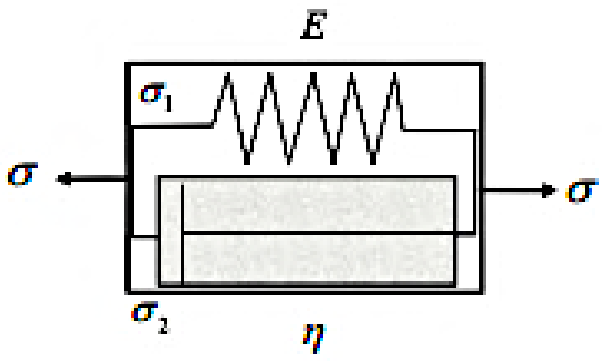


Figure 9. Schematic of Kelvin-Voigt Model [13]

Corresponding models [2],

For linear spring,

$$\sigma_1 = \varepsilon_1 E$$

For linear dashpot,

$$\sigma_2 = \eta \frac{d\varepsilon_2}{dt}$$

Taking both together as a parallel arrangement, no bending is supposed to happen and so there will be the same strain experienced in both dashpot and spring. However, stress will be divided partially by dashpot and the spring. Represented as,

$$\varepsilon = \varepsilon_1 = \varepsilon_2$$

$$\sigma = \sigma_1 + \sigma_2$$

Combining all these equations above and replacing the values [2],

$$\boldsymbol{\sigma = \varepsilon E + \eta \frac{d\varepsilon}{dt}} \qquad \text{Eq. 7}$$

2.3.4.1 Creep-Recovery

A creep curve, followed by a recovery is shown in figure 10 below. In the figure 10, x-axis represents time and y-axis represents rate of strain due to an applied initial stress [14].

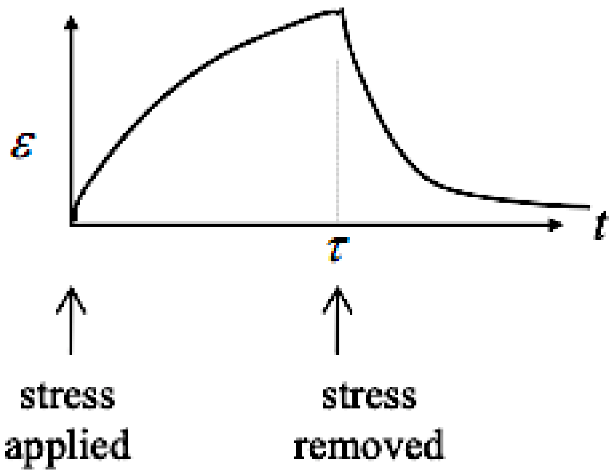


Figure 10. General creep and recovery curve of Kelvin Voigt model [14]

When a load σ_0 is applied, all the initial stress will be carried by the dashpot or a viscous part and no instantaneous strain will be seen in spring or a dashpot. So, the creep curve will have an initial slope of $\frac{\sigma_0}{\eta}$. Slowly, the viscous element starts to elongate, the strain starts to increase in a decreasing rate and the stress begins to transfer from dashpot to spring but still dashpot will have more stress than spring. So, at the moment, creep curve will have a slope of $\frac{\sigma_2}{\eta}$.

As the decrease of stress in the dashpot continuous, it will reach a point where spring will carry all the stress, i.e. σ_2=0 and reaches a maximum strain at $\frac{\sigma_0}{E}$. Then the permanent deformation starts to begin. [12]

Solving $\sigma = \varepsilon E + \eta \frac{d\varepsilon}{dt}$, as a first order non-homogeneous differential equation under initial stress σ_0 and $\varepsilon(0) = 0$. The expression for Kelvin Voigt model is [15],

$$\boldsymbol{\varepsilon(t) = \frac{\sigma_0}{E}\left(1 - e^{-\left(\frac{E}{\eta}\right)t}\right)} \qquad \text{Eq. 8}$$

Can also be written as,

$\varepsilon(t) = \sigma_0 J(t)$

Where,

$J(t)$ is a creep compliance function, given by $J(t) = \frac{1}{E}(1 - e^{-\frac{t}{t_r}})$

t_r is a retardation time for creep strain to accumulate, given by $t_r = \frac{\eta}{E}$

After unloading, even though the dashpot holds the spring, the spring will pull the dashpot back to its original position and full recovery with time is often expected. This indicates that there will not be any permanent strain in the system. [14]

2.3.4.2 Stress relaxation

During the stress relaxation test, a material is subjected to a constant strain ε_0 at $t = 0$ gives $\frac{d\varepsilon}{dt} = 0$. This reduces the model to $\sigma = E\varepsilon_0$, which shows that stress is carried by the spring not the dashpot and is constant. As the stress is constant, there is no stress relaxation over time. This model is assumed to be good with modelling a creep, but modelling a relaxation is difficult and less accurate. [7]

2.3.5 Standard linear solid model

In 1985, an American physicist, Clarence Melvin Zener defined a new model by combining Maxwell and Kelvin-Voigt model called Zener's model or SLS model of viscoelasticity. It is a model of three parameters: two springs elements and one dashpot. This is one of the simplest forms of all the models that describes both creep-recovery and stress relaxation very well. [16]

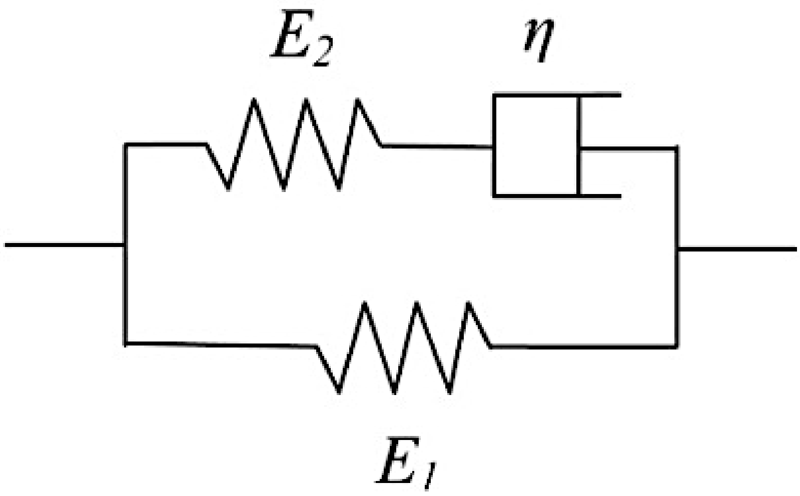


Figure 11. Schematic of Standard linear solid model [17]

Here, an additional linear elastic spring is added to a Maxwell model in parallel series [17]. To make it easy, the SLS model can be divided into two parts: equilibrium arm (spring) and a Maxwell arm (spring and dashpot). Let, overall stress and strain be σ and ε, stress and strain in spring part be $\sigma_1 and$ $\varepsilon_1$with a stiffness E_1. And in Maxwell part, overall stress and strain be $\sigma_2 and$ ε_2 with a stiffness E_2 and coefficient of viscosity η. [12]

As the maxwell and an equilibrium arm are in parallel sequence. When the stress is applied, total stress is divided into the maxwell arm and equilibrium arm.

$$\sigma = \sigma_1 + \sigma_2 \qquad \text{Eq. 9}$$

But a same amount of strain or elongation will be recorded in both maxwell and equilibrium arm.

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \qquad \text{Eq. 10}$$

In spring or equilibrium arm,

$$\sigma_1 = E_1 \varepsilon_1 = E_1 \varepsilon \qquad \text{Eq. 11}$$

In Maxwell arm,

Maxwell arm consist of a spring and a dashpot in a straight-line series. Let, stress and strain be $\sigma_{21}\ and\ \varepsilon_{21}$ (spring) and $\sigma_{22} and\ \varepsilon_{22}$ (dashpot).

Here a total strain is equal to the amount of strain observed in spring and a dashpot,

$$\varepsilon_2 = \varepsilon_{21} + \varepsilon_{22} \qquad \text{Eq. 12}$$

And, when stress is applied, both a spring and a dashpot will receive a same amount of stress.

$$\sigma_2 = \sigma_{21} = \sigma_{22} \qquad \text{Eq. 13}$$

Strain in spring part is given as,

$$\varepsilon_{21} = \frac{\sigma_{21}}{E_2} = \frac{\sigma_2}{E_2} \qquad \text{Eq. 14}$$

The function of strain on dashpot part is given by,

$$\varepsilon_{22} = \frac{\sigma_{22}}{\eta} t = \frac{\sigma_2}{\eta} t \qquad \text{Eq. 15}$$

From equation 12, 13, 14, and 15.

$$\varepsilon_2 = \frac{\sigma_2}{E_2} + \frac{\sigma_2}{\eta} t$$

Differentiating,

$$\dot{\varepsilon} = \frac{\dot{\sigma_2}}{E_2} + \frac{\sigma_2}{\eta}$$

Simplifying,

$$\dot{\varepsilon} = \sigma_2 \left(\frac{\frac{d}{dt}\eta + E_2}{E_2 \eta} \right)$$

Can be written as,

$$\sigma_2 = \left(\frac{\dot{\varepsilon} E_2 \eta}{\frac{d}{dt}\eta + E_2} \right) \quad \text{Eq. 16}$$

From equation 9, 11, and 16.

$$\sigma = E_1 \varepsilon + \left(\frac{\dot{\varepsilon} E_2 \eta}{\frac{d}{dt}\eta + E_2} \right)$$

Simplifying,

$$\sigma \left(\frac{d}{dt}\eta + E_2 \right) = E_1 \varepsilon \left(\frac{d}{dt}\eta + E_2 \right) + \dot{\varepsilon} E_2 \eta$$

Simplifying again,

$$\sigma E_2 + \dot{\sigma}\eta = E_1 E_2 \varepsilon + E_1 \eta \dot{\varepsilon} + \dot{\varepsilon} E_2 \eta$$

Dividing both side by E_2,

$$\boldsymbol{\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1 \varepsilon + \frac{\eta(E_1 + E_2)}{E_2}\dot{\varepsilon}} \quad \text{Eq. 17}$$

is the constitutive equation of SLS model. [18]

2.3.5.1 Creep-recovery

When an initial stress (σ_0) is applied, initially all the stress is carried by both the spring and an instantaneous strain of $\varepsilon_0 = \frac{\sigma_0}{E_1 + E_2}$ will be recorded. A creep curve will have an initial slope of $\frac{\sigma_0}{E_1 + E_2}$. [14] Slowly, the stress starts to transfer to a viscous element or a dashpot and starts to elongate. Over time a rate of strain starts to increase in a decreasing rate and at this point a dashpot will start to have a more stress than spring. Before recovery, the spring in spring arm will stretch completely and reaches its maximum at $\frac{\sigma_0}{E_1}$. [19]

After removing the stress, a quick recovery on both springs will be seen. As the springs are set in a parallel series, a recovery of $\frac{\sigma_0}{E_1}$ will take place but a dashpot part will remain unrecovered and a creep deformation will be realized. [19]

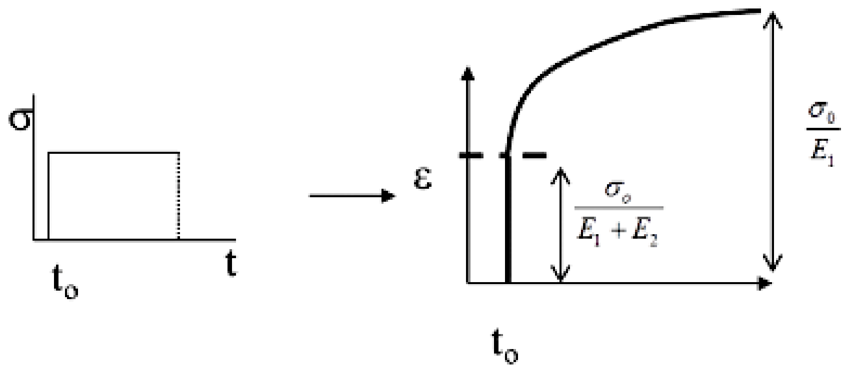


Figure 12. General creep response under SLS model [19]

During the test, stress will be kept at σ_0, and the stress is constant throughout the whole test i.e., $\dot{\sigma} = 0$. And there will be an instantaneous strain recorded in both the springs at time t_0. Applying this condition, constitutive equation of SLS model can be simplified as [16],

$$\boldsymbol{\sigma_0 = E_1\varepsilon + \frac{\eta(E_1+E_2)}{E_2}\dot{\varepsilon}}$$

A creep compliance or a strain response is given by using a Laplace transformation and generalizing the kelvin -Voigt model as,

$$\boldsymbol{\varepsilon(t) = \frac{\sigma_0}{E_1}\left(1 - \frac{E_2}{E_1+E_2}\, e^{-\left(\frac{t}{t_r}\right)}\right) = \sigma_0 J(t)} \qquad \text{Eq. 18}$$

And compliance function is given by,

$J(t) = \frac{1}{E_1}\left(1 - \frac{E_2}{E_1+E_2}\ e^{-\left(\frac{t}{t_r}\right)}\right)$ [18]

Where,

$$J(t)\ is\ a\ function\ of\ creep\ compliance$$

$$Retardation\ time, (t_r) = \frac{E_1 + E_2}{E_1 E_2}\eta$$

2.3.5.2 Stress relaxation

When a certain strain (ε_0) is applied, an instantaneous or initial stress on both of the spring will be seen as, $\sigma_0 = \varepsilon_0(E_1 + E_2)$. After holding a same strain continuously, a dashpot will gain some stress and will start to stretch as well to a certain time. Due to this stress will decrease over time and the relaxation occurs. [7]

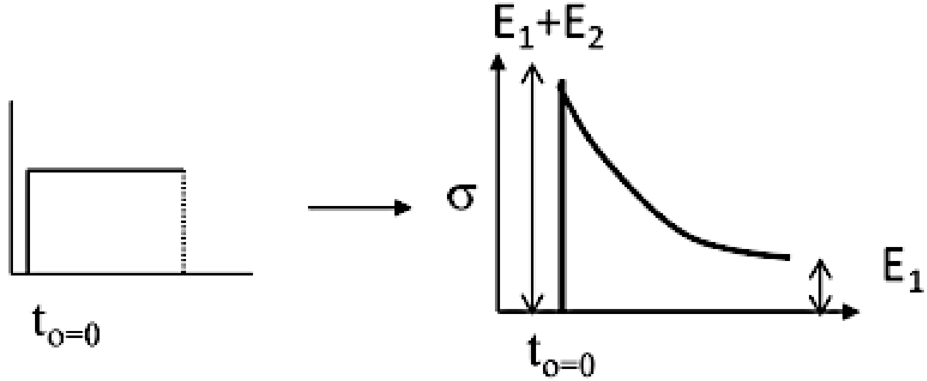


Figure 13. General stress relaxation response of SLS model [19]

During the test, strain will be kept constant at $\sigma_0 = \varepsilon_0(E_1 + E_2)$ and $\dot{\varepsilon} = 0$. Applying this condition in constitutive equation of SLS can be simplified as [16],

$$\boldsymbol{\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1\varepsilon_0} \qquad \text{Eq. 19}$$

Relaxation modulus or a stress function can be derived by using Laplace transformation and generalizing Maxwell model as [20],

$$\boldsymbol{\sigma}(\boldsymbol{t}) = \boldsymbol{\varepsilon_0}\left(\boldsymbol{E_1} + \boldsymbol{E_2}\, \boldsymbol{e}^{-\frac{t}{t_R}}\right) = \boldsymbol{\varepsilon_0}\boldsymbol{E}(\boldsymbol{t}) \qquad \text{Eq. 20}$$

and the relaxation function is given by,

$$\boldsymbol{E}(\boldsymbol{t}) = \left(\boldsymbol{E_1} + \boldsymbol{E_2}\, \boldsymbol{e}^{-\frac{t}{t_R}}\right)$$ [18]

Where,

$E(t)\ is\ a\ relaxation\ function$

$Relaxation\ time, t_R = \frac{\eta}{E_2}$

3 METHOD

3.1 Material

A pristine polypropylene copolymer (PP- BJ368MO) was used to injection mould a dog-bone sample pieces for the test. The injection moulding parameters and some physical characteristics from a manufacturer are shown in table 1 and 2 respectively.

Figure 14. Polypropylene dog-bone sample

Table 1. Moulding parameters

Melt temperature	210 – 260 °C
Holding pressure	200 – 500 bars
Mould temperature	10 – 30 °C
Injection speed	High

Table 2. Physical properties

Property	Typical value	Test method
Density	905 kg/m³	ISO 1183
Melt Flow Rate (230 °C/2,16 kg)	70 g/10min	ISO 1133
Flexural Modulus	1.500 MPa	ISO 178
Tensile Modulus (1 mm/min)	1.500 MPa	ISO 527-2
Tensile Strain at Yield (50 mm/min)	4 %	ISO 527-2
Tensile Stress at Yield (50 mm/min)	25 MPa	ISO 527-2
Heat Deflection Temperature	102 °C	ISO 75-2
Charpy Impact Strength, notched (23 °C)	5,5 kJ/m²	ISO 179/1eA
Charpy Impact Strength, notched (-20 °C)	3,5 kJ/m²	ISO 179/1eA

3.2 Experiment

Testometric material testing machine available in Arcada UAS was used for an experiment. With the help of a software called wintest analysis, step tensile test was created. The parameters were set accordingly to calculate the creep and stress relaxation of polypropylene. Several test for time dependent creep and stress relaxation under a room temperature was performed to ensure a better result.

Figure 15. Testometric Machine

3.2.1 Creep-recovery

With an applied stress, an instantaneous strain was recorded as a stretch in both the spring. Then a stress started to transfer to the dashpot and strain curve started to increase in decreasing rate over time. A quick recovery after 100 minutes was completed, and the elastic spring part was recovered while the viscous dashpot was left unrecovered as a creep deformation.

In a WinTest analysis software, a step tensile test with a test definition was created. In the definition a command of "Go to Stress and hold" was set as described in SLS model of viscoelasticity. Several tests were done for 100 minutes. Strain rate over time was observed to get a creep curve. During an experiment when a load or stress applied was 20 MPa or more, a fracture on sample piece was realized. So, most of a test were experimented with a stress of less than 20 MPa. But even with the load applied being less than 20 MPa, few of the sample piece broke during the process. A material failure during injection moulding was assumed to be a most possible reason for a sample piece to be broken during the test.

3.2.2 Stress relaxation

Using the same step tensile, three tests with an initial strain of 1.8 %, 1.6 % and 1.5 % was conducted. A test definition was created and a command of "Go to strain and hold" was set. Each test lasting for an hour. The strain was held and kept constant throughout the test. Initially, due to the elastic spring, an instantaneous rise in stress was recorded and with a transfer of stress from spring to a viscous dashpot, a gradual drop in level of stress is recorded. The amount of stress required to keep the same strain rate was observed over time to find a stress relaxation curve.

4 RESULT

4.1 Creep under SLS model of linear viscoelasticity

Several tests were experimented using step tensile method on Testometric machine and a most relevant and successful test is selected and calculated below.

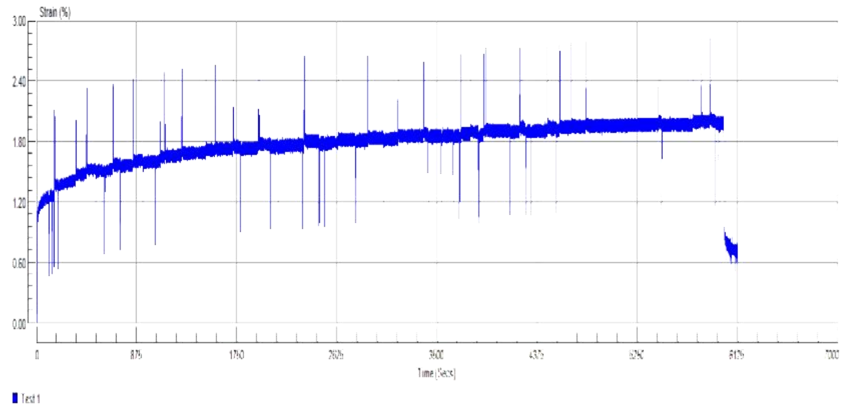
Testometric
materials testing machines

winTest™ Analysis

Creep Analysis_test1

Ref 1 :	Test Name : Nir_thesis
Ref 2 :	Test Type : Step Tensile
Ref 3 :	Test Date : 1/20/2021 4:34 PM
Ref 4 :	Pretension : Off
	Width : 13.000 mm
	Thickness : 3.000 mm
	Sample Length : 97.000 mm

Test No	Strain @ 1.000 Mins (%)
1	1.228



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Unit 1 Lincoln Business Park Lincoln Close, Rochdale, Lancashire, England OL11 1NR

Tel: (44) (0)1706 654039 Fax: (44) (0)1706 646089
Email: info@testometric.co.uk website: www.testometric.co.uk

Figure 16. Creep curve (raw data)

Initial stress of 10 MPa was applied to a polypropylene dog bone sample piece having a dimension of $(\,97\ mm \times 13mm \times 3mm)$ and the stress was held at a same level throughout the test for 100 minutes. Recovery of the creep was done by decreasing a load by 10 MPa. The rate of strain over time was recorded. A recorded creep curve as a function over time is shown in fig 14. A recorded creep curve is not smooth and lots of ups and down is realized due to the small disturbance in stress during the test.

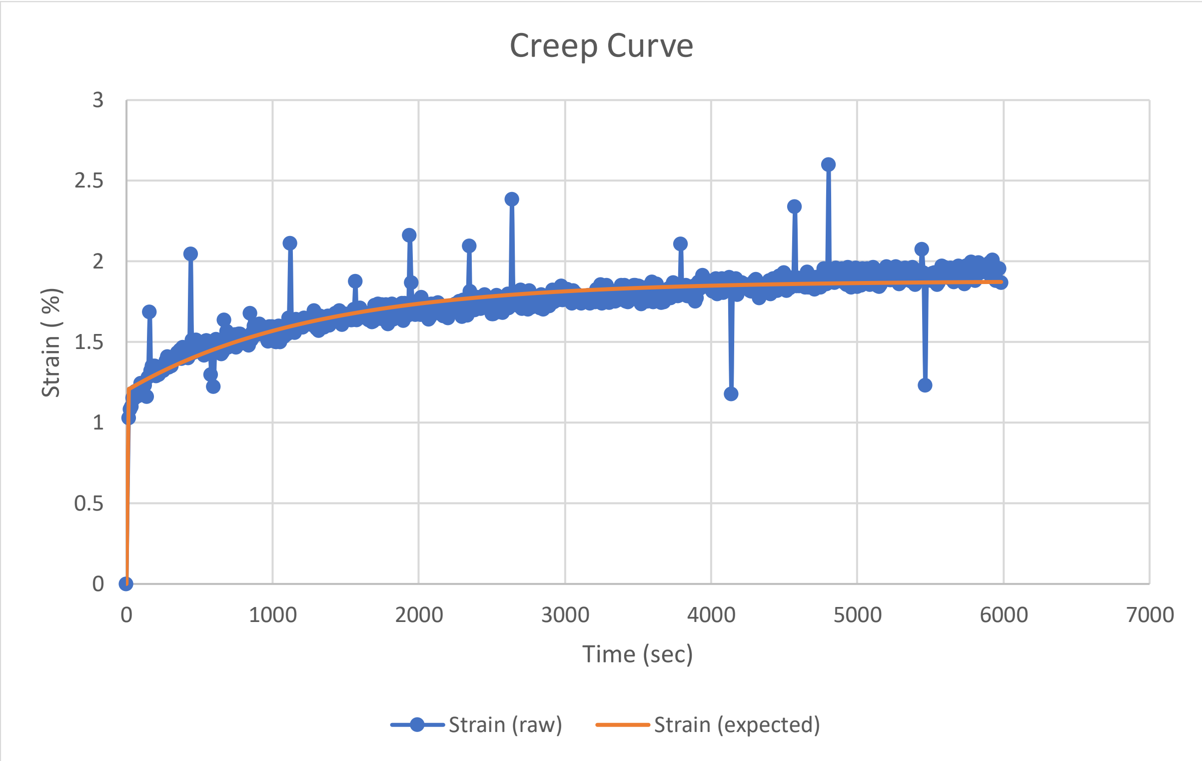

Figure 17. Creep curve (expected and raw)

In the above fig 15, blue curve represents a raw strain date having lots of disturbance. Referencing the raw curve, a regression curve is created, which is represented by orange line.

Calculation of creep modulus as a function of time:

Total test time = 100 min = 6000 sec

Initial stress $(\sigma_0) = 10\ MPa$

Stress is kept constant $\dot{\sigma} = 0$

Initial strain recorded $(\varepsilon_0) = 1.028\% = 0.01028$

Strain at the end $(\varepsilon_\infty) = 1.1954\% = 0.01954$

From the theory the function of strain in given by,

$$\varepsilon(t) = \frac{\sigma_0}{E_1}\left(1 - \frac{E_2}{E_1+E_2}\ e^{-\left(\frac{t}{t_r}\right)}\right)$$

At the end of the test,

$$\varepsilon_\infty = \frac{\sigma_0}{E_1} \qquad \text{As, } e^{-\left(\frac{t}{t_r}\right)} = 0$$

Replacing values,

$$0.01954 = \frac{10}{E_1}$$

Gives,

$\boldsymbol{E_1 = 511\ MPa}$

Applying the initial condition of SLS model, as initially the stress is consumed only by two elastic springs.

$$\varepsilon_0 = \frac{\sigma_0}{E_1+E_2}$$

Replacing values,

$$0.01028 = \frac{10}{511+E_2}$$

Gives,

$\boldsymbol{E_2 = 461\ MPa}$

To find the value of $(t_r) = \frac{E_1+E_2}{E_1E_2}\eta$, a non-linear regression analysis of a raw strain curve was conducted on Microsoft excel. A formula of $\varepsilon(t) = \frac{\sigma_0}{E_1}\left(1 - \frac{E_2}{E_1+E_2}\ e^{-\left(\frac{t}{t_r}\right)}\right)$ was created in excel to find a time dependent strain, keeping the constant as, $E_1 = 511\ MPa$, $E_2 = 461\ MPa$, $(\sigma_0) =$

10 MPa. The value of eta (η) was changed until the new curve overlapped exactly to an original curve and the best suitable and suited value was picked as,

$\boldsymbol{\eta = 98000\ MPa.sec}$

$\boldsymbol{t_r = 404\ sec}$

Substituting the calculated values, to find the constitutive equation of time dependent creep under SLS model in $\varepsilon(t) = \frac{\sigma_0}{E_1}\left(1 - \frac{E_2}{E_1+E_2}\ e^{-\left(\frac{t}{t_r}\right)}\right)$. Gives,

$$\boldsymbol{\varepsilon(t) = 0.0195\left(1 - 0.4742\ e^{-\left(\frac{t}{404}\right)}\right)}$$

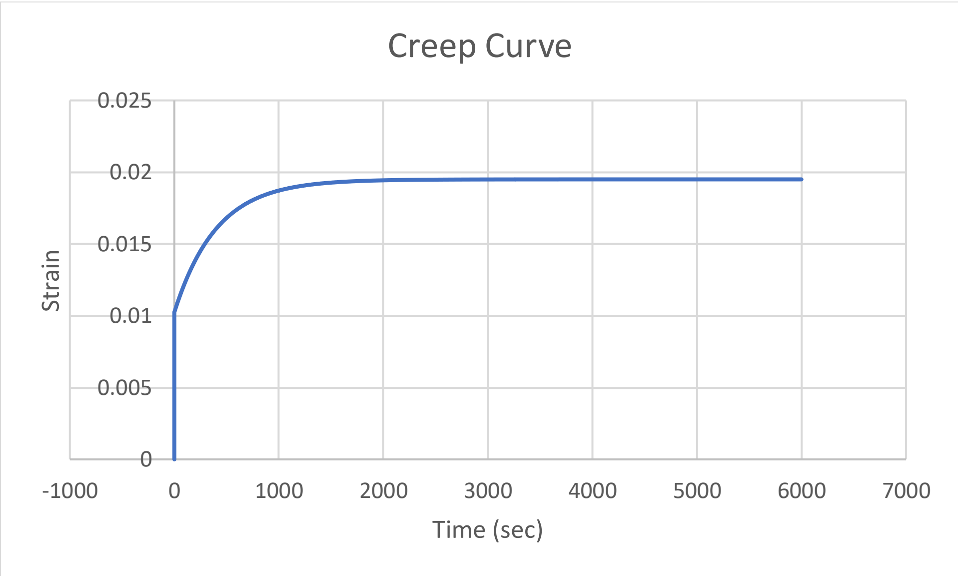

Figure 18. Creep Curve (as per constitutive equation)

Using an above equation $\boldsymbol{\varepsilon(t) = 0.0195\left(1 - 0.4742\ e^{-\left(\frac{t}{404}\right)}\right)}$ a creep curve is drawn over time period of 6000 second. In the fig 16. x-axis represents time (in seconds) and y-axis represents strain. Strain is kept constant at 10 MPa and a strain rate as a function of time is recorded to find a creep curve. Initial strain of 1.028 % was recorded and a gradual increase in strain is observed. As the time reaches to infinity the rate of strain increases to 1.95 % and it does not change much, then a creep or deformation occurs.

4.2 Stress relaxation under SLS model of linear viscoelasticity

Using a wintest analysis in a Testometric material testing machine, stress relaxation of polypropylene under a room temperature was conducted at Arcada UAS. Numerous tests were performed using different strain rate to determine a time dependent stress. The most appropriate and relevant data is evaluated further.

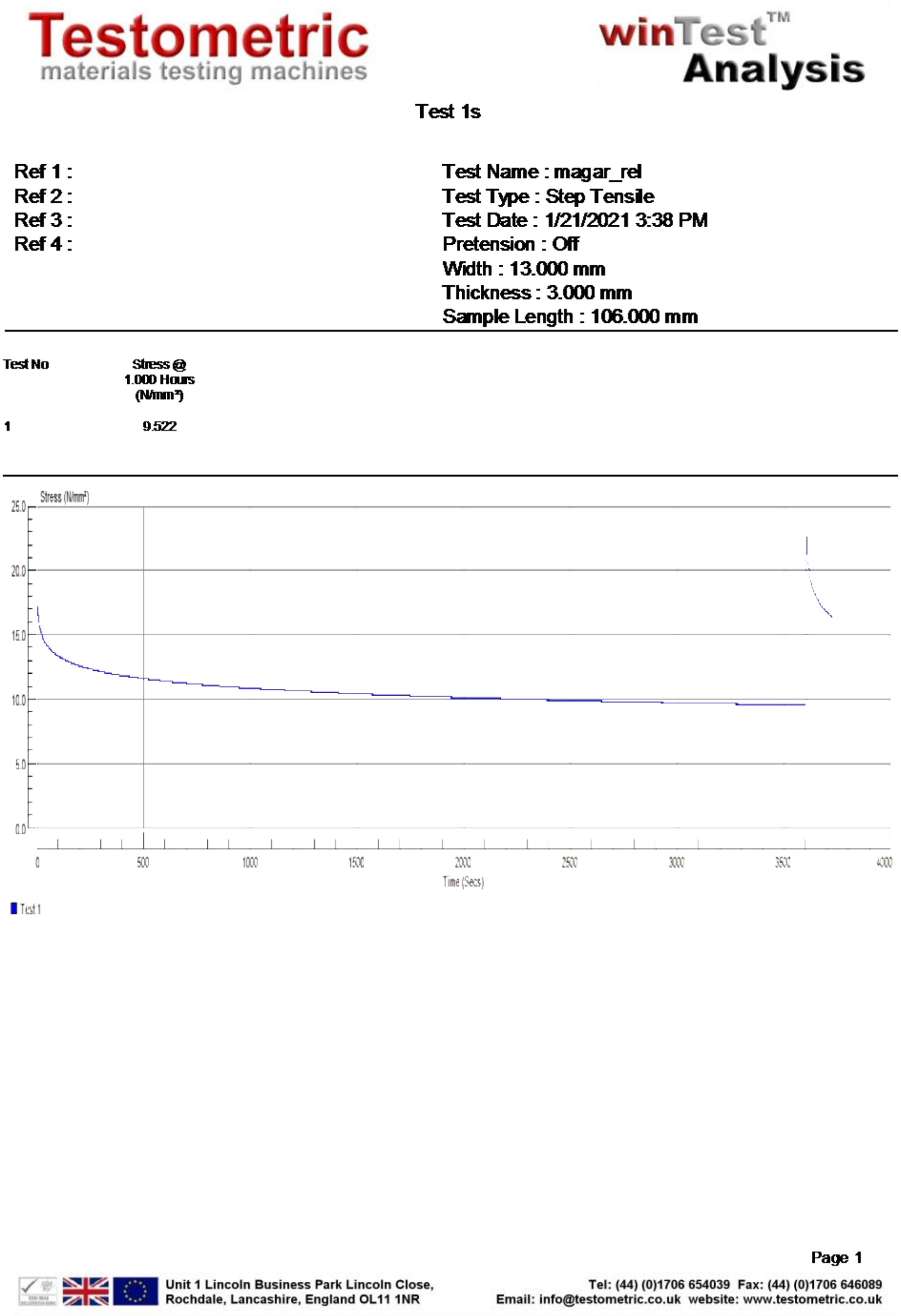

Figure 19. Stress Relaxation Curve (raw data)

Polypropylene dog bone sample piece with a dimension $(106\,mm \times 13\,mm \times 3\,mm)$ was examined for an hour. 1.6 % strain was applied throughout an hour and a stress was recorded over time. Initial stress reached to 17 MPa and a gradual decrease in stress with the increase in time, is recorded. As the time reaches to infinity, rate of change of stress decreases and after an hour, recorded stress reaches to 9.5 MPa. The recorded data is shown as a stress relaxation curve in above fig 17.

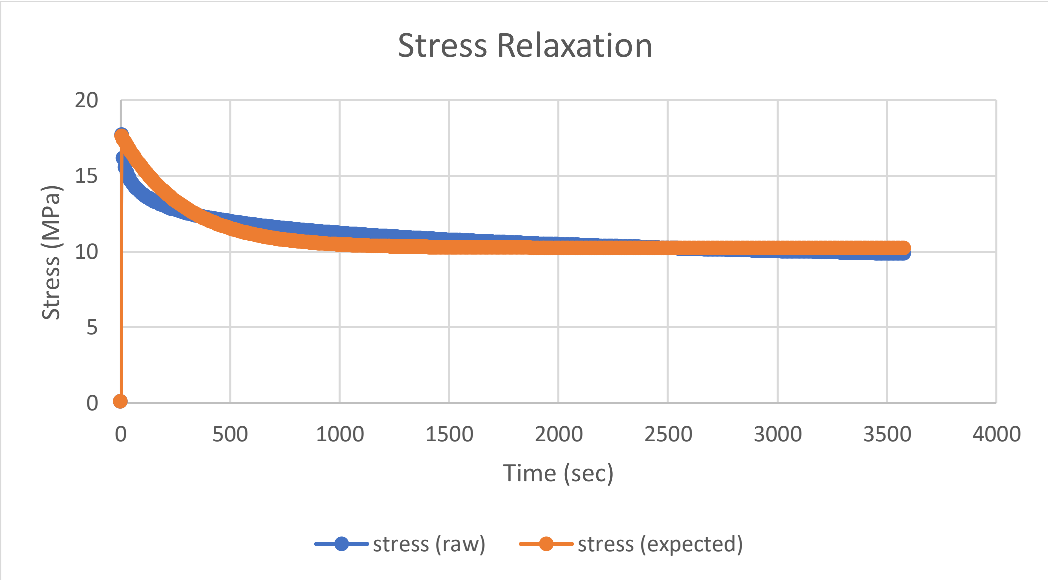

Figure 20. Stress relaxation curve (compared)

Figure 18. shows a comparison between a raw or actual relaxation curve (blue) and its regression curve (orange).

Calculation of relaxation modulus as a function of time:

Total test time $= 1\ hr = 3600\ sec$

Initial strain $(\varepsilon_0) = 1.6\ \% = 0.016$

Strain is kept constant $(\dot{\varepsilon}) = 0$

Stress recorded at the end $(\sigma_\infty) = 9.5\ MPa$

Initial stress recorded $(\sigma_0) = 17\ MPa$

The function of time dependent stress is given by,

$$\sigma(t) = \varepsilon_0 \left(E_1 + E_2\, e^{-\frac{t}{t_R}} \right)$$

At the end of the test,

$(\sigma_\infty) = \varepsilon_0 E_1$ As, $\left(e^{-\frac{3600}{t_R}} = 0 \right)$

Replacing values,

$\boldsymbol{E_1 = 593\ MPa}$

At the beginning of the test, initial stress is given by,

$\sigma_0 = \varepsilon_0\, (E_1 + E_2)$ As, $(e^{-0} = 1)$

Replacing values,

$\boldsymbol{E_2 = 469\ MPa}$

To find the value of stiffness (η) and relaxation time, a regression analysis of a stress curve was conducted on Microsoft excel. In a time-dependent stress column, a formula was created using

$\sigma(t) = \varepsilon_0 \left(E_1 + E_2\, e^{-\frac{t}{t_R}} \right)$. A constant value was kept as $(\varepsilon_0) = 1.6$ % , $E_1 = 593\ MPa$, $E_2 = 469\ MPa$ and a value of eta (η) was changed until the new curve overlapped exactly with an original curve. Through that the best suited value was noted as,

$\boldsymbol{\eta = 133410\ MPa.sec}$

$\boldsymbol{t_R = \frac{\eta}{E_2} = 284\ sec}$

Back substituting the values in constitutive equation of stress relaxation of SLS model,

$$\boldsymbol{\sigma(t) = 0.016\left(593 + 469\, e^{-\frac{t}{284}}\right)}$$

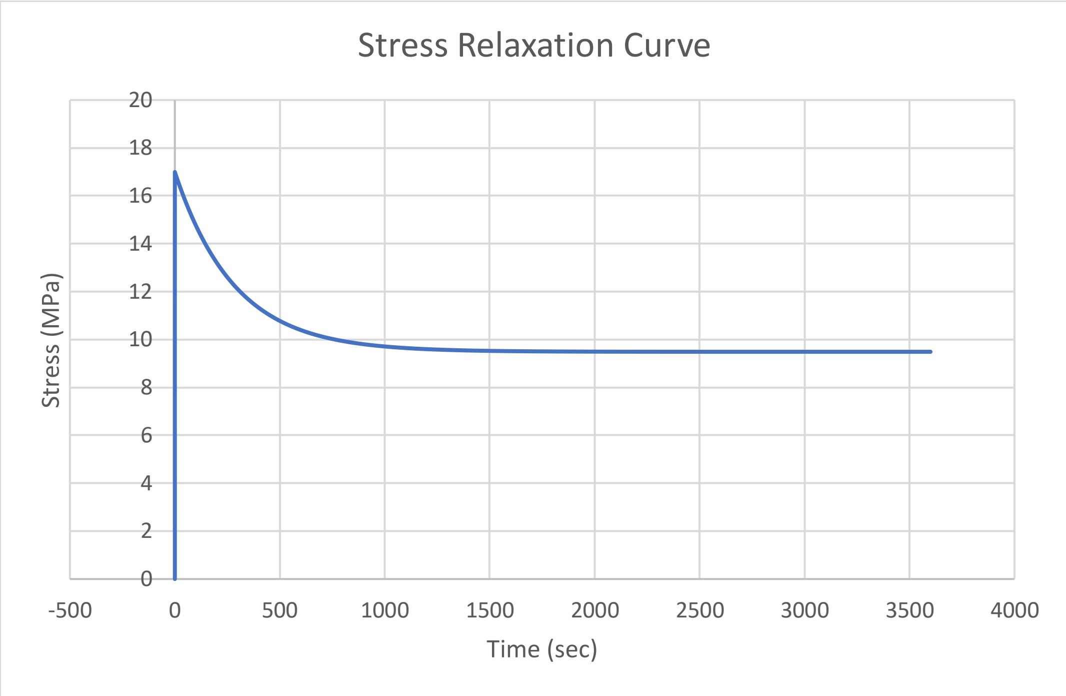

Figure 21. Relaxation curve (as per the constitutive equation)

The constitutive equation $\boldsymbol{\sigma(t) = 0.016\left(593 + 469\, e^{-\frac{t}{284}}\right)}$ of stress relaxation is used, and a relaxation curve is drawn over 3600 seconds. In the fig 19. x-axis represents time (in second) and y-axis represents stress (MPa). During the calculation initial strain of 1.6 % is applied and kept steady for 1hour. Initial stress of 17 MPa is recorded and there is gradual decrease in stress over time. As the time reaches to infinity, almost a constant rate of stress is realized, and a permanent deformation is likely to occur.

5 DISCUSSION

As stated, creep is a long term and a gradual deformation. Finding an actual deformation under room temperature may take several seconds to years, depending on a material and the amount of load. The experiment was performed based on Boltzmann superposition principle, for a time period of 2 hours (each test) and creep deformation was evaluated accordingly to viscoelastic theory of SLS model. According to the theory and the properties of polypropylene creep deformation was expected to rise with time and creep curve was expected to have a linear increment. But the creep curve achieved from the experiment was not showing that much of an increment (had a rise but not much). It resulted a calculated time dependent function of strain, deformation and a creep curve not to have an expected rise. However, some of the past research and experiments also had a similar result of not having a huge deformation or increment on a creep curve [22] [23]. So, despite of not having a huge rise in a creep curve, it is acknowledged as satisfactory to have such result. Still, it is anticipated that the amount of load applied with respect to time to be the possible reason for not having a better result or expected rise in deformation. It is anticipated, if the test was done for more time period with more stress, a deformation may be realized clearer with a more rise in creep curve.

The data accumulated from stress relaxation test with different initial strain rate was very reliable. The raw or the original curve appeared as expected to the theory of viscoelasticity. The calculated time dependent function of stress from the result also showed a steady decrease in stress. A continues decay of stress over time is achieved as well in the calculated result or relaxation curve. So, both resulted data or experiment is supposed to be very applicable to estimate the viscoelastic behaviour, creep deformation and stress relaxation of polypropylene as thermoplastic material.

6 CONCLUSION

The experiment, calculation and results conducted with PP under a room temperature has resulted a following conclusions:

- Using PP as a thermoplastic material, creep varied linearly with stress under a room temperature.
- The principle of a time dependent theory is very applicable to estimate a creep, relaxation and viscoelasticity of a thermoplastic material.
- Zenner or SLS model of viscoelasticity (Maxwell representation) can be used to calculate both creep and relaxation of a thermoplastic material, which completely fits an experimental deformation over time.

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