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# Buckling Check Tools for Plate Panel and Column Structures

Helsinki Metropolia University of Applied Sciences

Bachelor of Engineering

Automotive and Transport Engineering

Thesis

7 May 2014

# PREFACE

I want to thank Ilari Silvola, Area Manager at Sweco Industry Oy, for providing the interesting subject, the necessary tools and working space, and, for believing in me. I also want to thank Jyrki Kullaa at Metropolia University of Applied Sciences for supervising my thesis and my instructor from Sweco Oy, Structural Analyst Samuli Riihimäki. His patience together with his wide knowledge of the field made this a true learning journey.

Last but not least I want to thank my family. Your support is worth more than anything.

Helsinki 7.5.2014

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| Author(s)<br>Title<br><br>Number of Pages<br>Date | Pia Vuorela<br>Buckling check Tools for Plate Panel and Column Structures<br><br>54 pages<br>7 May 2014 |
| Degree | Bachelor of Engineering |
| Degree Programme | Automotive and Transport Engineering |
| Specialisation option | Automotive Design |
| Instructor(s) | Samuli Riihimäki, Structural Analyst<br>Jyrki Kullaa, Principal Lecturer |

Buckling is, besides yielding, one of the major causes of failures in structures, and buckling checks are therefore an integral part of strength analyses. Checks can be performed either with methods requiring heavy numerical calculations or with equations based on rules and recommendations from a classification society. Numerical calculations are extremely time-consuming and commercial buckling check programs are expensive. There is therefore a need for buckling check tools based on the recommendations of a classification society that are fast to use and easy to share.

The work presented in this thesis is based on the rules and recommendations of DNV (Det Norske Veritas). Eight Excel-based buckling tools to be used for stability analyses of plated and column structures were created. They cover the flexural buckling of columns, buckling of unstiffened plates and buckling of plates stiffened in one direction. The tools for unstiffened and stiffened plates cover the load cases of uniform and linear varying longitudinal and transverse compression, lateral load, shear and biaxially loaded with shear. The effective width method is presented, because the rules and recommendations of DNV are based on this theory. The theory part of the thesis also covers the Finite Element Method (FEM), as the results from the created tools were compared to those from FEM-analyses.

The results from the tools were verified with hand calculations and tested to be in line with certain basic results of buckling of plates and columns known to be true. In addition, the results from the tool for stiffened plates were compared with those from an old project. These comparisons showed that the results from the FEM-analyses were generally more conservative for slender plates whereas the results from the buckling check tools were generally more conservative for thicker plates. The buckling resistance calculated by the tools did not exceed the yield strength. While the results from the tests and comparison where mostly in line with what was expected, a few unforeseen outcomes are presented, and reasons for them discussed. As future work it would be recommended to expand the tools to cover plates with cutouts as they could not be included in this thesis due to currently unreliable rules. Additional tools for varying boundary conditions as well as plates stiffened in two directions would also be welcomed.

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| Keywords | stability, buckling, DNV, effective width method |

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| Tekijä(t)<br>Otsikko<br><br>Sivumäärä<br>Aika | Pia Vuorela<br>Stabiliteettianalyysityökalujen luonti pilari ja levyrakenteille<br><br>54 sivua<br>7.5.2014 |
| Tutkinto | Insinööri (AMK) |
| Koulutusohjelma | Auto- ja kuljetustekniikka |
| Suuntautumisvaihtoehto | Tuotetekniikka |
| Ohjaaja(t) | Rakenneanalyytikko Samuli Riihimäki<br>Yliopettaja Jyrki Kullaa |




Lommahdus ja nurjahdus ovat myötäämisen ohella yleisimpiä rakennevaurioihin johtavia syitä, ja tästä syystä ne muodostavatkin tärkeän osan lujuusanalyyseja. Analyysit voidaan suorittaa joko vaativin numeerisin laskentamenetelmin tai luokituslaitosten sääntöihin ja suosituksiin perustuvien yhtälöiden avulla. Numeeristen laskentamenetelmien käyttö on äärimmäisen aikaa vievää, kun taas kaupalliset lommahduslaskentaohjelmat ovat kalliita. Tästä syystä helposti jaettavissa olevalle, nopeakäyttöiselle, luokituslaitoksen sääntöihin ja suosituksiin perustuvalle laskentaohjelmalle on tarvetta.

Opinnäytetyössä luotiin kahdeksan Excel-pohjaista levy- ja pilarirakenteiden stabiilisuusanalyyseissa käytettävää laskentaohjelmaa. Laskentaohjelmat kattavat pilarin nurjahduksen, jäykistämättömän levykentän lommahduksen sekä yhteen suuntaan jäykistetyn levykentän lommahduksen. Levyrakenteiden laskentaohjelmien kattamat kuormitustyypit ovat tasainen ja lineaarisesti vaihtuva pitkittäinen ja poikittainen kuormitus, leikkaus, sivuttainen kuormitus sekä yhtäaikaisesti vaikuttavat poikittainen kuormitus, pitkittäinen kuormitus ja leikkauskuormitus.

Työssä esitetyt laskentaohjelmat pohjautuvat DNV:n (Det Norske Veritas) sääntöihin ja suosituksiin. Tehollisen leveyden menetelmä on esitelty siksi, että DNV:n säännöt ja suositukset perustuvat tähän teoriaan. Työn teoriaosuus kattaa myös elementtimenetelmän (FEM), koska luotujen työkalujen tuloksia verrattiin FEM-analyyseista saatuihin tuloksiin.

Laskentaohjelmien antamat tulokset verifioitiin käsin laskemalla ja varmistamalla, että ne noudattavat tiettyjä lommahduksen ja nurjahduksen tunnettuja perussääntöjä. Jäykistetyille levykentille luodun ohjelman antamia tuloksia verrattiin myös vanhasta projektista saatuihin tuloksiin. Nämä vertailut osoittivat, että FEM-analyysit antoivat yleisesti ottaen konservatiivisempia tuloksia hoikille levykentille, kun taas laskentaohjelmat antoivat yleisesti ottaen konservatiivisempia tuloksia paksummille levykentille. Laskentaohjelmien antamat lommahduskestävyydet eivät ylittäneet myötörajaa. Testien ja vertailujen tulokset olivat odotettuja, muutamaa yllättävää tapausta lukuun ottamatta. Nämä tapaukset, mukaan lukien mahdolliset syyt, on käsitelty tässä työssä.

On suositeltavaa että tulevaisuudessa laskentatyökalut laajennettaisiin kattamaan myös reiälliset levykentät, joita ei voitu käsitellä tämän työn yhteydessä epäluotettavia tuloksia antavien sääntöjen takia. On myös suotavaa laajentaa laskentaohjelmia kattamaan erilaisia reunaehtoja ja kahteen suuntaan jäykistettyjä levykenttiä.




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| Avainsanat | stabiilius, lommahdus, nurjahdus, DNV, tehollinen leveys |



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## Contents



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## List of Symbols and Abbreviations

| | |
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| $b_{eff}$ | effective width |
| $\sigma_{Rd}$ | buckling resistance |
| $\sigma_{Sd}$ | design stress |
| $\tau_{Rd}$ | buckling resistance, shear |
| $\tau_{Sd}$ | design stress, shear |
| $\lambda$ | slenderness |
| $psd$ | lateral load |
| DNV | Det Norske Veritas. A classification society providing rules and recommendation for e.g. ships and offshore structures. |
| FEM | Finite element method. A numerical method used for solving field problems. |
| usage factor | The ratio between the design stress/force and the critical stress/force. |

# 1 Introduction

Buckling is, besides yielding, one of the major causes of failures in structures. When a structure buckles it loses its stability. Buckling does usually happen without fracture or separation to the material, or at least prior to it. As buckling of load bearing parts of the structure can have devastating consequences, buckling checks are an integral part of strength analysis of the structure.

Buckling is divided into different sorts of buckling depending on the structure. This thesis focuses on the buckling of columns and plates. The risk of buckling is especially high if a structure is slender. By a slender structure it is taken to mean a structure which cross-sectional dimensions are small compared to its length (columns) or which thickness is small compared to its width (plates). These kinds of structures are commonly used in marine and offshore structures. Figure 1 illustrates stiffened panels used in ship hull structures.

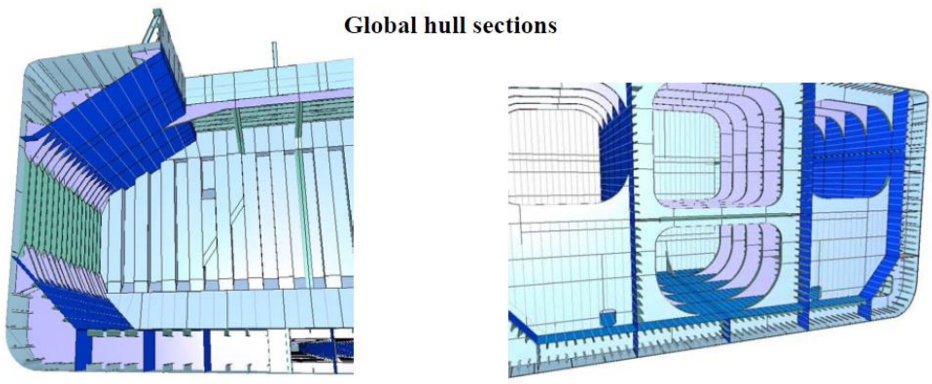



**Figure 1. Stiffened panels used in ship hull structures [1, p. 30].**

The aim of the study was to create and verify excel-based tools to be used for stability analysis of plated and column structures for the Offshore Department of Sweco Industry Oy. Sweco Industry Oy is a part of the Sweco Group, an engineering office with local presence in 12 countries and with around 9000 employees. The Sweco Group offers services within the fields of consulting engineering, environmental technology and architecture. These tools are to be based on the rules and recommendations of the classification society Det Norske Veritas (DNV).

Buckling checks play a significant role in the strength analyses performed within the Offshore Department of Sweco Industry Oy. Currently the analysts create their own tools used for buckling checks based on the needs of the project at hand. While these tools fulfill their purpose for the analyzing of the problem in question, they do not form a common functional tool. This practice might significantly lengthen the time needed for each analysis.

User friendly buckling check tools covering the most commonly used structures and loading situations would not only shorten the time needed for an analysis, but also reduce the risk of errors caused by possible mistakes done by individual programmers. Also, the analysts would not need to familiarize themselves with the rules and recommendations applicable for the problem in question to the same extent, as the tools would provide easy access to results approved by the classification society. Commercial buckling check tools are expensive and therefore not a relevant alternative.

The tools created for the study will cover the most commonly needed structures such as columns, unstiffened plates and plates that are stiffened in one direction, and are based on the rules and recommendations of DNV. DNV was chosen to be the classification society on which rules and recommendations the created tools would be based on, because it is the most commonly used classification society within offshore projects. DNV does also have rules and recommendations for a wide range of structures, which allows for a somewhat easy expanding of the tools for future needs.

The original assignment did also include the creation of a buckling check tool for plates with cut-outs. However, it was decided not to cover plates with cut-outs in this thesis, as the calculation rules recommended by DNV were not considered fully reliable.

The decision to create the buckling check tools as Excel spreadsheets was based on the fact that Excel is a widely known program. As it is a standard program in almost all office environments, the tools are also immediately ready for distribution, which would not be the case if using some kind of programming language like e.g. Octave. Excel also allows for an easy creation of a, albeit somewhat crude, graphical interface. The programming of a similar interface on some programming language would have considerably lengthened the time required to finish the project.

The buckling check tools for plates needed to cover the following loading alternatives:

- uniform longitudinal compression
- uniform transverse compression
- shear
- linear varying longitudinal compression
- linear varying transverse compression
- biaxial uniform loading with shear

This thesis can be roughly divided into three sections. The first section is a theory section introducing the concepts of stability and buckling as well as the concept and theory of the effective width method. The DNV´s approach to the problem of buckling of stiffened plates is addressed and in the end of the section there is a brief introduction of the Finite Element Method (FEM).

The second section introduces the buckling check tools created for the study. The restrictions of the tools, the inputs they require and the interpretation of the given outputs are described, as well as how the tools have been verified.

In the third section the results got from the created tools with the results got from a traditional Finite Element Method analysis are compared. Also, a closer look is taken at some features of the rules and recommendations of DNV that arouse questions.

## 2 Buckling

As stated in the introduction, when a structure buckles it loses it stability, i.e. its state of equilibrium changes. Therefore introducing the three different types of equilibrium is in order. The fundamental concepts of buckling and stability are approached through the examination of an idealized structure. The different buckling modes of a real life structure, as well as the effective width method used for buckling checks, are presented.

## 2.1 Equilibrium Types

Suppose there is a ball on a smooth surface (there is no rolling resistance). If the surface is concave upward, the ball will always return to the lowest point when disturbed. This type of equilibrium is called stable. If the surface is convex upward, the ball will roll away when disturbed. This type of equilibrium is called unstable. If the surface is perfectly flat, the ball remains wherever it is placed. This type of equilibrium is called neutral. Figure 2 illustrates the three different types of equilibrium.

**Figure 2 The equilibrium types; stable, unstable and neutral [2, p. 735].**

The next chapter will demonstrate how these different equilibrium types occur in a structure.

## 2.2 Buckling and Stability

To start, an idealized structure is examined, as it will help to understand the fundamental concepts of buckling and stability. The following chapter is based on the work of Gere & Timoshenko [2, pp. 732-735].

This hypothetical structure consists of two rigid bars AB and BC, each of length L/2. They are joined at B by a pin connection and held in a vertical position by a rotational spring having the stiffness $\beta_R$. These two bars are perfectly aligned and the axial load $P$ affects along the longitudinal axis (cf. Figure 3a). The spring is unstressed and the bars are in direct compression.

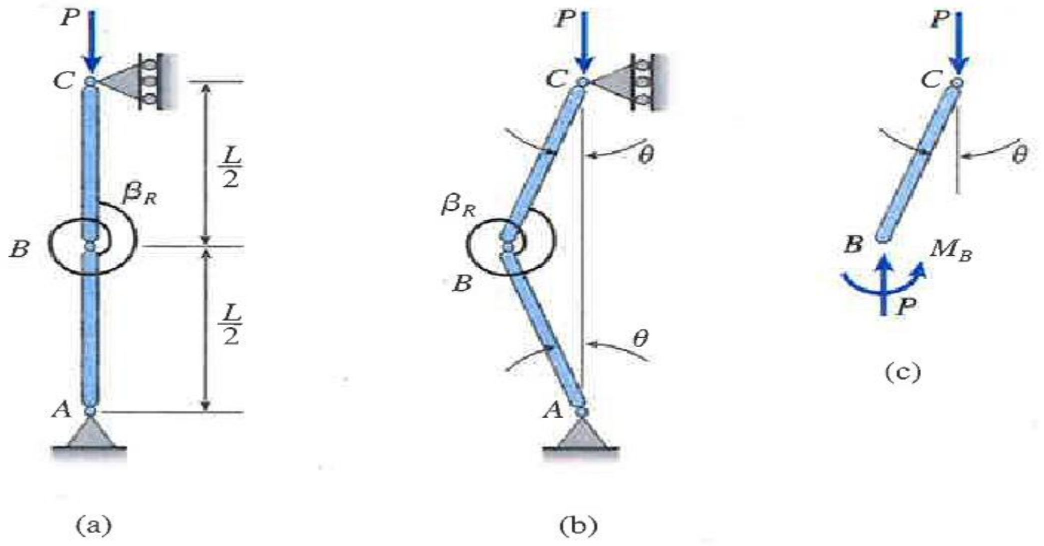



**Figure 3. Buckling of an idealized structure consisting of two rigid bars and a rotational spring [2, p. 732].**

The structure is now disturbed by adding an external force which will move point B a small distance laterally (cf. Figure 3b). This makes the rigid bars to rotate through small angles $\theta$ which develop a moment in the spring. This moment tends to return the structure into its original straight position and is therefore called a restoring moment. The axial compressive force will simultaneously try to increase the lateral displacement, and thus there are two actions going on at the same time; the restoring moment which tends to decrease the displacement and the axial force $P$ which tends to increase it.

What happens if the disturbing external force is removed? If the axial force $P$ is relatively small, the action of the restoring moment will predominate over the action of the axial force. In this case the structure will return to its initial straight position and is said to be stable. If the axial force $P$ is large, the lateral displacement of point B will increase and the bars will rotate through larger and larger angels until the structure collapses. The structure is unstable and ultimately fails by lateral buckling.

The transition between the stable and unstable conditions occurs at a specific value of the axial force $P$. This force is called the critical load and is denoted by the symbol $P_{cr}$. The critical load can be determined by investigating the equilibrium of the disturbed structure (cf. Figure 3c). First, the entire structure is considered as a free body and sum moments at support C. This leads to the conclusion that there is no horizontal reaction at support C. Second, bar BC is considered as a free body and it is noted that it is subjected to the action of the axial forces $P$ and the moment $M_B$, where $M_B$ is the rotation stiffness $\beta_R$ times the angle of rotation $2\theta$ of the spring:

$$M_B = 2\beta_R\theta \tag{1}$$

Since the angle $\theta$ is small, the lateral displacement of point B can be denoted $\frac{\theta L}{2}$. As a result one gets the equation of equilibrium about point B:

$$M_B - P\left(\frac{\theta L}{2}\right) = 0 \tag{2}$$

which can be written in another form using Equation 1:

$$\left(2\beta_R - \frac{PL}{2}\right)\theta = 0 \tag{3}$$

Equation 3 has two solutions. One solution is when $\theta = 0$, which means that the structure is in equilibrium when it is perfectly straight. In this solution the magnitude of the force $P$ does not matter. The second solution is found by setting the term in the parenthesis equal to zero and then solving the critical load $P_{cr}$:

$$P_{cr} = \frac{4\beta_R}{L} \tag{4}$$

At this value of the load the structure is in equilibrium at any magnitude of the angle $\theta$, as long as the angle is sufficiently small to fulfill the assumption made in Equation 2.

At the solved value of the load the effect of the restoring moment equals the buckling effect of the axial force, making this the only load where the structure is in equilibrium in the disturbed position. The critical load therefore represents the boundary between the stable and unstable conditions. If the axial load $P$ is smaller than the critical load $P_{cr}$, the structure returns to its straight position. If again the axial load $P$ is larger than the critical load $P_{cr}$, the structure buckles. These situations are referred to as stable and unstable equilibriums respectively. If the axial load $P$ equals the critical load $P_{cr}$, the structure is in equilibrium even when the point B is displaced laterally, in other words, the structure is in equilibrium with any small angle $\theta$, including $\theta = 0$. This state of equilibrium is called neutral equilibrium.

The equilibrium states of an idealized structure in a graph of axial load $P$ versus angle of rotation $\theta$ are presented in Figure 4.

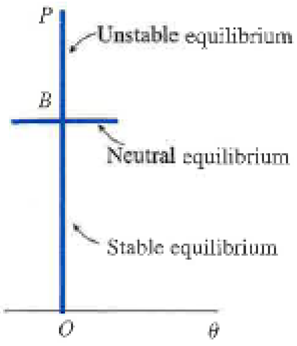



**Figure 4. An equilibrium diagram for buckling of an idealized structure. The diagram branches at the bifurcation point (B). [2, p. 735]**

The vertical and horizontal lines in Figure 4 represent the equilibrium conditions. The point where the diagram branches, point B, is called the bifurcation point. The horizontal line representing the state of neutral equilibrium extends both to the left and to the right (the angle $\theta$ can be to the both directions) and it is quite short because, as earlier assumed, the angle $\theta$ is small.

The loss of stability in an idealized structure has now been addressed. However, a real life structure can lose its stability in more than one way.

## 2.3 Snap-through Buckling and Bifurcation Buckling

There are two main ways a structure can lose its stability. The first type is called snap-through buckling. The load-deflection curve for snap-through buckling is shown in Figure 5.

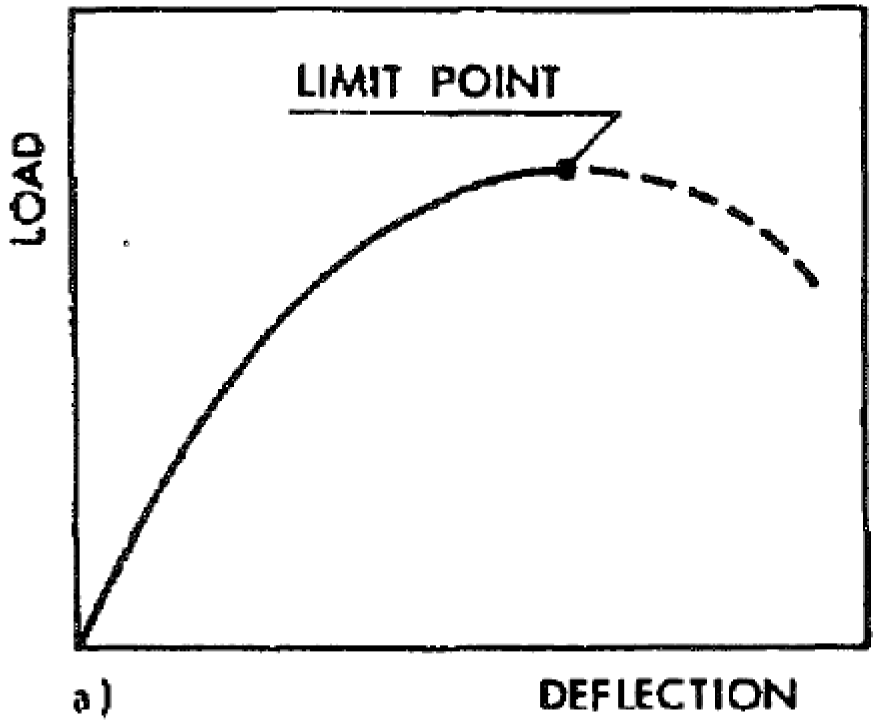



Figure 5. The load-deflection curve for snap-trough buckling [3, p. 4].

The limit point shown in Figure 5 is the maximum on the load-displacement curve. When the load increases over the limit point the structure collapses. Snap-through buckling is an example of nonlinear buckling.

The second type of buckling is called classical or bifurcation buckling. The concept of the bifurcation point was introduced in the previous chapter. The load-deflection graph for bifurcation buckling consists of two paths; the primary and the secondary part, both visible in Figure 6.

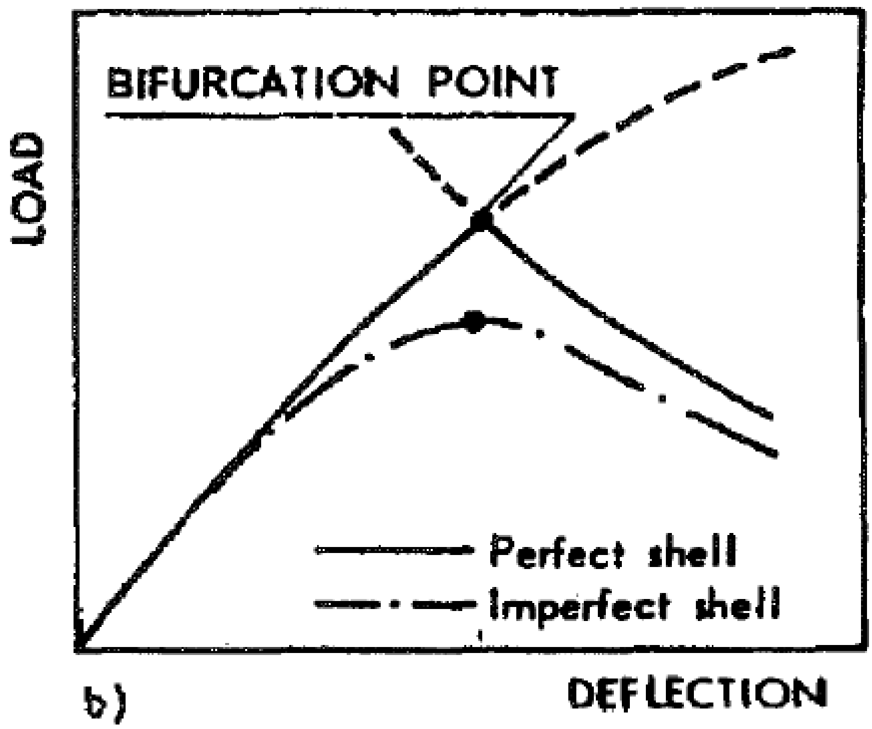



Figure 6. The load-deflection curve for bifurcation buckling [3, p. 4].

The primary path is the original load-displacement line and its extension, while the secondary path is the alternative path that originates from when the critical load is reached. The two paths intersect at the so called bifurcation point. The primary path is unstable past the bifurcation point. The structure can follow the secondary path instead. If the secondary path rises, the structure is said to have post-buckling strength. [3, p. 4; 4, p. 642.]

If a structure has imperfections, as realistic structures usually do, it may lose its stability at a limit point that is reached at a lower load than the bifurcation point. This is illustrated in Figure 6 by the line labeled “Imperfect Shell”. [3, p. 4.]

## 2.4 Effective Width Method

The DNV´s recommendations for buckling of unstiffened plates are based on the so called effective width method. The phenomenon of buckling of plates can be examined by using the theory of the buckling of columns presented in the previous chapter.

Let us examine a rectangular flat plate with boundary conditions equal to those of a plate simply supported on all four sides and affected by a compression stress on the short sides of the plate. The plate can be thought to consist of longitudinal and transversal stripes. The compression stress affects only the longitudinal stripes, which tend to buckle as columns, while the transversal stripes function as springs which try to resist the buckling. As a consequence, a plate like this can carry more compression stress than a series of loose stripes. The shorter the transversal stripe, the more it will give resistance. Figure 7 illustrates a simply supported plate subjected to compression.

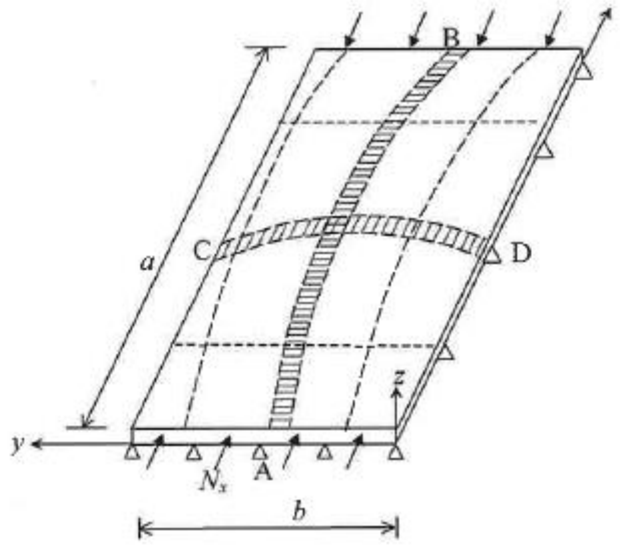



**Figure 7. A simply supported plate subjected to compression. Stripe AB tends to buckle as a column while stripe CD try to resist the buckling [5].**

The stress achieved by the critical load $P_{cr}$ is called the critical buckling stress. When the compressive stress reaches the critical buckling stress the center of the plate will buckle. At this point the longitudinal stripes located in the center of the plate are longer than the stripes close to the longitudinal sides of the plate (the boundary conditions prevent the deflection in the z-direction for the edges parallel to the x-axis, and therefore they remain straight). This leads to a non-uniform stress distribution along the short sides of the plate, where the stress affecting the middle part of the plate is smaller than the stress affecting the edges of the plate. If the stress affecting the edges of the plate is still below the yield stress of the material, the plate will continue to carry load even that the middle part of the plate has buckled. However, when the stress affecting the edges of the plate reaches the yield stress of the material the plate loses its stability. [6, p. 17.] Figure 8 shows how the actual distribution of the stress (on the left) is assumed to be distributed when using the effective width method (on the right).

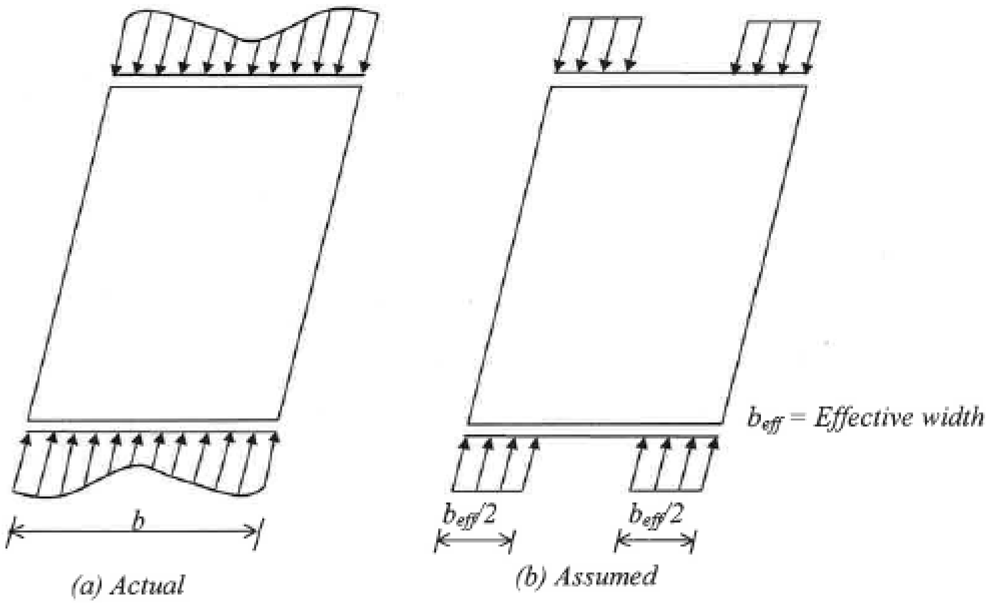



**Figure 8. The actual (a) and the assumed (b) distribution of the stress [5].**

The concept of the effective width method is used when one wants to determinate the load carrying capacity of the plate in the post-buckling range. In the effective width method the non-uniform stress distribution across the width of the buckled plate is replaced with uniform stress blocks equal to it. This uniform stress is assumed to affect over the width of $\frac{b_{eff}}{2}$ on either side where $b_{eff}$ is called the effective width of the plate (Figure 8). [5]

## 2.5 Buckling of Stiffened Plates

The buckling strength of a plate can be improved by stiffening, which is primarily done through the use of longitudinal stiffeners. Longitudinal stiffeners divide the plate into thinner segments which prevents the plate from buckling. Transversal stiffeners again shorten the buckling length of the longitudinal stiffeners. [6, p. 19.] In the Recommended Practice DNV-RP-C201 [1] the buckling problem of a stiffened panel is transformed to a buckling problem of a beam column as illustrated in Figure 9.

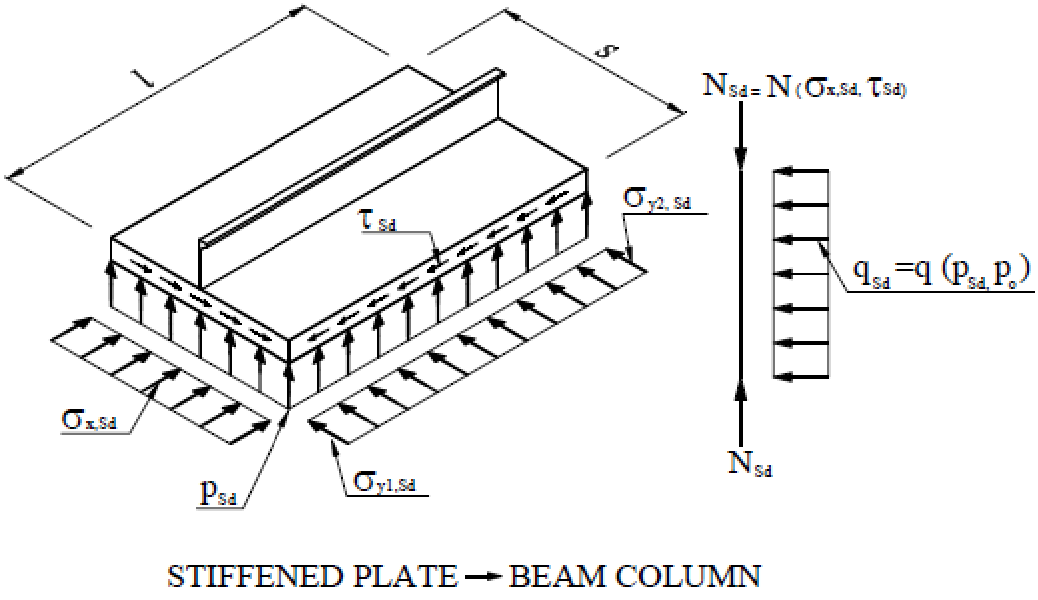



**Figure 9. The buckling problem of a stiffened panel is transformed to a buckling problem of a beam. The stresses affecting the stiffened panel are taken into account by equivalent forces and loads affecting the beam. [7]**

The longitudinal compression and shear affecting the stiffened panel are taken into account by transferring and combining them to an equivalent axial force. This axial force is represented by $N_{Sd}$ in Figure 9.

The transverse compression affecting the stiffened panel is taken into account by analyzing the beam column for an equivalent lateral pressure $p_0$ that is proportioned to give yield in extreme fiber for the transverse buckling stress. See [7, p. 5] for a more detailed definition of the lateral pressure $p_0$.

The lateral pressure $p_0$ and the design lateral pressure $p_{sd}$ are taken into account by combining them to an equivalent lateral line load. This lateral line load is represented by $q_{Sd}$ in Figure 9.

## 3 Finite Element Method (FEM)

Finite element method (FEM) is a numerical method used for solving field problems. FEM can be used for solving many kinds of engineering problems, for example stress analysis, heat transfer, vibration analysis or buckling analysis. FEM can be used for solving problems which involve complicated geometries, loadings or material properties, usually characterized by an ordinary or partial differential equation. Due to the complicity of the problems, there is typically no analytic solution available. The solution of the equation in question can however be obtained numerically by formulating the problem to a system of simultaneous algebraic solutions. A structure analyzed with FEM is divided into small pieces, *finite elements*. Two or more elements connect at points called *nodes*, and the arrangement of the elements is called a *mesh*. Figure 10 shows a mesh created for the analysis of a gear tooth.

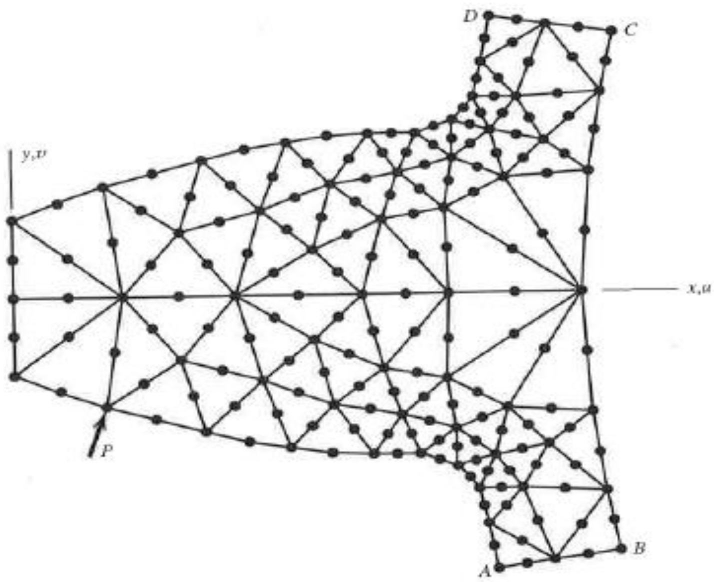



**Figure 10. A two-dimensional model of a gear tooth. All nodes and elements lie in the plane of the figure. Supports are not shown. [4, p. 2.]**

Numerically, the mesh is represented by a system of algebraic equations to be solved for unknowns at nodes. Instead of solving the problem for the whole structure in one operation FEM is solving the problem in a piecewise fashion, element by element. The equations for each element are combined to obtain the solution for the whole structure. The outcome of an analysis performed by FEM is always an approximation, never accurate. By using a more dense mesh (more elements presenting the structure) the outcome gets more accurate. The drawback, however, is a significantly lengthened time needed for the analysis.

FEM can be used for both linear and nonlinear stability analyses. In a linear stability analysis it is assumed that the structure behaves linearly up to the point that it loses its stability. The stresses in the structure are directly proportional to the load affecting it and the distribution of the stress is independent of the magnitude of the load. The affecting load is applied in its entirety before the beginning of the analysis. A linear elastic analysis does not take into account the yield stress of the material.

In the nonlinear stability analysis the affecting load is not applied in its entirety but step by step. The effect of the applied load is analyzed after each step. The next step is always based on the analysis of the previous step. In this way the stresses and deformations formed in the structure under the loading will be taking into account.

## 3.1 Geometric Stiffness Matrix

FEM uses matrix methods for simplifying the formulation of element equations. Matrix notation represents a simple notation for writing and solving sets of simultaneous algebraic equations. A nodal force and the corresponding nodal displacement are related through the stiffness of the material. This is represented by the static equilibrium equation:

$$\{F\} = [K]\{U\} \tag{5}$$

where $\{F\}$ is a row vector of size N for external nodal forces

$[K]$ is the stiffness matrix of size N x N for the structure

$\{U\}$ is a row vector of size N for nodal displacements of the structure

Here the integer N corresponds to the number of unknowns. In this fashion the stiffness equations for the single elements can be combined to a stiffness equation for the whole structure. [8, pp. 28-33.] As mentioned earlier, the FEM first solves the unknown nodal variables. With the help of these nodal variables one can then compute the rest of the unknown quantities.

A real life structure is never idealistic and does always contain some imperfections. Due to these imperfections a structure affected by compression will also start to bend. The lateral deflections caused by this bending in turn leads to membrane stresses. Bifurcation buckling occurs when the membrane strain energy in a structure converts into bending strain energy without any changes in the load affecting it. In some structures, e.g. columns and thin plates, the membrane stiffness is much greater than the bending stiffness. In these types of structures a significant amount of membrane strain energy can therefore be stored with only small deformations. The releasing of this membrane strain energy leads to large bending deformation in order to absorb all the released energy. This is when buckling occurs.

These membrane stresses are taken into account by a so called geometric stiffness matrix, usually noted $[K_g]$. The geometric stiffness matrix is determined by an elements geometry, displacement field and state of membrane stress. The total stiffness matrix $[K]$ for an element is a sum of the conventional stiffness matrix $[K_t]$ and the geometric stiffness matrix $[K_g]$:

$$[K] = [K_t] + [K_g] \tag{5}$$

Because the geometric stiffness matrix is a dependent on the load, a positive tensile stress increases the stiffness when again a negative compression decreases it. When the compression force grows large enough the structure loses its stiffness and buckles. [4, pp. 639, 648.]

## 3.2 Determination of Critical Load

The total potential energy $\Pi$ of a structure is a sum of the inner strain energy $U$ and the potential energy of applied loads $V$.

$$\Pi = U + V \tag{6}$$

Let us return for a second to the three different types of equilibrium; stable, unstable and neutral. In the stable equilibrium type the ball was on a smooth surface that concaved upwards. If the ball is disturbed from its lowest point its total potential energy will increase. Therefore, when a structure is stable its total potential energy is at its minimum. In the unstable equilibrium type the ball was on a smooth surface that was convex upwards. If the ball is disturbed it will roll away and its total potential energy will decrease. Therefore, when a structures potential energy is at its maximum its equilibrium state is unstable. In the neutral equilibrium type the ball was on a perfectly flat surface. If the ball is disturbed it does not return back to its initial position but it does not roll away either. It stays where it is. The level of the potential energy remains unchanged.

A change in a systems total potential energy, $\Delta\Pi$, can be examined mathematically with the help of the Taylor series. This gives the result:

$$\Delta\Pi = \delta\Pi + \frac{1}{2}\delta^2\Pi + 0(h^3) \tag{7}$$

where $\delta\Pi$ is the first order Taylor term, $\delta^2\Pi$ the second order Taylor term and $0(h^3)$ represents the higher order terms. In equilibrium the first variation, $\delta\Pi$, vanishes and the second variation $\frac{1}{2}\delta^2\Pi$ determinates the equilibrium type. If the second variation is positive the total potential energy is at its minimum. The equilibrium state is stable because more energy is needed to deflect the system. If the second variation is negative the total potential energy is at its maximum and the equilibrium state is unstable. The system can give up energy. If the second variation is zero, the equilibrium state is neutral. [8, pp. 175-177.]

Earlier the concept of the critical load, $P_{cr}$ , was introduced. This was the load were the transition between the stable and unstable condition of a structure occurred, or in other words, when the equilibriums state of the structure is neutral. The determination of the critical load begins with loading the structure with an arbitrary load $P_0$ and performing a standard linear static analysis. This analysis determinates the membrane stresses for the load in question.

The geometric stiffness matrix for the structure loaded with $P_0$ is $[K_g]_0$. It can be determined that for some other load level $P = \lambda P_0$ the geometric stiffness matrix is $[K_g] = \lambda[K_g]_0$, where $\lambda$ is the load coefficient. So a load multiplied by $\lambda$ also multiplies the corresponding stress by $\lambda$ without altering the distribution of the stress. As the loading does not alter the conventional stiffness matrix $[K_t]$, the total stiffness matrix can now be written

$$[K] = [K_t] + \lambda[K_g] = [K_t + \lambda K_g]. \tag{8}$$

[4, p. 648.]

From the principle of minimum total potential energy presented in the previous section it is now known that when the structure loses its stability both the first and second variation of the total potential energy is zero [8, p. 450].

$$\begin{cases} \delta \Pi = 0 \\ \delta^2 \Pi = 0 \end{cases} \tag{9}$$

This leads to [8, p. 451]

$$\delta^2 \Pi = \{\delta U\}^T [K] \{\delta U\} = 0 \tag{10}$$

from which follows that

$$\left[K_t + \lambda K_g\right]\{\delta U\} = 0 \tag{11}$$

For the existence of a non-trivial solution to Equation 11

$$|[Kt] + \lambda[Kg]| = 0 \tag{12}$$

From this equation one can determine the lowest critical load coefficient $\lambda_1$, and the critical load $P_{cr}$ is then given by

$$P_{cr} = \lambda_1 P_0 \tag{13}$$

The corresponding buckling mode $\{U\}$ can then be determined from the Equation 11, where the lowest buckling mode $\{U_1\}$ corresponds to $\lambda_1$.

These types of problems are often referred to as eigenvalue problems. [8, pp. 457-458.]

# 4 Classification Societies

The International Association of Classification Societies (IACS) defines the purpose of a classification society as following:

> The purpose of a Classification Society is to provide classification and statutory services and assistance to the maritime industry and regulatory bodies as regards maritime safety and pollution prevention, based on the accumulation of maritime knowledge and technology [9].

The tools created for this thesis are based on the rules and recommendations of the classification society Det Norske Veritas (DNV).

While writing this thesis, DNV merged with another classification society, Germanischer Lloyd (GL), and is now called DNV GL Group. According to DNV GL, this merger made them the world´s leading classification society, with a 24 percent market share of the world´s classed ships and mobile offshore units.

DNV GL Group is one of the twelve members of the IACS (International Association of Classification Societies). The other eleven members are:

- American Bureau of Shipping (ABS)
- Bureau Veritas (BV)
- China Classification Society (CCS)
- Croatian Register of Shipping (CRS)
- Indian Register of Shipping (IRS)
- Korean Register of Shipping (KR)
- Lloyd´s Register (LR)
- Nippon Kaiji Kyokai (NK/ClassNK)
- Polish Register of Shipping (PRS)
- Registro Italiano Navale (RINA)
- Russian Maritime Register of Shipping (RS)

Different classification societies have rules and recommendations for different kinds of structures. Some of these rules and recommendations are presented in Table 1, which contains the revisions available in May 2013. DNV and Germanischer Lloyd are presented separately since the merger had not yet taken place.

**Table 1. Example of classification societies along with their rules and applicable structures.**

| Classification society | Rules (revison) | Structures |
|---|---|---|
| **DNV** | Recommended practice DNV-RP-C201<br>Buckling strenght of plated structures<br>October 2010 ((October 2008) October 2002) | Offshore<br>- unstiffened plates<br>- stiffened plates<br>- girders |
| | Recommended practice DNV-RP-C202<br>Buckling Strenght of Shells<br>January 2013 (October 2010) | Cylindrical shells<br>- unstiffened<br>- stiffened<br>Conical shells<br>- unstiffened |
| | Classification Notes - No. 30.1<br>April 2004 (July 1995) | - bars<br>- frames<br>- unstiffened spherical shells |
| | Rules for classifications of ships<br>Part 3 Chapter 1<br>January 2013 (July 2012) | Hull structural design<br>- ships with length 100m and above |
| | Rules for classifications of ships<br>Part 3 Chapter 2<br>July 2012 (January 2012) | Hull structural design<br>- ships with length less than 100m |
| **Lloyd´s** | Rules and Regulations for the<br>Classification of Naval Ships,<br>Volume 1, Part 6<br>January 2009 | Naval ships<br>- plate panels |
| **ABS** | Guide for Buckling and Ultimate<br>Strenght Assessment for Offshore Structures<br>March 2005 | Offshore<br>- individual structural members<br>- plates<br>- stiffened panels<br>- corrugated panels |
| **Germanischer Lloyd** | Rules for Classification and<br>Construction, Ship Technology<br>Part 1, Chapter 1<br>Edition 2013<br><br>The calculation method<br>is based on<br>DIN-Standard 18 800 | Seagoing ships<br>- plates |

The information presented in Table 1 might be incomplete due to the fact that the details of rules and recommendations of some classification societies are not available free of cost.

# 5 Buckling Check Tool for Flexural Buckling of Columns

The buckling check tool for flexural buckling of columns presented in this chapter is based on the DNV Classification Note No.30.1, Revision April 2004 (hereinafter referred to as “Classification Note”) [3].

## 5.1 Restrictions

The above-mentioned buckling check tool only covers the cases of flexural buckling of columns leaving out all other possible buckling modes for bars. It is assumed that the cross-section of the member under consideration has at least one axis of symmetry (the z-axis). The tool does not cover buckling checks of members with an arbitrary cross-section.

The Classification Note defines the following concepts:

a *column* is a bar that is subjected to pure compression.

*flexural buckling of columns* is bending about the axis of least resistance.

Also, in the tool it is assumed that the column is subjected to longitudinal compression only and that the compressive force is centric. According to the Classification Note, flexural buckling may be the critical mode of a slender column of doubly symmetrical cross-section or one which is not susceptible to, or is braced against twisting.

## 5.2 Inputs and Outputs

The required input data consist of:

- the yield stress of the material [N/mm$^2$]
- the modulus of elasticity [N/mm$^2$]
- the Poisson´s ratio

- the area of the section being directed to compression [$mm^2$]

- the moment of inertia about the weak axis [$cm^4$]

- the length of the column [mm].

In addition the user has to enter the **length factor**, the **column curve**, the **axial force of the column** and the **maximum allowable usage factor**.

The length factor determinates the buckling length of the column, which again depend on the end fixity of the column. The length factor can be found in the Table 2-2 located next to the input area. The factor to be entered is the recommended design value (i.e. the value on the bottom line of the Table). The column curve depends on the shape of the section and the axis it tends to buckle around. The column curve corresponding to the column in question is chosen from the column selection chart located to the far right. The axial force is the axial force affecting the column [kN]. The maximum allowable usage factor is the highest accepted ratio between the design axial force and the allowable axial force.

The output data consists of the allowable axial force of the column and the ratio between the design axial force and the allowable axial force. This ratio is expressed both verbally (OK / NOT OK) and numerically (the usage factor). In addition, the outcomes are indicated with red or green depending on if they are satisfying or not. The design axial force has to be lower or equal to the allowable axial force and the usage factor has to be lower or equal to the maximum allowable usage factor.

## 5.3 Tool for Flexural Buckling of Columns

The following buckling check tool is for flexural buckling of columns. A screenshot of the program is presented in Figure 11.

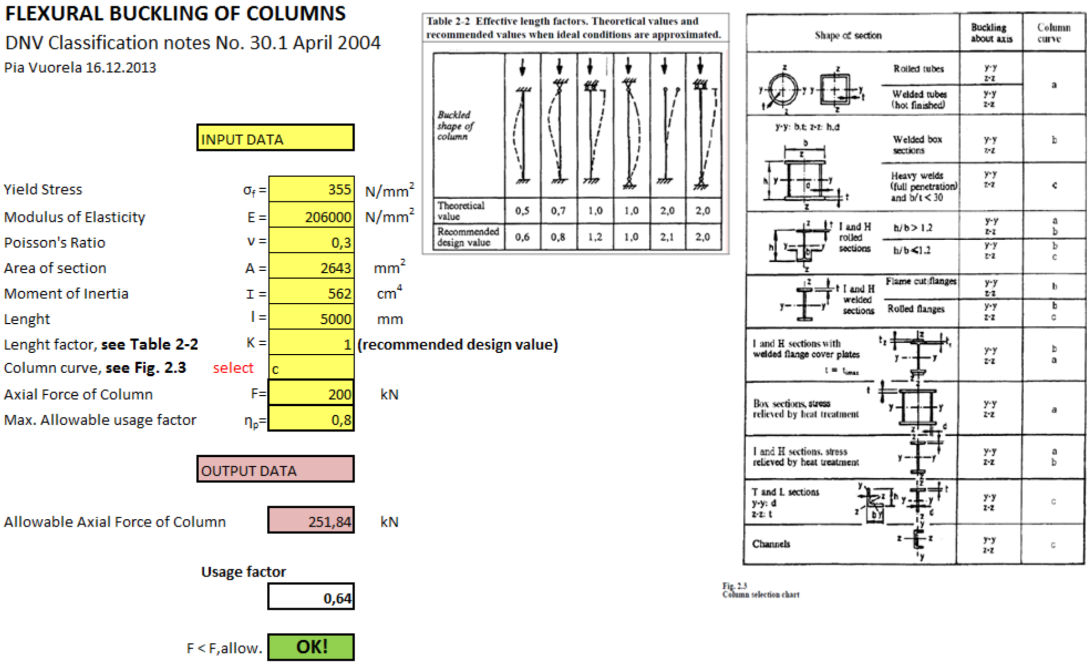



Figure 11. Screenshot of the application. The axial force is within the required limits.

The program calculates the allowable axial force $F$ from the critical buckling stress $\sigma_{cr}$, where $\sigma_{cr}$ is a function of the slenderness λ, the given yield stress $\sigma_f$ and coefficients depending on the shape of the section.

## 6 Buckling Check Tools for Unstiffened Plates

The buckling check programs for unstiffened plates, regardless of the load type, are based on the Recommended Practice, DNV-RP-C201, October 2010 Revision (hereinafter referred to as "Recommended Practice") [1].

The layouts of the programs are built to fit all the relevant input and output data on one page. This allows for easy printing or screen capturing of the data when necessary. Some of the programs print out intermediate results which are not counted as official output data. These intermediate results are not designed to fit on the front page but are visible on the page underneath. The screen captures displayed within this work does not include intermediate results.

The programs are designed to prevent the user from giving the input data incorrectly. They alert if the user e.g. have given the input data for the width and the length of the plate the wrong way around.

The numbers next to the outputs (e.g. Figure 12) or intermediate results refer to the equations in the Recommended Practice. The Recommended Practice uses a material factor $\gamma_M$ of 1.15.

The necessity of a buckling check is determined with the help of the slenderness of the plate. The slenderness of the plate is defined as the ratio $\frac{s}{t}$ where $s$ stands for the width and $t$ for the thickness of the plate. Under the heading of each program the limit value for a required buckling check for the load type in question is defined. After the user has given the input data for the plate thickness and plate width the program will notify if a buckling check is necessary or not.

To prevent possible confusion it is worth mentioning the compression stresses are always taken as positive while tension stresses again are negative. Furthermore, a stress is always assumed to vary in a linear fashion.

Plates subjected to lateral pressure also have to fulfill the requirements of other buckling checks, and the tool automatically determines and performs the appropriate secondary buckling check without multiple inputs from the user.

The Recommended Practice defines the buckling strength $\sigma_{Rd}$ for unstiffened plates to be

$$\sigma_{Rd} = C\left(\frac{\sigma_f}{\gamma_M}\right) \tag{14}$$

where $C$ is a coefficient that depends on e.g. $\lambda$, $\sigma_f$ is the yield stress and $\gamma_M$ is the material factor. In situations where the structure is affected by shear, $\tau_{Rd}$ is calculated in a similar fashion instead of $\sigma_{Rd}$.

An exception is made when calculating the buckling strength of biaxially loaded unstiffened plates, where the following requirement has to be fulfilled:

$$\left(\frac{\sigma_{x,Sd}}{\sigma_{x,Rd}}\right)^2 + \left(\frac{\sigma_{y,Sd}}{\sigma_{y,Rd}}\right)^2 - c_i \left(\frac{\sigma_{x,Sd}}{\sigma_{x,Rd}}\right)^2 \left(\frac{\sigma_{y,Sd}}{\sigma_{y,Rd}}\right)^2 + \left(\frac{\tau_{Sd}}{\tau_{Rd}}\right)^2 \leq 1.0, \quad (15)$$

where $\sigma_{Sd}$ and $\tau_{Sd}$ is the design stress and design shear respectively, $c_i$ is a coefficient depending on the dimensions of the plate. The notations x and y refer to the direction of the stress, and the other variables are as defined above.

## 6.1 Inputs and Outputs

The input data can be divided into three groups; material data, dimensions of the plate and load data. The material data include the yield stress and the modulus of elasticity of the material. The dimensions of the plate include the thickness, width and length of the plate. The load data include the quantity of the load or loads in question.

As output data the characteristic buckling strength and the usage factors for the loads are given. The design stress has to be lower or equal to the characteristic buckling strength. The output data consist of the characteristic buckling strength of the considered structure and the ratio between the design stress and the characteristic buckling strength. This ratio is expressed both verbally (OK / NOT OK) and numerically (the usage factor). In addition the outcomes are indicated with red or green depending on if they are satisfying of not. The maximum allowable usage factor is assumed to be 1.00.

## 6.2 Restrictions

The tools are based on the theory of the effective width method, and the adjoining structures will therefore need to be checked on the basis of the same model. The unstiffened plates are assumed to have boundary conditions equivalent to those of a simply supported plate. With simply supported is understood the boundary conditions for all four sides where the translations in the directions of the normal of the plate are locked and the rest of the degrees of freedom are free.

The slenderness, i.e. the width to thickness ratio of the plate shall be less than 120, and the plate is assumed to be rectangular with $l > s$, where $l$ is the length of the plate and $s$ the width of the plate.

## 6.3 Tool for Uniform Longitudinal Compression

The following tool is for buckling checks of unstiffened plates subjected to uniform longitudinal compression. Figure 12 shows a screenshot from the application.

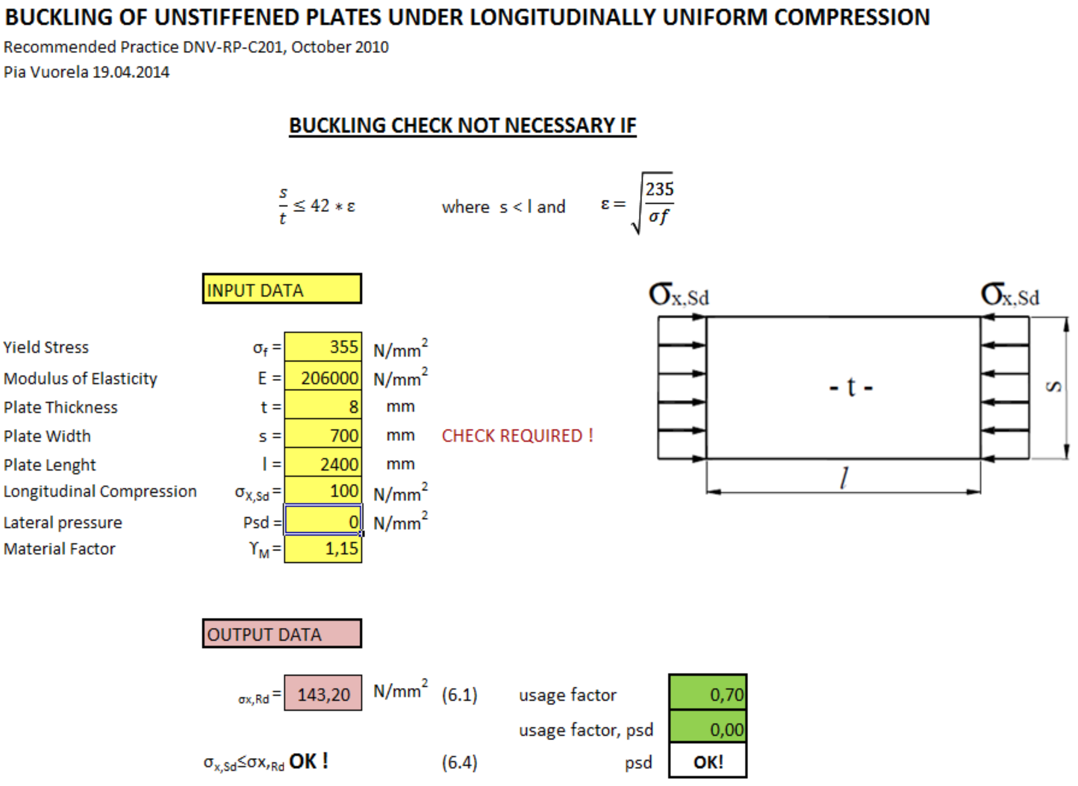



**Figure 12. Screenshot from the application. The results from the required check indicates that the structure will not buckle.**

The buckling resistance for unstiffened plates subjected to uniform longitudinal compression $\sigma_{x,Rd}$ is defined as:

$$\sigma_{x,Rd} = C_x \frac{\sigma_f}{\gamma_M}, \tag{16}$$

where $C_x$ is a coefficient that is a function of the reduced slenderness $\overline{\lambda_p}$ defined as

$$\overline{\lambda_p} = 0{,}525 \cdot \frac{s}{t} \sqrt{\frac{\sigma_f}{E}}. \tag{17}$$

In Equation 17 $s$ is the width and $t$ is the thickness of the plate, $\sigma_f$ is the yield stress and $E$ is the modulus of elasticity.

## 6.4 Tool for Uniform Transverse Compression

The following tool is for buckling checks of unstiffened plates subjected to uniform transverse compression. Figure 13 shows a situation where the actual buckling check due to the transverse compression is satisfactory, but the check performed due to the lateral load is not.

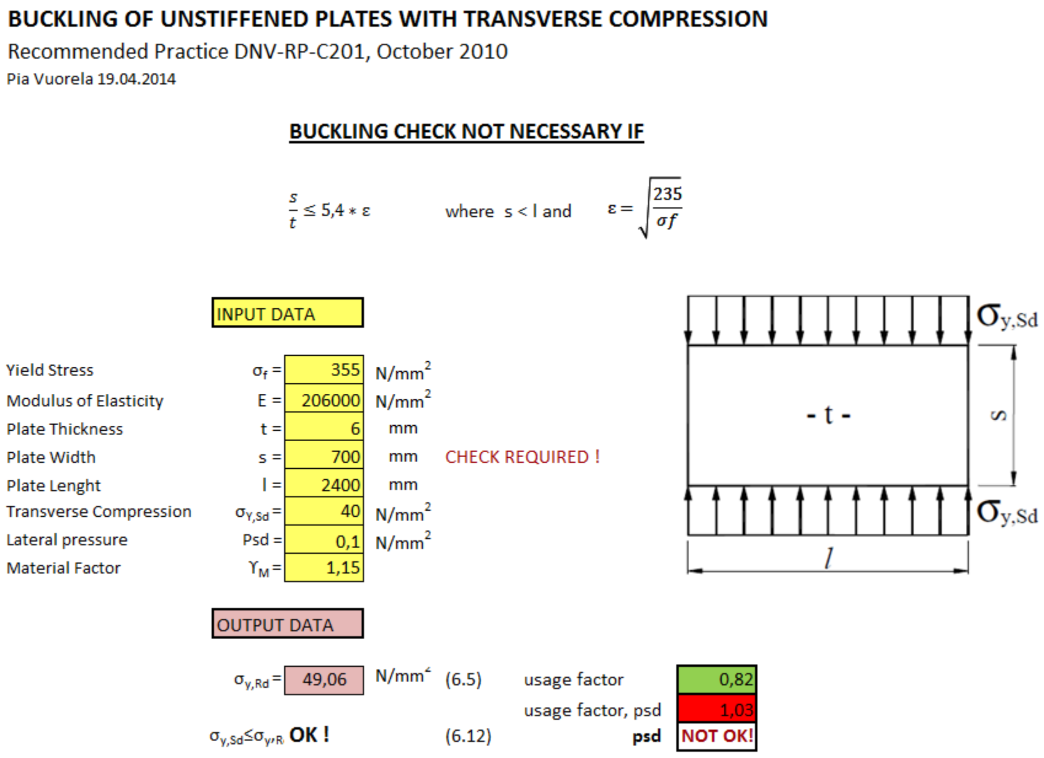



Figure 13. The result of one of the two required checks is not satisfactory.

The buckling check formulas for plates subjected to uniform transverse compression does take into account the reduction of the buckling strength due to a simultaneous lateral load. However, there are occasions where the check made for lateral load only is not satisfactory even though the check made for combined uniform transverse compression and lateral load is. The loads affecting the plate have to fulfill the requirements of both individual checks.

The buckling resistance for unstiffened plates subjected to uniform transverse compression $\sigma_{y,Rd}$ is defined as:

$$\sigma_{y,Rd} = \frac{\sigma_{y,R}}{\gamma_M}, \tag{18}$$

where

$$\sigma_{y,R} = \left[\frac{1.3t}{l}\sqrt{\frac{E}{\sigma_f}} + \kappa\left(1 - \frac{1.3t}{l}\sqrt{\frac{E}{\sigma_f}}\right)\right]\sigma_f k_p. \tag{19}$$

The coefficients are functions $\kappa(\overline{\lambda_c})$ and $k_p(p_{sd}, s, t)$. The reduced slenderness $\overline{\lambda_c}$ is defined as:

$$\overline{\lambda_c} = 1.1\frac{s}{t}\sqrt{\frac{\sigma_f}{E}}. \tag{20}$$

As before, $s$ is the width and $t$ is the thickness of the plate, while $\sigma_f$ is the yield stress and $E$ is the modulus of elasticity.

## 6.5 Tool for Shear Stress

The following tool is for buckling checks of unstiffened plates subjected to shear. Figure 14 shows a situation where the program alerts due to unsatisfactory inputs.

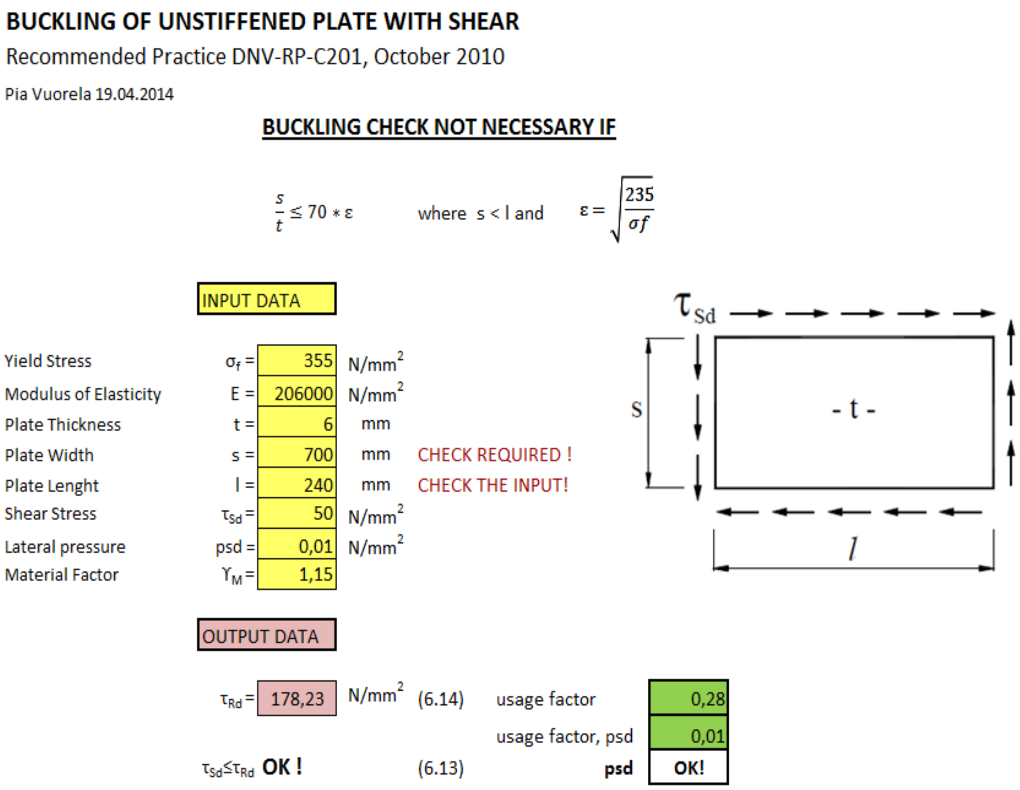



**Figure 14. The program alerts the user to check the input, because the given width is greater than the given length.**

The buckling resistance for unstiffened plates subjected to shear stress $\tau_{Rd}$ is defined as:

$$\tau_{Rd} = \frac{C_\tau}{\gamma_M} \frac{\sigma_f}{\sqrt{3}}, \tag{21}$$

where the coefficient $C_\tau$ is a function of the reduced slenderness $\overline{\lambda_w}$, defined as

$$\overline{\lambda_w} = 0.795 \cdot \frac{s}{t} \sqrt{\frac{\sigma_f}{E \cdot k_l}}. \tag{22}$$

The coefficient $k_l$ is dependent on the dimensions of the plate.

## 6.6 Tool for Varying Longitudinal Stress

The following tool is for buckling checks of unstiffened plates subjected to linear varying longitudinal compression. Figure 15 shows a situation where the program alerts, as the slenderness of the plate is not within the required limits.

**BUCKLING OF UNSTIFFENED PLATES WITH VARYING LONGITUDINAL STRESS**

Recommended Practice DNV-RP-C201, October 2010

Pia Vuorela 19.04.2014

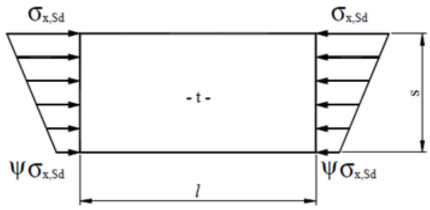
**BUCKLING CHECK NOT NECESSARY IF**

$\frac{s}{t} \leq 42 * \varepsilon$ where s < l and $\varepsilon = \sqrt{\frac{235}{\sigma f}}$

INPUT DATA

| | | | |
|---|---|---|---|
| Yield Stress | $\sigma_f$ = | 355 | N/mm² |
| Modulus of Elasticity | E = | 206000 | N/mm² |
| Plate Thickness | t = | 5 | mm |
| Plate Width | s = | 700 | mm |
| Plate Lenght | l = | 2400 | mm |
| Max Longitudinal Compression | $\sigma_{x,Sd}$ max | 100 | N/mm² |
| Min Longitudinal Compression | $\sigma_{x,Sd}$ min | 0 | N/mm² |
| Lateral pressure | psd = | 0,01 | N/mm² |
| Material Factor | $\gamma_M$ = | 1,15 | |

PLATE IS TOO SLENDER!




OUTPUT DATA

| | | | | | |
|---|---|---|---|---|---|
| $\sigma_{x,Rd}$ = | 131,55 | N/mm² | (6.21) | usage factor | 0,76 |
| | | | | usage factor, psd | 0,15 |
| $\sigma_{x,Sd} \leq \sigma_{x,Rd}$ | OK ! | | (6.25) | psd | OK! |

**Figure 15. In this example the given input data does not fulfill the conditions given for the required plate slenderness. The produced results are therefore not valid and should not be used.**

The buckling resistance for unstiffened plates subjected to linear varying longitudinal compression $\sigma_{x,Rd}$ is defined as:

$$\sigma_{x,Rd} = C_x \frac{\sigma_f}{\gamma_M}, \tag{23}$$

where $C_x$ is a coefficient that is a function of the reduced slenderness $\overline{\lambda_p}$ defined as

$$\overline{\lambda_p} = \frac{s}{t} \cdot \frac{1}{28.4\varepsilon\sqrt{k_\sigma}}. \tag{24}$$

The coefficient $\varepsilon$ is defined as $\sqrt{\frac{235}{\sigma_f}}$ and $k_\sigma$ is a function of the stress ratio $\psi = \frac{\sigma_2}{\sigma_1}$, where $\sigma_1$ is the largest compressive stress.

## 6.7 Tool for Varying Transverse Stress

The following tool is for buckling checks of unstiffened plates subjected to linear varying transverse compression. Figure 16 presents a situation where the program does not give any outputs due to unsatisfactory load data.

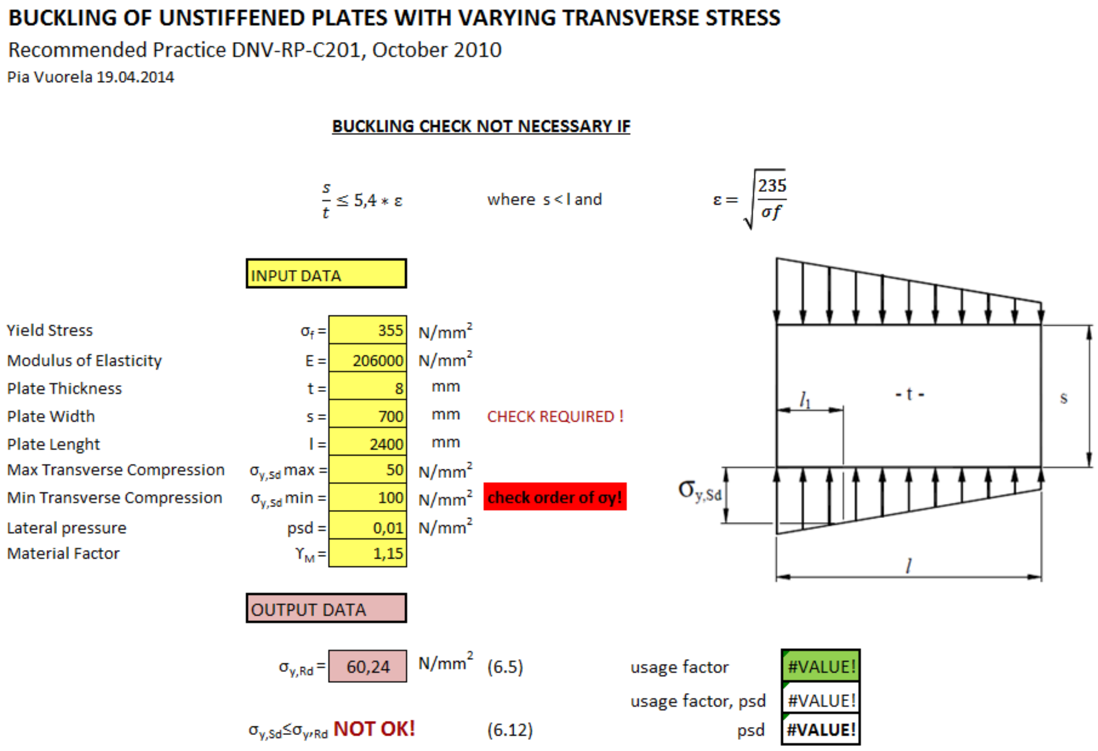



**Figure 16. The program alerts and asks the user to check the order for the given data for $\sigma_{y,Sd}$, as the minimum and maximum values are given the wrong way around. The program does not give out any usage factors before the order of the data is corrected.**

The buckling resistance $\sigma_{y,Rd}$ for unstiffened plates subjected to linear varying transverse compression is defined as:

$$\sigma_{y,Rd} = \frac{\sigma_{y,R}}{\gamma_M}, \tag{25}$$

where $\sigma_{y,R}$ is as in section 6.4 (Tool for Uniform Transverse Compression).

The Recommended Practice process the buckling check for linear varying transverse stress in a very similar way than the buckling check for uniform transverse stress. The characteristic buckling strength $\sigma_{y,Rd}$ is the same for the both cases. However, the design stress value $\sigma_{y,Sd}$ that it is compared to differs. The design stress value used in the buckling check for linear varying transverse compression is the stress value at distance $l_1$ (illustrated in Figure 16). The distance $l_1$ is defined as

$$l_1 = \min\{0.25 \cdot l,\ 0.5 \cdot s\} \tag{26}$$

## 6.8 Tool for Biaxially Loaded with Shear

The following tool is for buckling checks of biaxially loaded unstiffened plates with shear. Figure 17 illustrates a situation, where the simultaneously acting loads lead to buckling.

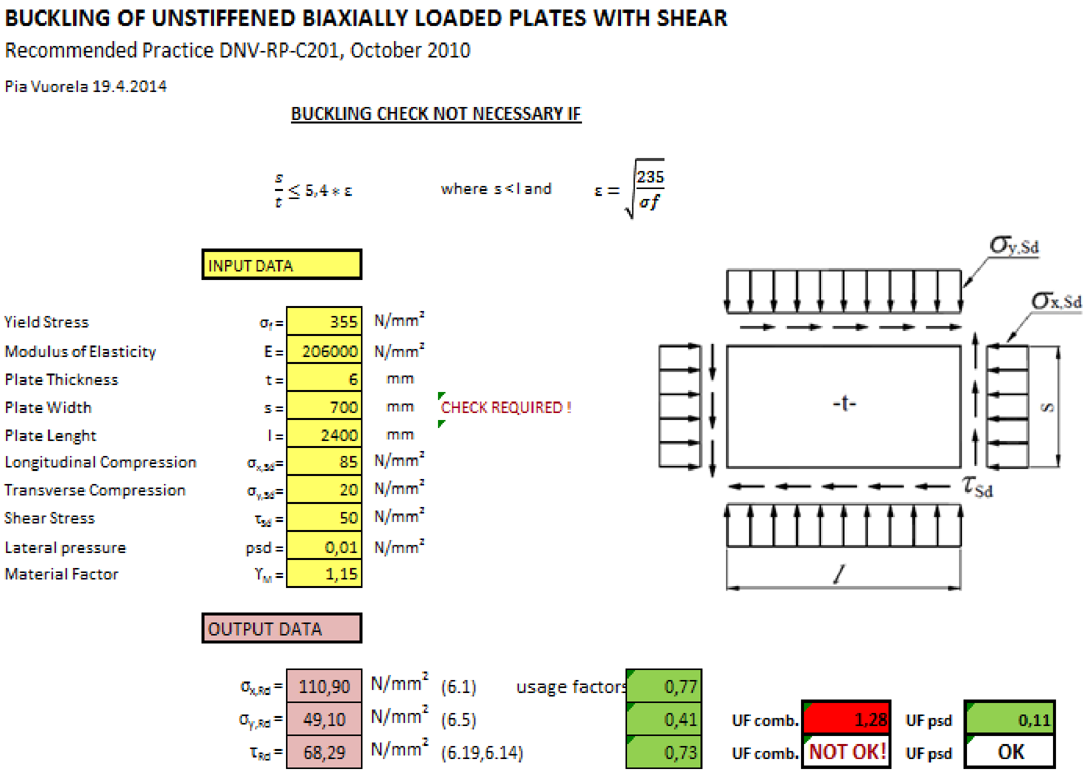



**Figure 17. The program calculates separate usage factors for each load type. The usage factors are then combined into the relevant usage factor by Equation 27. As this example shows, the loads can be within the required limits individually but not fulfill the requirements when acting together.**

The plate needs to fulfill the following requirement:

$$\left(\frac{\sigma_{x,Sd}}{\sigma_{x,Rd}}\right)^2 + \left(\frac{\sigma_{y,Sd}}{\sigma_{y,Rd}}\right)^2 - c_i\left(\frac{\sigma_{x,Sd}}{\sigma_{x,Rd}}\right)^2\left(\frac{\sigma_{y,Sd}}{\sigma_{y,Rd}}\right)^2 + \left(\frac{\tau_{Sd}}{\tau_{Rd}}\right)^2 \leq 1.0 \quad (27)$$

The coefficient $c_i$ depends on the dimensions of the plate. The buckling resistances $\sigma_{x,Rd}$ and $\sigma_{y,Rd}$ are given by Equations 16 and (18) respectively. If $\sigma_{x,Sd}$ $(\sigma_{y,Sd})$ are in tension, Equation 16 (18) shall be taken as $\frac{\sigma_f}{\gamma_M}$. The shear buckling resistance $\tau_{Rd}$ is given by Equation 21. If $\sigma_{y,Sd}$ is in tension, an identical coefficient $C_\tau$ is used. Otherwise the coefficient is calculated slightly different, but is still a function of the reduced slenderness $\overline{\lambda_w}$ given by Equation 22.

## 7 Buckling Check Tool for Plates Stiffened in One Direction

The buckling check tool for plates stiffened in one direction is based on the Recommended Practice, DNV-RP-C201, October 2010 Revision (hereinafter referred to as "Recommended Practice") [1]. The buckling check tool is programmed with the data for a range of bulb flat stiffeners only. The dimension and section properties for the bulb flats in the range 120x6 to 430x20 were taken from a table of "British Standards for bulb flats" that is used at Sweco Oy. An exception was made for the bulb flat sizes 100x6, 100x7 and 100x8 as they were not featured in the table. These dimensions and properties where instead approximated using Finnpro, which is an element model program for calculating quantities and strains for the profiles cross sections. A drawing of a bulb flat stiffener is presented in Figure 18.

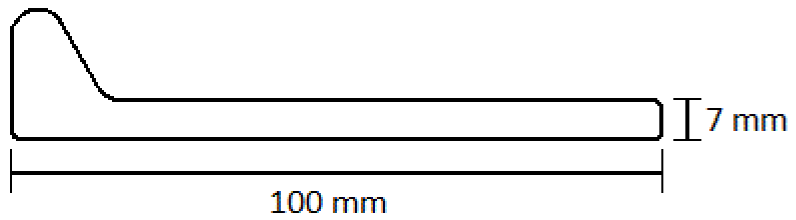



Figure 18. A bulb flat sized 100X7.

The program was limited to only one kind of stiffeners. The bulb flat stiffener was chosen due to its wide use in offshore structures.

## 7.1 Restrictions

The spacing between the stiffeners is assumed to be uniform. The stiffeners are assumed to be bulb flat stiffeners within the size range from 100x6 to 430x20. Tension field action is not allowed. More information about this restriction can be found in the Analysis chapter.

## 7.2 Inputs and Outputs

The input data is divided into the following sections:

The **Input** section contains the material data that consists of the yield stress for the plate and for the stiffener, the modulus of elasticity of the plate and the material factor.

The **Plate** section contains the thickness and width of the stiffened plate as well as the stiffener length, the torsional buckling length (the distance between sideways supports of the stiffener) and the girder length.

The **Stresses** section contains the load data. The stresses in the longitudinal and transverse direction are given as stresses at two corners. The locations of the corners are illustrated next to the input area. This allows the input stress to be non-uniform. If the stresses at the two corners differ from each other the assumption is made that the stress is linear varying between the two corners. The direction of the lateral load is given by choosing the correct alternative from the “Overpressure may occur on”-menu. The alternatives are both sides / plate side / stiffener side.

In the **Stiffener** section the user enters the stiffener data: the size, the type (continuous / sniped) and the fabrication method (welded / rolled).

The **Tension field action** is always off. More information of this restriction can be found in the Analysis chapter.

The **z* optimizing** can be chosen either to be On or Off. The distance z* stands for optimum eccentricity. In the optimum eccentricity method the working point of the axial force is optimized in order to find the maximum capacity of the structure. The concept of the optimum eccentricity is illustrated in Figure 19. If the z* optimizing is off, the working point of the axial force is assumed to be at the neutral axis.

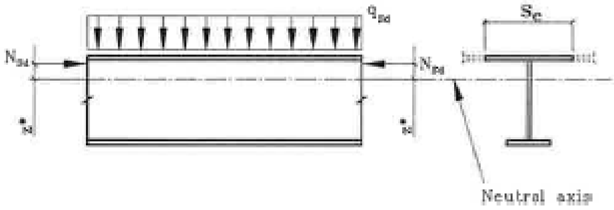



Figure 19. A stiffened panel viewed from the side (left) and from the end (right). The optimum eccentricity z* is the distance between the neutral axis and the position of the axial force $N_{sd}$. [7]

The dropdown menus are marked with the word “select”, as no arrow is visible in the menu prior to its activation by the user.

The output data is given as a series of **Usage factors**. The usage factors numbered 1, 2, 3 and 4 represent the results of interaction equation of four different locations. These locations are illustrated in Figure 20 and also in the graph on the right in Figure 21. The locations 1 and 2 are at the stiffener support and the locations 3 and 4 are at the mid span.

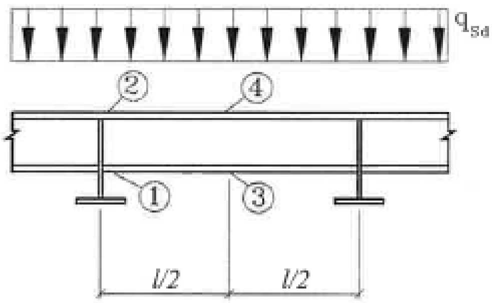



Figure 20. The different locations on a stiffened panel where the usage factor is calculated. [7]

These four usage factors are given out separately for lateral pressure on plate side and for lateral pressure on stiffener side. This is to avoid any mistakes caused by the signs of the stresses. For sniped stiffeners the only points that need checking are the ones located at the mid span. Therefore the program gives out only two usage factors for a plate stiffened with sniped stiffeners. The usage factors of $\sigma_y$, $\tau$, **shear** and **psd** are ratios between the given design stress and the characteristic buckling strength for the respective stress / force.

The program does also do a **local buckling check** for the stiffener flange and the stiffener web. The outcome of this check is given in the form OK / NOT OK. On the bottom of the page the maximum usage factor is shown along with the indication OK / NOT OK. The maximum allowable usage factor is assumed to be 1.00.

## 7.3 Tool for Plates Stiffened in One Direction

The following tool is for buckling checks of plates stiffened in one direction, a screenshot of the tool is presented in Figure 21.

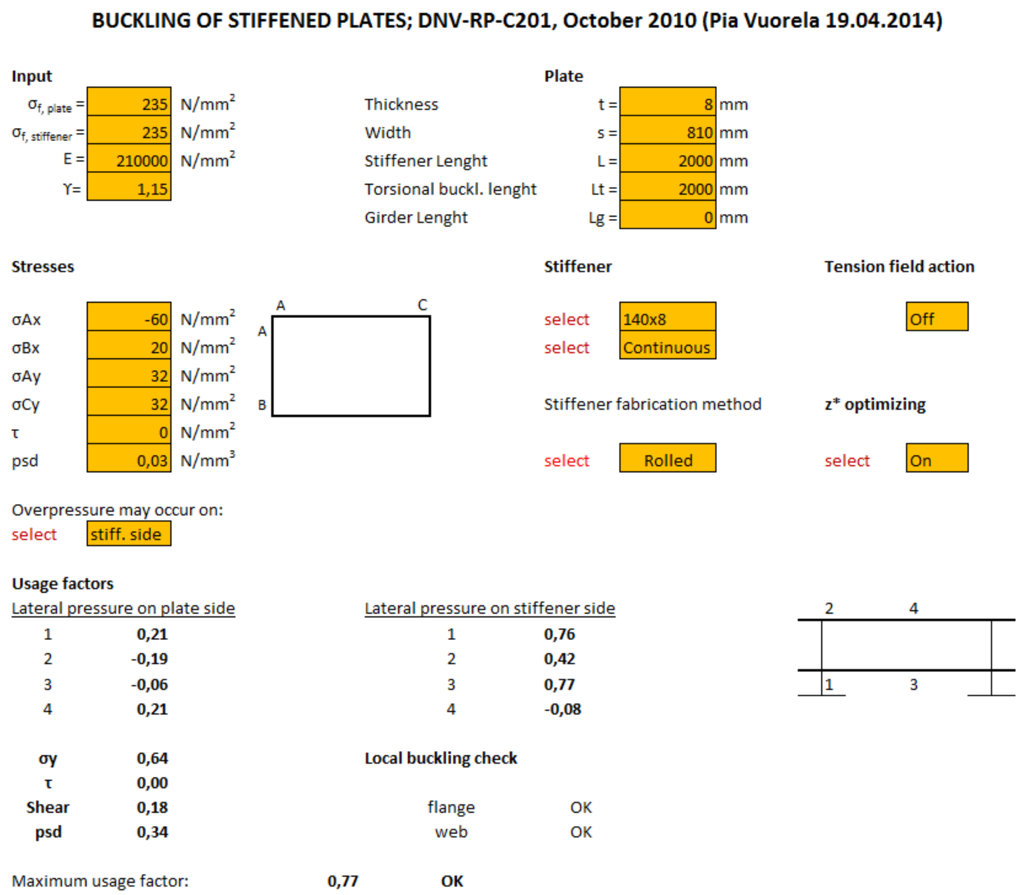



**Figure 21. Screenshot from the tool for plates stiffened in one direction. The negative values for the usage factors represents situations of tension.**

The tool calculates usage factors for four different locations. Each location must fulfill the interaction equation for lateral pressure on both the plate and the stiffener side. As an example the interaction equation for location 3 for lateral pressure on the stiffener side is given in Equation 28.

$$\frac{N_{Sd}}{N_{ks,Rd}} \cdot \frac{M_{2,Sd} - N_{Sd} \cdot z^*}{M_{s2,Rd}\left(1 - \frac{N_{Sd}}{N_E}\right)} + u \leq 1 \tag{28}$$

The variables $N$ represent forces and the variables $M$ represent moments affecting location 3. The coefficient $u$ is calculated using the design shear stress and shear buckling resistance

$$u = \left(\frac{\tau_{Sd}}{\tau_{Rd}}\right)^2, \tag{29}$$

and $z^*$ is the optimum eccentricity defined in section 7.2.

# 8 Verification

The verification of the buckling check tools was performed, besides careful hand calculations, by studying e.g. the effect of changes made to the thickness of a plate on the results and thus making sure that the values produced by the tools do not contradict what is commonly known about the behavior of the structure. One check was also to make sure that the results produced by the tools never exceeded the known yield strength of the material. However, a few situations were found where this was not the case. These cases will be discussed in Chapter 10. In addition, the results from the buckling check tool for stiffened plates was compared to results from an old project. The calculations for the old project were made with a program that also was based on the DNV's rules and recommendations. The results from the created tools coincided well with the results from the old project, but the comparison is not presented in more detail.

The programs were tested in order to check if the programs produce results compatible with certain basic results about buckling of plates and columns which are known to be true. The properties assumed to be known were:

- the buckling resistance of a plate is reduced when the width of the plate is increased

- the buckling resistance of a plate is increased if an uniform compression is changed to a linear varying compression

- the buckling resistance of a plate subjected to longitudinal stress in higher than the one of a plate subjected to shear stress, that is in turn higher than the one of transverse stress.

- the buckling resistance of a plate subjected to longitudinal stress is reduced if the plate is simultaneously subjected to transverse stress

- the allowable axial force for a column is reduced when the length of the column increases

- the buckling resistance of a stiffened plate is increased when the stiffener size is increased.

All of these assumptions were tested and some of the results are presented below. Firstly, the buckling resistances for different load types and for varying plate thicknesses (from 6 mm to 20 mm) were calculated. This test was run to make sure that the results follow the assumption made about the buckling resistance dependence on the load type. The results are shown In Figure 22.

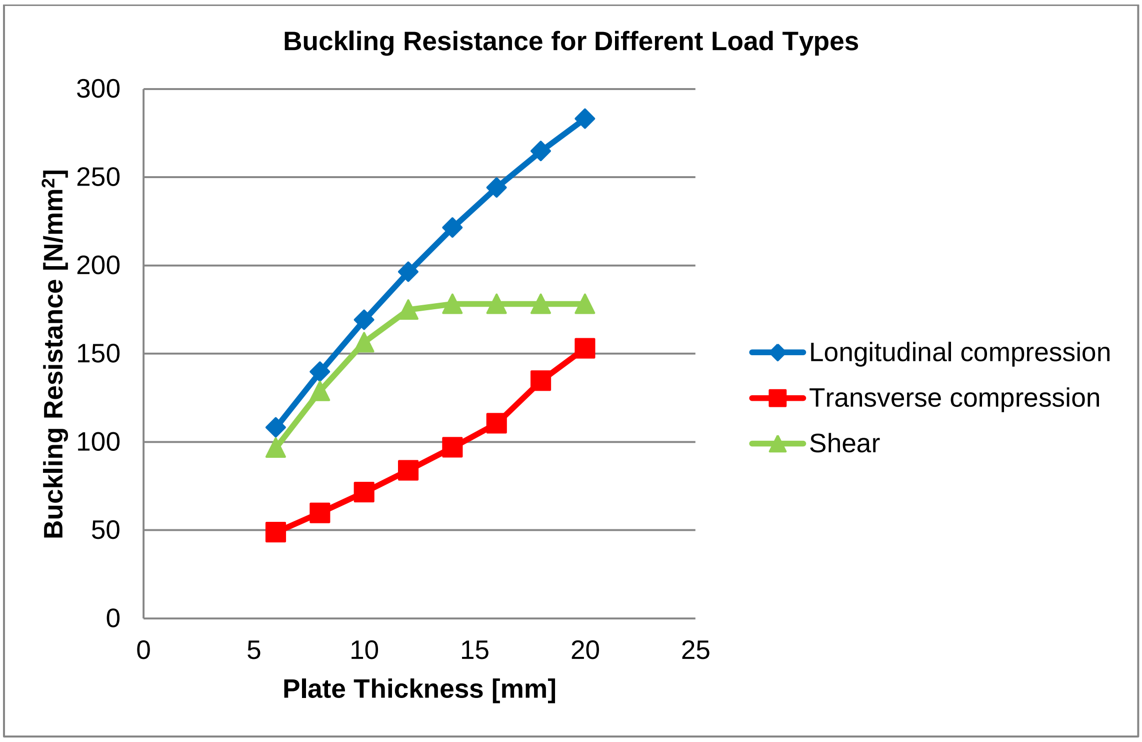



**Figure 22. The buckling resistance of unstiffened plates for different load types. The properties of the plate were $\sigma_f$=355 N/mm$^2$ and E=206 000 N/mm$^2$, and the dimensions were s=720 mm (width) and l=2400 mm (length).**

As shown in Figure 22, the highest buckling resistance was obtained with a longitudinal compression, whereas the transverse compression produced the lowest buckling resistance. For the case of shear the buckling resistance was limited by the shear strength, as illustrated by the graph. The shear strength of a plate with a yield stress of 355 N/mm$^2$ and using a material factor of 1.15 can be calculated as $\frac{355\,{}^{N}/_{mm^2}}{1.15\cdot\sqrt{3}} = 178.23\,{}^{N}/_{mm^2}$.

Secondly, the buckling resistance for plates with varying widths (from 500mm to 1000 mm) was calculated. This test was run to check that the buckling resistance of a plate is reduced when the width of the plate is increased. Figure 23 illustrates how the widening of the plate affects the buckling resistance.

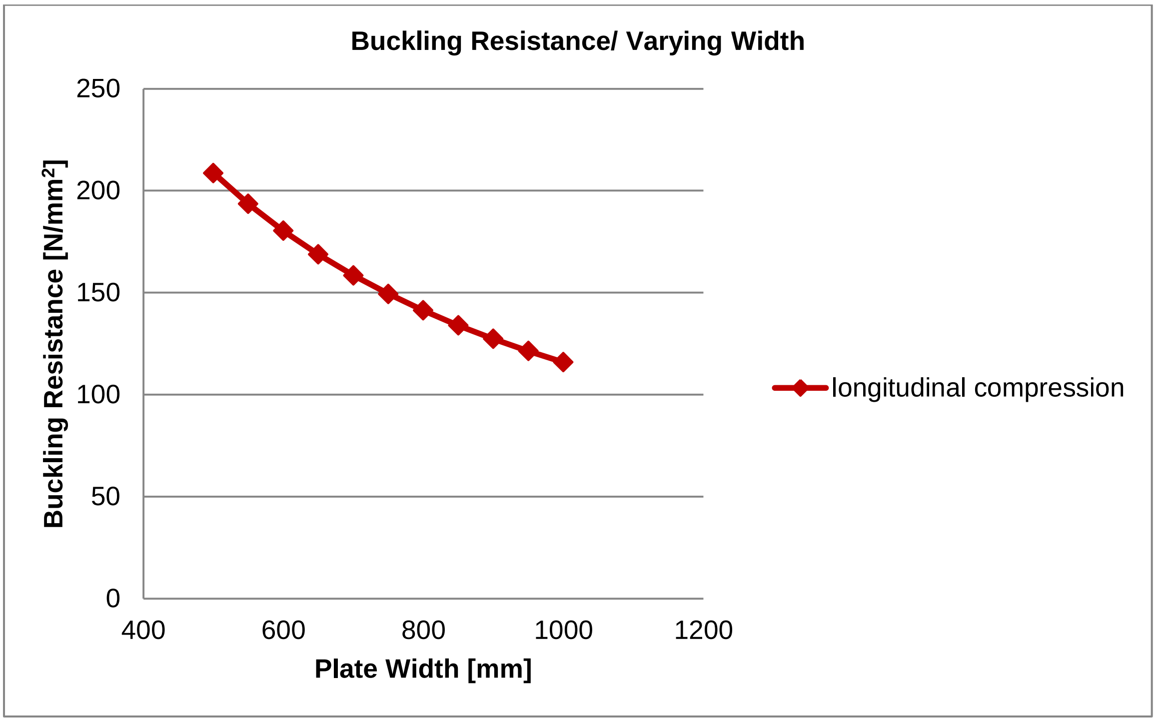



**Figure 23. The buckling resistance of unstiffened plates with varying plate widths. The properties of the plate were $\sigma_f$=355 N/mm$^2$ and E=206 000 N/mm$^2$, and the dimensions were t=9 mm (thickness) and l=2400 mm (length).**

The buckling resistance is reduced from 208,63 N/mm$^2$ to 115,9 N/mm$^2$ when the width of the plate increases from 500 mm to 1000 mm, which is what was expected.

Furthermore, the assumption that the allowable axial force for a column is reduced when the length of the column increases was checked, and the results are shown in Figure 24 below.

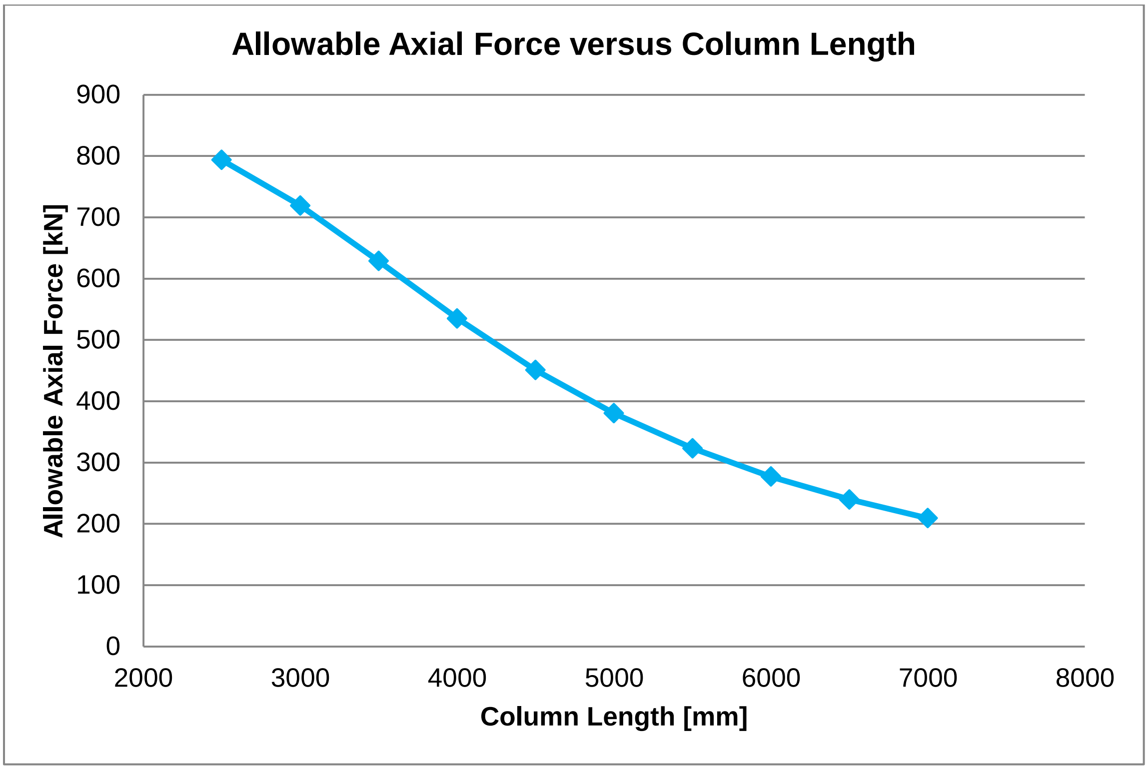



**Figure 24. The allowable axial force for columns with varying length. The properties of the column are $\sigma_f$=355 N/mm$^2$ (yield strength), E=206 000 N/mm$^2$(modulus of elasticity), v=0.3 (Poisson´s ratio), A=2643 mm$^2$ (area of section) I=562 cm$^4$ (moment of inertia). The length factor K=1, the buckling curve=a and the maximum allowable usage factor $\eta_p$=1.**

The length of the column varied between 2000 mm and 7000 mm. These results were also in line with the assumption being tested.

Lastly is presented the results of the test made to check the assumption that the buckling resistance of a stiffened plate is correlated to the stiffener size. Figure 25 illustrates how the maximum usage factor for a stiffened plate subjected to a uniform longitudinal compression of $\sigma_x$= 100 N/mm$^2$ can be reduced by increasing the size of the bulb flat.

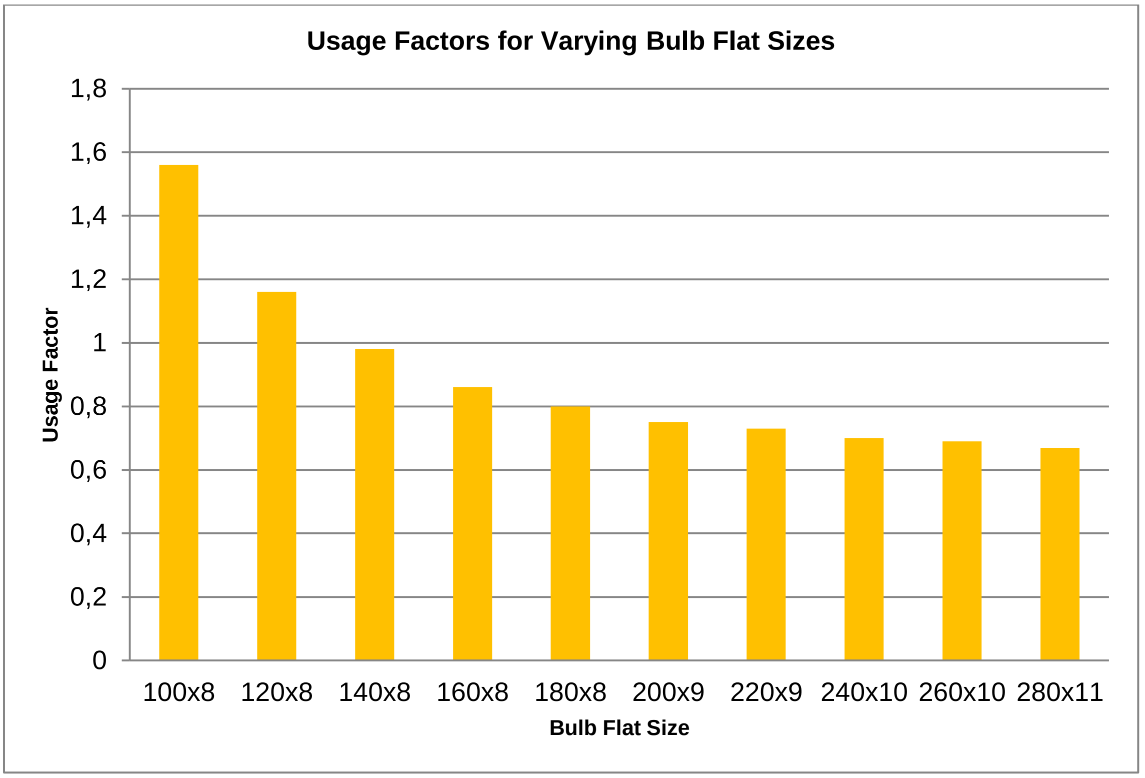



**Figure 25. The usage factors for a panel stiffened with different bulb flat sizes. The properties of the plate were $\sigma_f$=235 N/mm$^2$ (yield strength) and E=206 000 N/mm$^2$ (modulus of elasticity), s=800 mm (width), l=2800 mm (length) and t=12 mm (thickness). The properties for the stiffeners were $\sigma_f$=235 N/mm$^2$ (yield strength) and $l_t$=2800 mm (torsional stiffener buckling length). The stiffeners were continuous and fabricated by welding. z* optimizing was enabled.**

As with all the previous assumptions, also this one was confirmed by the results.

The hand calculations were performed by choosing arbitrary inputs and then completing the calculations following the equations for the load case in question given by DNV.

## 9 Analysis

The results were mainly analyzed by comparison to results of a FEM analysis. The FEM analysis was performed with a FEM program called Finnsap. The main reason for choosing this program was that there was a license available. The simple nature of the case for unstiffened panels allowed for the performing of both a linear and non-linear analysis. When running a linear FEM analysis one must keep in mind that for the purpose of the analysis it is assumed that the material is linearly elastic and that deflections are small compared to the thickness of the plate. The validity of these assumptions is however not checked by the program. The analyses were performed by changing the thickness of the plate while keeping other dimensions constant. The elements chosen for the analyses were 4-node thin shell elements.

The boundary conditions in the linear and non-linear analysis for the unstiffened plate were equivalent to those of a simply supported plate. The definition of simply supported can be found in chapter 6.2. The unstiffened plate was subjected to all of the load types than available in the created tools. The yield strength in the non-linear analysis was assumed to be 355 N/mm$^2$. The validity of the results from the linear analysis was confirmed by hand calculation based on the linear buckling theory [10, pp. 855-863].

The model used in the analysis of a stiffened plate was created in the x y-plane with the z-axis in the direction normal to the plate. The stiffened plate was assumed to be symmetrical with the respect to the x- and the y-axis. Therefore two sides had symmetrical boundary conditions, i.e. the translation in the direction of the axis transverse to the side as well as the rotations in the directions of the two remaining axis were locked, while the rest of the degrees of freedom were free. The two remaining sides had boundary conditions where the translation in direction and rotation around the z-axis, as well as the rotation around the axis transverse to the side in question were locked, while the rest of the degrees of freedom were free. The stiffened plate was subjected separately to in plane uniform longitudinal compression, in plane uniform transverse compression and shear.

The use of symmetrical boundary conditions ignores some of the buckling modes, because the shape of the buckling also has to be symmetrical with respect to axis in question. The lowest buckling mode, i.e. the shape of half a sine wave, fulfills this requirement whereas the second buckling mode, a full sine wave, does not. The third buckling mode, i.e. one and a half sine waves, would again be allowed, but for a stability analysis only the lowest buckling mode is relevant.

## 9.1 Comparison between DNV and FEM, Unstiffened Plates

According to the DNV Recommended Practice, analyses based on the effective width method may lead to more efficient structures. This was for the most part confirmed by the comparisons.

### 9.1.1 Uniform Longitudinal Compression

The results from the linear and non-linear analysis, as well as the results given by DNV for a plate subjected to uniform longitudinal compression, are presented in Table 2. The DNV-results are given both for a plate with the yield strength 235 N/mm$^2$ and 355 N/mm$^2$. The DNV-results are calculated with the material factor 1 in order to enable a direct comparison.

**Table 2. The buckling resistance [N/mm$^2$] for plates subjected to uniform longitudinal compression. The properties of the plate are s=720 mm (width), l=2400 mm (length) and E=206 000 N/mm$^2$ (modulus of elasticity).**

| t [mm] | Linear | Non-Linear | DNV/235 | DNV/355 |
|---|---|---|---|---|
| 6 | 52,22 | 51,67 | 99,03 | 124,33 |
| 8 | 92,78 | 93,75 | 126,95 | 160,69 |
| 10 | 144,86 | 144,00 | 152,26 | 194,51 |
| 12 | 208,43 | 208,33 | 175,20 | 225,80 |
| 14 | 283,41 | 280,00 | 195,52 | 254,56 |
| 16 | 369,74 | 354,06 | 213,31 | 280,77 |
| 18 | 467,36 | 355,00 | 228,55 | 304,45 |
| 20 | 576,18 | 355,00 | 235,00 | 325,60 |

The results presented in Table 2 are illustrated in Figure 26. It is worth observing that the non-linear FEM analyses were conducted using a yield strength of 355 N/mm$^2$.

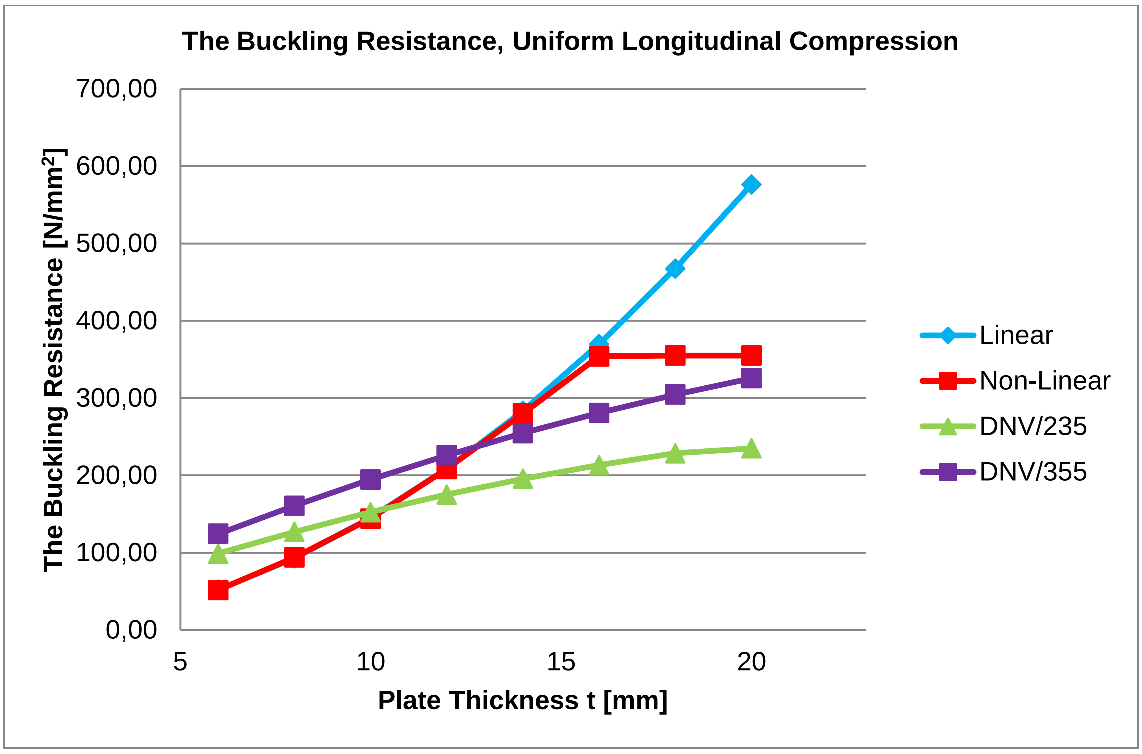



**Figure 26. The buckling resistance of plates with varying thickness subjected to uniform longitudinal compression. The results are presented from analyses based on linear FEM (blue) and non-linear FEM with yield strength 355 N/mm$^2$ (red), and DNV using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple).**

The results from the linear and non-linear analysis follow almost the exact same path up to the point where the yield strength 355 N/mm$^2$ is reached. In contrast to the non-linear analysis, the linear analysis does not take into account the yield strength of the material. The results from the non-linear analysis does again not exceed the given yield strength. FEM gives more conservative results for slender plates, but the roles are reversed at a plate thickness of approximately 13 mm.

### 9.1.2 Uniform Transverse Compression

We now move on to consider plates subjected to a uniform transverse compression, and the results are presented in Table 3. As before, the DNV-results are given for yield strengths of 235 N/mm$^2$ and 355 N/mm$^2$ with a material factor 1.

**Table 3. The buckling resistance [N/mm$^2$] for plates subjected to uniform transverse compression. The properties of the plate are s=720 mm (width), l=2400 mm (length) and E=206 000 N/mm$^2$ (modulus of elasticity).**

| t [mm] | Linear | Non-Linear | DNV/235 | DNV/355 |
|---|---|---|---|---|
| 6 | 15,48 | 15,33 | 42,83 | 56,14 |
| 8 | 27,50 | 27,25 | 53,65 | 68,72 |
| 10 | 42,96 | 42,50 | 65,29 | 82,20 |
| 12 | 61,84 | 61,67 | 77,60 | 96,49 |
| 14 | 84,14 | 82,86 | 96,95 | 111,47 |
| 16 | 109,84 | 108,13 | 114,18 | 127,04 |
| 18 | 138,93 | 137,78 | 131,09 | 154,86 |
| 20 | 171,41 | 170,00 | 147,18 | 176,02 |

The results presented in Table 3 are illustrated in a graphical form in Figure 27.

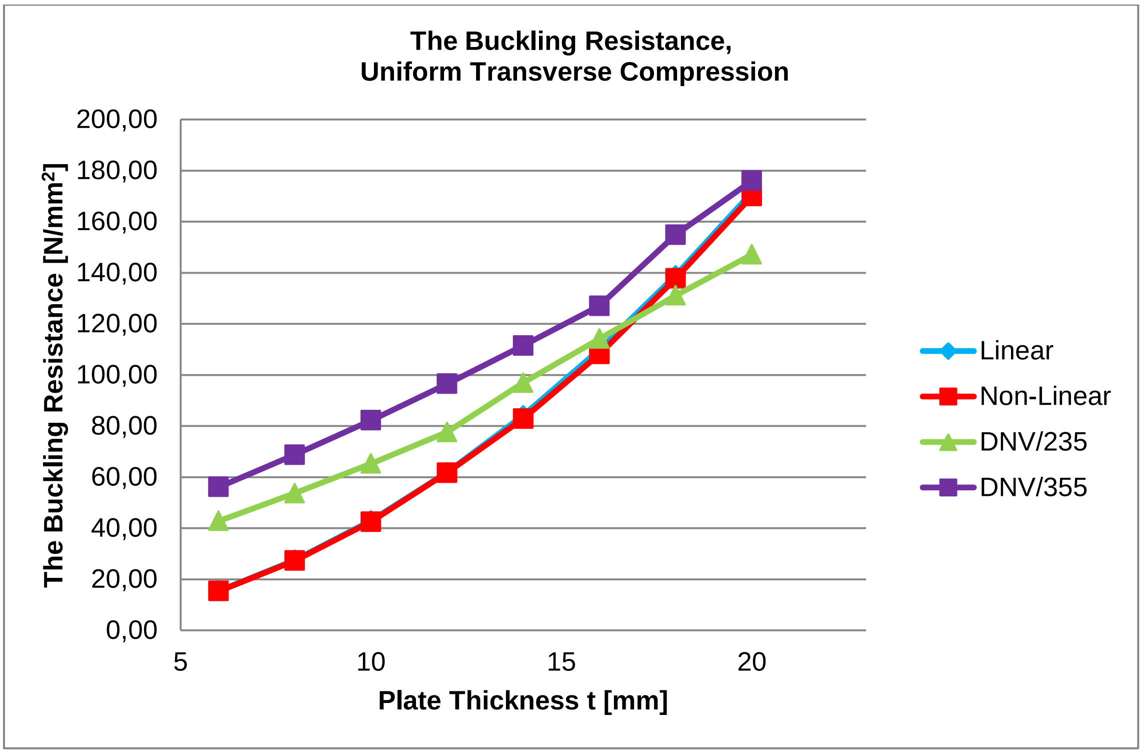



**Figure 27. The buckling resistance of plates with varying thickness subjected to uniform transverse compression. The results are presented from analyses based on linear FEM (blue) and non-linear FEM with yield strength 355 N/mm$^2$ (red), and DNV using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple). The linear and the non-linear FEM analysis coincides as the yield strength is not reached. The results of the linear analysis are therefore barely visible.**

FEM gives more conservative results for plates of all thickness. Because the yield strength is not reached for any of the plates, the results from the linear and non-linear analysis follow almost exact the same path.

### 9.1.3 Shear Stress

The final comparison is for plates subjected to shear stress. The results are presented in Table 4. The comparison was made between the results from linear and non-linear FEM analyses as well as the results from DNV-analyses using yield strengths of both 235 N/mm$^2$ and 355 N/mm$^2$. The DNV-results are calculated with the material factor 1.

**Table 4. The buckling resistance [N/mm$^2$] for plates subjected to shear stress. The properties of the plate are s=720 mm (width), l=2400 mm (length) and E=206 000 N/mm$^2$ (modulus of elasticity).**

| t [mm] | Linear | Non-Linear | DNV/235 | DNV/355 |
|---|---|---|---|---|
| 6 | 75,18 | 93,33 | 90,48 | 111,21 |
| 8 | 133,51 | 133,75 | 117,68 | 148,27 |
| 10 | 208,37 | 205,00 | 134,85 | 179,94 |
| 12 | 299,63 | 205,00 | 135,68 | 201,19 |
| 14 | 407,14 | 205,00 | 135,68 | 204,96 |

The results presented in Table 4 are illustrated in Figure 28.

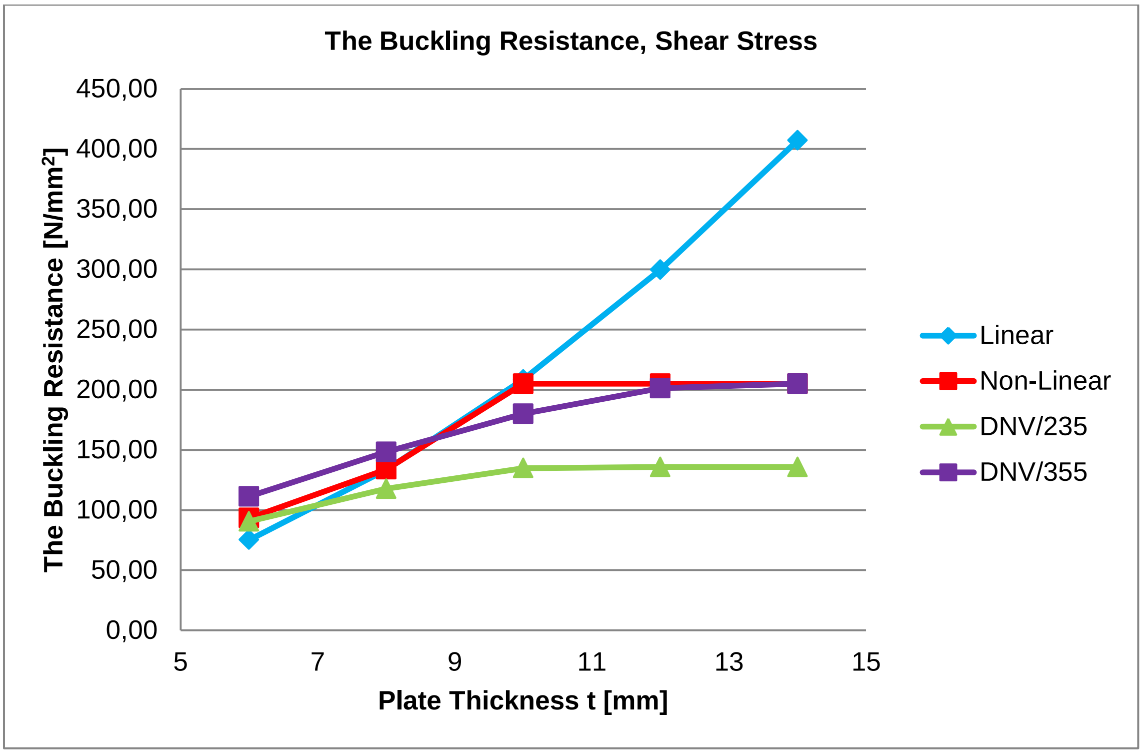



**Figure 28. The buckling resistance of plates with varying thickness subjected to shear stress. The results are presented from analyses based on linear FEM (blue) and non-linear FEM with yield strength 355 N/mm$^2$ (red), and DNV using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple).**

The differences between the results from the non-linear FEM analysis and the results given by DNV using a yield strength of 355 N/mm$^2$ are smaller than in the previous comparisons. The root mean square error for the results in case of shear stress was 15.31 N/mm$^2$, compared to 52.51 N/mm$^2$ and 30.98 N/mm$^2$ for the cases of longitudinal and transverse compression. The results are limited by, not the yield strength, but by the yield strength divided by $\sqrt{3}$ (i.e. the shear strength). As the DNV-results for shear actually reached the limiting value, in contrast to the cases of longitudinal and transverse compression, a natural result is a smaller deviation between the two analyses.

## 9.2 Comparison between DNV and FEM, Stiffened Plates

The stiffened plate was subjected separately to in plane uniform longitudinal compression, in plane uniform transverse compression and shear. The computing time of the FEM-analyses for stiffened plates was much higher than those for unstiffened ones. The non-linear analysis for stiffened plates was too time consuming, and could not be performed with the available resources. As a result, the number of cases was kept to a minimum. The cases were limited to those of most interest; the analyses were done for plates with a slenderness ratio under 120, but that were still slender enough to fail by buckling instead of yielding. With these restrictions, the results obtained by the linear FEM-analysis are expected to be very similar to those of a non-linear analysis.

The results given by Finnsap for stiffened plates subjected to shear stress did not seem reliable. No comparison for the cases of shear stress has therefore been done.

### 9.2.1 Uniform Longitudinal Compression

The results from the linear analyses and the results given by DNV for a stiffened plate subjected to uniform longitudinal compression are presented in Table 5. The DNV-results are given both for a plate with the yield strength 235 N/mm$^2$ and 355 N/mm$^2$. The DNV-results are calculated with the material factor 1 in order to enable a direct comparison.

To get more comparison without having to perform additional time consuming FEM-analyses, an assumption was made, that plates with a thickness greater than 14 mm would have reached the yield strength 355 N/mm$^2$.

**Table 5 The buckling resistance [N/mm$^2$] for stiffened plates subjected to uniform longitudinal compression. The properties of the plate are s=700 mm (width), l=2400 mm (length) and E=206 000 N/mm$^2$ (modulus of elasticity). The size of the stiffener is 300x11. z* optimizing was enabled for the DNV-analyses. The results in italic are approximations.**

| t [mm] | Linear | DNV / 235 | DNV/ 355 |
|---|---|---|---|
| 6 | 88,60 | 146,05 | 175,95 |
| 8 | 145,04 | 154,10 | 186,30 |
| 10 | 205,89 | 165,60 | 201,25 |
| 12 | 273,38 | 183,66 | 218,50 |
| 14 | 342,28 | 202,40 | 256,45 |
| 16 | *355,00* | 215,05 | 279,45 |
| 18 | *355,00* | 224,25 | 296,70 |
| 20 | *355,00* | 227,70 | 310,50 |

The results presented in Table 5 are illustrated in a graphical form in Figure 29.

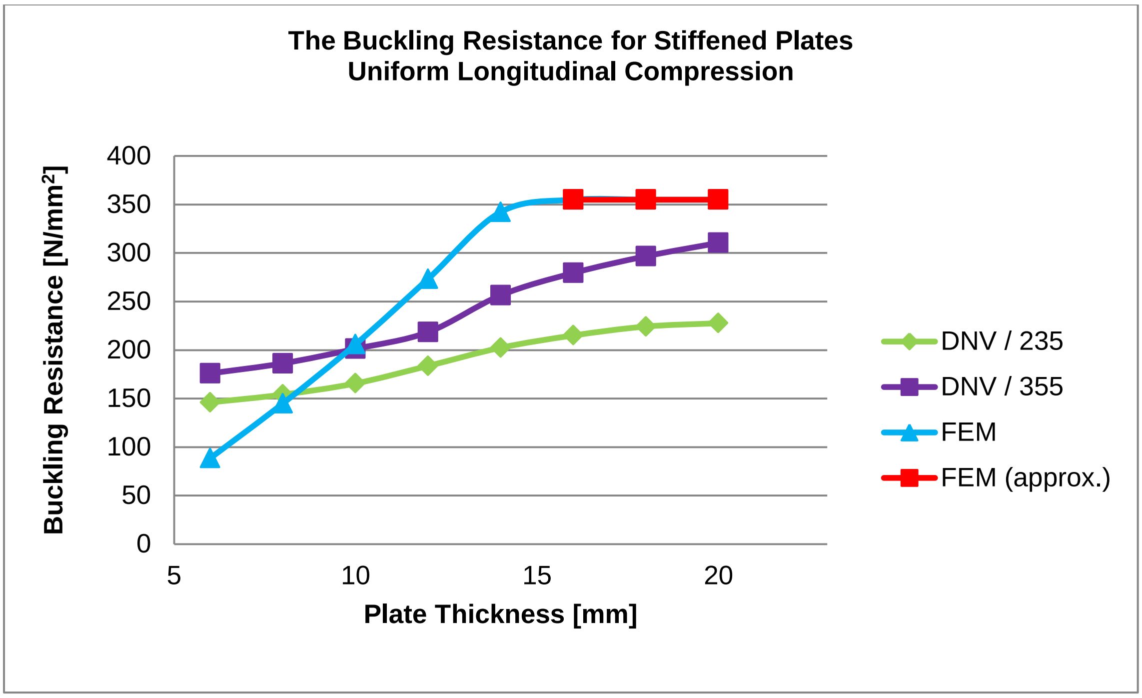



**Figure 29. The buckling resistance for stiffened plates with varying thickness subjected to uniform longitudinal compression. The results are presented from analyses based on linear FEM (blue) and DNV using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple). The results presented in red are approximations based on the assumption that the yield strength is reached.**

The common phenomenon through all the comparisons have been that the results from the analysis based on FEM grow in a steeper manner then the results from the analysis based on DNV, something that was most clearly seen in this last comparison. The average change for the results were 31.71 Nmm$^{-2}$/mm (10.75 Nmm$^{-2}$/mm) for the FEM analysis (DNV-analysis with yield strength 355 N/mm$^2$). The similar numbers for the unstiffened plates were 28.9 Nmm$^{-2}$/mm and 15.64 Nmm$^{-2}$/mm. The results for the stiffened (unstiffened) plates were calculated for a plate size of 700mmx2400mm (720mmx2400mm). The small difference in size should not affect the results significantly. Assuming that the rules and recommendations of DNV represent a desirable structure, a buckling check performed by FEM would in most cases lead to either too conservative or non-conservative structures. The almost similar results for a plate with the thickness of 10 mm are more of a coincidence.

### 9.2.2 Uniform Transverse Compression

Next, plates subjected to a uniform transverse compression are considered. The results are presented in Table 6, and again, the DNV-results are given for yield strengths of 235 N/mm$^2$ and 355 N/mm$^2$ with a material factor 1.

**Table 6. The buckling resistance [N/mm$^2$] for stiffened plates subjected to uniform transverse compression. The properties of the plate are s=700 mm (width), l=2400 mm (length) and E=206 000 N/mm$^2$ (modulus of elasticity). The size of the stiffener is 300x11. z* optimizing was enabled for the DNV-analyses.**

| t [mm] | Linear | DNV / 235 | DNV/ 355 |
|---|---|---|---|
| 6 | 36,38 | 43,13 | 56,35 |
| 8 | 48,71 | 54,05 | 69,00 |
| 10 | 62,89 | 66,13 | 82,80 |
| 12 | 80,60 | 78,78 | 97,75 |
| 14 | 102,21 | 98,90 | 112,70 |
| 16 | 127,75 | 117,30 | 136,85 |
| 18 | 157,15 | 134,55 | 158,70 |
| 20 | 190,33 | 150,65 | 180,55 |

The results presented in Table 6 above are illustrated in Figure 30 below.

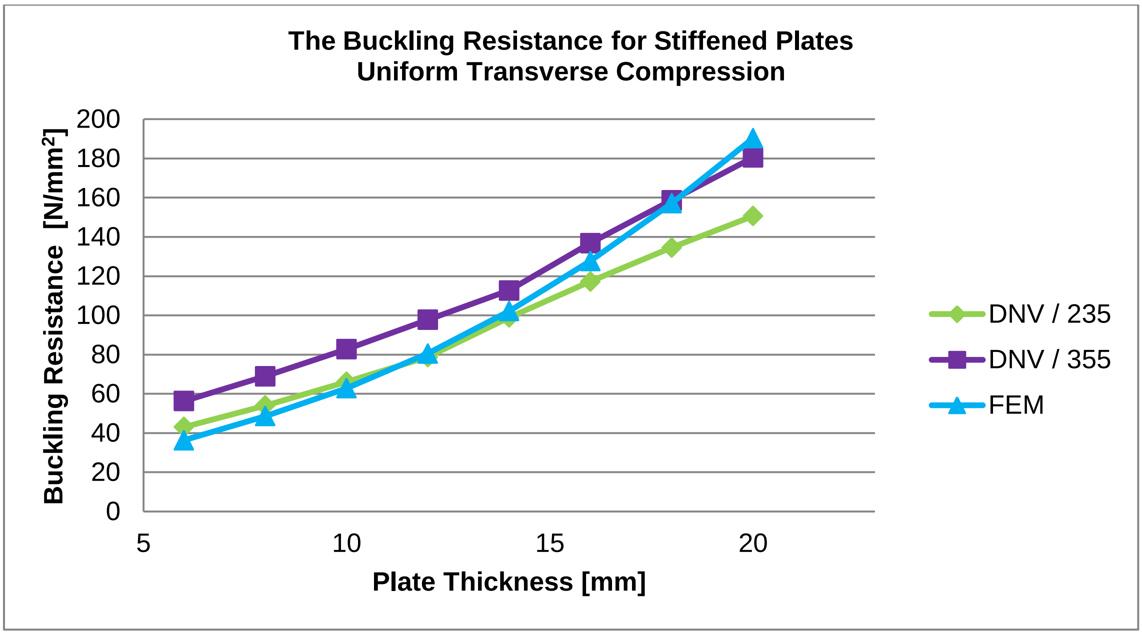



**Figure 30. The buckling resistance for stiffened plates with varying thickness subjected to uniform transverse compression. The results are presented from analyses based on linear FEM (blue) and DNV using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple).**

The FEM- and the DNV-analysis behaved quite similarly. The FEM-analysis did, however, give somewhat more conservative results on average, especially for slender plates.

## 9.3 Contradicting Results

For the most part the outcomes were in line with what was expected, but there were nevertheless a few surprising results. These contradictions did not have a significant impact on the usability of the results, but in some cases they limited the options available in the tools.

### 9.3.1 Cases of Buckling Resistance Exceeding Yield Strength

The definition of the buckling strength for unstiffened plates subjected to varying longitudinal stress was introduced in Chapter 6.6. The buckling resistance is calculated as:

$$\sigma_{x,Rd} = C_x \frac{\sigma_f}{\gamma_M}, \tag{30}$$

where the coefficient $C_x$ is a function of the reduced slenderness $\overline{\lambda_p}$.

The precise way to calculate the value of the coefficient is as follows:

$$C_x = \begin{cases} 1 & , when\ \overline{\lambda_p} \leq 0.673 \\ \frac{\overline{\lambda_p} - 0{,}055(3+\psi)}{\overline{\lambda_p}^2} & , when\ \overline{\lambda_p} > 0.673 \end{cases} \tag{31}$$

where $\psi$ is the stress ratio as defined in Chapter 6.6.

The rules do not give any further limitations for the value of $C_x$. Still, considering the nature of the coefficient, it is reasonable to assume that it should not exceed the value of 1. This was, however, not always the case, which lead to situations where the buckling resistance exceeded the yield strength. This is clearly not physically possible. One of these situations is illustrated in Figure 31.

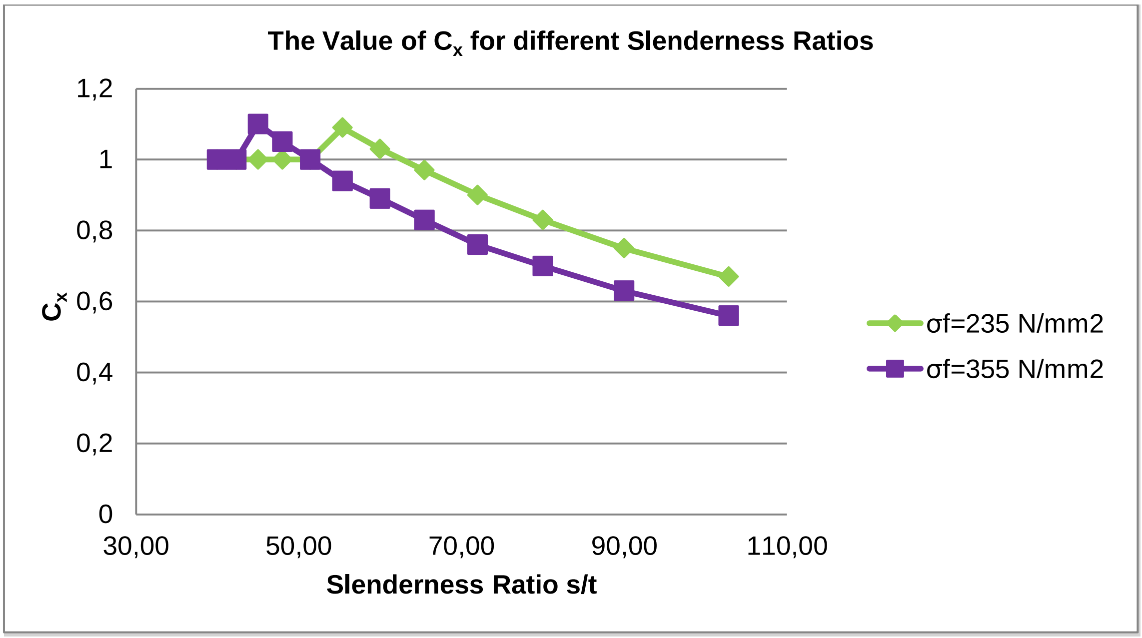



**Figure 31. Values of the coefficient $C_x$ for different slenderness ratios (plate width / plate thickness). The values are calculated using yield strengths of 235 N/mm$^2$ (green) and 355 N/mm$^2$ (purple). The stress ratio $\psi$=0.**

It can be seen that the coefficient exceed the value 1 with certain slenderness ratios, depending on the yield strength of the material. Preliminary testing indicates, that these types of situations might occur in cases where the compression gradient is large, i.e.

the stress ratio $\psi$ is small. To prevent unphysical results, the value of the coefficient $C_x$ was restricted to a maximum of 1 in the buckling check tool.

### 9.3.2 Tension Field Action

According to the Recommended Practice, by tension field action is understood the load carrying actions in slender webs beyond the elastic buckling load.

There was quite little information to be found about this phenomenon. The results given when tension field action was allowed indicates that when plates subjected to shear stress buckle, the strength of the plate grows.

The usage factors given as results from buckling checks of a stiffened plate affected to varying shear stress are represented in Table 7. The usage factors under the heading “UF/On” are results from a buckling check were tension field action was allowed whereas the usage factors under the heading “UF/Off” are results from a buckling check were tension field action was not allowed.

**Table 7. Usage factors from buckling checks of a stiffened plate subjected to varying shear stress. The properties of the plate are s=700 mm (width), t= 6mm (thickness), l= 3000 mm (length), E=210 000 N/mm$^2$ (modulus of elasticity) and $\sigma_f$=355 N/mm$^2$ (yield strength). The size of the stiffener is irrelevant for the results.**

| τ [N/mm$^2$] | UF / On | UF /Off |
|---|---|---|
| 60 | 0,79 | 0,79 |
| 62 | 0,85 | 0,85 |
| 64 | 0,9 | 0,9 |
| 66 | 0,96 | 0,96 |
| 67 | 0,99 | 0,99 |
| 68 | 0,29 | 1,02 |
| 70 | 0,3 | 1,08 |
| 72 | 0,32 | 1,14 |
| 74 | 0,33 | 1,21 |
| 76 | 0,34 | 1,27 |

The results presented in Table 7 are illustrated in Figure 32.

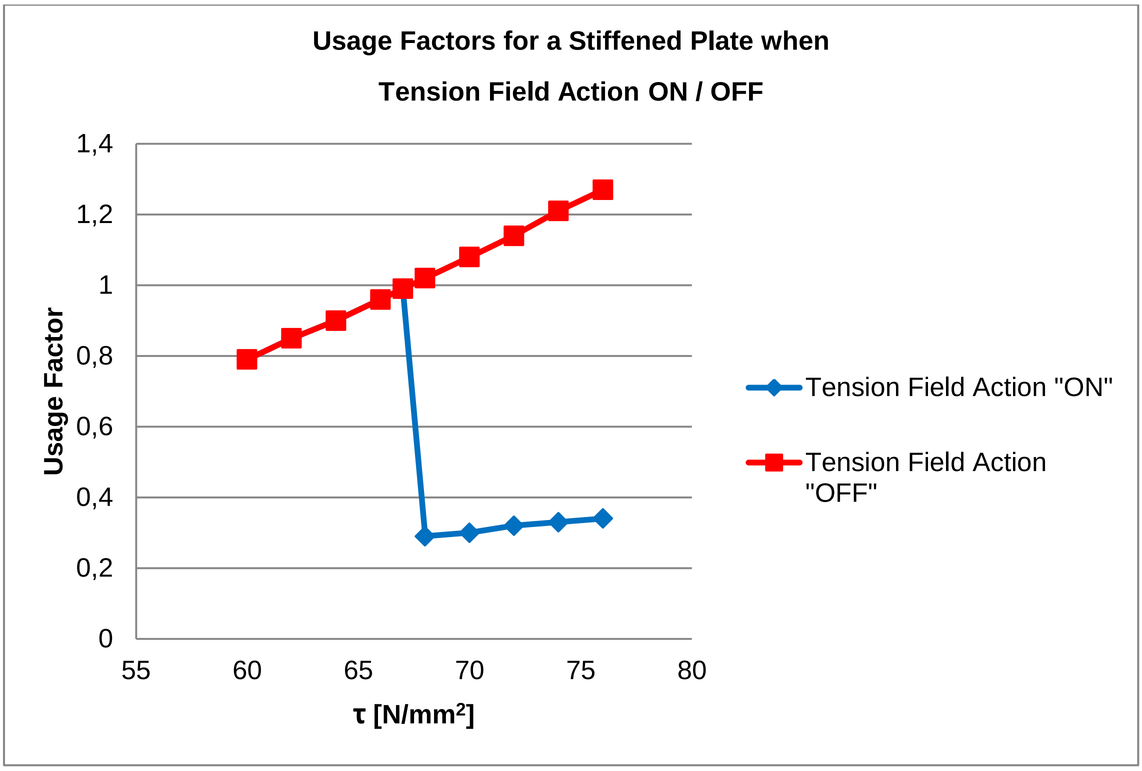



**Figure 32. Usage factors from buckling checks of a stiffened plate subjected to varying shear stress when tension field action is allowed (blue) and tension fiel action is not allowed (red).**

It can be seen that up to the point where the critical buckling strength is reached (UF 0.99) the usage factors are the same regardless if the tension field action is allowed or not. The big difference is after the plate has supposedly buckled; if tension field action is allowed the usage factor drops radically. When the plate is subjected to a shear stress of the magnitude 74 $N/mm^2$ it is either clearly over the safe limit (tension field action not allowed, usage factor 1.21) or it still have two thirds of its capacity left (tension field action allowed, usage factor 0.33).

Because of the incomplete understanding of the phenomenon and the way it is accounted for in the rules and recommendation of DNV together with the huge impact it had on the results, the alternative of allowing tension field action was decided to be left out from the buckling check tool.

## 10 Discussion and Conclusions

Eight tools for buckling checks based on the rules and recommendations of DNV were created. The results were then verified by hand calculating and by checking that the results produced were in line with known results of buckling when e.g. the thickness of the plate was varied. A FEM analysis for unstiffened and stiffened plates was performed and the results were compared with results from the appropriate tools.

It was shown that the tools produce results consistent with what is to be expected based on the theory of buckling. The results differed from the results acquired by the FEM analysis, which was expected due to the different theoretical backgrounds of the two approaches. The FEM results were consistently more conservative when regarding slender plates, although the roles were reversed when moving towards thicker plates as the FEM results reached yield stress.

Excel might be considered for these types of projects as long as the formulas needed to program stay simple. The use of Excel gave rise to problems in situations where the formulations of the coefficients were subjected to several conditions. The use of a proper programming language might be beneficial in these types of projects.

The study revealed that the recommendations of the DNV are in no way exhaustive, and common sense and caution have to be used as it is e.g. possible for the buckling resistance to exceed the yield strength.

As future work, a closer examination of the behavior of the tension field action might be in order. Assuming any problems regarding the reliability of this property is solved, the implementation of the tension field action into the tool might be considered. Plates with cutouts had to be omitted from this work due to suspicions of unreliability in the rules and recommendations. As these types of structures are, however, common, the implementation would be recommended when the rules and recommendations are updated. The use of boundary conditions other than the simply supported would also be a welcomed addition to the tools, along with plates stiffened in two directions. The rules and recommendations for these types of structures exist, so the implementation is already possible.

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