YRKESHÖGSKOLAN NOVIA

# Effect of Semi-Rigid Joints in Structural Steel Design

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Degree Thesis

Thesis for a Bachelor of Engineering – degree

Construction and Civil Engineering

Vaasa 2022

**DEGREE THESIS**


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| Specialization: | Structural Engineering |
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Title: Effect of Semi-Rigid Joints in Structural Steel Design


Date: 14.5.2022 Number of pages: 58 Appendices: 1



**Abstract**

This thesis is written on behalf of the Civil Engineering department at Citec Oy Ab, with the purpose of expanding steel design knowledge. More specifically to identify the rules and benefits of using semi-rigid joints compared to the limitations and risks of using traditional pinned- and rigid joints.

To gather necessary information for the subject a literature review has been done, which covers pinned-, rigid- and semi-rigid joints in elastic global analysis targeting joint stiffness and classification, economic aspect, and definition of global analysis.

The semi-rigid joint effect has been investigated through analysis of bending moment distribution and displacement in RFEM, and by buckling length calculations with RF-Stability and Designers guide to EC3, along with buckling theory.

The results consist of a compilation of traditional and theoretical joints and their effect on frame behavior, together with use, rules, and limits with semi-rigid joints. How semi-rigid joints affect the buckling length has been discovered, in addition to a relationship between the buckling length factor and joint initial stiffness.



Language: English
Key Words: semi-rigid joint, frame analysis, rotational stiffness, buckling length

**EXAMENSARBETE**


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| Inriktning: | Konstruktion |
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Titel: Effekten av eftergivliga anslutningar inom stålkonstruktion

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Datum: 14.5.2022 Sidantal: 58 Bilagor: 1


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## Abstrakt

Arbetet har gjorts på uppdrag av Citec Oy Ab:s byggnadskonstruktionsavdelning, med syftet att utöka kunskaper inom stålkonstruktion. Mera specifikt innebär arbetet att identifiera befintliga regler, samt lyfta fram fördelar med användningen av eftergivliga anslutningar. Detta i jämförelse med begränsningar och risker som tillkommer med traditionella ledade- och momentstyva anslutningar.

En litteraturöversikt har gjorts i samband med inhämtning av nödvändig information om ämnet. Den innehåller teori om ledade-, momentstyva- och eftergivliga anslutningar, dimensionering enligt elastisk global analys med inriktning på rotationsstyvhet och klassificering. Även ekonomiska aspekter för anslutningar och definitionen av global analys har redogjorts för.

Effekten av eftergivliga anslutningar har undersökts genom analys av momentfördelning och deformationer i RFEM, samt knäcklängdsberäkningar med RF-Stability och Designers guide to EC3. Allmänt om knäckningsteori för stålkonstruktioner har också lyfts fram.

Resultaten består av en sammanställning av traditionella och teoretiska anslutningar, och deras effekt på ramverksbeteende. Risker och begränsningar som tillkommer med användningen av traditionella anslutningar lyfts fram. Förutom hur eftergivliga anslutningar påverkar knäcklängder, har även ett samband mellan anslutningars initiella styvhet och knäcklängdsfaktorn påvisats.


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Språk: engelska
Nyckelord: eftergivlig anslutning, ramverksanalys, rotationsstyvhet, knäcklängd

**OPINNÄYTETYÖ**


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Nimike: Osittain jäykkien liitosten vaikutus rakenneterässuunnitteluun

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Päivämäärä: 14.5.2021 Sivumäärä: 58 Liitteet: 1


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**Tiivistelmä**

Tämä opinnäytetyö on kirjoitettu Citec Oy Ab:n rakennesuunnitteluosaston toimeksiannosta tarkoituksena laajentaa terässuunnittelun osaamista. Tarkemmin sanottuna osittain jäykkien liitosten käytön sääntöjen ja hyödyn tunnistaminen verrattuna perinteisten nivel- ja jäykkien liitosten käytön rajoituksiin ja riskeihin.

Aiheeseen tarvittavan tiedon keräämiseksi on tehty kirjallisuuskatsaus, joka kattaa nivel-, jäykät ja osittain jäykät liitokset elastisessa globaalissa analyysissä, ja kohdistuu nivelten jäykkyyteen ja luokitukseen, sekä taloudelliseen näkökulmaan ja globaalin analyysin määritelmään.

Osittain jäykkä liitosvaikutus on tutkittu analysoimalla taivutusmomenttijakaumaa ja taipumaa RFEM:ssä sekä nurjahduspituuslaskelmilla RF-Stabilitylla ja Designers guide to EC3:lla. Aiheeseen liittyvä nurjahdusteoria on myös esitelty.

Tulokset ovat kooste perinteisistä ja teoreettisista liitoksista ja niiden vaikutuksista kehärakenteen käyttäytymiseen sekä puolijäykkien liitosten käytöstä, säännöistä ja rajoituksista. Nurjahduspituustekijän ja liitoksen jäykkyyden välisen suhteen lisäksi on selvitetty, miten osittain jäykät liitokset vaikuttavat nurjahduspituuteen.


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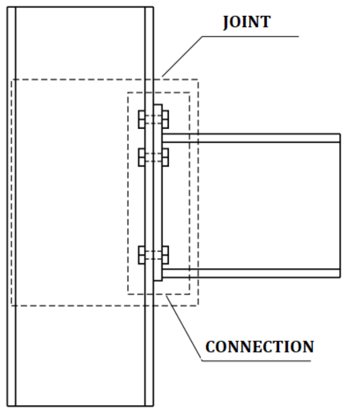
Kieli: englanti

Avainsanat: osittain jäykkä liitos, teräsrakenteen kehäanalyysi, kiertymäjäykkyys, nurjahduspituus

**TERMINOLOGY**

Joint – Refers to any kind of connection of members, or members that are designated pinned or rigid depending on their ability to transfer moment.

Connection – The connection itself, including end-plate, bolts and welds, not considering the connected members.




Semi-rigid joint – A resilient joint, resilient in terms of a joint that transfers bending moments.

Knee-joint - Beam to top of column.

FEA - Finite element analysis is a numerical method used to solve partial differential equations with softwares.

Component method - Where a connection is split into simple basic components and form a basic model. The strength and stiffness can then be calculated separately for each basic component according to the Eurocode.

CBFEM - Component based finite element method. A synergy between the finite element method and the component method.

ECCS - European Convention for Constructional Steelwork.

Depending on the type of global analysis used for a structure, different names are used for the joints. In this thesis rigid, pinned and semi-rigid are mostly used when referring to any joint type.

## Foreword

When I first started as a structural engineering trainee at Citec, I was assigned to model a pipe rack in RFEM. I struggled with getting the supports correct and was not aware of how large an impact different joint solution had on a structure. After this I started to do some own research, only to straighten things out for myself, and I realized that this is something I want, and need, to learn more about. A couple of months later when discussing thesis topics with my supervisor Ronny Engblom, he introduced me to several different topics. I thought, that if I choose to study semi-rigid joints, it will give me a lot of necessary knowledge and a great base for my future engineering career.

The key interest of this subject is how semi-rigid joints can impact a structure's behaviour. With my colleagues at Citec, we have discussed the topic of joint modelling regarding how the visual appearance is far from the truth. I have learned that there are more than bending, shear and tension resistance to be considered in joint modelling.

True behaviour of a frame with semi-rigid joints is something I think most of my Citec steel design colleagues are interested in. I hope that I will bring them useful data, or insight for them to apply to their daily structural engineering tasks.

*Staffan Fagerudd*

# Table of Contents

# 1 Introduction

Modelling joints in steel structures is an essential part of steel structural design. In frame analysis joints are generally simplified as either rigid or pinned depending on their ability to transfer moment. By utilizing the more complex semi-rigid joints in the frame analysis, it is possible to optimize capacity, economics, or find true frame behaviour.

Traditionally, a standard approach for pinned- and rigid joints is that the bolt row is located either inside or outside the beam flanges. This is illustrated in Figure 1.

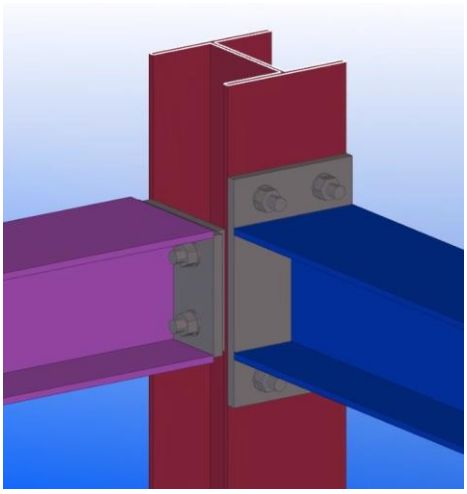


**Figure 1. Standard approach for pinned- and rigid joint.**

However, if the joints are not further analysed, the structure can behave differently than expected, and cause risks. These traditionally used joints are modelled and used in this thesis as semi-rigid joints, hence the semi-rigid joints are not half stiff, but instead a rigid or a pinned counterpart.

In Figure 2, a frame, with a uniform load on the beam, is presented with bending moment distribution diagrams for different knee-joints.

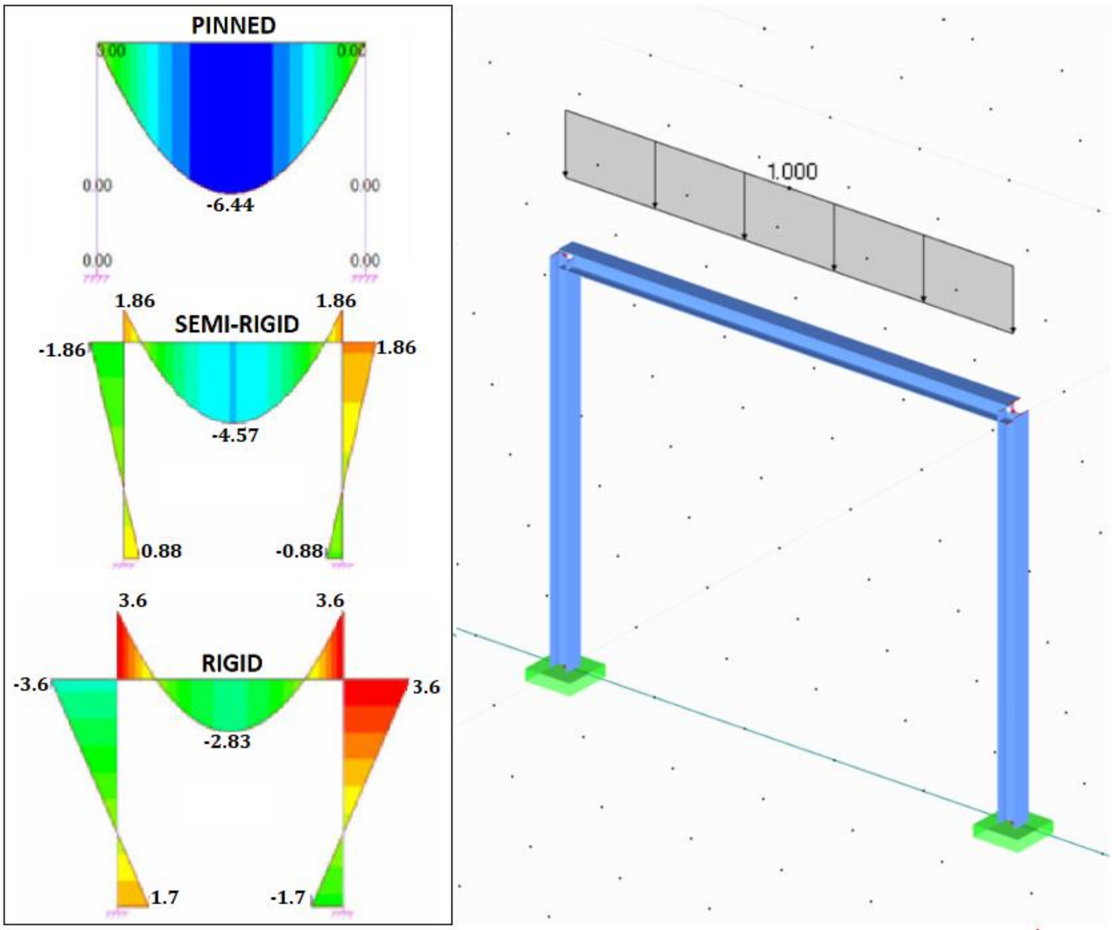



**Figure 2. Bending moment diagrams for frame structure with three different knee-joint conditions. (Graitec, n.d.).**

This simplification illustrates well how different joint conditions can affect the bending moment distribution.

## 1.1 Background

Semi-rigid joints have been researched, at least, since 1930-1940, and the first study considering the flexibility of riveted structural connections was made by W.M. Wilson and H.F. Moore, "*Tests to Determine the Rigidity of Riveted Joints of Steel Structures",* 1917 (Jones, 1980, pp. 7-8). Proper software for stiffness analysis became available around 2011 with Dlubal RF-joints (Dlubal, 2011). In 2014 the IDEAStatiCa-team together with technical universities created the method CBFEM, that they shortly thereafter implemented in their software IDEAStatiCa. Through that, the whole approach to connection design changed, and design limitations to calculation templates decreased (Fridrich, 2019).

## 1.2 Purpose

This thesis is written on behalf of the Steel & Concrete team at Citec Oy Ab:s Civil Engineering department. The focus is on semi-rigid joints, and the risks with traditionally considered rigid- and pinned joints in frame structures. Citec is a multidisciplinary engineering company that wants to expand knowledge in steel structural design, find extra capacity when needed, and true frame behaviour.

The main purpose is to increase knowledge and understanding of the benefits and ways of working with semi-rigid joints, and rotational stiffness of joints and to compare the standard approach joint frames to actual rigid- and pinned joint frames. The work is a “proof of concept” and should interpret general concepts instead of high complexity calculations and details.

Three comparisons are done to see how different joint configurations affect frames with the same dimensions and profiles. The joint type determines the internal force distribution and is essential in the displacement calculation of a structure. This will lead to getting insight into whether it is necessary to spend time calculating a joint stiffness and avoid risk, or only using simpler assumptions. The main research questions are:

- How do semi-rigid joints affect the frame behaviour in terms of bending moment distribution and horizontal displacements?
- What impact can it have on a structure buckling length if the joint stiffness is calculated?
- What are the risks of using common pinned and rigid joint solutions in frame structures?
- What are the rules and limits of joints in steel structures?

## 1.3 Method

A literature study of steel joints in elastic global analysis targeting joint stiffness and classification is presented as a review. The literature review is followed by the method part where joint stiffness and comparisons are performed and investigated. The joint stiffness is determined for traditionally considered joints, that is, semi-rigid joints, and joints that are classified as pinned or rigid. The effects of semi-rigid joints in bending moment distribution and displacement in frame structures are investigated. The effect on buckling length and the relationship between initial stiffness and buckling length factor is also analysed.

Two comparisons are done by investigating how frame structures behave differently when using pinned and rigid joints in relation to frames with semi-rigid joints. The first considers bending moment distribution and displacement, and the second considers buckling length. To visualise the behaviour of the compared frames, 3D-structural analysis and design software RFEM and CBFEM-software IDEAStatiCa are used. For the buckling length analysis, the add-on module for RFEM, RF-Stability is used.

Lastly, a comparison considering semi-rigid beam-to-column joint, with a continuous beam is performed. The joint rotational stiffness is calculated by a theoretical approach following the guidelines in EN-1993-1-8. This is done to find similarities, insight and to see how it differs from a costly simulation technique using IDEAStatiCa.

## 1.4 Restriction

The work does not invent any new interpretation of FEA but focuses on identifying and comparing established methods. Base-joints will be rigid, and the behaviour is not considered. An optimisation of the frame structures to get a higher utility ratio, regarding internal force distribution or displacement, is not done.

Elastic 1$^{st}$ order analysis will be used for all calculations, 2$^{nd}$ order analysis, and plastic approach, will be mentioned but not considered. Since elastic analysis is performed in this thesis, only initial stiffness will be calculated, non-linearities are not considered.

The joints modelled are up-down symmetrical joints. This is done to avoid different rotational stiffnesses when a bending moment is applied in different directions.

## 1.5 Disposition

The first chapter is an introduction that highlights the key parts of this topic. It also brings forward the customer and its needs for this thesis. Purpose, method, goal and restriction are also presented.

The second chapter is a literature review of the most relevant concepts, including an economic aspect of joint modelling and a definition of global analysis with all its fundamental elements. It continues to describe pinned, rigid and semi-rigid joints, including ductility requirements, moment-rotation characteristics and upper/lower limits.

The third chapter explains joint classification by stiffness, with the classification boundaries and initial/secant stiffness. It continues with determining stiffness for common pinned and rigid joints, that is, semi-rigid, as well as theoretical pinned and rigid joints. It concludes with a simplification of joint effect in a beam structure.

The fourth chapter contains an analysis of how the semi-rigid joint effect contributes to the distribution of bending moment and horizontal displacement in simple frame structures.

The fifth chapter brings up concepts of buckling and buckling length and evaluates different approaches to determine buckling length. It also analyses the relationship between the initial stiffness of the end-conditions and the buckling length factor. It concludes with semi-rigid joint effect on the buckling length in frame structures.

The sixth chapter evaluates the joint stiffness calculation by comparing IDEAStatiCa approach to the Eurocode approach.

The seventh chapter presents a contextualization of the results.

The eight chapter concludes the thesis, and the research questions are answered.

# 2 Literature review

Joints are from a static point of view classified as rigid, pinned or semi-rigid. The classification of a joint is made through either its rotational stiffness or strength, and sometimes both. The method of global analysis, elastic or plastic, used for a structure determines the classification approach. (SBI, 2013, pp. 7-8). A global frame analysis and checking of the individual cross-section and member design are part of a two-step procedure for the design of a frame and its components.

When loads are applied on a frame structure, the global frame analysis may be used for obtaining the possible internal forces, moments and displacements in the structure. The component behaviour, elastic or plastic, and the geometrical response, first- or second-order analysis, of the frame is the base for global analysis. The design checks of all the frame components can be performed when the global frame analysis is complete. (Jaspart & Weynand, 2016, p. 47).

Modelling of joints considering frame analysis is depending on the flexibility of the joint. The joints must be modelled since their behaviour affects the structural frame response (Jaspart & Weynand, 2016, pp. 51-52). The effects of the behaviour of the joints, on the distribution of internal forces and moments within a structure, and on the overall deformations of the structure are significant in the case of semi-rigid joints. (Jaspart & Weynand, 2016, p. 159).

## 2.1 Economic aspect of joint modelling

Rigid joints are more expensive than pinned joints due to the costs of extra stiffeners, cutting, welding, and inspections (Brown & Iles, 2013, p. 2). Since inherent stiffness in pinned joints can be utilized, a semi-rigid counterpart could be used in the frame analysis model, to keep a required span moment capacity and save steel tonnage and extra expenses. (Haapio & Heinisuo, 2010, pp. 10-11).

In the article Economy Studies of Steel Building Frames with semi-rigid joints, by K. Weynand, J-P. Jaspart & M. Steenhuis (1998), two strategies are highlighted when the minimum costs of steel structures are of interest. The first one considers unbraced frames where the joints transfer significant moments, traditionally rigid joints. There a simplification of the joint detailing and fabrication cost is stated as a strategy to economic benefit. The second strategy is to reduce profile dimensions and by that reduce material cost, typically relevant for braced frames with simple joints, because of the inherent stiffness.

In the same article two strategies for savings of fabrication and erection costs are pointed out. The first strategy regards rigid joints to be designed as close to semi-rigid but still classified as rigid: *"..omit some of the stiffeners while still fulfilling the criterion for rigid joints".* For the second strategy it is stated: *"Use semi-rigid joints in order to have any freedom to optimize the global frame and joint design"*. Thus, more economical solutions can be considered by selecting a semi-rigid joint alternative instead of rigid one. Another statement in the article is that, pinned joints are normally more economic in braced frames, while semi-rigid and rigid are usually better in unbraced frames. (Weynand, Jaspart & Steenhuis, 1998).

## 2.2 Global analysis

When determining structural model analysis, the structure should be based upon the calculation models that are suitable for the structure for the limit state that is considered. The calculation model and basic assumptions made for the structure shall be in line with the structural behaviours in a relevant limit state, with enough accuracy. It will also reflect the expected behaviours of the cross-sections, joints and bearing. The method of analysis used will comply with the assumed design conditions. (EN 1992-1-1, 5.1.1).

Elastic global analysis is that, regardless of stress level, material stress-strain behaviour is linear. While effects of material non-linearity are used in plastic global analysis in design of a such structure. For plastic global analysis the joint behaviour is modelled either with elastic-plastic analysis or rigid plastic analysis. (EN 1993-1-8, 5.1).

In elastic analysis the components are described by their stiffness and the strength is checked. The joints are thus classified according to their stiffness. In rigid plastic analysis the components are described by strength and deformation capacity, alternatively the rotational capacity is checked. The joints are thus classified according to their strength. In elastic-plastic analysis components are described by all the three above, deformation, strength, and stiffness. Here, both stiffness and strength determine the type of joint. (Norlin, Veljkovic & Husson, 2015).

Joints classified by stiffness are described as rigid, pinned and semi-rigid in elastic global analysis, that has been considered in this thesis. How joints are described according to the elastic- rigid-plastic, and elastic-plastic global analysis is shown in Table 1.

**Table 1. Type of joint model**

| Method of global analysis | Classification of joint | | |
|---|---|---|---|
| Elastic | Nominally pinned | Rigid | Semi-rigid |
| Rigid-Plastic | Nominally pinned | Full-strength | Partial-strength |
| Elastic-Plastic | Nominally pinned | Rigid and full-strength | Semi-rigid and partial-strength<br>Semi-rigid and full-strength<br>Rigid and partial-strength |
| Type of joint model | Simple | Continuous | Semi-continuous |

(EN1993-1-8, Table 5.1).

Which type of joint model the joint belongs to is also shown in Table 1, and follows:

- Simple, no bending moment is distributed through the joint.
- Continuous, where joint behaviour can be assumed to have no effect on the global analysis.
- Semi-continuous, the joint behaviour needs to be considered in the global analysis.

(EN 1993-1-8, 5.1.1(2)).

### 2.2.1 Elastic or plastic analysis

In EN1993-1-1:5.4.1 it is stated that the internal forces and moments may be determined using either elastic, or plastic global analysis (EN 1993-1-1, 2005). In (Jaspart & Weynand, 2016, p. 48) it is explained that the class of the connected member cross-sections determines if it is elastic or plastic analysis that needs to be selected for the design process: "*A class expresses the way on how the possible local plate buckling of cross-section walls subjected to compression may or may not affect the resistance or the ductility of the cross-section*".

Cross section classes from 1 to 4 defines whether the cross-section walls subjected to compression, due to possible local plate buckling, affect cross-section resistance or ductility. Classes 2, 3 and 4 requires elastic analysis for a frame, class 1 allows a plastic analysis. Cross section class also determines the actual cross section and member design checks. (Jaspart & Weynand, 2016, p. 48). Table 2 gives examples of different possibilities in cross section class selection.

**Table 2. Selection between elastic and plastic analysis**

| Cross section class | Frame analysis | Cross section/member verifications | Designation of the global approach (E = elastic; P = plastic) |
|---|---|---|---|
| Class 1 | Plastic | Plastic | P-P |
| | Elastic | Plastic | E-P |
| | Elastic | Elastic | E-E |
| Class 2 | Elastic | Plastic | E-P |
| | Elastic | Elastic | E-E |
| Class 3 – Class 4 | Elastic | Elastic | E-E |

(Jaspart & Weynand, 2016, p. 49).

Equilibrium between external and internal forces of a frame is a definition of a frame structural analysis. Structure geometry as reference for this equilibrium gives two analysis options, shown in Table 3. $1^{st}$ order analysis, reference to initial undeformed shape, and $2^{nd}$ order analysis, reference to actual deformed shape. (Jaspart & Weynand, 2016, p. 49). The two options are available for both elastic and plastic analysis.

**Table 3. Four main approaches for frame analysis**

| | | Assumed material law (linear or non-linear) | |
|---|---|---|---|
| | | Elastic | Plastic |
| Assumption in terms of geometrical effects (linearity or non-linearity) | $1^{st}$ order | Linear elastic first order analysis | Plastic (or elasto-plastic) first order analysis |
| | $2^{nd}$ order | Linear elastic second order analysis | Plastic (or elasto-plastic) second order analysis |

(Jaspart & Weynand, 2016, p. 48).

### 2.2.2 1st or 2nd order elastic global analysis

1st order elastic theory is the traditional analysis method for load-bearing structures. It is used widely, but today more often with FE-methods. The starting point is that the material is elastic, regardless of the stress level, and the forces are acting on the undeformed structure. Thus, the relation between loads and deformation is linear, a linear system. Superposition law can here be used to sum different loadings. In an elastic analysis the stiffness of the structure is essential for determining the force distribution. Varying stiffness must be considered. Normally stiffness of a nominal cross-section is used. The 1st order elastic analysis can be used in many practical cases, for instance, in continuous beams and frames with prevented lateral displacement. If there are both large normal forces and large deformations, or slender frames with free joints, sway frames, 2nd order analysis would be preferred. (Johansson, 2006, pp. 3-4).

In other words, if deformations cause internal forces or moments to increase their impact on the structure, second order analysis is preferred. If the increase can be neglected, first order analysis shall be used. In EN 1993-1-1, 5.2.1(5.1) a criterium for this can be used:

$$a_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 10 \text{ for elastic analysis.} \tag{2.1}$$

$$a_{cr} = \frac{F_{cr}}{F_{Ed}} \geq 15 \text{ for plastic analysis.} \tag{2.2}$$

Where $F_{cr}$ is critical load for global buckling of the whole structure and $F_{Ed}$ is design load. However, it can be optimized to model a frame that does not require a 2nd order analysis. If 2nd order effects are negligible, the design process can be based on a strain calculation and maximum strain is restricted to the design value of yield line strength. This has for long been a traditional approach but is more and more compensated by design criteria for average results. This amounts to using plastic moment resistance for cross-section classes 1-2, even in elastic analysis. It may seem like an inconsistency, but it is irrelevant when 1st order theory is applicable. In a 2nd order analysis, the increasing deformations are of concern. (Johansson, 2006, p. 4).

### 2.2.3 Frame stability

Geometrical non-linearities can cause local or global instability phenomena because of the members response or full frame response. Instability phenomena locally means for instance, flexural buckling, torsional buckling and lateral torsional buckling and global sway instability regarding global instability phenomena. (Jaspart & Weynand, 2016, p. 49).

## 2.3 Joint classification

Joints are not classified by their visual appearance. As mentioned in the introduction, a standard approach for pinned- and rigid joints is that the bolt row is located either inside or outside the beam flanges. This is illustrated in Figure 3, however, this visual classification is not accurate. Along with stiffness and strength of a joint, length of the connected member matters. If the length changes, the type of joint will also get a different classification, regardless of the visual appearance of the joint. However, rotational stiffness, determined by other parameters, is not affected by a longitudinal increase of a connected member. (Idea StatiCa & Construsoft, 2020, p. 9).

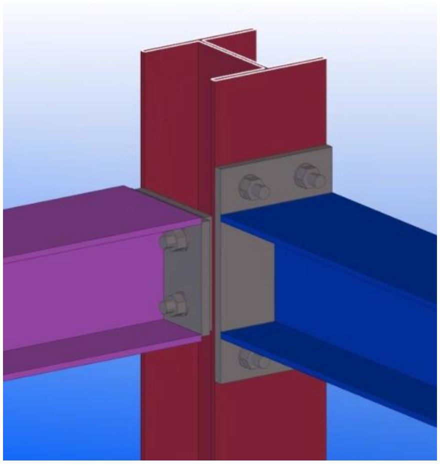


**Figure 3. Standard approach design of pinned- and rigid joint.**

## 2.4 Pinned- and Rigid joints

In theory, rigid joint means that no relative rotation is happening between the connected members during an applied loading. A pinned joint is not transferring bending moments and is often referred to as a perfect hinge between the connected members. Apart from this simplification, joints, whether if it is a pinned or rigid, an actual stiffness can be modelled. In daily practise, a rigid joint is never fully rigid, nor is a pinned joint never fully hinged.

Rigid joints can be designed either as a bolted and/or welded connection. Pinned joints are almost always designed as bolted connections, because that enables enough flexibility of the joint. Welds could prevent a necessary rotation in a pinned joint. (Jaspart & Weynand, 2016, p. 51).

When modelling joints according to strength, the design resistance of a rigid joint should not be less than that of the connected members (EN 1993-1-8, 5.2.3). A rigid, full strength, joint is stronger than the weaker of the connected members, when in a semi-rigid joint, partial strength, it is not.  (Jaspart & Weynand, 2016, p. 52).

To model a perfectly frictionless hinged joint would be practically unnecessary and expensive. That is why pinned joints for many years have been modelled base on this exact assumption but with a simpler design. That way a pinned joint can transfer a small, but often neglected, moment. There are many cases in structural design when these small transferred moments can be neglected. In EN 1993-1-8, 5.2.3.2 it is stated that a pinned joint must be able to transfer forces, without giving origin to large moments that negatively could affect the connected members or the structure as a whole. It is important in pinned joints that all the components, such as steel plates, welds and bolts, can transfer the design load effects. In some cases, a single bolt that is misplaced, suffers larger loads than estimated because of the rotation of the joint. That is why rotation capacity must be checked for pinned joints. (Norlin, Veljkovic & Husson, 2015, pp. 43–44).

Rigid joints are designed to take bending moments. Deformations caused by the joint must be limited to not affect the distribution of internal forces and not cause issues with the deformations of the structure. Joints that are classified as rigid are assumed to have enough rotational stiffness, that allows for an overall analysis based on the continuity of the structure. (Ongelin & Valkonen, 2010, p. 302). In a Swedish handbook for structural steel design, *Modul 10*, written by Bert Norlin, Milan Veljkovic and Wylliam Husson, it is expressed that an optimal design, regarding economy and function, of a rigid joint is when the bolts are acting in tension. The number of bolts increases when shear loads are acting on them, also the stiffness decreases, especially if the joint is not prestressed. (Norlin, Veljkovic & Husson, 2015 , p. 45).

Beam connections with pinned joints are easy and cheap to execute and usually give weak columns as a result. However, a rigid joint will make it possible to use a smaller beam cross-section, than a pinned joint. It is stated in the section *Balk-pelarinfästning* in the book *Detaljhandboken*, by SBI that the additional cost of the connection is often larger than savings in material costs of the beam. Especially in smaller buildings, stabilization through a frame structure can be an alternative. In higher buildings trusses or bracings are often used instead (SBI, 2013, p. 7). It is assumed that pinned joints don't transfer any bending moments and will be designed accordingly. However, in plane structures they should be designed against shear and axial forces (Jaspart & Weynand, 2016, p. 153).

### 2.4.1 Ductility Requirement

A certain amount of bending moment is always transferred through a connection, even if it's a pinned joint. If a semi-rigid joint is to be utilized in a calculation-model, it is necessary to get information about its moment-rotation characteristics. It is possible to use semi rigid joints, that have a smaller moment resistance than the beam-member in the connection (Demonceau, et al., 2009, pp. 17-19).

Together with rotation capacity, enough ductility requirements are necessary for ensuring safe modelling of a pinned joint. In theory, a pinned joint is only subjected to a shear force. In daily practise, a shear force and a bending moment are together acting on it. Bending moment in a pinned joint will lead to tension forces in the bolts. (Demonceau, et al., 2009, pp. 25-27). The utilization of tension capacity in the bolts is dependent on the available rotation: "*To enable rotation without increasing too much the bending moment which develops into the joint, contact between the lower beam flange and the supporting member has to be strictly avoided*" (Demonceau, et al., 2009, p. 27).

If contact between the lower beam flange and the supporting member appears, a compression force in the contact zone and bolt tension forces creates a significant bending moment in the joint. It is crucial that web $d_b$ of supported beam is larger than the height of the end plate $h_p$, as illustrated in Figure 4.

This will allow rotation in the joint, along with bolt tension capacity. The required rotation, which depends on the structural system and loading, should be smaller. than the available.

Available rotation of the joint is rotation utilizing allowed tension of the bolts before causing a compression force on the supporting member with lower beam flange.

In (Demonceau, et al., 2009, pp. 26–27) it is stated that before any calculaton is made, the designer can check with two rough assumptions, to see if the risk of contact can be neglected:

- *Supporting elements remains un-deformed;*
- *The centre of rotation of the beam is located at the lower extremity of the header plate.*

With these assumptions, a safe estimation of the available rotation can be calculated by:

$$\Phi_{available} = \frac{t_p}{h_e}, \tag{2.3}$$

The values are described in Figure 4. Here the deformation of the supporting elements has been neglected, also beam centre of rotation is set to be at the lower extremity of the header plate.

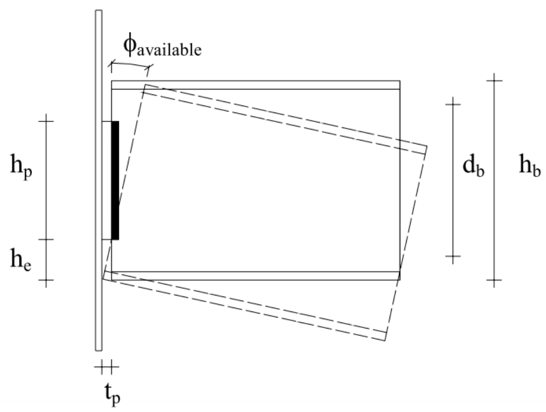



**Figure 4. Header plate joint (Demonceau, et al., 2009, p. 28).**

The required rotation capacity for a simply supported beam subjected to a uniform load can be calculated as:

$$\Phi_{required} = \frac{\gamma p \times L}{24EI} \tag{2.4}$$

Where, EI is the bending stiffness of the beam, L is the length of the beam, and $\gamma p$ is the factored uniform load. (Demonceau, et al., 2009, p. 28).

When bending moments develop in a joint, both tension and shear forces are acting in the connection. That a yield lines mechanism develops in the plate, before the strength of the bolts is exhausted, is a criterion (Demonceau, et al., 2009, p. 29). In J.P Jasparts doctoral degree thesis, *Recent Advances in the Field of Steel Joints*, appendix 2, chapter 10, it is stated that bolts being weaker than the plate in a beam-to-column connection should be avoided. Weynand, Jaspart and Steenhuis explains in *Economy Studies of Steel Building Frames with Semi-Rigid Joints,* that to receive a more ductile behaviour of a steel connection, thinner end plates should be used (Weynand, Jaspart & Steenhuis, 1998).

Premature bolt failure can occure due to tension forces. To avoid this atleast one of the following inequalities has to be satisfied:

$$\frac{d}{t_p} \geq 2.8\sqrt{\frac{f_{yp}}{f_{ub}}} \tag{2.5}$$

$$\frac{d}{t_{cf}} \geq 2.8\sqrt{\frac{f_{ycf}}{f_{ub}}} \tag{2.6}$$

Where,

$d$ is the nominal diameter of the bolt shank;

$t_p$ is the thickness of the header plate;

$t_{cf}$ is the thickness of the supporting column flange;

$f_{yp}$ is the yield strength of the end plate;

$f_{ycf}$ is the yield strength of the supporting column flange;

$f_{ub}$ is the ultimate strength of the bolt.

A reduction of the shear capacity may be used, since the whole tension capacity cannot be considered when ensuring the yield strength of the joint. (Demonceau, et al., 2009, p. 29–30).

### 2.4.2 Upper/lower limits

A joint that does not meet the criteria of a rigid or pinned joint should be classified as a semi-rigid joint. This applies to all global analysis methods, and order of theory. (EN 1993-1-8, 5.2.2.4 & 5.2.3.4).

## 2.5 Semi-rigid joints

When using semi-rigid joints in a calculation model, the stiffness of the joint needs to be calculated and the properties of the basic components needs to be considered. The basic components are end-plate stiffness, bolt capacity, length of connected member, and welds. Semi-rigid joints transfer moment between the connected members, but their deformation is not negligible. They can be modelled as rotational springs, that are elastic or elastic plastic. (Johansson, 2006, p. 1). Depending on what type of analysis is to be performed, semi-rigid joints rotational stiffness, design bending resistance and shear resistance have to be modelled and the actual behaviour of the joint has to be taken into consideration in the structural analysis and the design phase (Jaspart & Weynand, 2016, p. 160).

In EN1993-1-8, 5.2 it is stated that semi-rigid joints should be capable of transmitting the internal forces and moments. According to the ECCS, when joints are designed, in daily practise, to transfer internal forces, semi-rigid joints are used. Rigid joints resist the full capacity of the connected members but still transfer internal forces. Rigid joints assure a safe structure in a displacement point of view, but this assurance is not always needed. Then it would be optimal to model a semi-rigid joint instead of a rigid one. (Jaspart & Weynand, 2016, p. 52).

### 2.5.1 Design moment-rotation characteristic

Bending capacity, rotational stiffness and rotation ability are all main structural features that are considered in the moment-rotation characteristics (EN 1993-1-8, 6.1.2.1). In EN1993-1-8, 6.1.2 & 6.1.3 it is stated that the design moment-rotation characteristics of a joint should depend on the properties of its basic components: *"A joint may be represented by a rotational spring connecting the members at their intersection point. The properties of the spring can be expressed in the form of a design moment-rotation characteristic that describes the relationship between the bending moment and the corresponding rotation in the joint".* The design moment-rotation characteristic is generally non-linear. It is also stated that, moment resistance, rotational stiffness and rotation capacity should be defined by the design moment-rotation characteristic. (EN 1993-1-8, 2005).

By clarifying the design moment-rotation behaviours of a joint, it will provide a certain level of interaction between the members connected (Demonceau et al., 2009, p. 18). The ECCS states that, rotational actions of the joints can affect the local and/or global structural response of the frames. According to them, the rotational stiffness, resistance, and the rotation capacity of the joint might be affected by the axial and/or shear forces acting in the joint. However, these forces usually are neglected due to their slight effect on the frame response, concerning rotational deformability. (Jaspart & Weynand, 2016, pp. 15-16).

# 3 Joints classified by stiffness

A classification of a joint can be made through its rotational stiffness. This varies depending on if it is a rigid, pinned or semi-rigid joint. In *Modul 10* it is stated that when all the components of a beam-to-column joint have sufficient rotational stiffness, both the column and the beam will have an almost identical rotation at the intersection point. Then a joint is assumed as rigid. It is further explained that when a joint has a very small rotational stiffness, the beam will act as a simply supported beam independently of how the column deform. Then the relative rotation between column and beam at the intersection point, will be the same as in a simply supported beam-end.

The stiffness of a beam varies only with its length, since the bending stiffness is constant throughout the whole length. Further explained in *Modul 10*, when a beam is relatively long and flexible, the joint will function as close to rigid, and if the beam is relatively short, stiff, the joint will be pinned. (Norlin, Veljkovic & Husson, 2015, pp. 53–55).

Classification boundaries, zones illustrated in Figure 5:

Zone 1: Rigid, if $S_{j.ini} \geq k_b EI_b/L_b$, where $k_b = 8$ for frames where the bracing system reduces the horizontal displacement by at least 80 %. $k_b = 25$ for other frames provided that in every storey $K_b/K_c \geq 0.1$*)

*) For frames where $K_b/K_c < 0.1$ the joints should be classified as semi-rigid.

Zone 2: All joints in zone 2 should be classified as semi-rigid. Joints in zone 1 or 3 may optionally also be treated as semi rigid.

Zone 3: Nominally pinned if $S_{j.ini} \leq 0.5EI_b/L_b$

Where: $K_b$ and $K_c$ is mean value of moment of inertia, $I$ divided by length of member, $L$) $I/L$. Index c or b refers to the member that is either column or beam. (EN 1993-1-8, 5.2.2, Figure 5.4).

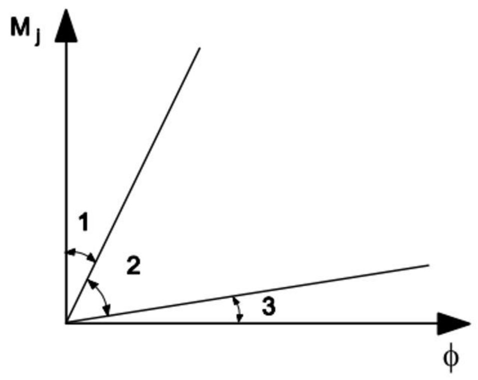



**Figure 5. Classification of joints by stiffness (EN 1993-1-8, 5.22, Figure 5.4).**

When the rotational stiffness is known it enables determination of the buckling length and calculation on how large bending moment that are transferred through the joint (EN 1992-1-1, 5.8.3.2).

## 3.1 Initial/secant stiffness

Depending on the global analysis joints can be classified by their stiffness. As explained in the restrictions chapter, initial stiffness is considered in the method part and non-linearities are not.

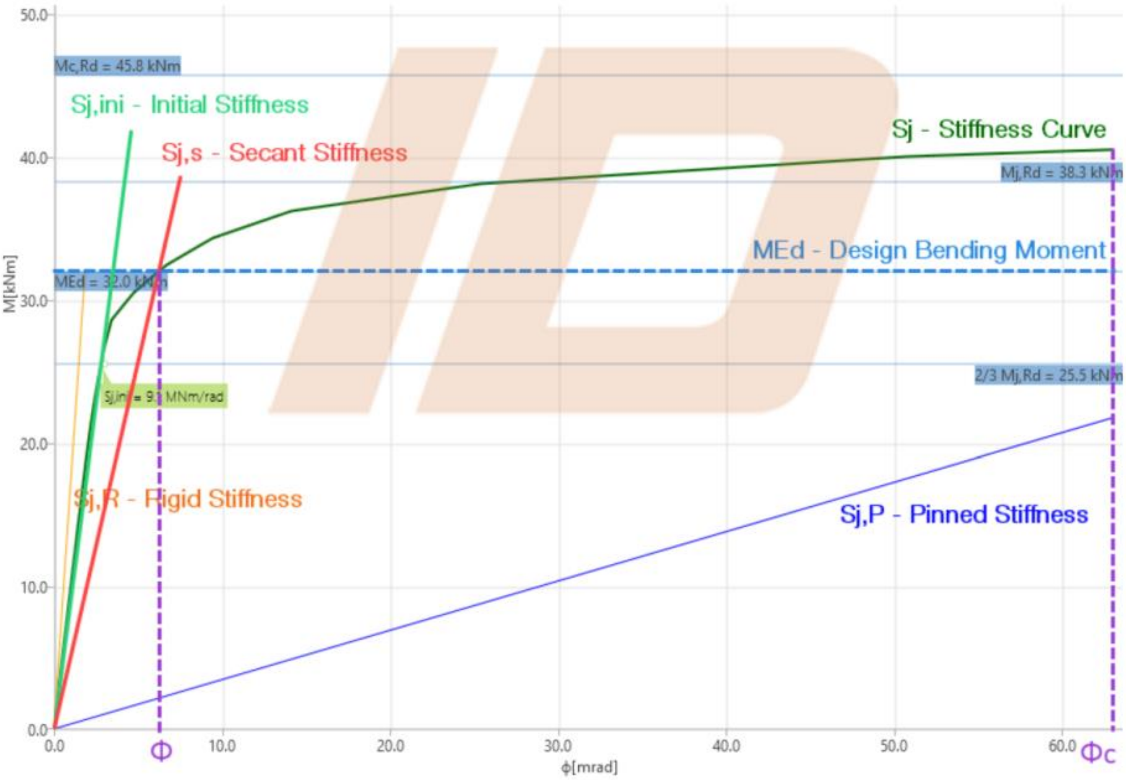



**Figure 6. Detailed moment-rotation diagram showing the relationship of secant- and initial stiffness to bending resistance, obtained from a stiffness analysis (Idea StatiCa & Construsoft, 2020, p. 12).**

The secant stiffness value is defined as the intersection of the design bending moment, $M_{Ed}$, and the stiffness curve of the connection $S_j$, see Figure 6. Initial rotational stiffness is linear, assumed up to 2/3 of moment capacity, and secant stiffness is the non-linearity when the material is plasticizing, where the green curve slackens in Figure 6. (Idea StatiCa & Construsoft, 2020, pp. 12-13). The secant stiffness is calculated by inserting a specific value for the design bending moment, while initial stiffness is constant.

## 3.2 Determining stiffness

In this chapter, rigid-, pinned- and semi-rigid joints are modelled in IDEAStatiCa and are used throughout the comparisons in this thesis. The joint classification is based on the classification boundaries in EN1993-1-8, 5.2.2.5 and are presented in the next sub-chapter. For the modelled joints, IDEAStatiCa gives an estimation of the production cost.

The values for the semi-rigid joint rotational stiffness are used in RFEM as rotation springs later. Whenever pinned or rigid joints are modelled in RFEM, no use of rotation spring, only free to rotate around y-axis and respectively rigid.

### 3.2.1 Classification boundaries

Boundaries depend on beam profile and its length. HEA200 used for the equations:

Modulus of elasticity, $E\ =\ 210\ 000$ N/mm², 
moment of inertia, $I_b = 36.92 \times 10^6$ N/mm², 
beam span, theoretical length, $L_b = 5000$ mm, 
sway factor, $k_b = 25$.

Classification boundaries for a pinned, rigid and semi-rigid joint according to EN1993-1-8, 5.2.2.5, Fig. 5.3:

Pinned if,

$$S_{j.ini} \ \leq\ 0.5EI_b/L_b = 0.77\ MNm/rad \tag{3.1}$$

Rigid if,

$$S_{j.ini} \geq k_bEI_b/L_b = 38.77\ MNm/rad \tag{3.2}$$

Semi rigid if,

$$0.5 \times EI_b/L_b \geq S_j \geq\ k_bEI_b/L_b \tag{3.3}$$

### 3.2.2 Pinned and semi-rigid counterpart

The joints are presented in Figure 7 and Figure 9, and rotational stiffness in Figure 8 and Figure 10.

#### Theoretical pinned joint

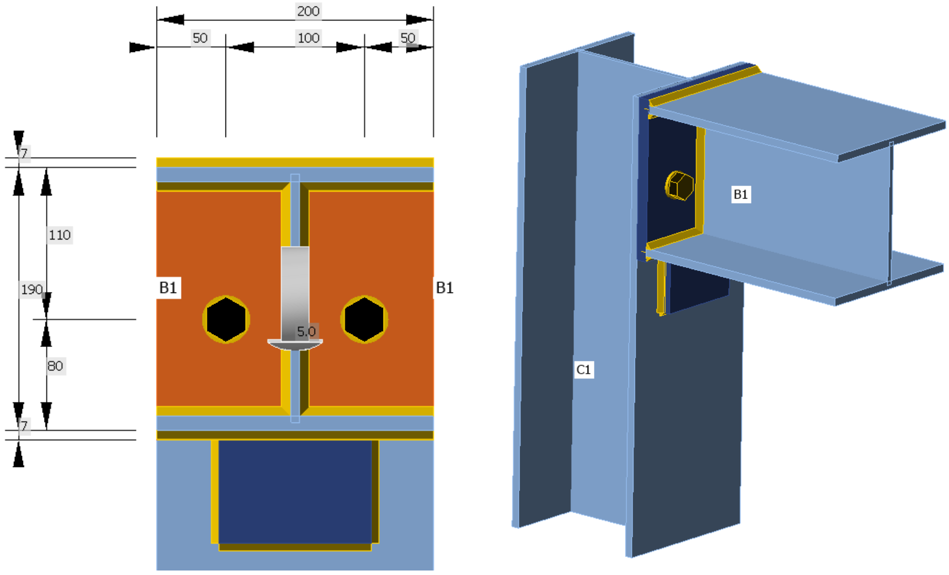



**Figure 7. Pinned joint for Test 1 made in IDEAStatiCa.**

Rotational stiffness of joint component

| | Item | Comp. | Loads | MEd [kNm] | Mj,Rd [kNm] | Sj,ini [MNm/rad] | Sjs [MNm/rad] | ϕ [mrad] | ϕc [mrad] | L [m] | Sj,R [MNm/rad] | Sj,P [MNm/rad] | Class |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| > | B1 | My | LE1 | 5.0 | 15.9 | 0.77 | 0.87 | 5.8 | 123.5 | 5.00 | 38.75 | 0.77 | Pinned |

**Figure 8. Rotational stiffness values for joint in Figure 7.**

The pinned joint presented in Figure 8 has an initial stiffness of $S_j = 0.77\ MNm/rad$, which fulfils the classification boundaries of a pinned joint. As can be seen, the secant stiffness $S_{j.s}$ is larger than the initial stiffness. As shown previously in the theory part, chapter 3.1, Figure 6 , the initial stiffness is constant up to a value of $2/3\ M_{j.Rd}$. When the stiffness becomes non-linear, the stiffness curve slackens, but can still increase until failure. According to IDEAStatiCa the production cost for this joint is 27 €.

## Traditional pinned joint – Semi-rigid joint

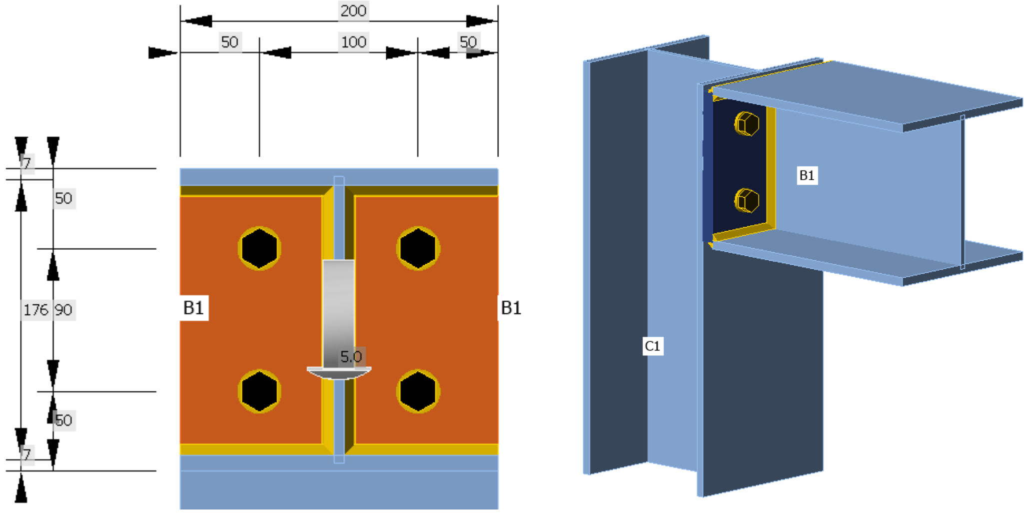



**Figure 9. Semi-rigid joint.**

Rotational stiffness of joint component

| | Item | Comp. | Loads | MEd [kNm] | Mj,Rd [kNm] | Sj,ini [MNm/rad] | Sjs [MNm/rad] | ϕ [mrad] | ϕc [mrad] | L [m] | Sj,R [MNm/rad] | Sj,P [MNm/rad] | Class |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| > | B1 | My | LE1 | 5.0 | 20.9 | 1.47 | 1.68 | 3.0 | 93.4 | 5.00 | 38.75 | 0.77 | Semi-rigid |

**Figure 10. Rotational stiffness values for joint in Figure 9.**

The semi-rigid joint presented in Figure 9 has an initial stiffness of $S_j = 1.47\ MNm/rad$. The rotational spring in RFEM is adjusted to 1470 $kNm/rad$, whenever this joint is considered. According to IDEAStatiCa the production cost for this joint is 22 €.

### 3.2.3 Rigid and semi-rigid counterpart

A theoretical example of a rigid joint is presented Figure 11, with rotational stiffness in Figure 12. The semi-rigid joint is presented in Figure 13 and its rotational stiffness in Figure 14.

#### Theoretical rigid joint

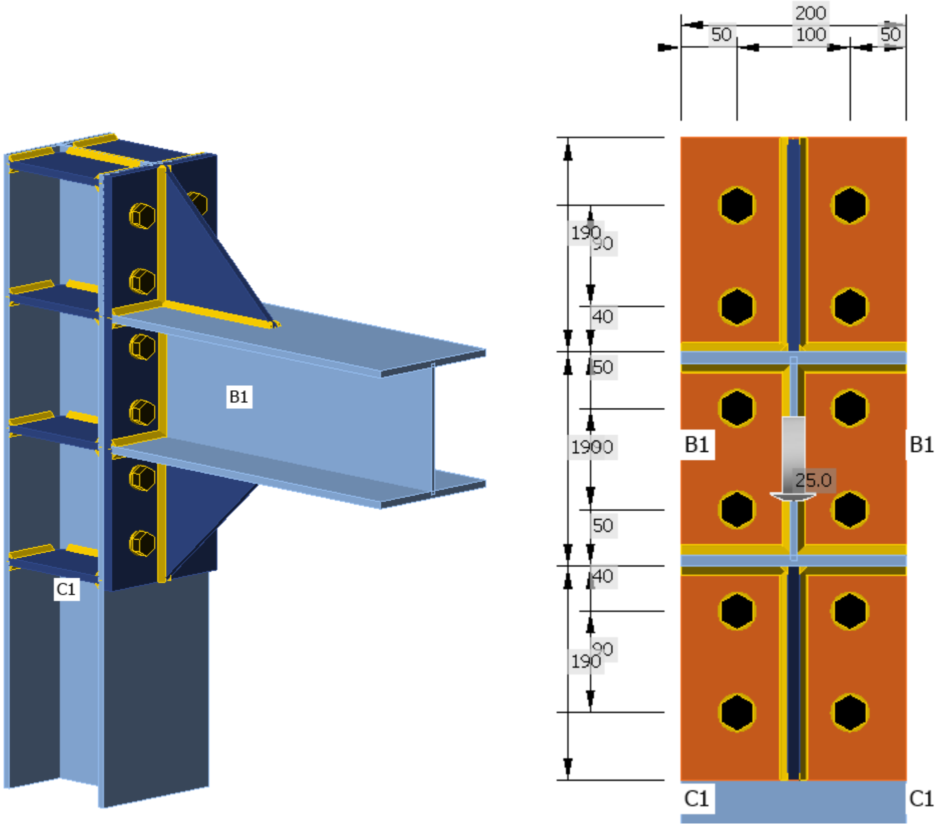



**Figure 11. Bolted rigid joint.**

Rotational stiffness of joint component

| | Item | Comp. | Loads | MEd [kNm] | Mj,Rd [kNm] | Sj,ini [MNm/rad] | Sjs [MNm/rad] | ϕ [mrad] | ϕc [mrad] | L [m] | Sj,R [MNm/rad] | Sj,P [MNm/rad] | Class |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| > | B1 | My | LE1 | 25.0 | 113.1 | 40.62 | 42.68 | 0.6 | 117.7 | 5.00 | 38.75 | 0.77 | Rigid |

**Figure 12. Rotational stiffness values for the bolted rigid joint.**

This joint would require a production cost of 141 € according to IDEAStatiCa. The initial stiffness is $S_j = 40.62\ MNm/rad$. This is not an ideal design, only a visualization of a joint that is actual rigid.

## Traditional rigid joint – Semi-rigid joint

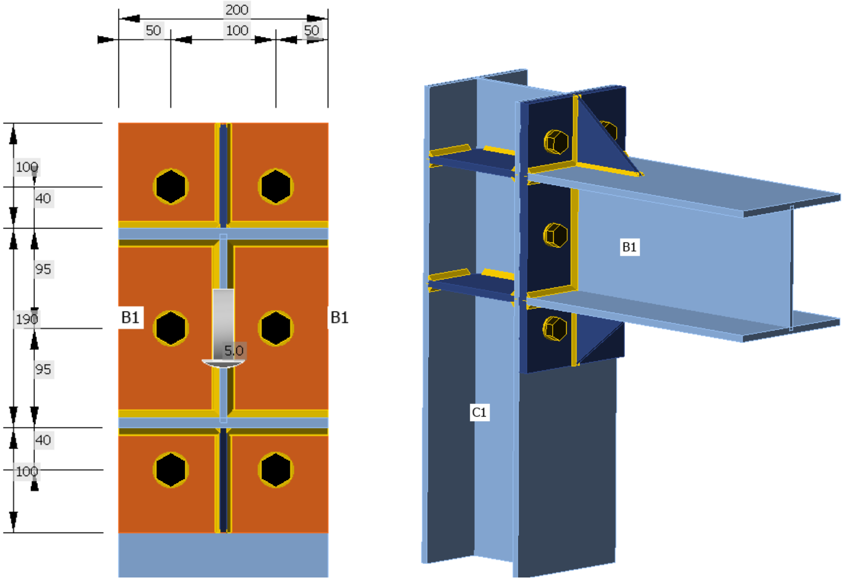



**Figure 13. Semi-rigid joint.**

Rotational stiffness of joint component

| | Item | Comp. | Loads | MEd [kNm] | Mj,Rd [kNm] | Sj,ini [MNm/rad] | Sjs [MNm/rad] | ϕ [mrad] | ϕc [mrad] | L [m] | Sj,R [MNm/rad] | Sj,P [MNm/rad] | Class |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| > | B1 | My | LE1 | 5.0 | 69.6 | 11.42 | 11.84 | 0.4 | 50.6 | 5.00 | 38.75 | 0.77 | Semi-rigid |

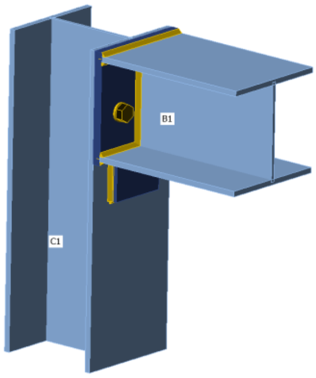
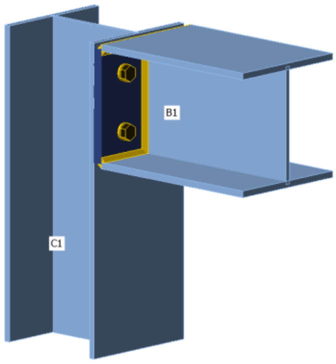
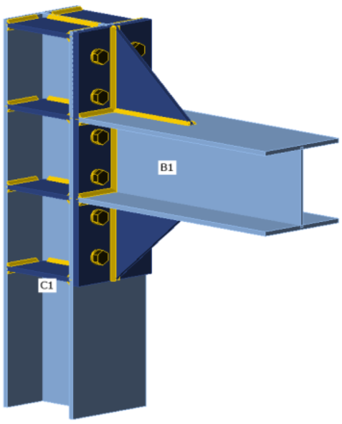
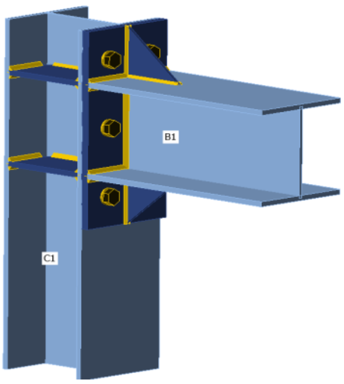
**Figure 14. Rotational stiffness values for the joint in Figure 13.**

This joint has an initial stiffness of $S_j = 11.42\ MNm/rad$. The rotational spring in RFEM is adjusted to 11420 $kNm/rad$, whenever this joint is considered. According to IDEAStatiCa, the production cost is 71 €.

## 3.3 Stiffness conclusion

A summary of joint types including classification, initial stiffness, production and bending resistance is presented in. Whether the joint is traditional, or theoretical is also stated.

**Table 4. Summary of calculated joint types**

| | | |
|---|---|---|
| Pinned and semi-rigid counterpart |  |  |
| **Joint** | Theoretical | Traditional |
| **Classification** | Pinned | Semi-rigid |
| **Initial stiffness (MNm/rad)** | 0.77 | 1.47 |
| **relative production cost** | 27 € | 22 € |
| **Bending resistance (kNm)** | 15.9 | 20.9 |
| Rigid and semi-rigid counterpart |  |  |
| **Joint** | Theoretical | Traditional |
| **Classification** | Rigid | Semi-rigid |
| **Initial stiffness (MNm/rad)** | 40.62 | 11.42 |
| **relative production cost** | 141 € | 71 € |
| **Bending resistance (kNm)** | 113.1 | 69.6 |

The semi-rigid joint is much cheaper than the rigid joint, half the production cost. Savings regarding the pinned and semi-rigid joint is not significant, only 5 € differs.

## 3.4 Joint effect simplified

In this chapter, a member load, $Q = 10\ kN/m$, is applied on a beam that is modelled with different joint types, to visualise how semi-rigid, rigid or pinned joint types can affect the bending moment distribution.

In Figure 15 three cases are shown on how a beam can behave with different joint solutions. A beam with pinned supports has the largest span moment compared to the others. For the rigidly supported beam, the bending moment is large at the support. For a semi-rigid supported beam, the so-called, rotation springs can be adjusted to receive a certain bending moment diagram.

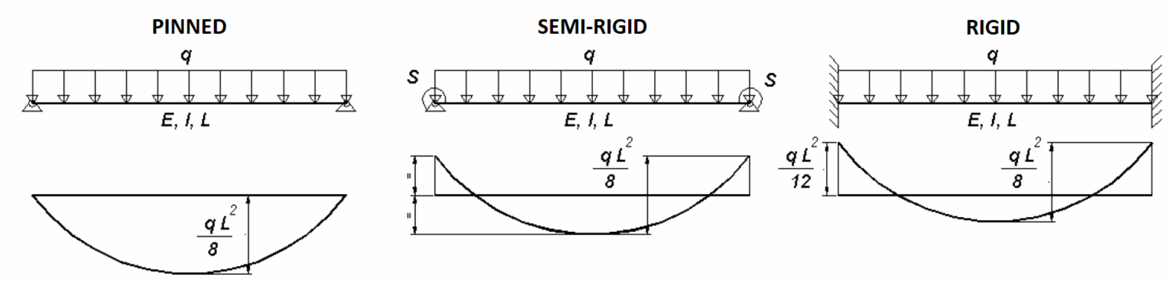



**Figure 15. Bending moments: pinned-, semi-rigid- and rigid supported beam (Haapio & Heinisuo, 2010, p. 2).**

### 3.4.1 Comparison

A HEA200 beam is modelled with pinned, rigid and semi-rigid joints, to illustrate how the bending moment distribution is affected.

**Rigid beam**

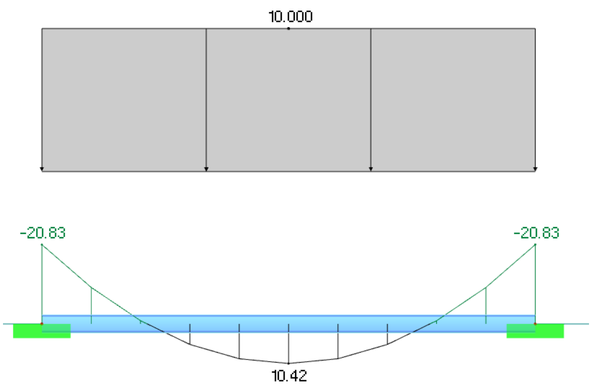



**Figure 16. Beam modelled with rigid supports in RFEM.**

In Figure 16. Beam modelled with rigid supports the beam is modelled with rigid supports. As can be seen, it corresponds to rigid case in Figure 15, where maximum bending moment is at the supports, $M_{Ed.j} = 20.83\ kNm$. The span moment is nearly half of $M_{Ed.j}$, $M_{Ed.s} = 10.42\ kNm$.

### Pinned beam

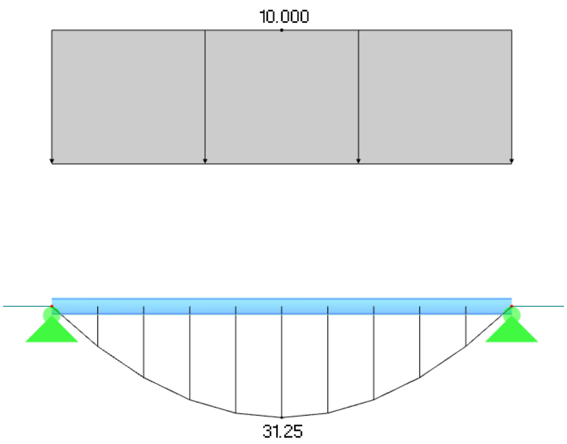



**Figure 17. Beam modelled with pinned supports in RFEM.**

In Figure 17. Beam modelled with pinned supports in RFEM. the beam is modelled with pinned supports. The bending moment is maximum at the middle of the span, $M_{Ed.s} = 31.25\ kNm$, in accordance with pinned case in Figure 15.

### Semi-rigid beam

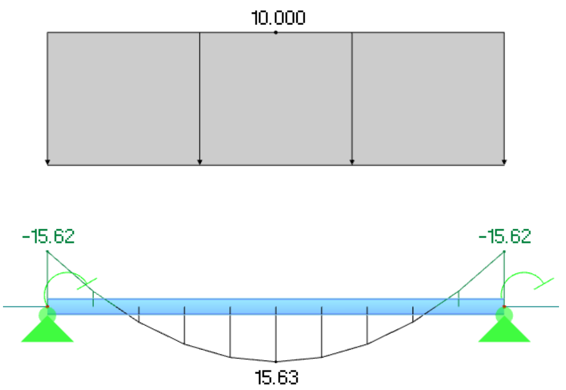



**Figure 18. Beam modelled with semi-rigid rotation springs in RFEM.**

In Figure 18 the rotation springs are adjusted that the bending moment is almost same size at the support as at the span, $M_{Ed.j} = 15.63\ kNm$. The stiffness of the rotation spring is formed by iteration, $S_{j.ini} =$ 9,30 MNm/rad.

### 3.4.2 Manual calculation

For the beam with rigid supports the formula for span bending moment and bending moment at support can be taken from Figure 15, rigid case.

At the support: $M_{Ed.j} = \frac{q \times L^2}{12} = \frac{10\ kN/m^2 \times (5\ m)^2}{12} = 20.83\ kNm$ (3.4)

Span moment: $M_{Ed.s} = \frac{q \times L^2}{8} - \frac{q \times L^2}{12} = \frac{q \times L^2}{24} = \frac{10\ kN/m^2 \times (5\ m)^2}{24} = 10.42\ kNm$ (3.5)

For the semi-rigid supported beam, the stiffness of the rotation spring is determining how much of the bending moment that is distributed to the support. For instance, using the joint classification boundaries for a certain profile to find out how close to rigid or pinned it is. For the bending moment to transfer equally through the beam, the formula for both the span bending moment and bending moment at the support is given in Figure 15, semi-rigid case:

$$M_{Ed} = \frac{q \times L^2}{8} / 2 = \frac{q \times L^2}{16} = \frac{10 kN/m^2 \times (5m)^2}{16} = 15.63\ kNm \qquad (3.6)$$

For the beam with pinned supports, where moment at the support is 0, span moment is calculated by the formula given in Figure 15, pinned case:

$$M_{Ed.s} = \frac{q \times L^2}{8} = \frac{10\ kN/m^2 \times (5\ m)^2}{8} = 31.25\ kNm \qquad (3.7)$$

### 3.4.3 Result evaluation

The bending moment is distributed more evenly in the semi-rigid beam than in the beams with common supports. Smaller cross-sections could be used for the semi-rigid supported beam than for the other two cases, especially the pinned supported beam. Also, less amount of steel is needed for a connection that requires less bending resistance, when comparing semi-rigid case to rigid case. However, this is only an illustration on how different joint solutions affect the bending moment distribution for a single beam.

# 4 Joint effect in bending moment distribution and displacement

Frames to be compared are modelled in RFEM with two columns and one beam, forming a typical simple frame. Two frames are designed in two different ways with varying joint solutions to find the semi-rigid joint effect on bending moment distribution and horizontal displacement. All frames are considered as sway-frames. Same loading, and dimensions are applied on the frames. Steel grade S355 is used for all components and HEA200-profiles are used as cross sections for the beams and columns in the frame structures.

The joint effect for two frames, with rigid and pinned knee-joints, are compared to their semi-rigid counterpart, joint stiffness calculated earlier can be reviewed in Table 5. The comparison regards steel usage, distribution of bending moment and displacement. The comparison is divided into two Tests, presented in Figure 20 and Figure 21. Symbols for the knee-joint conditions are presented in Figure 22. Loads applied are vertical uniform load, $Q = 10\ kN/m$ and lateral point load, $F = 15\ kN$.

**Table 5. Rigid and pinned joints along with their semi-rigid counterpart, based on results in 3.2**

| JOINTS | RIGID JOINT | SEMI-RIGID COUNTERPART | PINNED JOINT | SEMI-RIGID COUNTERPART |
|---|---|---|---|---|
| STIFFNESS (MNm/rad) | ≥ 38.77 | 11.42 | ≤0.78 | 1.62 |

Rotation springs with the considered counterpart stiffness is inserted in RFEM for the semi-rigid joint frames.

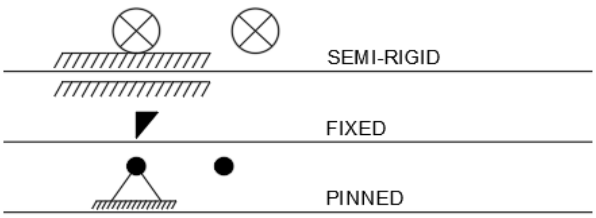



**Figure 19. Symbols for joint types drawn in Autocad.**

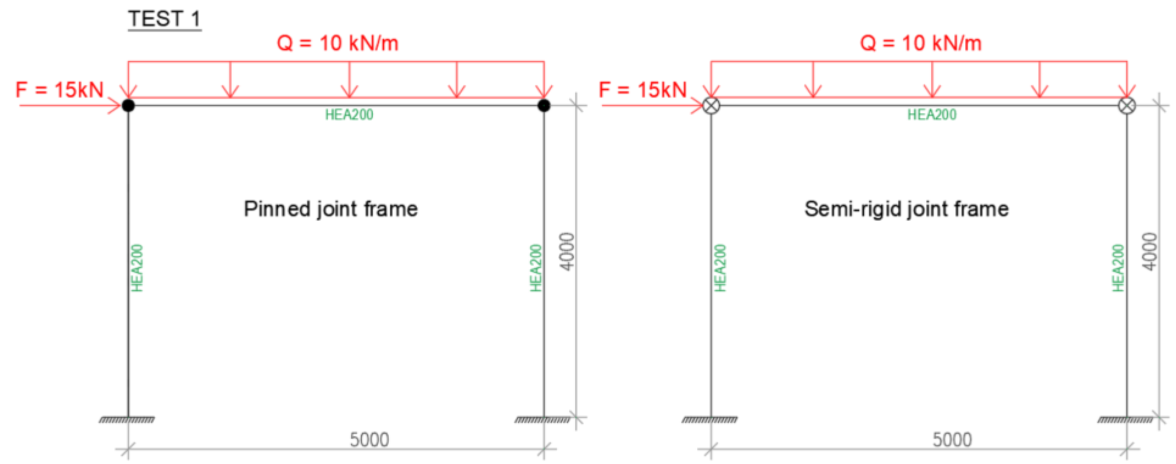



**Figure 20. Frame structures for Pinned vs. Semi-rigid.**

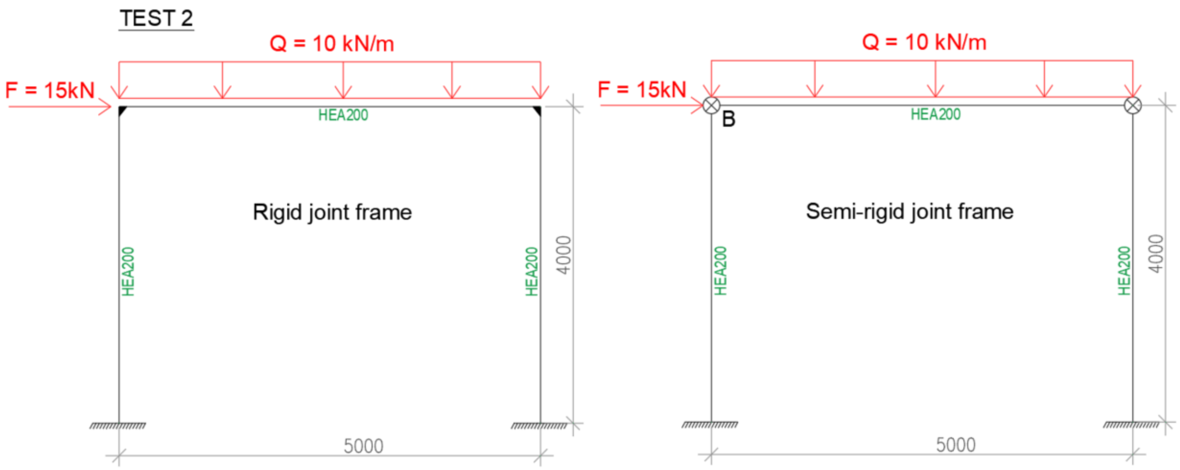



**Figure 21. Frame structures for Rigid vs. Semi-rigid.**

## 4.1 Pinned vs Semi-rigid

A frame with semi-rigid knee-joints is compared to a frame with pinned knee-joints. The stiffness values calculated in IDEAStatiCa is inserted in the RFEM-model as rotation springs for the frame with semi-rigid joints.

### 4.1.1 Comparison

Two frame structures are compared in terms of bending moments, Figure 22 and horizontal displacement, Figure 23.

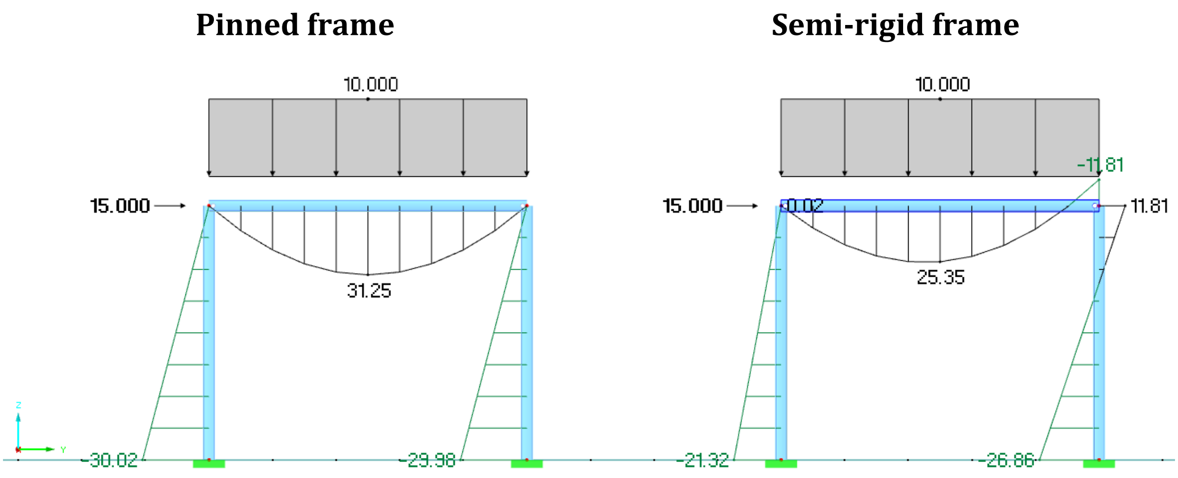



**Figure 22. Moment diagrams for compared frames in RFEM.**

As can be seen in Figure 22, larger bending moments occurs in the frame with pinned knee-joints, $M_{Ed} = 31.25\ kNm$. Bending moment is transferred through the semi-rigid joints and distributed among the members, and provides a smaller span moment, $M_{Ed.s} = 25.35\ kNm$ . Pinned joints cannot transfer bending moments, thus the distribution does not occur for the frame to the left.

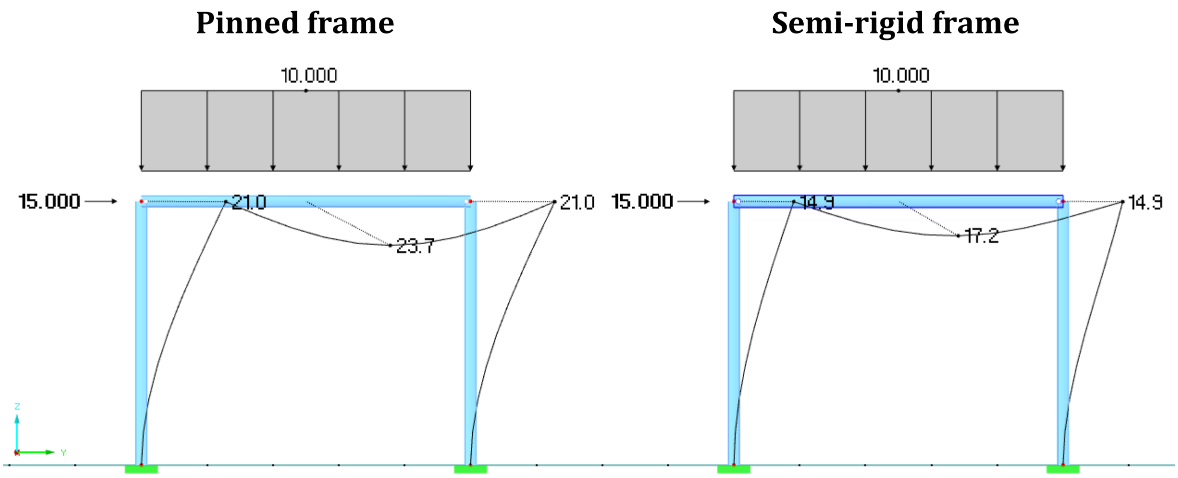



**Figure 23. Displacement curves for compared frames in RFEM.**

As can be seen in Figure 23, the pinned joint frame deflects more than the semi-rigid frame.

By following the horizontal displacement boundary H/150 for 1-storey buildings, according to the Finnish National Annex, (NAFI, 2019), the utility ratio for displacement is calculated:

$$u = \frac{4000\ mm}{150} = 26.67 \text{ mm}, \quad (4.1)$$

where $u$ is the displacement limit.

The displacement utility ratio for the frames are:

Semi-rigid frame, $\frac{14.9\ mm}{26.67\ mm} \times 100\ \% \approx 56\ \%$. (4.2)

Pinned frame, $\frac{21\ mm}{26.67\ mm} \times 100\ \% \approx\ 79\ \%$. (4.3)

### 4.1.2 Risks

Because of large deformation when the frame is modelled as pinned, the structure might get oversized. Columns can be strengthened, or using bigger profile sizes, to manage large deformations. Large span moment, for the frame with pinned joints, leads to oversized beam. As can be seen, in Figure 22, the moment that is transferred from column to base joint is larger for the frame with pinned knee-joints. Frames built with traditionally designed pinned joints may not exactly match the model for the pinned joint frame presented previously, if the rotational stiffness is not considered.

### 4.1.3 Savings

Smaller beam and column cross-sections for the semi-rigid joint frame than in the pinned joint frame, that would lead to less steel tonnage. This can be visualized by calculating the span moment in percentage of the two frames:

Percentage of span moment in pinned frame, 31.25 $kN/m\ =\ 100\ \%$.

Percentage of span moment in semi-rigid, 25.35 $kN/m\ =\ \frac{25.35}{31.25} \approx\ 81\ \%$ less steel.

## 4.2 Rigid vs Semi-rigid

A frame with semi-rigid knee-joints is compared to a frame with rigid knee-joints. The stiffness value calculated with IDEAStatiCa is inserted in the RFEM-model as rotation springs for the frame with semi-rigid joints.

### 4.2.1 Comparison

Two frame structures are compared in terms of bending moments, Figure 24 and horizontal displacements, Figure 25.

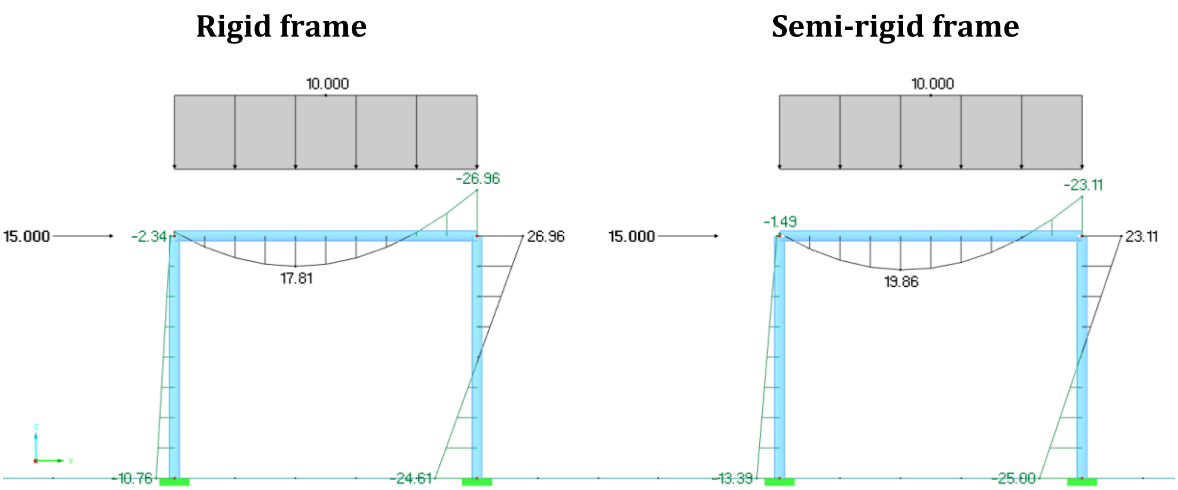



**Figure 24. Moment diagrams for compared frames in RFEM.**

In Figure 24, larger bending moments are transferred through the joint and distributed to the members in the rigid joint frame, than in the semi-rigid joint frame. This leads to large bending moments at the joint for the rigid joint frame, $M_{j.Ed} = 26.96\ kNm$. In the semi-rigid joint frame, bending moments are distributed more equally, $M_{j.Ed} = 23.09\ kNm$. The bending moment that occurs in the base joint in the semi-rigid joint frame is not significantly larger than in the rigid joint frame.

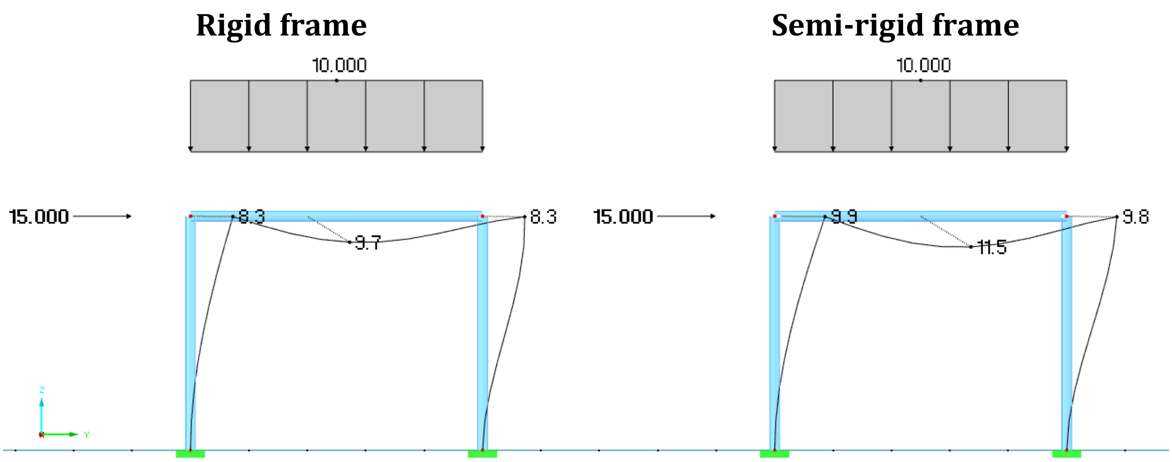



**Figure 25. Displacement curves for compared frames in RFEM.**

As can be seen in Figure 25, none of the frames are close to the displacement boundary. However, the semi-rigid joint frame is closer, and will deflect more than the rigid joint frame. The displacement utility ratio for the frames can be calculated in the same way as before:

$$u = \frac{4000\ mm}{150} = 26.67 \text{ mm} \tag{4.4}$$

Semi-rigid frame, $\frac{9.8\ mm}{26.67\ mm} \times 100\ \% \approx\ 37\ \%.$ (4.5)

Rigid frame, $\frac{8.3\ mm}{26.67\ mm} \times 100\ \% \approx\ 31\ \%.$ (4.6)

### 4.2.2 Risks

The usage of rigid joint can cause an oversized structure due to bending moment, but for displacement, may be checked, otherwise the structure might be undersized. If a commonly engineered rigid joint is in fact semi-rigid, displacement is a risk factor. The bending moment transferred to the base joint is larger in the semi-rigid joint frame. This can lead to having less capacity in the foundation than needed, if a frame structure is modelled with commonly engineered rigid joints.

### 4.2.3 Savings

Smaller cross-sections can be used for the beam in the semi-rigid joint frame. Because of the larger bending moment at the support for the semi-rigid joint frame, larger cross-section for the column, than for the other frame, would be necessary.

Percentage of span moment in rigid joint frame, 26.96 $kN/m\ =\ 100\ \%$.

Percentage of span moment in semi-rigid, 23.11 $kN/m\ =\ \frac{23.11}{26.96} \approx\ 86\ \%$ less steel.

# 5 Buckling

There are two types of buckling deformation, flexural buckling and torsional buckling. Buckling is due to geometrical instability of a structure, not material failure, as yielding or fracture. The different buckling types refers to displacement and deformation of the member, not to loads applied on the structure. Torsional buckling is mostly occurring in open cross-sectional profiles, such as IPE- or HEA-profiles, because of their small torsional stiffness. The cutting centre is in the centre of gravity of a double symmetric or a point symmetric cross-section. Potential buckling types here are flexural buckling, in either major or minor axis direction, and pure torsional buckling. Pure torsional buckling is when cross-section rotates around its torsion centre. (Ongelin & Valkonen, 2010, p. 107). In Mechanics of materials 8$^{th}$ edition by R.C. Hibbeler, 2011 it is stated that, the *critical load* is the maximum axial load that a column can support when it is on the verge of buckling. This loading represents a case of *neutral equilibrium*. A pinned joint column will buckle about the principal axis of the cross-section having the smallest moment of inertia. (R.C. Hibbeler, 2011, p. 660).

Buckling of a column occurs when a critical load is applied on the member. If a compressive force is large, and the member affected by it is slender and long, the member can displace laterally or sideways. This can lead to a sudden and dramatic failure of a subjected member. Thus, structures that are subjected to these types of loadings, must be designed against buckling.

The critical load can be calculated using Euler's formula. Where, length *L* represents the unsupported distance between the points of zero moments. This distance is explained as the *effective length,* same as buckling length, in Mechanics of materials 8$^{th}$ edition by R.C. Hibbeler, 2011, and can be calculated and used for other types of end-conditions as well, to determine the critical load. A column buckling length is depending on the members actual length, the end-conditions, and possible sideway-supports or bracings in non-sway frames (Ongelin & Valkonen, 2016, p. 400). The buckling length is determined by multiplying the buckling length factor, $\beta$, with the actual length of the member. (R.C. Hibbeler, 2011, p. 667).

Euler's formula: $$P_{cr} = \frac{\pi^2 \times EI}{(\beta L)^2} \tag{5.1}$$

The buckling of a column or beam can be simplified as only bending or buckling in one plane and considering one type of joint. This is only a simplification to understand how different joint types affects the buckling length and the behaviour of the column. A more realistic analysis can be done with RFEM and RS-Stability when rotational stiffness is known.

## 5.1 Buckling length

The Eurocode 3 does not provide comprehensive guidance on buckling lengths for compressive members with different end-conditions. According to Designers guide to EC3, the reason for this lack of guidance is partly based on disagreements: "..*because no common consensus between the contributing countries could be reached."* (Gardner & Nethercot, 2011, p. 60).

In SSAB Domex Tube - Rakenneputket, written by Ongelin & Valkonen, formulas for buckling length of a continuous column is described and how to consider semi-rigid end-joints. An equivalent to the traditional buckling table regarding semi-rigid joints is not considered (Ongelin & Valkonen, 2016, p. 404). In the concrete eurocode EN 1992-1-1, 5.8.3.2, Fig. 5.7, the traditional buckling table with consideration of semi-rigid end joint is given. However, this might not be relyable to use for steel structures.

The Finnish building calendar, 2020-2021, provides four buckling cases and belonging theoretical values for the factor $\beta$ in the first row of values in Figure 26. The buckling lengths for steel structures are shown in the second row. Only pinned-, rigid- and free end-conditions are considered for the buckling lengths here.

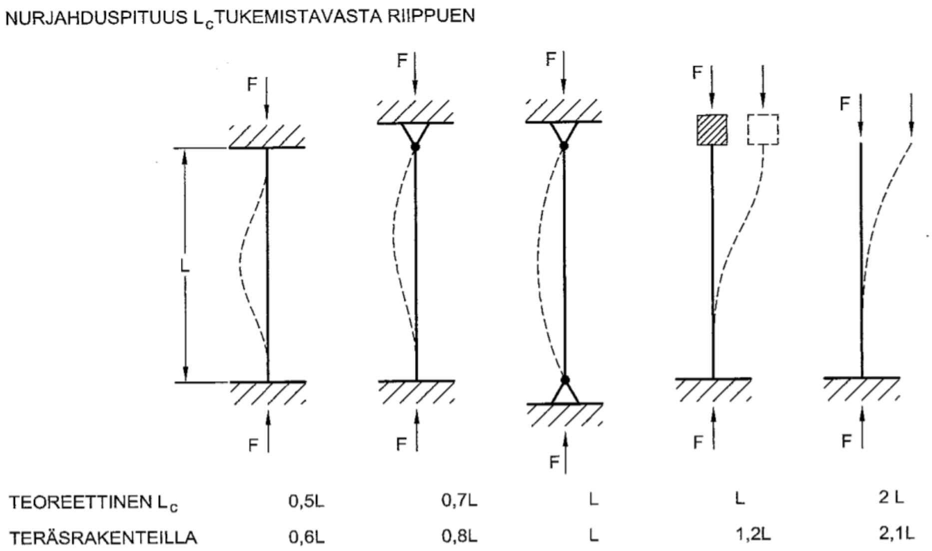



**Figure 26. Buckling shapes and effective lengths (RKL & SBIF, 2020-2021, s. 79).**

According to the first case in Figure 26, the theoretical buckling length factor, for a 4-m-long steel column, with rigid end-joints, is $\beta = 0.6$. The theoretical buckling length is then calculated by:

$$L_{cr} = 0.6 \times 4\, m = 2.4\, m \tag{5.2}$$

In a handbook for steel structural engineers, *Modul 6*, written by Torsten Höglund it stated: *"In EN-1993-1-1 nothing is mentioned regarding semi-rigidity in column joints, which can be interpreted as that theoretical values for $\beta$ may be used"*. $\beta$ is the theoretical factor for buckling lengths, the factor is depending on the end-joint conditions of the analysed column. In *Modul* 6, some approaches for modelling frame deformability is presented, but nothing is mentioned about semi-rigid end-conditions. (Höglund, 2006, pp. 12-15). Because of the lack of information in Eurocode 3, Designers guide to EC3 refers to the British Standard for design, fabrication and erection of structural steelwork, BS 59050. There it is offered buckling lengths that are less optimistic than the theoretical values, and can be adopted when determining buckling length of a compressive member, to reach a value for $\beta$ closer to the truth.

These buckling lengths are presented in Figure 27 below. (Gardner & Nethercot, 2011, pp. 60-61).

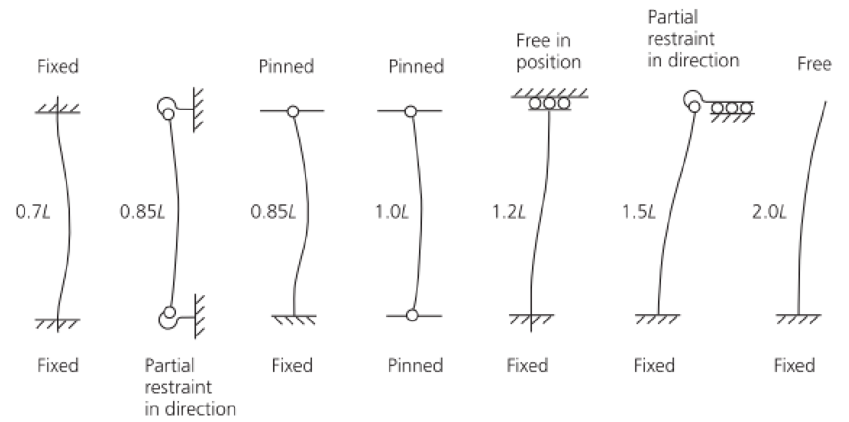



**Figure 27. Designers guide nominal buckling length table (Gardner & Nethercot, 2011, p. 61).**

According to the first case in Figure 27, the theoretical buckling length factor, for a 4-m-long column, with rigid end-joints, is $\beta = 0.7$. The theoretical buckling length is then calculated by:

$$L_{cr} = 0.7 \times 4\ m = 2.8\ m \tag{5.3}$$

It can be shown that shorter buckling length gives less risk of buckling by calculating the critical load, using Euler's formula, (5.1), for a rigid-rigid column with different buckling lengths. First buckling length is according to the Finnish building calendar and the second is according to Designers guide to EC3. The bending stiffness value for a HEA200 profile is used, $EI = 7.753\ \times 10^3\ kNm^2$.

Critical load with buckling length according to the Finnish building calendar:

$$P_{cr} = \frac{\pi^2 \times 7.753\ \times 10^3 kNm^2}{(0.6\ \times 4\ m)^2} = 13\ 284\ \text{kN} \tag{5.4}$$

Critical load with buckling length according to Designers guide to EC3:

$$P_{cr} = \frac{\pi^2 \times 7.753\ \times 10^3 kNm^2}{(0.7 \times 4\ m)^2} = 9760\ \text{k}N \tag{5.5}$$

### 5.1.1 Buckling of compressive member in RF-Stability

In RF-Stability, different eigenvalues, or eigenvectors for buckling, are calculated. Henceforth I will refer to them as E-vectors. The E-vectors considers different types of buckling cases. The stronger direction of the cross-section, around y-axis, and standard buckling is considered, local axis is shown in Figure 28.

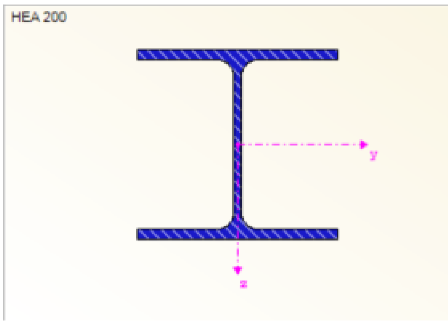



**Figure 28. Local axis system for a HEA200 column in RFEM/RF-Stability.**

In the following two Figures, the stability of a column exposed to an axial nodal force is checked with two different end-joint conditions. In Figure 29 rigid in both ends, and semi-rigid counterparts in Figure 30. For the rigid supported column, E-vector 6 regards buckling in the stronger direction, and for the semi-rigid, E-vector 3. The buckling length factor and the buckling length is referred to as $k_{cr.y}$ and *Effective length*.

2.2 Effective Lengths and Critical Loads

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Member No. | Node No. Start | End | Length L [m] | E-vector No. | Effective Length [m] $L_{cr,y}$ | $L_{cr,z}$ | Effective Length Factor [-] $k_{cr,y}$ | $k_{cr,z}$ | Critical Load $N_{cr}$ [kN] |
| 1 | 1 | 2 | 4.000 | 1 | 3.387 | 2.037 | 0.847 | 0.509 | 6670.530 |
| | | | | 2 | 3.387 | 2.037 | 0.847 | 0.509 | 6670.530 |
| | | | | 3 | 2.763 | 1.662 | 0.691 | 0.416 | 10022.000 |
| | | | | 4 | 2.392 | 1.439 | 0.598 | 0.360 | 13377.000 |
| | | | | 5 | 2.392 | 1.439 | 0.598 | 0.360 | 13377.000 |
| | | | | 6 | 2.221 | 1.336 | 0.555 | 0.334 | 15514.000 |
| | | | | 7 | 1.941 | 1.168 | 0.485 | 0.292 | 20311.300 |
| | | | | 8 | 1.721 | 1.035 | 0.430 | 0.259 | 25850.100 |

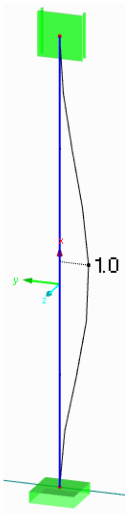



**Figure 29. Buckling lengths and critical loads for a rigid supported column in RF-Stability.**

The initial stiffness value, for the semi-rigid, rigid counterpart, calculated in chapter 3.2.3 is used for the semi-rigid joints, $S_j = 11.42\ MNm/rad$:

2.2 Effective Lengths and Critical Loads

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Member No. | Node No. Start | End | Length L [m] | E-vector No. | Effective Length [m] Lcr,y | Lcr,z | Effective Length Factor [-] kcr,y | kcr,z | Critical Load Ncr [kN] |
| 1 | 1 | 2 | 4.000 | 1 | 3.387 | 2.037 | 0.847 | 0.509 | 6670.530 |
| | | | | 2 | 3.387 | 2.037 | 0.847 | 0.509 | 6670.530 |
| | | | | 3 | 2.763 | 1.662 | 0.691 | 0.416 | 10022.000 |
| | | | | 4 | 2.392 | 1.439 | 0.598 | 0.360 | 13377.000 |

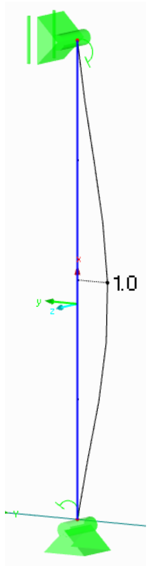



**Figure 30. Buckling lengths and critical loads for a semi-rigid supported column in RF-Stability.**

Similar buckling length and critical load results for the semi-rigid supported column in RF-stability as for the rigid supported column calculated according to Designers guide, (5.3). The buckling length factor being, $\beta = 0.691$ and $\beta = 0.7$, demonstrates that the Designers guide can be used, when considering buckling length for a rigid supported column.

The buckling length factor for a semi-rigid supported column, according to Designers guide, where the buckling length factor is $\beta = 0.85,$ is a conservative option, that could be used without risking an undersized structure.

## 5.2 Buckling length for compressive members in frame structures

In this chapter a buckling length comparison is carried out by comparing a rigid joint frame to a semi-rigid joint frame, using the semi-rigid counterpart to rigid calculated in 3.2, see Table 6. This is done with the RF-stability, to find out risks for buckling modelling when frame joints are considered as rigid, instead of using their actual stiffness.

**Table 6. Rigid joint and semi-rigid counterpart based on results in chapter 3.2**

| JOINTS | RIGID JOINT | SEMI-RIGID COUNTERPART |
|---|---|---|
| STIFFNESS (MN/mrad) | ≥ 38.77 | 11.42 |

Figure 31 presents the buckling behaviour for the rigid joint frame, and Figure 32 semi-rigid joint frame. The frame members are numbered in accordance to RF-Stability, from 1-3. For the both frames, E-vector 4 regards buckling in the stronger direction.

### Rigid frame

2.2 Effective Lengths and Critical Loads

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Member No. | Node No. Start | End | Length L [m] | E-vector No. | Effective Length [m] $L_{cr,y}$ | $L_{cr,z}$ | Effective Length Factor [-] $k_{cr,y}$ | $k_{cr,z}$ | Critical Load $N_{cr}$ [kN] |
| 1 | 1 | 2 | 4.000 | 1 | 18.936 | 11.391 | 4.734 | 2.848 | 213.409 |
| | | | | 2 | 14.262 | 8.579 | 3.565 | 2.145 | 376.201 |
| | | | | 3 | 6.426 | 3.866 | 1.606 | 0.966 | 1853.130 |
| | | | | 4 | 5.455 | 3.282 | 1.364 | 0.820 | 2571.210 |
| 2 | 3 | 4 | 4.000 | 1 | 15.735 | 9.466 | 3.934 | 2.366 | 309.053 |
| | | | | 2 | 11.851 | 7.129 | 2.963 | 1.782 | 544.804 |
| | | | | 3 | 5.340 | 3.212 | 1.335 | 0.803 | 2683.650 |
| | | | | 4 | 4.533 | 2.727 | 1.133 | 0.682 | 3723.560 |
| 3 | 4 | 2 | 5.000 | 1 | 24.506 | 14.741 | 4.901 | 2.948 | 127.423 |
| | | | | 2 | 18.457 | 11.103 | 3.691 | 2.221 | 224.624 |
| | | | | 3 | 8.316 | 5.003 | 1.663 | 1.001 | 1106.480 |
| | | | | 4 | 7.060 | 4.247 | 1.412 | 0.849 | 1535.240 |

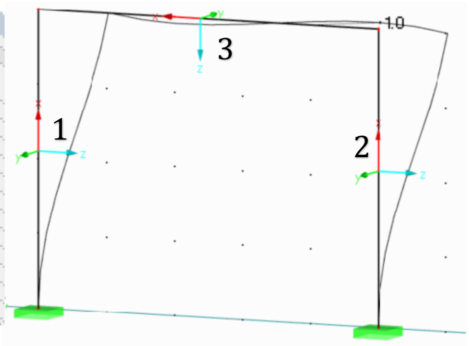



**Figure 31. Frame with rigid joints in RF-Stability.**

### Semi-rigid frame

2.2 Effective Lengths and Critical Loads

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| Member No. | Node No. Start | End | Length L [m] | E-vector No. | Effective Length [m] $L_{cr,y}$ | $L_{cr,z}$ | Effective Length Factor [-] $k_{cr,y}$ | $k_{cr,z}$ | Critical Load $N_{cr}$ [kN] |
| 1 | 1 | 2 | 4.000 | 1 | 18.388 | 11.061 | 4.597 | 2.765 | 226.319 |
| | | | | 2 | 14.131 | 8.501 | 3.533 | 2.125 | 383.189 |
| | | | | 3 | 6.118 | 3.680 | 1.529 | 0.920 | 2044.630 |
| | | | | 4 | 5.839 | 3.513 | 1.460 | 0.878 | 2244.360 |
| 2 | 3 | 4 | 4.000 | 1 | 15.636 | 9.406 | 3.909 | 2.352 | 312.973 |
| | | | | 2 | 12.017 | 7.229 | 3.004 | 1.807 | 529.905 |
| | | | | 3 | 5.202 | 3.129 | 1.301 | 0.782 | 2827.480 |
| | | | | 4 | 4.965 | 2.987 | 1.241 | 0.747 | 3103.680 |
| 3 | 4 | 2 | 5.000 | 1 | 24.996 | 15.036 | 4.999 | 3.007 | 122.475 |
| | | | | 2 | 19.210 | 11.556 | 3.842 | 2.311 | 207.367 |
| | | | | 3 | 8.316 | 5.003 | 1.663 | 1.001 | 1106.480 |
| | | | | 4 | 7.937 | 4.775 | 1.587 | 0.955 | 1214.560 |

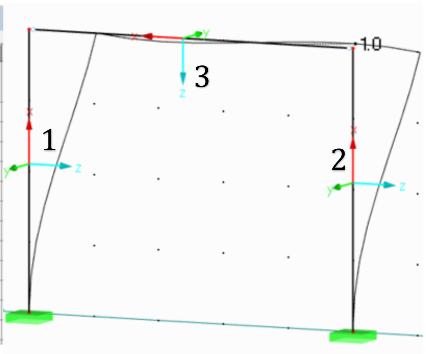



**Figure 32. Frame with semi-rigid joints in RF-Stability.**

Values for the buckling length factor and critical load for each compressive member of the two frames are shown in Table 7:

**Table 7. Result of joint effect buckling length factor in compared frames**

| Frames | Buckling factor 1 | Critical load 1 | Buckling factor 2 | Critical load 2 |
|---|---|---|---|---|
| RIGID | 1.4 | 2571 kN | 1.1 | 3724 kN |
| SEMI-RIGID | 1.5 | 2244 kN | 1.2 | 3104 kN |
| **Difference** | 0.1 | Δ327 kN | 0.1 | Δ620 kN |

As can be seen, the small difference, 0.1, of the buckling length factor, still causes significant differences in the critical loads. This indicates on the rigid case being more optimistic than the semi-rigid.

### 5.2.1 Relationship to initial stiffness

In this chapter, an evaluation of the relationship between joint initial stiffness, $S_{j.ini}$, and buckling length factor, $\beta$, is performed. The analyzed member joint stiffness varies from pinned, to rigid requirement, with semi-rigid joints with increasing stiffness in between. Theoretical values for pinned = 0 and rigid = 100 are also included. The column checked is the first column of the compared frames, which means only the knee-joint condition is varying. The base joint remains rigid. Calculated values for the buckling length factors and ratios of initial stiffness and rigid requirement for the sway compressive member, with different joint conditions, are presented in Table 8:

**Table 8. Buckling length factor ß for non-sway HEA200 column**

| RF-Stab/IDEA | STIFFNESS | RELATION | **SWAY** |
|---|---|---|---|
| End-condition(s) | $S_{j.ini}$ (MN/mrad) | $S_{j.ini}$ / $S_{j.rigid.req}$ | $ß_{n-s}$ |
| Pinned -- | 0.00 | 0.00 | 2.01 |
| Pinned | 0.77 | 0.02 | 1.85 |
| Semi-rigid | 1.54 | 0.04 | 1.75 |
| Semi-rigid | 3.08 | 0.08 | 1.64 |
| Semi-rigid | 7.10 | 0.18 | 1.52 |
| Semi-rigid | 13.44 | 0.35 | 1.46 |
| Semi-rigid | 19.77 | 0.51 | 1.44 |
| Semi-rigid | 26.10 | 0.67 | 1.43 |
| Semi-rigid | 32.44 | 0.84 | 1.42 |
| Rigid | 38.77 | 1.00 | 1.41 |
| Rigid ++ | 100.00 | 2.58 | 1.38 |

As can be seen in Table 8, a stiffer knee-joint will provide a smaller buckling length factor, which will lead to a more optimistic design.

A method to calculate buckling factor straight from a known stiffness could be received by plotting the relation and buckling factor values to find a correlation. This is done inFigure 33. However, the theoretical stiffness values are skipped to find a more practical result:

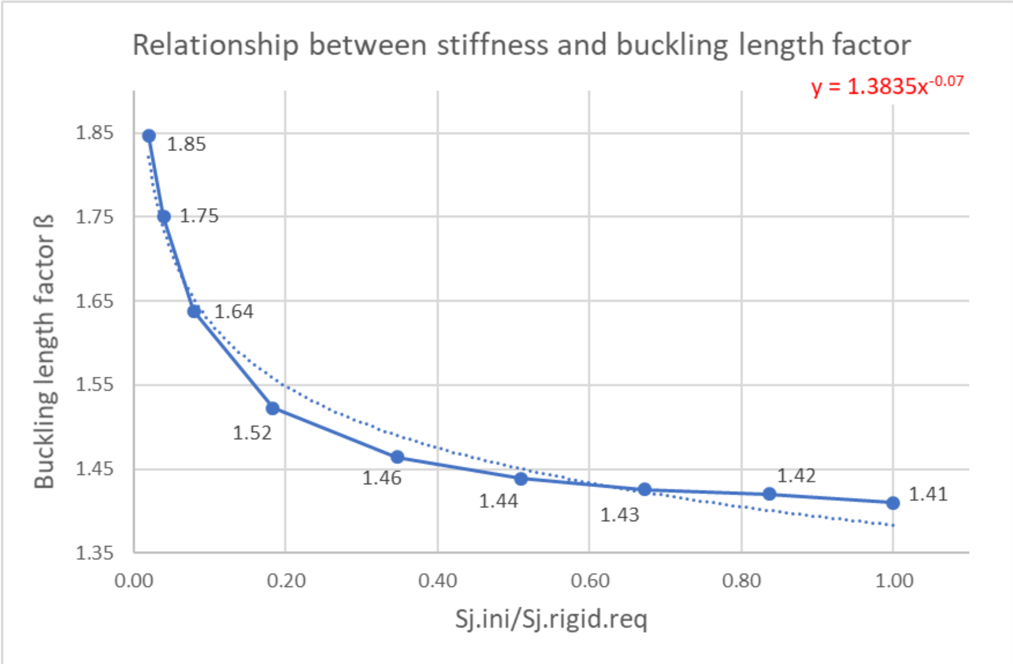



**Figure 33. Relationship of buckling length factor to joint stiffness.**

Here is found an equation that works quite well, but this needs to be tested for other column profiles as well before it is applicable in daily practice.

Deviation between the buckling length factor calculated with excel, to exact values received from RF-Stability is shown in Table 9:

**Table 9. Calculation of y-value and difference to actual value**

| | y=1.4x^-0.07 | **Deviation** |
|---|---|---|
| **x-values** | 1.84 | 0.005 |
| | 1.75 | -0.005 |
| | 1.67 | -0.034 |
| | 1.58 | -0.054 |
| | 1.51 | -0.044 |
| | 1.47 | -0.029 |
| | 1.44 | -0.013 |
| | 1.42 | 0.002 |
| | 1.40 | 0.010 |
| | 1.31 | 0.070 |
| | | -0.090 |

# 6 Stiffness calculation with IDEAStatiCa compared to EC3

In the following chapters, will be carried out rotational stiffness calculation evaluation by a theoretical approach following the guidelines in EN-1993-1-8 and a component based finite element analysis, IDEAStatiCa, on joint modelling. The results of the EC3 calculation, that is created to get insight in rotational stiffness calculation, is compared to an IDEAStatiCa calculation of an identical connection. Initial stiffness of a joint will be calculated based on EN1993-1-8. In Appendix 1 the equation parameters for the Eurocode calculation are explained step by step.

## 6.1 Geometry

Geometry of the joint is presented in Figure 34. Bolts are 8.8 strength, end plates S355, profiles S355, HEA200 profiles for both column and beam.

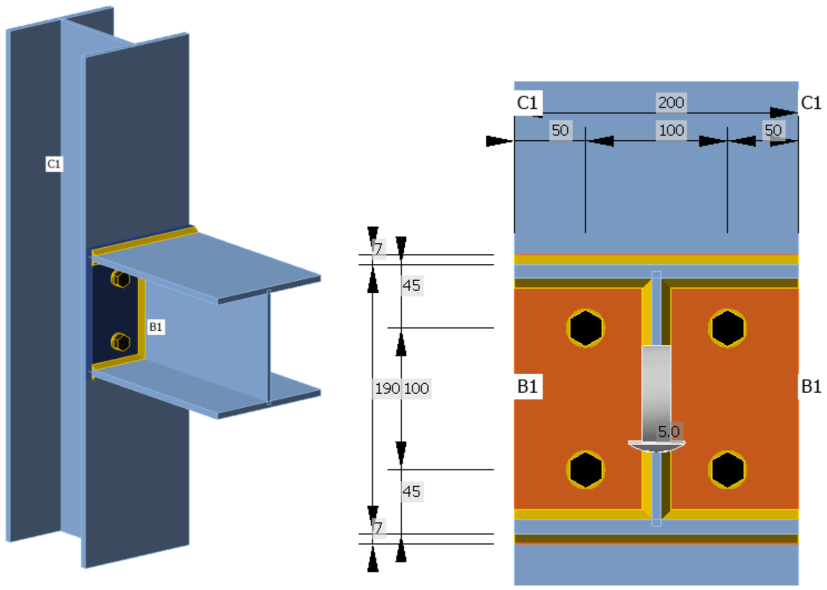



**Figure 34. Joint type studied, modelled with IDEAStatiCa.**

## 6.2 Initial stiffness according to EC3-1-8

To reach values for initial stiffness and design moment resistance, the joint will be modelled as an assembly of basic components. The design moment resistance, $M_{j.Rd}$ of a beam-to-column joint is calculated according to EN1993-1-8, 6.2.7.2 (6.25):

$$M_{j.Rd} = \sum h_r F_{tr.Rd} \tag{6.1}$$

Where:

$F_{t.Rd}$ is the effective design tension resistance of bolt-row r;

$h_r$ is the distance from bolt-row r to the centre of compression, Figure 35;

$r$ is the bolt row number. (EN-1993-1-8:6.2.7.2).

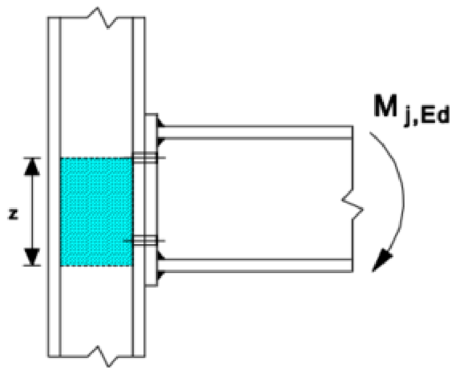



**Figure 35. Bolted end-plate connection with only one bolt-row active in tension $h_r$ = z.**

According to EN1993-1-8:6.2.7.2(6), the effective design tension resistance $F_{t,Rd}$ of bolt-row $r$, taken as an individual bolt-row, should be taken as the smallest value of the design tension resistance for an individual bolt-row of the following basic components:

- the column web in tension $F_{t,wc,Rd}$
- the column flange in bending $F_{t,fc,Rd}$
- the end-plate in bending $F_{t,ep,Rd}$
- the beam web in tension $F_{t,fc,Rd}$

The initial rotational stiffness is calculated according to EN1993-1-8, 2005, 6.3 (6.27):

$$S_j = \frac{Ez^2}{\mu \sum_i \frac{1}{k_i}} \tag{6.2}$$

Where:

- $k_i$ is the stiffness coefficient for basic joint component i;
- $z$ is the lever arm, see Figure 35;
- $\mu$ is the stiffness ratio $S_{j,ini}/S_j$, for initial stiffness $\mu = 1$

(EN1993-1-8(6.28a)).

The stiffness coefficients considered is presented in Table 10.

**Table 10. Joints with bolted end-plate connections**

| Beam-to-column joint with bolted end-plate connections | Number of bolt-rows in tension | Stiffness coefficients $k_i$ to be taken into account |
|---|---|---|
| Single-sided | One | $k_1$; $k_2$; $k_3$; $k_4$; $k_5$; $k_{10}$ |
| | Two or more | $k_1$; $k_2$; $k_{eq}$ |
| Double sided – Moments equal and opposite | One | $k_2$; $k_3$; $k_4$; $k_5$; $k_{10}$ |
| | Two or more | $k_2$; $k_{eq}$ |
| Double sided – Moments unequal | One | $k_1$; $k_2$; $k_3$; $k_4$; $k_5$; $k_{10}$ |
| | Two or more | $k_1$; $k_2$; $k_{eq}$ |

(EN1993-1-8: Table 6.10).

### 6.2.1 Stiffness coefficients

The stiffness coefficients to be taken into account is depending on the geometry of the joint, which leads to what basic joint components are to be considered. The basic joint components that determine the design moment-rotation characteristics of a beam to column joint with bolted end plate connections are:

- Column web panel in shear
  In Table 10, $k_1$ stands for column web panel in shear, which stiffness coefficient is taken from EN 1993-1-8:6.3.2.

- Column web in compression
  In Table 10, $k_2$ is the stiffness coefficient for column web in compression, which is taken from EN 1993-1-8:6.3.2.

- Column web in tension
  In Table 10, $k_3$ is the stiffness coefficient for column web in tension, which is taken from EN 1993-1-8:6.3.2.

- Column flange in bending
  In Table 10, $k_4$ is the stiffness coefficient for column flange in bending, which is taken from EN 1993-1-8:6.3.2.

- End-plate in bending
  In Table 10, $k_5$ is the stiffness coefficient for end-plate in bending, which is taken from EN 1993-1-8:6.3.2.

- Bolts in tension
  In Table 10, $k_{10}$ is the stiffness coefficient for bolts in tension, which is taken from EN 1993-1-8:6.3.2.

### 6.2.2 Calculation conclusion

All calculations are presented step by step in Appendix 1.

**Calculation of the joint rotational stiffness using formula (6.3):**

$$S_j = \frac{Ez^2}{\mu \times (\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} + \frac{1}{k_5} + \frac{1}{k_{10}}} = 4.75\ MNm/rad \tag{6.3}$$

where,

$$k_1 = \frac{0.38 \times A_{vc}}{\beta \times z} = 4.91\ mm \tag{6.4}$$

$$k_2 = \frac{0.7 \times b_{eff.c.wc} \times t_w}{d_w} = 6.29\ mm \tag{6.5}$$

$$k_3 = \frac{0.7 \times b_{eff.t.wc} \times t_w}{d_w} = 6.52\ mm \tag{6.6}$$

$$k_4 = \frac{0.7 \times l_{eff.cf} \times {t_f}^3}{m^3} = 5.10\ mm \tag{6.7}$$

$$k_5 = \frac{0.9 \times l_{eff.ep} \times {t_p}^3}{m^3} = 18.24\ mm \tag{6.8}$$

$$k_{10} = \frac{1.6 \times A_{sb}}{L_b} = 10.05\ mm \tag{6.9}$$

The rotational stiffness calculated with IDEAStatiCa is $S_j = 2.02\ MNm/rad$, see Appendix 1. This does not match with the results of the theoretical approach.

**Calculation of the design bending resistance:**

$$M_{j.Rd} = h_r \times F_{t.Rd} = 21.165\ kNm \tag{6.10}$$

where,

$$F_{t.Rd} = min(F_{t.wc.Rd}, F_{t.fc.Rd}, F_{t.ep.Rd}, F_{t.wb.Rd}) = 15.177\ kN \tag{6.11}$$

The design bending resistance calculated with IDEAStatiCa, $M_{j.Rd} = 30.0\ kNm$, see Appendix 1, does not match with the results of the theoretical approach.

# 7 Results

In this chapter, results of the thesis are presented and how the results are correlated to the concepts of the literature review is described.

## 7.1 Determining stiffness

It is proven that common pinned or rigid joints may possess an initial stiffness other than the classification requirements. Results of stiffness determination are presented in Table 11. The table consists of calculated stiffness, stiffness in percentage for each joint, a relative price and the price deviation of theoretical joint to the semi-rigid counterpart.

**Table 11. Result of calculated joints**

| **JOINTS** | RIGID JOINT | SEMI-RIGID COUNTERPART | PINNED JOINT | SEMI-RIGID COUNTERPART |
|---|---|---|---|---|
| STIFFNESS (MNm/rad) | ≥ 38.77 | 11.42 | ≤0.78 | 1.62 |
| STIFFNESS IN PERCENTAGE | 100% | **29%** | 100% | **210%** |
| IDEAStatiCa price | 141 € | 71 € | 27 € | 22 € |
| RELATIVE PRICE | 100% | 50% | 100% | 81% |
| PRICE DEVIATION | **70 €** | | **5 €** | |

As can be seen in Table 11, the semi-rigid counterpart to pinned is twice as stiff and the counterpart to rigid is less than 1/3 stiff. Brown & Iles states in *"Joints in Steel Construction"*, 2013 that rigid joints are more expensive than pinned joints due to the costs of extra stiffeners, cutting, welding, and inspections. This statement is true when comparing the relative prices of each joint.

However, as stated in *Economy Studies of Steel Building Frames with Semi-Rigid Joints*, 1998, by Weynand, Jaspart & Steenhuis, pinned joints are normally more economic in braced frames, while semi-rigid and rigid usually are better in unbraced frames. The results of the semi-rigid joint effect in unbraced frames are presented in the following chapters.

## 7.2 Bending moment distribution

Results of joint effect in bending moment distribution are presented in Table 12. The table consists of max span moment and max joint moment for each compared frame.

**Table 12. Result of joint effect on bending moment distribution in frame structures**

| Frames | Max span moment | Max joint moment |
|---|---|---|
| PINNED | 31.5 kNm | - |
| SEMI-RIGID | 25.4 kNm | 11.8 kNm |
| **Deviation** | 6.1 kNm | - |
| RIGID | 17.8 kNm | 27.0 kNm |
| SEMI-RIGID | 19.9 kNm | 23.1 kNm |
| **Deviation** | 2.1 kNm | 3.9 kNm |

As can be seen in Table 12, it has been discovered that rigid joints lead to large joint reactions, which leads to a higher amount of steel tonnage in both the jointed member and the connection. For pinned cases, it has been proven that a large span moment occurs if joint stiffness is not considered, which leads to oversized structure. As confirmed by the comparisons, a semi-rigid counterpart, would lead to a better distribution of the bending moment.

According to *Economy Studies of Steel Building Frames with Semi-rigid Joints*, 1998, by Weynand, Jaspart & Steenhuis, semi-rigid joints should be used in frame structures, since it provides freedom to optimize the global frame and joint design. This correlates to my results that semi-rigid joints can distribute the bending moment more equally and lead to cheaper and simpler joints.

Another dictum that meets my results is in "*Minimum Cost Steel Beam using Semi-rigid Joints*", by Haapio & Heinisuo, 2010, that inherent stiffness in pinned joint can be utilized to keep a required span moment capacity and save steel tonnage.

## 7.3 Displacement

Results of joint effect for displacement are presented in Table 13. The table consists of max horizontal displacement for each compared frame.

**Table 13. Results of joint effect for displacement in compared frames**

| FRAMES | PINNED | SEMI-RIGID COUNTERPART | RIGID | SEMI-RIGID COUNTERPART |
|---|---|---|---|---|
| Max displacement | 23.7 mm | 17.2 mm | 9.7 mm | 11.5 mm |
| Displacement percentage | 100% | 72.6% | 100% | 118.6% |

As can be seen in Table 13, the displacement of the pinned frame is larger than the semi-rigid. This can lead to an oversized structure due to displacement, if not the joint stiffness is considered.

Smaller displacement occurs for the rigid frame than in the semi-rigid. If the actual behaviour is according to the semi-rigid, it can lead to risks with too optimistic design.

## 7.4 Buckling length

Through the analysis of the semi-rigid joint effect on buckling length, it is proven that structures with common rigid joints will have shorter buckling length than expected, if not the joint stiffness is considered. By calculating the critical load using different buckling lengths, it has been found that shorter buckling lengths will give a larger critical load. This means that structures with common rigid joints, that is semi-rigid, are designed optimistically in relation to the true behaviour, if the joint stiffness is not considered.

Results of joint effect for the buckling length factor are presented in Table 14. The table consists of calculated buckling length factors and critical loads for the rigid- and semi-rigid frame.

**Table 14. Result of joint effect buckling length factor in compared frames**

| Frames | Buckling factor 1 | Critical load 1 | Buckling factor 2 | Critical load 2 |
|---|---|---|---|---|
| RIGID | 1.4 | 2571 kN | 1.1 | 3724 kN |
| SEMI-RIGID | 1.5 | 2244 kN | 1.2 | 3104 kN |
| **Difference** | 0.1 | Δ327 kN | 0.1 | Δ620 kN |

Even if the difference between the buckling length factor is small, it still gives a significant deviation for the critical load. As can be seen in Table 14, the rigid frame buckling length behaviour is more optimistic than in the semi-rigid case. This can lead to risk of buckling failure, if the joint stiffness is not considered.

A relationship of the buckling length factor to the initial stiffness has been demonstrated by calculating buckling length factor for a compressive member in a sway frame structure with varying knee-joint conditions.

Relationship between joint stiffness and buckling length factor calculated using RF-Stability and excel is presented in Figure 36:

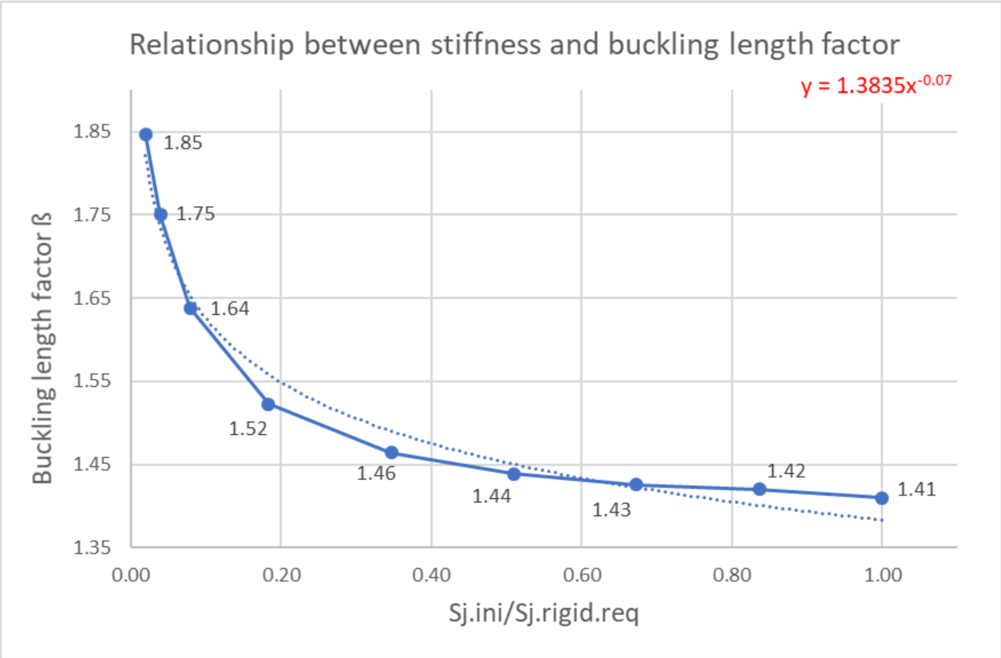



**Figure 36. Result of buckling relation to stiffness.**

The equation (7.1), gives a value for the buckling length factor when the initial stiffness relation to required stiffness, for a HEA200-profile, is known. This could ease buckling length calculations for structures when rotational stiffness is determined. The equation is simplified as:

$$y = 1.4x^{-0.07} \tag{7.1}$$

By using the formula, (7.1), given in excel a deviation of 0.09 is received for the buckling length factor compared to RF-Stability values. The deviations received are not significant. However, to find a true relationship between the buckling length factor and the initial stiffness, more steel profiles with varying lengths should be considered.

## 7.5 Initial stiffness calculation in IDEAStatiCa compared to EC3-1-8

Difference between the numerical approach and IDEAStatiCa approach for joint rotational stiffness calculation is presented in Table 15 below.

**Table 15. Result of initial stiffness calculation**

| APPROACH | Initial stiffness | Moment resistance |
|---|---|---|
| IDEAStatiCA | 2.0 MNm/rad | 30 kNm |
| EC3-1-8 | 4.8 MNm/rad | 21.2 kNm |
| Difference | 2.8 MNm/rad | 8.8 kNm |

The calculation evaluation of the numerical approach according to EN 1993-1-8 and the IDEAStatiCa approach did not result in a favourable way. The gap between the rotational stiffness received for each method is too big for it to be considered as one of them being conservative or specific. However, the process of the numerical method, as well as using IDEAStatiCa, gave insight in the component method and usage of the Eurocodes when determining rotational stiffness. The calculation process of the joint rotational stiffness according to the Eurocode needs to be further investigated.

# 8 Conclusion

In this thesis it has been proven that by calculating rotational stiffness for common pinned and rigid joints, the effect on steel structures should be considered as that joints are in fact semi-rigid. According to the results I presented in the previous chapter, semi-rigid joint effect does have an impact on the true behaviour of a steel frame structure. If common engineering joints do not have the required stiffness for an assumed joint type, risks regarding both displacements, bending moment distribution and buckling may occur.

**How do semi-rigid joints affect the frame behaviour in terms of bending moment distribution and horizontal displacements?**

The bending moment distribution for rigid frames causes large joint moment and small span moment. If a semi-rigid joint is utilized instead, the bending moment distribution is more even between the span moment and the joint moment. Since the beam profile for a frame structure is designed according to the maximum of joint- and span moment, semi-rigid joints may be used to optimize the capacity.

The horizontal displacement has been investigated, and found that if using semi-rigid joints, the frame will displace more than when using theoretically rigid. For the pinned case it is the other way around, when comparing pinned frame to the frame with semi-rigid counterpart joints.

**What impact can it have on a structure buckling length if the joint stiffness is calculated?**

By comparing the theoretically rigid jointed frame to the frame with semi-rigid counterpart joints, the buckling length factor causes differences in the critical load calculation. The buckling length factors calculated for the semi-rigid frame are larger than for the rigid, which leads to longer buckling length, that causes a smaller critical load. This means that a smaller load can lead to buckling if semi-rigidity is considered.

**What are the risks of using common pinned- and rigid joint solutions in frame structures?**

Risks have been found for both the pinned- and the rigid frame in bending moment distribution, if the stiffnesses of the joints are not considered. The risk for pinned frames is that they might be designed according to a larger span moment, that leads to an unnecessary conservative design for the beam. The same applies to the rigid frame, and here also a large amount of steel material would be required to fulfil the rigid classification boundary.

When it comes to displacement, assumed rigid joints will lead to small displacement and pinned to large. When using a semi-rigid counterpart for the rigid case, risks with a too optimistic design is avoided. On the other hand, when using semi-rigid counterpart for the pinned case, a too conservative design can be avoided. It is easy to underestimate displacement in rigid joints, if the joint is not actually rigid. Displacement might be overestimated when modelling pinned joints that actually are semi-rigid.

The buckling length design for rigid frames is more optimistic than when the joint stiffness is considered. This can lead to an overestimation of the frame capacity for the critical load and lead to failure.

**What are the rules and limits with joints in steel structures?**

It has been mentioned earlier in this thesis that joints must be modelled since their behaviour affects the structural frame response. Rules and limits that have been discovered, for joints studied in the thesis, are as follows:

For all joints,

- Strength of the bolts in joints should not be exhausted before a yield line mechanism in the plate occurs.

For pinned,

- Transfers in theory no bending moment, only shear and normal forces. Must be able to transfer these forces without giving origin to large moments.
- Enable rotation by freeing bottom flange by a gap, to avoid compressive load on the column due to rotation.
- In larger buildings it is necessary to use pinned joints and use trusses and bracings for stabilization.

For rigid,

- Joints classified as rigid are assumed to have enough rotational stiffness, that allows for an overall analysis based on the continuity of the structure.
- Bolts acting in tension is an optimal design approach regarding economy and function.
- Shear loads acting on a rigid joint cause loss of stiffness and requires more bolts.
- Stabilization with rigid jointed frames can be used in smaller buildings.

As per the results received in this thesis it could be an advantage to use semi-rigid joints in daily structural engineering practise. If proper tools to determine stiffness and buckling length factor are available, semi-rigid joints should be used in the global analysis model. However, if time is key and knowledge is limited, usage of rigid and pinned joints may be preferable. Then, based on the effect of common pinned and rigid joints, the designer can use engineering judgement and not rely fully on the FEM-design results. If software for FEM-design is not affordable, it is more sensible to continue with only regarding rigid or pinned joint conditions.

## 8.1 Personal reflection

The work that I have put in has been very rewarding, in terms of self-development. Throughout the process I have had issues with both IDEAStatiCa and RFEM, as well as with the stiffness calculation according to the Eurocode. Unfortunately, the Eurocode based calculation result was not favourable, and needs to be further studied.

However, I still have gained a lot of experience and knowledge of studying the Eurocode, EN1993-1-8 as well as EN1993-1-1 and learned a lot about beam-to-column connection design. In addition to learning good structural engineering theory, I am nowadays more confident in using RFEM, and IDEAStatiCa. Nevertheless, the work of this thesis has most certainly given me a great base for my future engineering career.

## 8.2 Suggestions for further research

Further research regarding this topic would be to do more advanced comparisons and calculations and consider plastic global analysis and 2$^{nd}$ order analysis. It would be interesting to find a correlation between joint stiffness and the stiffness of the connected member, for a semi-rigid joint beam where the bending moments in both span and joint are equal. A tangent topic for a higher research level would be to perform a study on parametric modelling and AI learning for steel connection design.

## 8.3 Acknowledgements

I would like to acknowledge and give my warmest thanks to my supervisors from Citec Oy Ab: Manager Ronny Engblom, for his great guidance and insight, and Lead Engineer Mikael Renlund, for his expertise and knowledge in the subject. They have given me many helpful advices and pushed me in the right direction throughout the process.

I also want to direct a special thanks to my supervisor from Novia University of Applied Science, Senior Lecturer Anders Borg, who has encouraged me to test my limits and helped me to increase my knowledge in the subject.

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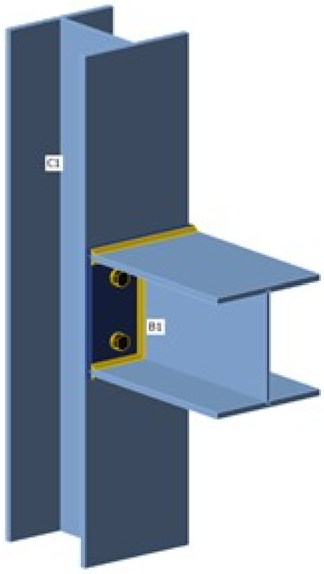
**Rotational stiffness and moment resistance calculation template**

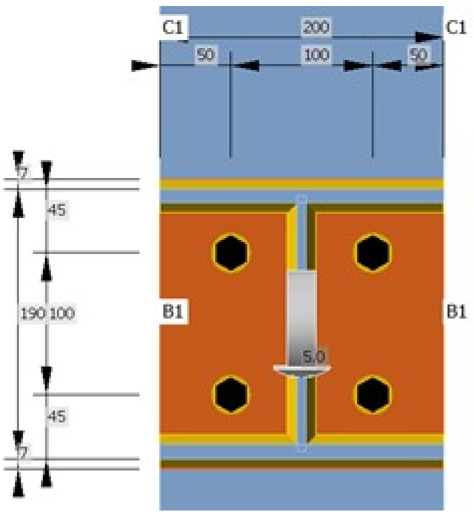
**According to EN1993-1-8**
The joint will be designed as a header plate connection between a beam and a column, both HEA200




**1. Joint material and properties**
modulus of elasticity $E:=210000\ MPa$
yield strength $f_y:=355\ MPa$
HEA200:

| | |
|---|---|
| width | $b:=200\ mm$ |
| height | $h:=190\ mm$ |
| root radius | $r:=18\ mm$ |
| flange thickness | $t_f:=10\ mm$ |
| web thickness | $t_w:=6.5\ mm$ |
| cross-section area | $A:=5383\ mm^2$ |
| | source: Stålbyggnadsfakta |

The joint will be designed according to figure below:




distance from bolt to edge of column flange $e_1:=50\ mm$
distance from bolt to edge of end plate $e_2:=50\ mm$
distance from bolt to edge of beam flange $e_3:=45\ mm$

$$e_{min}:=min\left(e_1, e_2\right)=50\ mm$$

End plate thickness: $t_p:=15\ mm$

welds (beam to end plate) $a:=5\ mm$

$$s:=r \quad s_p:=\frac{t_p}{\sin(45\ °)}=21.213\ mm \qquad [6.2.6.1(1)]$$

Bolts M16 8.8

| | | |
|---|---|---|
| yield strength | $f_{ub}:=800\ MPa$ | |
| design strength | $f_{yb}:=640\ MPa$ | |
| washer: | $w_t:=4\ mm$ | |
| nut: | $h_{nut}:=10\ mm$ | ISO 4017 |
| bolt head: | $h_{bh}:=10\ mm$ | |
| elongation length: | $L_b:=25\ mm$ | (-->Same as in IDEAStatiCa) |
| shear area: | $A_{sb}:=157\ mm^2$ | |

## 2. STIFFNESS COEFFICIENTS FOR THE BASIC COMPONENTS

from table 6.10 for a "single-sided Beam-to-column joint with bolted end-plate connections"

**Table 6.10** (part of)

| Beam-to-column joint with bolted end-plate connections | Number of bolt-rows in tension | Stiffness coefficients $k_i$ to be taken into account |
|---|---|---|
| Single-sided | One | $k_1$; $k_2$; $k_3$; $k_4$; $k_5$; $k_{10}$ |
| | Two or more | $k_1$; $k_2$; $k_{eq}$ |

### Column web panel in shear

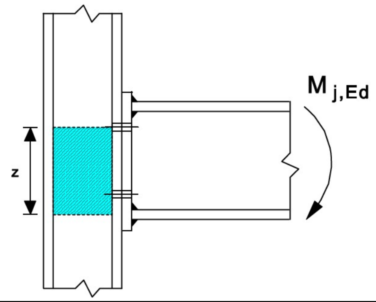
$$k_1 = \frac{0.38 \cdot A_{vc}}{\beta \cdot z}$$ (Table 6.11)

Shear area of the column $A_{vc} := A - 2 \cdot b \cdot t_f + (t_w + 2 \cdot r) \cdot t_f = (1.808 \cdot 10^3)\ mm^2$ [EN1993-1-1-6.2.6(3)]

transformation parameter $\beta := 1$ [5.3(7) & Table 5.4]

lever arm $z := 140\ mm$

c) Bolted end-plate connection with only one bolt-row active in tension




$$k_1 := \frac{0.38 \cdot A_{vc}}{\beta \cdot z} = 4.907\ mm$$

### Column web in compression

$$k_2 = \frac{0.7 \cdot b_{eff.c.wc} \cdot t_w}{d_c}$$ (Table 6.11)

web depth $d_w := h - t_f - t_f - r - r = 134\ mm$

effective width of column web in compression

$b_{eff.c.wc} := t_f + 2 \cdot \sqrt{2} \cdot a + 5 \cdot (t_f + s) + s_p = 185.355\ mm$ (6.2.6.2)

$$k_2 := \frac{0.7 \cdot b_{eff.c.wc} \cdot t_w}{d_w} = 6.294\ mm$$

### Column web in tension

$$k_3 = \frac{0.7 \cdot b_{eff.t.wc} \cdot t_w}{d_w}$$ (Table 6.11)

effective width $b_{eff.t.wc}$ 6.2.6.4/3

$$b_{eff.t.wc} = min(2 \cdot \pi \cdot m, 4 \cdot m + 1.25 \cdot e_{min})$$ (Table 6.4)

where $m := \frac{b}{2} - \frac{t_w}{2} - e_{min} - 0.8 \cdot r = 32.35\ mm$ and $e_{min}$ is according to Figure 6.8

$b_{eff.t.wc} := min\left(2 \cdot \pi \cdot m, 4 \cdot m + 1.25 \cdot e_{min}\right) = 191.9\ mm$

$$k_3 := \frac{0.7 \cdot b_{eff.t.wc} \cdot t_w}{d_w} = 6.516\ mm$$

## Column flange in bending

$k_4 = \dfrac{0.9 \cdot l_{eff.cf} \cdot {t_f}^3}{m^3}$ (Table 6.11)

effective length $l_{eff.cf} := min\left(2 \cdot \pi \cdot m, 4 \cdot m + 1.25 \cdot e_{min}\right)$ (Table 6.4)

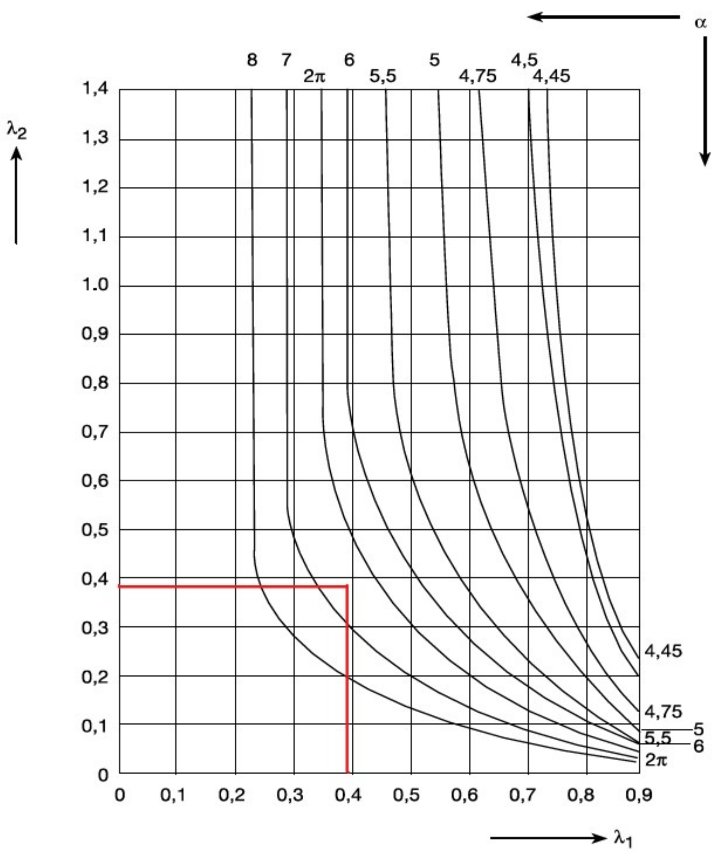
$$k_4 := \frac{0.9 \cdot l_{eff.cf} \cdot {t_f}^3}{m^3} = 5.101\ mm$$

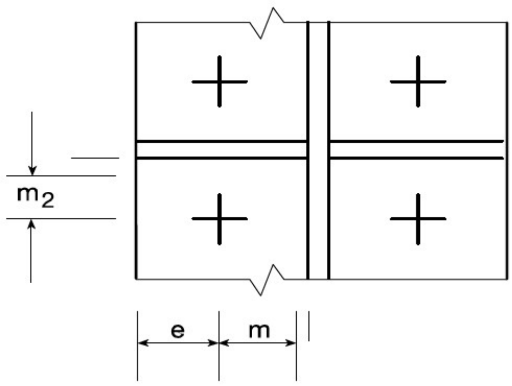
## End plate in bending

$k_5 = \dfrac{0.9 \cdot l_{eff.ep} \cdot {t_p}^3}{m^3}$ (Table 6.11)

effective length $l_{eff.ep} = min\left(2 \cdot \pi \cdot m, \alpha \cdot m\right)$ table 6.6
*where $m = 0.032\ m$ , same as for components above

$\alpha$ is taken from Figure 6.11




(part of figure 6.11)
where
(I interpret $m_2$ as the distance from bolt row in tension to middle of the beam-end-plate weld.)

$$m_2 := e_3 - \frac{t_f}{2} - 0.8\ a \cdot \sqrt{2} = 34.343\ mm$$

(part of figure 6.11)

$$\lambda_1 := \frac{m}{m + e_{min}} = 0.393$$

$\lambda_2 := \frac{m}{m_2 + e_{min}} = 0.384$ both formulas: (Figure 6.11)

According to Figure 6.11 $\alpha := \frac{(7 + 2 \cdot \pi)}{2} = 6.642$

$l_{eff.ep} := min(2 \cdot \pi \cdot m, \alpha \cdot m) = 203.261\ mm$

$k_5 := \frac{0.9 \cdot l_{eff.ep} \cdot t_p^{\ 3}}{m^3} = 18.237\ mm$

**Bolts in tension**

$k_{10} := \frac{1.6 \cdot A_{sb}}{L_b} = 10.048\ mm$ (Table 6.11)

$A_{sb} = 157\ mm^2$

### 3. ROTATIONAL STIFFNESS

According to [6.3]

$$S_j = \frac{Ez^2}{\mu \sum_i \frac{1}{k_i}} \qquad \text{... (6.27)}$$

where $\mu := 1.0$ for initial stiffness ..(6.28a)

$$S_j := \frac{E \cdot z^2}{\mu \cdot \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \frac{1}{k_4} + \frac{1}{k_5} + \frac{1}{k_{10}}\right)} = 4.75\ \frac{MN \cdot m}{rad}$$

Initial stiffness calculated with Ideastatica: $S_{j.in} = 2.02\ \frac{MN \cdot m}{rad}$

## 4. DESIGN MOMENT RESISTANCE

According to [6.2.7.2]

$$M_{j,Rd} = \sum_r h_r F_{tr,Rd} \qquad \text{... (6.25)}$$

where

$F_{tr.Rd}$ is the effective design tension resistance of bolt row r

$h_r$ is the distance of bolt row r to the centre of compression (same as lever arm in previous calulation)

$h_r := z$

$r$ is the bolt row number

$F_{tr.Rd}$ is calculated according to [6.2.7.2(6)]

where $F_{tr.Rd}$ should be taken as the smallest value of the following basic components:

- column web in tension $F_{t.wc.Rd}$
- column flange in bending $F_{t.fc.Rd}$
- end-plate in bending $F_{t.ep.Rd}$
- beam web in bending $F_{t.wb.Rd}$

Partial safety factors

$\gamma_{M0} := 1$

$\gamma_{M1} := 1$

$\gamma_{M2} := 1.25$

---

**Column web in tension** [6.2.6.3]

$$F_{t,wc,Rd} = \frac{\omega b_{eff,t,wc} t_{wc} f_{y,wc}}{\gamma_{M0}} \qquad \text{... (6.15)}$$

where,

$$\omega := \frac{1}{\left(\sqrt{1 + 1.3 \cdot \left(\frac{b_{eff.t.wc} \cdot t_w}{A_{vc}}\right)^2}\right)} = 0.786$$ is a reduction factor for shear (Table 6.3)

when the transformation parameter $\beta := 1$ [5.3(7)]

safety factor $\gamma_{M0} := 1$

$$F_{t.wc.Rd} := \frac{\omega \cdot b_{eff.t.wc} \cdot t_w \cdot f_y}{\gamma_{M0}} = 348.037 \ kN$$

---

**Column flange in bending** [6.2.6.4]

The design resistance of an unstiffened column flange in transversal bending is calculated by using an equivalent T-stub flange.

In case prying effect is to be considered, $F_{t.fc.Rd}$ is taken as the smallest value for the three possible failure modes 1, 2 and 3

<u>Check if prying forces may develop:</u> $L_b \le L_{b\#}$

$$L_{b\#} = \frac{8.8 \cdot m^3 \cdot A_{sb}}{\Sigma L_{eff.1} \cdot t_f^{\,3}} \qquad \text{(Table 6.2)}$$

where
effective length for mode 1 $\Sigma L_{eff.cf.1} := l_{eff.cf} = 191.9\ mm$
effective length for mode 2 $\Sigma L_{eff.cf.2} := 4 \cdot m + 1.25 \cdot e_{min} = 191.9\ mm$ (Table 6.4)

$$L_{b\#} := \frac{8.8 \cdot m^3 \cdot A_{sb}}{\Sigma L_{eff.cf.1} \cdot t_f^{\ 3}}$$

$L_b \le L_{b\#} = 1$ --> PRYING EFFECT!

$$F_{t.fc.Rd} = min\left(\frac{4 \cdot M_{pl.1.Rd}}{m}, \frac{2 \cdot M_{pl.2.Rd} + n \cdot \Sigma F_{t.Rd}}{m+n}, \Sigma F_{t.Rd}\right)$$ (Table 6.2)

where,

plastic bending resistance for mode 1 $M_{pl.1.Rd} := \dfrac{0.25 \cdot \Sigma L_{eff.cf.1} \cdot t_f^{\ 2} \cdot f_y}{\gamma_{M0}} = 1.703\ kN \cdot m$

plastic bending resistance for mode 2 $M_{pl.2.Rd} := \dfrac{0.25 \cdot \Sigma L_{eff.cf.2} \cdot t_f^{\ 2} \cdot f_y}{\gamma_{M0}} = 1.703\ kN \cdot m$

$n := min\left(e_{min}, 1.25\ m\right) = 50\ mm$

the design resistance of all bolts in the T-stub $\Sigma F_{t.Rd} := \left(\dfrac{0.9 \cdot f_{ub} \cdot A_{sb}}{\gamma_{M2}}\right) \cdot 2 = 180.864\ kN$

(Modul 10, p. 14 (2.6))

$$F_{t.fc.Rd} := min\left(\frac{4 \cdot M_{pl.1.Rd}}{m}, \frac{2 \cdot M_{pl.2.Rd} + n \cdot \Sigma F_{t.Rd}}{m+n}, \Sigma F_{t.Rd}\right) = 151.177\ kN$$

---

**End-plate in bending** [6.2.6.5]

The design resistance of an end plate in bending is calculated by using an equivalent T-stub flange.

Since prying effects may develop in the joint, same principle as above is followed.

where
effective length for mode 1 $\Sigma L_{eff.ep.1} := l_{eff.ep} = 203.261\ mm$
effective length for mode 2 $\Sigma L_{eff.ep.2} := \alpha \cdot m = 214.856\ mm$ (Table 6.4)

where

plastic bending resistance for mode 1 $M_{pl.1.Rd} := \dfrac{0.25 \cdot \Sigma L_{eff.ep.1} \cdot t_p^{\ 2} \cdot f_y}{\gamma_{M0}} = 4.059\ kN \cdot m$

plastic bending resistance for mode 2 $M_{pl.2.Rd} := \dfrac{0.25 \cdot \Sigma L_{eff.ep.2} \cdot t_p^{\ 2} \cdot f_y}{\gamma_{M0}} = 4.29\ kN \cdot m$

$$F_{t.ep.Rd} := min\left(\frac{4 \cdot M_{pl.1.Rd}}{m}, \frac{2 \cdot M_{pl.2.Rd} + n \cdot \Sigma F_{t.Rd}}{m+n}, \Sigma F_{t.Rd}\right) = 180.864\ kN$$

---

**Beam web in bending** [6.2.6.8]

$$F_{t,wb,Rd} = b_{eff,t,wb}\, t_{wb}\, f_{y,wb} / \gamma_{M0} \qquad \text{... (6.22)}$$

where

$b_{eff.t.wb} := l_{eff.ep}$ [6.2.6.8(2)]

$$F_{t.wb.Rd} := \frac{b_{eff.t.wb} \cdot t_w \cdot f_y}{\gamma_{M0}} = 469.025\ kN$$

---

$$F_{t.Rd} := min\left(F_{t.wc.Rd}, F_{t.fc.Rd}, F_{t.ep.Rd}, F_{t.wb.Rd}\right) = 151.177\ kN$$

since only one bolt row in tension:

Design moment resistance:

$$M_{j.Rd} := h_r \cdot F_{t.Rd} = 21.165\ kN \cdot m$$

Design moment resistance calculated with Ideastatica: $M_{j.Rd} = 30\ kN \cdot m$

Snapshot from IDEAStatiCa is shown below.

Rotational stiffness of joint component

| | Item | Comp. | Loads | MEd [kNm] | Mj,Rd [kNm] | Sj,ini [MNm/rad] | Sjs [MNm/rad] | ϕ [mrad] | ϕc [mrad] | L [m] | Sj,R [MNm/rad] | Sj,P [MNm/rad] | Class |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| > | B1 | My | LE1 | 5.0 | 30.0 | 2.02 | 2.53 | 2.0 | 95.7 | 5.00 | 38.75 | 0.77 | Semi-rigid |