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Faster Than Sound, Daredevil Parachute Jumps from the Edge of the Space

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Abstract

This article discusses the kinematics of a parachutist making a very-high-altitude jump. The effect of altitude on the density of air, on the gravitational field strength of the Earth, and on the atmosphere's temperature has been taken into account in our analysis. The well-known equations of classical mechanics governing the selected topic have been solved numerically by using the mathematical software Mathcad. Especially, the possibility of a person exceeding the speed of sound during their fall has been considered in our analysis. The effect of the sound barrier is taken into account so that the shape factor of the falling body is given as a speed-dependent function, which reaches its maximum value at Mach 1.0. The obtained results have been found to be highly consistent with the available experimental data on some high-altitude jumps. The data published on the famous jump of Captain Joseph Kittinger has been analyzed very carefully, and although our calculations reproduced the reported values for most parts, some interesting inconsistencies were also discovered. Kittinger jumped from a gondola attached to a helium-filled balloon from a record-high altitude of 102,800 ft, or 31,330 m, in August 1960. We also made numerical analysis on the high-altitude jump of Felix Baumgartner. He bailed out from his gondola at the record-high altitude of 39.0 km in October 2012.

Keywords

Parachute Jumps, Atmosphere, Numerical Modelling, Transonic Speeds

1. Introduction

In the early days of space era, scientists were extremely interested in the effects of super-high altitudes on human beings. Such knowledge was of high importance, since scientists and engineers were vigorously working for the first-ever manned flight around the Earth through the empty and freezing space. Engineers and scientists were also worried about the possibility of a pilot being forced to leave his aircraft at extremely high altitudes. They wanted to know if one can survive from such a high-altitude fall, and if so, with what kind of equipment.

At an altitude of about 30 km or more, the surroundings are very hostile to any living organisms: temperature may be as low as −70˚C, the pressure of the atmosphere is only about 1% of its value at the sea level, and the intensity of harmful ultraviolet radiation is dangerously high. So, why not send a person there! At the beginning of 1960's, a person was indeed sent there.

On August 16th 1960, Captain Joseph Kittinger climbed into a gondola, which was firmly attached to a vast helium-filled balloon. The balloon was released, and it started its vertical journey through the Earth's atmosphere. The balloon kept on rising for about an hour and a half before reaching the final ultimate altitude of about 102,800 ft. Kittinger was looking down (and up, toward the strikingly black empty space) for 12 minutes, before leaping out and starting his fall through the almost-zero-density and extremely cold air.

According to the authentic information given by Kittinger himself, he first fell freely for about 16 seconds and then opened a stabilization canopy parachute 1.8 m in diameter [1] [2]. After this, he continued falling for more than 4 minutes before opening the main parachute. The total time between the bail-out and the time Kittinger opened his main parachute was 4 min 38 s. Some sources of information claim that during his almost-free fall towards the Earth, Kittinger exceeded the speed of sound [3] [4]. However, this is in contradiction with the data given immediately after the jump [1] [2]. Hence, it would be interesting to show whether that Kittinger did travel faster than sound during his fall, or not.

Besides the valuable information for the space-travel designers and the U.S. Air Forces, Kittinger's jump had also other consequences: he inspired many other daredevils to try the same, or even more. The most famous of such individuals are an Australian parachutist Rodd Millner, a retired French parachute regiment officer Michael Fournier and a U.S. parachuting champion Cheryl Stearns. All of them have made serious plans for a jump from a record-high altitude of 40.0 km. They have claimed that they would definitely fall faster than the speed of sound during their almost-free fall. In addition to such plans, two persons have already made such a jump: an Austrian professional parachutist Felix Baumgartner made his celebrated world record jump on October 14th, 2012 [5], and an American senior vice president of engineering at Google Alan Eustace, who jumped from 41.4 km in October 2014 [6].

All this arises important questions that need to be answered. Is it possible to exceed the speed of sound when jumped at any altitude? How does the speed of a falling parachutist vary as a function of altitude, if all the present knowledge is taken into account? And, finally, did Joseph Kittinger and/or Felix Baumgartner fall faster than the speed of sound during their jumps? We wanted to develop a reliable and easy-to-understand numerical model for cases of this kind, in order to answer these questions, and many more. Our model can be used to plot the speed of the falling body as a function of time or altitude, the drag force acting on the falling body vs. time or vs. altitude, etc. We find our model relatively versatile, because it can be used for several different kinds of cases for a body with a given shape and speed travel through a given medium.

2. The Relevant Laws of Physics

Let us apply the Work-Energy Theorem for a parachutist jumping from a high altitude:

$$\sum W_{\text{ext}} = \Delta K \tag{1}$$

According to this result, the total amount of work done on the parachutist by all the external forces acting on him equals the change in his kinetic energy. The parachutist's kinetic energy depends on his mass, *m*, and speed, *v*:

$$K = \frac{1}{2}mv^2 \tag{2}$$

The relevant external forces acting of the falling person are the force of gravity, $F_{\text{g}}(h)$, and the drag force due to air resistance, $F_{\text{D}}(v,h)$:

$$\begin{cases} F_{\text{g}}(h) = G\dfrac{mM_{\text{E}}}{(R_{\text{E}}+h)^2} \\ F_{\text{D}}(v,h) = \dfrac{1}{2}C(v)A\rho(h)v^2 \end{cases} \tag{3}$$

In Equation (3), *h* stands for the altitude, *i.e.* the vertical distance from the surface of the Earth, *G* is the gravitational constant: $G = 6.672\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, M_{E} is the mass of the Earth: $M_{\text{E}} = 5.974\times10^{24}\ \text{kg}$, R_{E} is the Earth's average radius: $R_{\text{E}} = 6.371\times10^{6}\ \text{m}$, $C(v)$ is a dimensionless shape factor of the falling object, *A* is the projection of the object's area in the plane perpendicular to the direction of motion (*i.e.* horizontal), and $\rho(h)$ is the altitude-dependent density of the atmosphere.

When the speed of a falling object approaches that of sound, the value of $C(v)$ increases strongly. We have estimated that $C(v)$ is constant, $C_0 = 1.05$, for a person falling freely before opening the stabilization parachute. After the stabilization canopy parachute is opened, the value of the drag coefficient is estimated to be constant, $C_1 = 1.10$, until the parachutist's speed exceeds 70% of the speed of sound. After this limit, the magnitude of $C(v)$ increases smoothly, and reaches its maximum value at the speed of sound (Mach 1.0). At transonic speeds, $C(v)$ decreases smoothly, and saturates to another constant at about Mach 1.2 (**Figure 1**).

We have estimated the shape of our $C(v)$ function mainly by using two sources of information: 1) traditional data on a large number of airfoils tested in wind tunnels at the transonic region [7] [8] and 2) more recent results on 3D bodies tested in wind tunnels [9]. Such tests have revealed that the magnitude of the drag coefficient starts increasing smoothly as the speed of air around an obstacle exceeds some 70% of the speed of sound, or Mach 0.70. The value of $C(v)$ reaches its maximum at about Mach 1.0. Although the magnitude of the

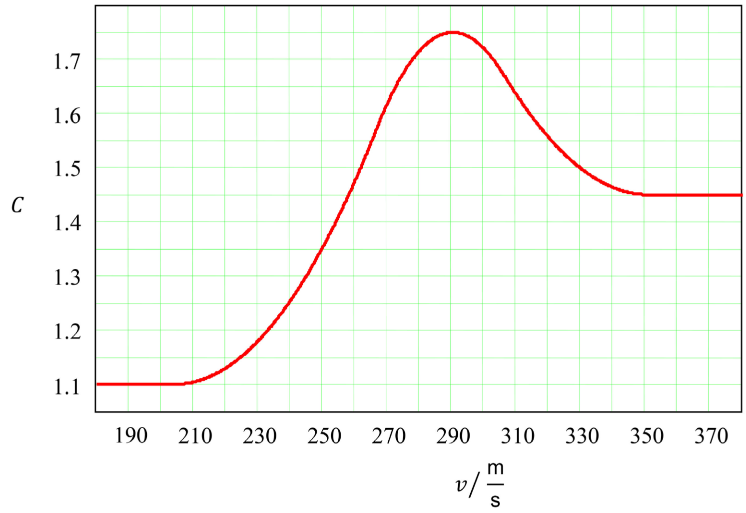

Figure 1. The shape factor of a falling body as a function of its speed. The figure corresponds to an altitude of 26.5 km.

drag coefficient for airfoils, cylinders and spheres has been found to increase by a factor of about 2.0…2.5 at the transonic region, we reckon the value of C for the parachutist-parachute system does not raise that much. Our suggestion is that for such a system, C increases from its initial value of 1.10 to about 1.75 as the speed of the falling system grows from Mach 0.7 to Mach 1.0. After this, the drag coefficient smoothly decreases to 1.45 as the speed increases from the speed of sound to about Mach 1.2. For speeds above this limit, C is estimated to have a constant value of 1.45 (**Figure 1**). The most important reason for not using values greater than 1.75 is that for any obstacle, C very seldom attains a value larger than this limit.

In extra-high-altitude parachute jumps, the parachutist and the stabilization parachute together form a system, which falls downward so that the person's feet break the air front first. The aerodynamic behaviour of such a system, especially when its speed approaches Mach 1.0, is very different from that of a parachute (and the parachutist) falling downward at a low speed. Hence, although a value of 1.2 is typically used for the shape factor of a low-porosity parachute [9], we suggest that this is too high for a parachutist-parachute system, which is falling at a very high speed. For this reason, we have estimated that $C_1 = 1.10$ for the system modelled in our calculations.

One should note that even though we call C a function of v only, it also depends on the altitude, since at every step of our numerical calculations, the speed of sound at the corresponding altitude affects the value of C, as explained above. The speed of sound, on the other hand, is a function of the atmospheric temperature (Equation (16)), which, in turn, depends on the altitude. In our calculations, we have used the temperature-vs-altitude function $T(h)$ as shown in **Figure 2**. The function is defined so that it fits well with the traditional atmospheric data [10] from 1962, as well as more recent experimental data [11] collected in 2002.

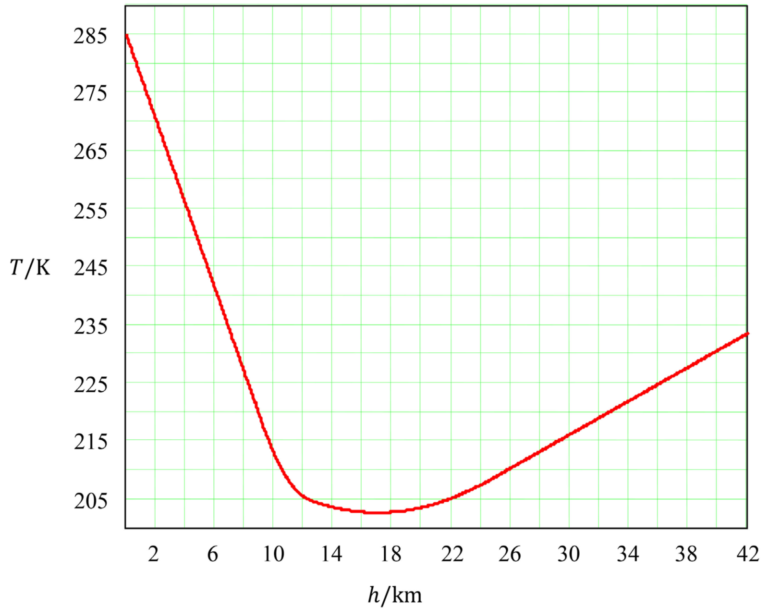

Figure 2. The temperature of the Earth's atmosphere as a function of altitude.

The altitude-dependent density of air can be calculated by using a result derived from the Ideal Gas Law:

$$\rho(h) = \frac{p(h)M}{RT(h)} \tag{4}$$

In this expression, $p(h)$ is the atmospheric pressure as a function of altitude, M is the average molecular mass of air, R is the universal gas constant: $R = 8.314\ \text{J/mol} \cdot \text{K}$, and T is the absolute temperature of air. It can be shown that the atmospheric pressure decreases exponentially as a function of altitude [12]:

$$p(h) = p_0 \mathrm{e}^{-mg(h)h/k_\mathrm{B}T(h)} \tag{5}$$

In this formula, p_0 is the atmospheric pressure at sea level, m is the average mass of a molecule in the atmosphere, $g(h)$ is the gravitational field strength as a function of altitude and k_B is Boltzmann's constant. The value of p_0, under normal circumstances, is 1.013×10^5 Pa, $k_\mathrm{B} = 1.38 \times 10^{-23}\ \text{J/K}$, and $g(h)$ can be calculated as:

$$g(h) = G\frac{M_\mathrm{E}}{(R_\mathrm{E} + h)^2} \tag{6}$$

The mass of one atmospheric molecule can be calculated from the average molecular mass of air, M, and Avogadro's number, $N_\mathrm{A} = 6.022 \times 10^{23}\ 1/\text{mol}$, which specifies the number of air molecules in one mole of air:

$$m = \frac{M}{N_\mathrm{A}} \tag{7}$$

The average molecular mass of air can be calculated from the relative amounts of different gases in air, and the result is 28.96 g/mol [13].

Combination of the results given above yields the following formula for the density of air vs. altitude:

$$\rho(h) = \frac{p_0 \mathrm{e}^{-Mg(h)h/N_\mathrm{A} k_\mathrm{B} T(h)} M}{RT(h)} \tag{8}$$

Boltzmann's constant, Avogadro's number and the universal gas constant are related with each other as follows:

$$R = N_\mathrm{A} k_\mathrm{B} \tag{9}$$

So, the atmospheric density can be given as:

$$\rho(h) = \frac{p_0 M}{RT(h)} \mathrm{e}^{-Mg(h)h/RT(h)} \tag{10}$$

In our calculations, we used the normal atmospheric pressure for p_0. We used 12˚C for the zero-altitude temperature, because this makes our $T(h)$ function yield −70˚C for the lowest temperature achieved in the altitude range of 0…39 km used in our analysis. This minimum temperature value is the same as the lowest temperature met by J. W. Kittinger during his ascent to the peak altitude of 102,800 ft, about an hour before his celebrated long, lonely leap [1] [2] (**Figure 3**).

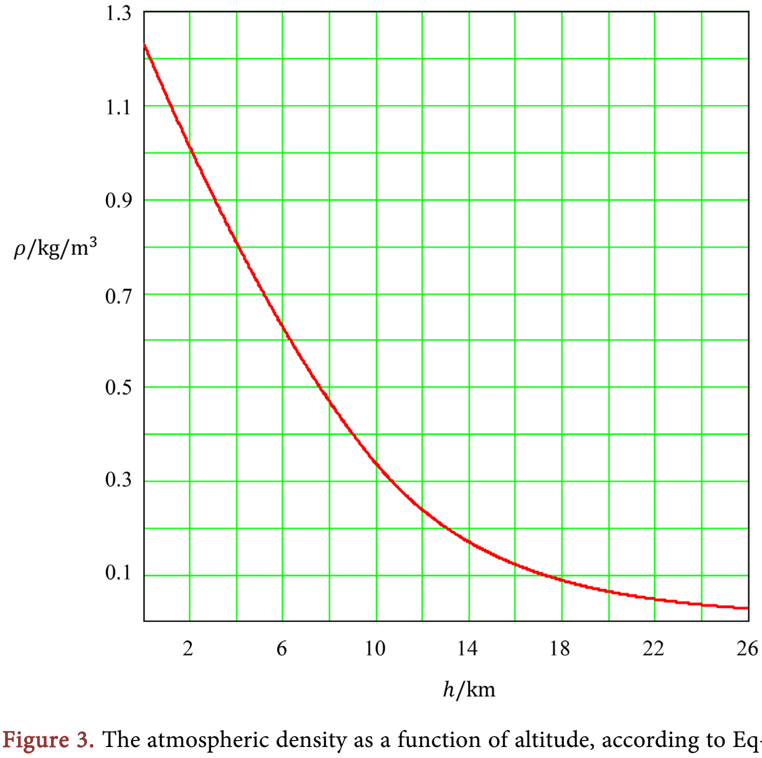

Figure 3. The atmospheric density as a function of altitude, according to Equation (10).

3. Numerical Analysis

Our analysis was carried out such that the parachutist was first given an initial altitude, e.g. 31.33 km, and an initial speed of 0 m/s. After this, the person was let to fall vertically for 1 m, and the computer program determined the total amount of external work done on the parachutist in this displacement. The corresponding change in the person's kinetic energy was then solved from the obtained external work. Finally, a new value for the person's speed was determined with a combination of Equations (1), (2), and (3).

The parachutist was then let to fall another vertical displacement of 1 m, and his new speed was determined in the same way. This procedure was repeated until the zero altitude was reached.

For a numerical procedure of this kind, the Work-Energy Theorem can be written as:

$$\begin{cases} F_{\mathrm{g}}\left(H_i\right)\cdot\Delta h-\frac{1}{2}C\left(v_i\right)A\rho\left(H_i\right)v_i^2\cdot\Delta h=\frac{1}{2}mv_{i+1}^2-\frac{1}{2}mv_i^2 \\ \Delta h=1\,\mathrm{m} \end{cases} \tag{11}$$

In this result, i is an index, which can be understood as follows: v_i is the magnitude of the person's vertical velocity after he has fallen vertically for $i\cdot\Delta h$ metres; H_i is the corresponding altitude.

Equation (11) can now be used in solving the speed after $i+1$ numerical steps, from the speed found after the ith step:

$$v_{i+1}=\sqrt{\frac{2}{m}\left\{\frac{1}{2}mv_i^2+F_{\mathrm{g}}\left(H_i\right)\cdot\Delta h-\frac{1}{2}C\left(v_i\right)A\rho\left(H_i\right)v_i^2\cdot\Delta h\right\}} \tag{12}$$

When Equation (3) is combined with this formula, one arrives at the following expression for the speed after the $(i+1)$st numerical step:

$$v_{i+1}=\sqrt{v_i^2\left\{1-\frac{C\left(v_i\right)A\rho\left(H_i\right)}{m}\cdot\Delta h\right\}+\frac{2GM_{\mathrm{E}}}{\left(R_{\mathrm{E}}+h\right)^2}\cdot\Delta h} \tag{13}$$

In our analysis, we supposed that the person first falls freely, or, to be exact, almost freely, for 16 seconds, after which he opens a small stabilization canopy parachute. This matches with the data known for Kittinger's celebrated jump [1]. At an altitude of 5.5 km, he opened the main parachute, and this was repeated also in our calculations.

The fall time was computed separately for each 1 m long vertical displacement. This was done by using the average value of the speeds corresponding to two consecutive steps of the procedure:

$$t_i=\frac{\Delta h}{\left(\frac{v_{i-1}+v_i}{2}\right)} \tag{14}$$

The total fall time is, naturally, the sum of the t_i values, when i ranges from 0 to the value corresponding to the altitude at which the main parachute is opened. In addition to this time, we also computed the time it takes the person to descend from the 5.5 km altitude to the ground.

Finally, we wanted to see how the acceleration of the falling person varies as a function of his altitude. The acceleration was determined separately for every 1 m long vertical displacement:

$$a_i = \frac{v_i - v_{i-1}}{t_i} \tag{15}$$

Using the obtained values of $i \cdot \Delta h$, v_i, a_i and t_i, we were able to plot the parachutist's speed and acceleration as a function of altitude, as well as the falling person's altitude, speed and acceleration as a function of time.

At each altitude separately, the speed of the falling person can be compared with the speed of sound, which can be calculated from the temperature of air:

$$v_s(h) = \sqrt{\frac{\gamma RT(h)}{M}} \tag{16}$$

In this result, γ is the adiabatic constant of air, $\gamma = 1.40$. Even though the speed of sound is fundamentally a function of temperature, in our case it appears to be merely a function of altitude.

4. The Long, Lonely Leap

In our analysis, we use the data published on Captain Kittinger's famous jump from an altitude of 31.33 km [1] [2]. The mass of the person and his gear was given to be 142 kg, as reported by Kittinger himself [1] [2]. Our calculations were done in three separate parts: 1) The person starts his vertical fall at an altitude of 31,330 m with zero initial speed and falls for 16 seconds. An estimated value of 0.60 m^2 is used for A, and C is reckoned to be 1.05. The final speed of the parachutist is determined, and this will be used as the initial speed in the second part of the calculations. 2) The 6-foot-wide stabilization canopy parachute is opened, and a value of 2.6 m^2 is used for A. The value of the shape factor, $C(v)$, remains constant at 1.10 until the speed of the falling person exceeds Mach 0.7. After this, the magnitude of $C(v)$ increases smoothly according to **Figure 1**, reaching the maximum value of 1.75 at Mach 1.0. If the speed happens to be even higher than this, the value of $C(v)$ decreases smoothly to another constant, 1.45, at about Mach 1.2. When the falling person has reached an altitude of 5.5 km, his speed is stored, and it will be used as the initial speed in the last part of the analysis. 3) The main 28 ft wide canopy parachute is opened, and the value of A is changed to 57.2 m^2. After this, the person descends slowly, until he meets the ground at the ultimate final speed.

The following results were found for Captain Kittinger's famous jump: After the 16-second-long almost-free fall, his speed has increased to 150 m/s and he has reached an altitude of 30.1 km (**Figure 4**). After having opened the stabilization parachute, his fall lasts for additional 4 min 37 s before the main parachute pops open. Hence, according to our calculations, Kittinger had been falling for 4 minutes and 53 seconds before he opened the main parachute. This is only 15 seconds longer than the total time of his almost-free fall reported by Kittinger himself [1] [2], 4 min 38 s. At this point, our calculations give him a speed of 38.4 m/s.

Finally, the main parachute is opened, and the virtual Kittinger descends for additional 13 min 17 s before hitting the ground at the final speed of 6.0 m/s. We

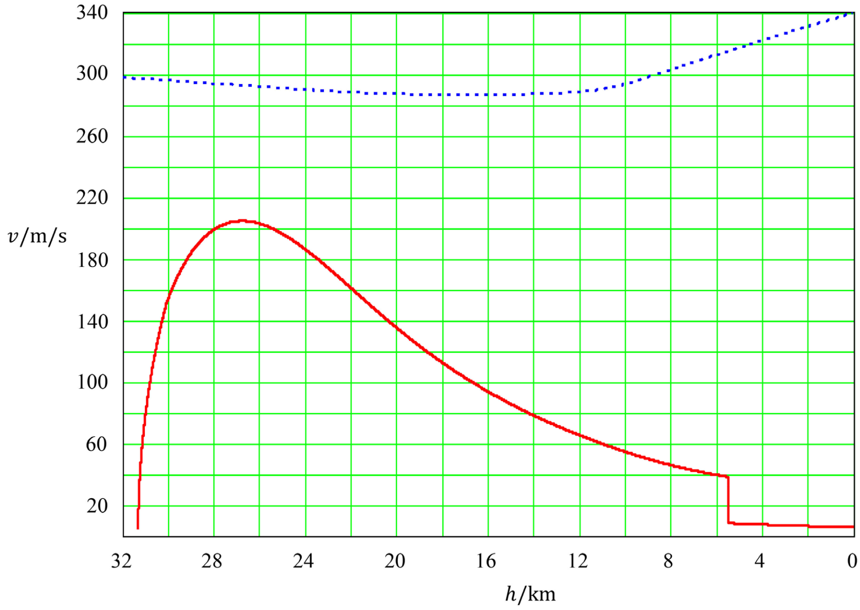

Figure 4. The velocity of Captain Kittinger (solid line) as a function of altitude in his famous high-altitude jump from 31.3 km. The dashed line shows the speed of sound at different altitudes.

may now compare the calculated zero-altitude speed with the magnitude of the terminal speed given by a formula, which assumes constant atmospheric density and constant gravitational field strength:

$$v_t = \sqrt{\frac{2mg}{CA\rho}} \tag{17}$$

If one uses the values corresponding to zero altitude in this formula, the magnitude of v_t is found to be 5.7 m/s. It is very interesting to note that almost the same value is obtained in our numerical calculations for zero altitude, even though we have used the Work-Energy Theorem.

One should note that a well-done numerical or analytical analysis never yields a time-independent or an altitude-independent terminal speed for a falling body. This fact is clearly seen in all our results, as the speed of the falling body depends strongly on the time/altitude.

Although we have found such a great consistence with the time interval between Kittinger's leap-out and the moment he opened the main parachute, our result for the total time of the parachutist's fall, 18 min 10 s, differs very much from the value given by Kittinger [1] [2], 13 min 45 s. If we believe in the latter value, we may calculate the average vertical velocity at which Kittinger descended after having opened the main canopy parachute:

$$v_{ave} = \frac{5500\ \text{m}}{(825 - 293)\text{s}} = 10.3\ \text{m/s}$$

In our opinion, this is an unreasonably high speed for a person who should meet the ground safely with a total mass of 142 kg. So, we suppose the data given by Kittinger immediately after his jump is incorrect in this sense. It is interesting to note, that the time we have found for the last part of his fall, with the main parachute open, is almost exactly the same as the time Kittinger has reported for the total time of the whole fall. Would it be possible that these two times have been mixed in the thrill of his heroic high-altitude jump?

It has been reported [1] [2] that Kittinger reached a maximum speed of 614 mi/h, or even 714 mi/h, during his fall [3] [4]. According to our results (**Figure 4**), his speed increased only up to 204 m/s, which corresponds to 457 mi/h. So, our result is only about 64%~74% of the value reported for Kittinger's maximum speed. At the moment, we want to believe in our numerical predictions, and hence we conclude that Kittinger's speed was not even close to the speed of sound during his fall from 31,330 metres.

Our numerical data shows that Kittinger reached the maximum speed at an altitude of 26.6 km (**Figure 4**). Kittinger himself has reported [1] [2], that he hit a peak speed at 27.4 km. The difference of 800 m is relatively small—it corresponds to a time interval of less than 4 seconds.

If **Figure 5**, we have plotted Kittinger's calculated vertical acceleration as a function of his altitude. The point at which the stabilization parachute is opened is clearly seen, as well as the point corresponding to the opening of the main parachute. The values begin at 9.70 m/s², and the minimum acceleration of −2.2 m/s² is reached at an altitude of 23.1 km.

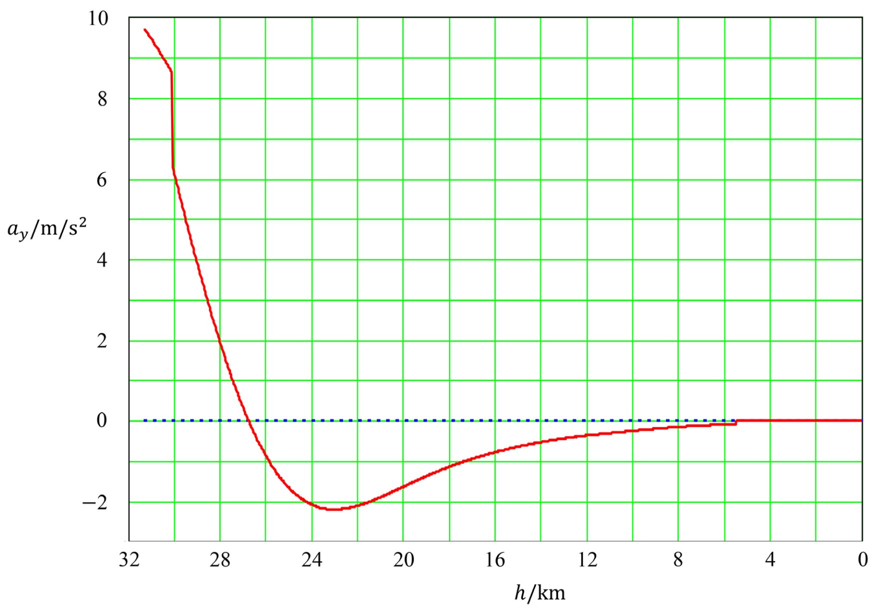

Figure 5. The vertical component of Captain Kittinger's acceleration vs. altitude in his high-altitude jump from 31.3 km.

5. Faster Than Sound; The Jump of Felix Baumgartner

We repeated the calculations for Felix Baumgartner's extremely-high parachute jump, which he made on October 14th 2012. He bailed out from his gondola during a live video stream at an altitude of 39.0 km. In contrast to Joseph Kittinger, Baumgartner did not use a stabilization canopy parachute, which made his jump extremely dangerous and life-threatening [14] [15].

In our calculations, some values were changed so that the data would correspond to Baumgartner's jump. Firstly, we used an estimated value of 110 kg for the total mass of the falling object, and we used 15˚C for the ground-level temperature of air. Secondly, the frontal area of the person was kept 0.60 m^2 until the main parachute was opened at 1.5 km. All the other variables were kept the same as in Kittinger's jump.

The vertical velocity of Felix Baumgartner, according to our calculations, is shown in **Figure 6**. We found that his free fall lasted for 4 minutes and 17 seconds; Baumgartner's own team has reported that he was falling freely for 4 minutes and 16 seconds [15]. We found that his maximum speed was 1253 km/h (Mach 1.18), which is about 7% less than the value reported by Baumgartner's team, 1350 km/h (Mach 1.25) [15]. According to our numerical calculations, Baumgartner was falling at transonic speeds for 32 seconds, between altitudes of 33.4 km and 22.9 km. Baumgartner's team reported that his speed exceeded the speed of sound at about 30 km, and he was falling faster than the speed of sound for 30 seconds. Comparison between the calculated and the measured values gives relatively strong support to the model used in our calculations.

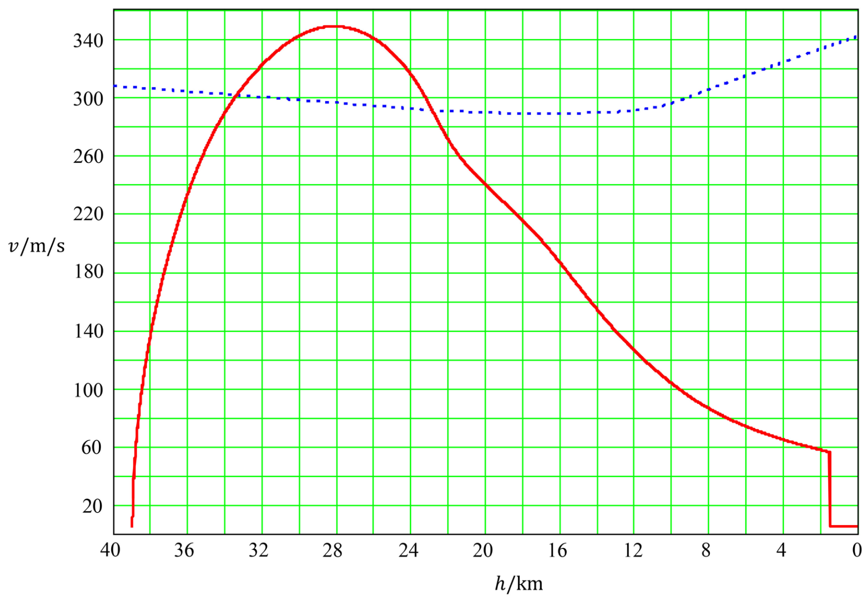

Figure 6. The velocity of Felix Baumgartner (solid line) as a function of altitude during his free fall from a record-high altitude of 39.0 km. The dashed line shows the speed of sound at different altitudes.

It is interesting to look at Felix Baumgartner's vertical acceleration vs. altitude, because he was falling without any stabilizing parachute. If one compares **Figure 5** for Kittinger with **Figure 7** for Baumgartner, one sees distinctly different behaviour between the two. The use of the stabilizing canopy parachute made Kittinger's fall much softer than that of Baumgartner. Actually, Baumgartner suffered from serious difficulties during his fall which is easy to understand by looking at **Figure 7**. When he was about to break the sound barrier (at about 33.4 km),

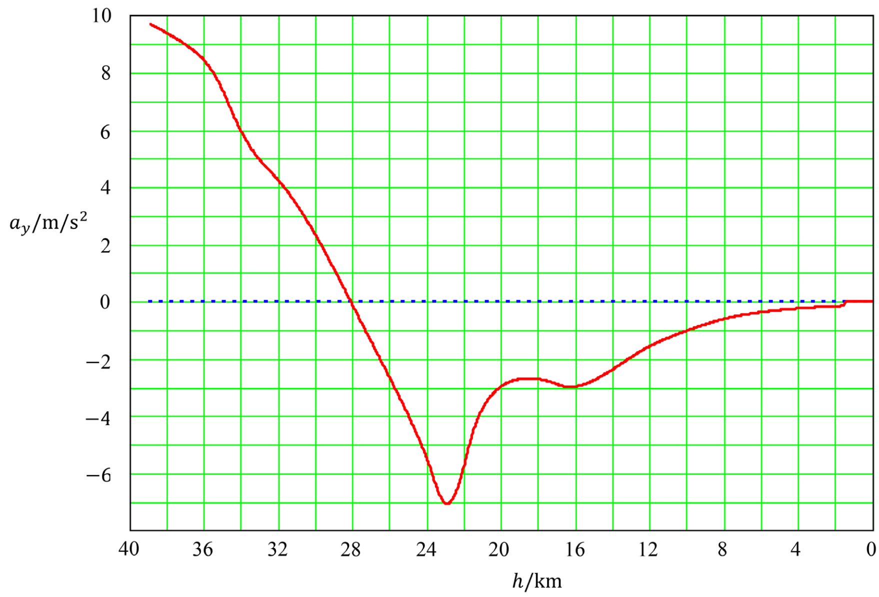

Figure 7. The vertical component of Felix Baumgartner's acceleration vs. altitude in his high-altitude jump from 39.0 km.

he started to undergo uncontrolled spinning, which might have developed fatal. Fortunately, this dangerous action ended at an altitude of about 19 km.

6. Conclusion

We have used the Work-Energy Theorem in our study on kinematics of very-high-altitude parachute jumps. Our approach is different from the earlier papers based on numerical analysis [16] or analytical calculations [17]. This paper gives a large number of numerical and, in our opinion, physically relevant data. Our analysis reveals that one person has experienced transonic speeds during his free fall. Had he used a stabilizing canopy parachute, the parachute might have behaved really dangerously at speeds close to Mach 1.0. According to **Figure 7** and the experience of Felix Baumgartner, the danger of serious entanglement seems to be highly possible at the moment when the falling body pierces through the sound barrier.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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