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Conversion of a belt-driven balancing system of a large-bore diesel engine to a gear-driven system

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Bachelor’s thesis | Abstract

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Conversion of a belt-driven balancing system of a large-bore diesel engine to a gear-driven system

This thesis was commissioned by Wärtsilä Finland Oy. The aim was to design a gear-driven balancing system to replace the belt-driven balancing system used in Wärtsilä's 46F and 46DF engines.

The design of the new gear-driven balancing system is a very extensive process and involves many different aspects. In this thesis, the topic is limited so that only the design and dimensioning of the gears and bearings were studied carefully. The gears were dimensioned according to the required transmission ratio and the strength of the gear wheels was examined using the KISSsoft simulation software. When selecting the appropriate bearing type, the load situation was considered and bearing solutions were taken into account in the corresponding balancing system of Wärtsilä's 46TS engine.

In addition to the theoretical calculation part, a 3D model of the new system was created, presenting the system as a whole at the concept level. The solutions made during the work are visible in the model and it serves as a starting point for further design of the system.

Keywords:

Gears, bearings, simulation, concept design

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Large bore -dieselmoottorin hihnavetoisen tasapainotusjärjestelmän muuntaminen hammaspyörävetoiseksi

Tämä opinnäytetyö on tehty toimeksiantona Wärtsilä Finland Oy:lle. Työn tavoitteena oli suunnitella hammaspyörävetoinen tasapainojärjestelmä Wärtsilän 46F- sekä 46DF -moottoreissa käytettävän hihnavetoisen tasapainotusjärjestelmä tilalle.

Uuden hammaspyörävetoisen tasapainotusjärjestelmän suunnittelu on hyvin laaja prosessi ja pitää sisällään monia eri osa-alueita. Aihe onkin rajattu siten, että työssä perehdyttiin tarkasti ainoastaan hammaspyörien sekä laakerien suunnitteluun ja mitoitukseen. Hammaspyörät mitoittiin vaaditun välityssuhteen mukaan ja pyörien lujuutta tarkasteltiin KISSsoft-simulointiohjelmalla. Sopivan laakerityypin valinnassa tarkasteltiin kuormitustilannetta ja otettiin huomioon laakeriratkaisut Wärtsilän 46TS-moottorin vastaavassa tasapainotusjärjestelmässä.

Teoreettisen laskentaosuuden lisäksi uudesta järjestelmästä luotiin 3D-malli, joka esittää järjestelmän kokonaisuudessaan konseptitasolla. Työn aikana tehdyt ratkaisut ovat näkyvillä mallissa ja se toimii lähtökohtana järjestelmän jatkosuunnittelulle.

Asiasanat:

Hammasvaihteet, laakerit, simulointi, konseptisuunnittelu

Appendices

Pictures

Tables

Used abbreviations and symbols

a	Acceleration
a_w	Center distance
a_r	Central acceleration
c	Tip clearance
D	Bearing outer diameter
d	Diameter
d_a	Tip diameter
d_b	Base circle
d_f	Root diameter
h	Whole depth
h_a	Addendum
h_f	Dedendum
i	Transmission ratio
J	Inertial moment
j	Backlash
j_n	Normal backlash
j_r	Central backlash
j_t	Circumferential backlash
j_θ	Angular backlash
L	Bearing length

M	Torque
m	Module
n	Rotational speed
P	Power
p_b	Bearing pressure
p	Circular pitch
p_{bt}	Base pitch
PV	PV value
SFS	Finnish Standards Association
S_F	Safety factor for bending strength
S_H	Safety factor for surface durability
U	Sliding speed
W_t	Section modulus
x	Addendum modification coefficient
z	Number of teeth
α	Pressure angle
ϵ_α	Transverse contact ratio
μ	Coefficient of friction
σ	Bending stress
σ_F	Actual tooth root bending stress
σ_{FG}	Permissible tooth root bending stress
σ_H	Tooth flank contact stress

σ_{HG}	Permissible tooth flank contact stress
ω	Angular velocity

1 Introduction

Power transmission from one shaft to another is one of the most essential aspects of mechanical engineering. For example, chains, different kinds of belts, and gears can be used to transmit power. Wärtsilä 46F and 46DF marine diesel engines with 12V, 14V and 16V cylinder configurations have a balancing system in place, with a toothed belt for power transmission. Balancing system is a device that neutralizes external moments that cause unwanted vibrations in the engine. However, the belt-driven system has proven unreliable, and the failure of the belt has caused problems in field use.

The purpose of this thesis is to design a gear-driven balancing system which could replace the current belt-driven system in Wärtsilä's 46F and 46DF large bore engines. The aim of the work is to design a functioning concept for a system whose detail-engineering would be possible in the future. The most significant requirements for the new system are easy installation to replace the old system, durability of gears and bearings, and maintaining the current level of balancing.

When transferring power from one rotary shaft to another, several different types of machine parts are usually involved, and each component is subject to different loads and stresses. The design of a complete power transmission system is a wide-ranging entity that includes several factors that affect in a different way. In terms of theoretical calculation, this thesis work focuses only on the design of gears and bearings. The lubrication of gears and bearings, the exact dimensioning of shafts, or the choice of materials are not considered in this work.

First, the existing theory of engine balancing and current balancing solutions in different engine models are examined. The actual design work will begin by getting acquainted with the gear geometry, as well as the different bearing types and their properties. After that, the dimensioning and selection of gears and bearings will begin. The work culminates when a 3D model is created based on the theoretical part, presenting the new balancing system as a whole at the

concept level. 3D-model includes the concept design of gears, bearings, shafts and the cover box. The cover box is a casing, inside which the balancing system operates.

Literature and standards of the field, engine product guides and some web pages and articles have been used as source. Some potential sources had to be excluded as they were based on different standards than ones used in this thesis.

2 Balancing system

2.1 Wärtsilä 46F and 46DF

Balancing system discussed in this thesis is used in Wärtsilä 46F and 46DF large bore engines. They are medium speed four-stroke diesel engines and typically used as the prime mover on cargo and passenger ships. They both can be installed for diesel electric propulsion and direct drive main engine applications. (Wärtsilä 2024.)

Wärtsilä 46F can be run on heavy fuel oil (HFO) or marine diesel oil (MDO). Also, light diesel can be used when being operated within strict coastal or port emissions areas. The 46DF design is based on the 46F which was already released in 2004. In addition to HFO and MDO, 46DF can also be run on natural gas. The switch between HFO/MDO and gas fuel operation can be done without loss of power or speed. (Wärtsilä 2024.) Table 1 shows different cylinder configurations and technical specifications of both engines.

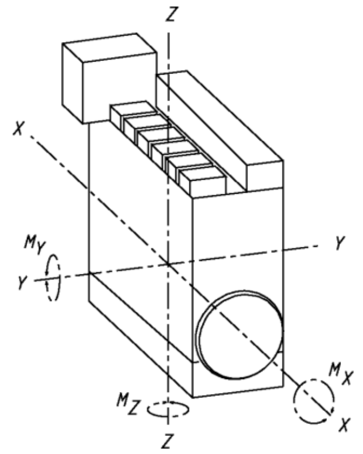
Table 1. Wärtsilä 46F/DF technical specification.

	Wärtsilä 46F	Wärtsilä 46DF
Speed	600 rpm	600 rpm
Cylinder output	1200 kW/cyl	1145 kW/cyl
Cylinder bore	460 mm	460 mm
Piston stroke	580 mm	580 mm
Available cylinder configurations	Inline engines: 6L, 7L, 8L, 9L, V engines: 12V, 14V, 16V	6L, 7L, 8L, 9L, 12V, 14V, 16V

2.2 Engine balancing

The crank mechanism of the internal combustion engine transmits the motion of the piston to the crankshaft. Due to the periodicity of the working method, mass forces are then generated due to the acceleration and deceleration phases of the masses moving back and forth. These forces have effects that can be divided into external and internal effects. The external effects of free forces and moments cause engine movements that are transmitted as vibration to the engine support structures. Depending on the frequency, the vibration affects the running vibrations as well as the running noise. Internal forces, on the other hand, cause varying loads on different parts of the engine, which should be considered in the design of these parts. (Horst 2003, 436.)

External effects are caused by forces and couples. These effects are formed by the revolving and reciprocating mass forces of the crank mechanism. Piston, piston pin and upper part of connecting rod create vertical variating force. Crankshaft, lower part of connecting rod and counterweights create rotating force. Horizontal and vertical forces are resultants of these forces. Mass moments can occur in multicylinder engines when the mass forces of the entire crank mechanism form a couple of forces (Horst 2003, 436—437). Picture 1 shows the main coordinates of mass forces in large bore diesel engine. External forces can be present in the direction of the z-axis and y-axis. External moments can be present around the z-axis and y-axis (My, Mz) and torque variation can occur in the x-axis (My).



Picture 1. External couples in Wärtsilä 46F and DF engines (Wärtsilä 2019).

External forces and moments are dynamic, and they can be classified by comparing their amplitude to the engine speed. The frequency of the first-order forces is the same as the engine speed. The frequency of the second-order forces is twice the engine speed, the third-order frequency is triple, and so on. In general, only the forces of the first and second order matter and the forces of the higher order are not considered. (Horst 2003, 437.) The external forces and moments present vary according to the number of cylinders. Picture 1 shows the external forces and moments in Wärtsilä 46F and 46DF engines.

Table 2. External forces and couples in Wärtsilä 46F and DF engines (Wärtsilä 2019).

Table 16-1 External forces

Engine	Speed [rpm]	Frequency [Hz]	F_Y [kN]	F_Z [kN]
8L46F	600	40	–	12.3

– forces are zero or insignificant

Table 16-2 External couples

Engine	Speed [rpm]	Frequency [Hz]	M_Y [kNm]	M_Z [kNm]	Frequency [Hz]	M_Y [kNm]	M_Z [kNm]	Frequency [Hz]	M_Y [kNm]	M_Z [kNm]
7L46F	600	10	63	63	20	104.2	– [1]	40	12.4	–
9L46F	**600**	**10**	**30**	**30**	**20**	**163**	**–**	**40**	**11**	**–**
14V46F	**600**	**10**	**103**	**103**	**20**	**155**	**86**	**40**	**5**	**13**
14V46F[2]	**600**	**10**	**–**	**–**	**20**	**155**	**86**	**40**	**5**	**13**

[1] zero or insignificant value marked as "-"
[2] balancing device adopted

When the engine is used in ship, the vibrations and noises are transmitted along engine foundation to the ship hull. As the passenger comfort plays a vital role in cruise ship industry as well as in cargo ships, it is clear that reducing vibrations is an essential part of engine design.

Balancing is a way to reduce engine running vibrations caused by the low frequency vibrations. It is done by neutralizing external forces and moments caused by the revolving and reciprocating masses. Revolving mass forces can be fully balanced by adding a counterweight rotating at the same speed, which has the opposite effect. The full balancing of the reciprocating forces can only take place by means of mass systems rotating in opposite directions. (Horst 2003, 439.) When talking about multi-cylinder engines, consideration should also be given to their natural balancing. With some engines, depending on the type of structure and ignition system, it is possible that the mass forces of the different cylinders balance each other. (Horst 2003, 441.) External moments therefore do not occur on all multi-cylinder configurations.

In large bore diesel engines, counterweights in crankshaft and separate masses attached to rotating components like flywheel and intermediate gear are used to achieve the wanted level of balancing. In addition, balancing masses can be placed on their own separate shafts.

The balancing system discussed in this thesis is a device that runs a separate balancing axis. It is mounted on the pump cover located at the free end of the engine. The pump cover is an engine part to which cooling water pumps and lubricating oil pump are attached. There are also routes for cooling water inside the pump housing.

As seen from the table 2, external moments in Wärtsilä 46F and DF are only present in 7L, 9L and 14V configurations. As mentioned before, only first and second order moments are relevant. However, when looking at ship hull vibrations, the frequency of 20 hertz is not in the same range as the most harmful specific frequencies. Balancing of the second order moments would

require masses that rotate at twice the engine speed and the construction of these kind of systems would be complicated.

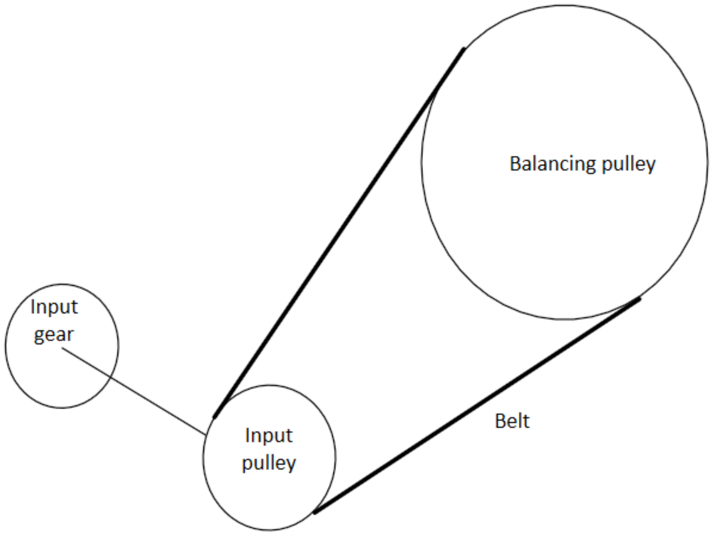
In Wärtsilä 46F and DF, only the external moments of the first order are balanced. The balancing system discussed is one of the components involved in balancing the first order moments in the engine. As seen from the table 2, the external moments are not present in the 12V and 16V configurations. However, when parts and tolerances are large, residual forces and moments may occur. Also, different crank stars and firing orders can affect the engine balance. Some customers want to achieve the maximum possible level of balancing and therefore the balancing system is also available for the 12V and 16V engines.

2.3 Existing solutions

2.3.1 Wärtsilä 46F/DF

Picture 2 shows an illustration of the Wärtsilä 46F/DF balancing system. Crankshaft is connected to balancing unit with spur gear, and it is called the input gear in the picture. Inside the balancing system, rotation between the input pulley and the balancing pulley is transmitted with belt drive. So only two shafts are included in the system and toothed belt connects them. Input shaft connects the balancing system to the crankshaft and transmits the rotation to the system. A pulley that rotates the belt is attached to one end of the shaft and the other end of the shaft is connected to the crankshaft with a spur gear. Balancing weights rotate on the balancing shaft. The belt and the balancing weights are attached to rotating pulley that is significantly bigger than the pulley on the input shaft.

The whole system operates inside a cover box and the balancing shaft is attached to the back of the cover box. The cover box itself is attached to the engine pump cover by bolts.



Picture 2. Illustration of the Wärtsilä 46F/DF balancing system.

Both shafts have two bearings. Input shaft has one spherical roller bearing at the input gear end and one ball bearing at the belt drive end. Balancing shaft has one spherical roller bearing on both sides of the rotating pulley. The input shaft has the gearwheels fixed into it and the shaft and gears rotate in one piece. The bearings supporting the input shaft are mounted to a bearing housing which is bolted to the cover box. Unlike input shaft, the balancing shaft is fixed to the covering box and the balancing gear rotates around it.

The balancing masses are mounted to the balancing gearwheel that has a shaped groove on which the weights settle. In addition to the tuneable masses used in 12V and 16V, the 14V engine has one bigger extra mass mounted on the other side of the balancing wheel. Balancing masses rotate in same speed and in opposite direction with the engine and therefore the total transmission ratio of the balancing system is one. Principal data of the system is listed in table 3.

Table 3. Principal data of balancing system for W46F/DF.

Balancing system W46F/DF	
Total transmission ratio	1
Transmission ratio (crankshaft – input gear)	0,2325
Transmission ratio (input shaft – balancing shaft)	4,3
Reference diameter input shaft pulley (mm)	101,86
Reference diameter balancing shaft pulley (mm)	438
Center distance (mm)	543

2.3.2 Wärtsilä 46TS

Unlike the 46F and 46DF engines, the Wärtsilä 46TS has a two-stage turbocharging system. It has cylinder output of 1300 kW/cyl which is slightly higher than in 46F and 46DF. The Wärtsilä 46TS has balancing system that uses gearing to transmit rotation from crankshaft to balancing gear. This system gives a lot of useful information to be used as reference data when re-designing the balancing system for the W46F/DF.

In the 46TS balancing system, the power from crankshaft to balancing shaft is transmitted with three shafts and four gearwheels. Basic principle is similar to the 46F/DF but instead of the belt, there is one additional gear wheel that transmits the rotation between the input shaft and the balancing shaft. Input shaft transmits power from crankshaft to balancing device itself and it consists of two gearwheels. One gear wheel transmits the rotation from the crankshaft, and another gear called the input gear transmits the rotation to the balancing system. Intermediate gear transmits power from input shaft to balancing shaft with which the balancing mass rotates.

In addition to drivetrain type, the main difference between 46TS balancing system and 46F/DF system is that 46TS uses plain bearings instead of roller bearings. Two types of bearings are used: journal bearings that receive radial

loads and thrust bearings that don’t take any radial loads but work as guiding elements. The balancing shaft and intermediate shaft are fixed into position and gears rotate around them and as in the 46F/DF system, the input shaft has the gears fixed into it and they rotate as one piece. Unlike 46F/DF system, the covering box of the 46TS system doesn’t have back panel and balancing and intermediate shafts are mounted straight to the pump cover.

One notable difference compared to the 46F/DF system is that the balancing masses rotate separately from the balancing gear, and they are mounted on a separate fixing element. Unlike in 46F/DF, the balancing shaft is supported from both ends. The 14-cylinder version of the 46TS engine also has one bigger extra mass mounted on the backside of the fixing element. The covering box is made by casting whereas in 46F/DF the box is welded from steel sheet.

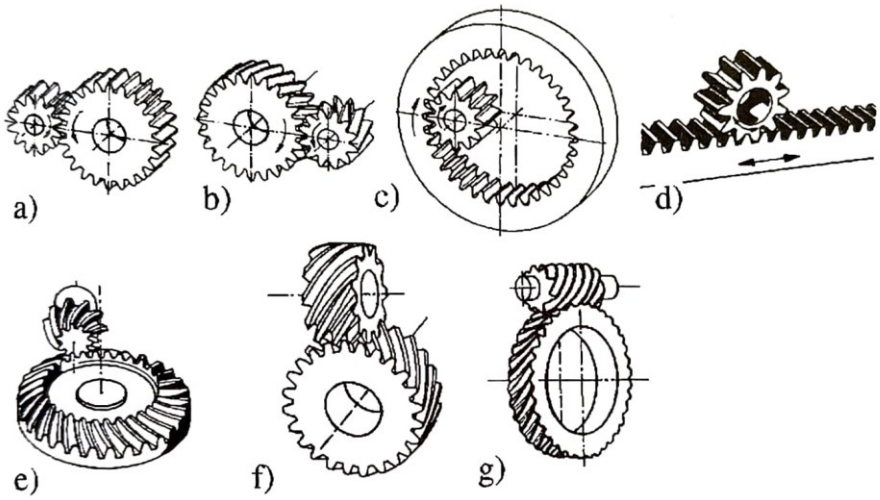
Transmission ratio between the input gear and the balancing gear is significantly smaller than in the 46F/DF system as the difference in size between the input gear and the balancing gear is much smaller. The total transmission ratio between crankshaft and the balancing gear is naturally 1 as the system is balancing the first order moment.

3 Gears

Gearwheels are used to transfer rotation and power from one shaft to another and at the same time, maintain a definite ratio between the velocities of the rotating shafts. Teeth of the driving gear push on the driven gear teeth transmitting torque and as the gear rotates, power is transferred. (Hamrock etc. 1999, 615.)

Gears are most rugged and durable compared to other torque transmitters and they can reach power transmission efficiency of 98% (Juvinall & Marshek 1999, 619). Disadvantage of gears is that they are usually more expensive than other power transmitters like chain drives and belt drives (Hamrock etc. 1999, 615).

Gears can be divided into different classes based on their way of function. Different classes are for example parallel-axis gears, nonparallel coplanar gears and nonparallel noncoplanar gears. (Hamrock etc. 1999, 615, 617.)



Picture 3. Gear types (Björk etc. 2014, 329).

Picture 3 shows different types of gears.

- Sections a-d: Parallel-axis
- Section e: nonparallel, coplanar
- Sections f-g: nonparallel, noncoplanar

The gearwheel itself can be either spur gear or helical gear. Picture 3 section a shows parallel-axis spur gear drive with teeth parallel to the cylinder axis. Spur gears are the most common and also the simplest gear type. (Hamrock etc. 1999, 615.)

Section b shows a helical gear drive. Unlike spur gears, the teeth in helical gears are not parallel to the axis but they have been cut on a spiral that wraps around a cylinder. Helical gears have some advantages compared to the spur gears. Helical gear teeth have smoother running than spur gear teeth because their oblique teeth enter the meshing zone progressively. Helical gears are also often quieter than spur gears. Another significant advantage is that helical gears can transmit larger loads than spur gears which means that gear life is also longer for the same load. (Hamrock etc. 1999, 615, 617.)

The main disadvantage of helical gears is that they generate an axial thrust force, and this end thrust might require additional components like thrust collars or ball bearings (Hamrock etc. 1999, 617). Because of the more complex structure, helical gears are also more expensive to manufacture than spur gears.

The tooth shape and size of gears is highly standardized. The American Gear Manufacturers Association (AGMA) publishes standards for English units (Hamrock etc. 1999, 615). For the SI units standards are published by International Organization for Standardization (ISO). ISO standards are now on used in this thesis work.

3.1 Spur gear geometry

Functions of a pair of gears are now on described using spur gear (picture 3 section a) as an example. Driving gear is marked with subscript 1 and driven gear is marked with subscript 2.

Transmission ratio of a pair of gears is constant and it is calculated with following formula:

$$i = \frac{\omega_1}{\omega_2} = \frac{n_1}{n_2} = \frac{d_2}{d_1} = \frac{z_2}{z_1} \tag{1}$$

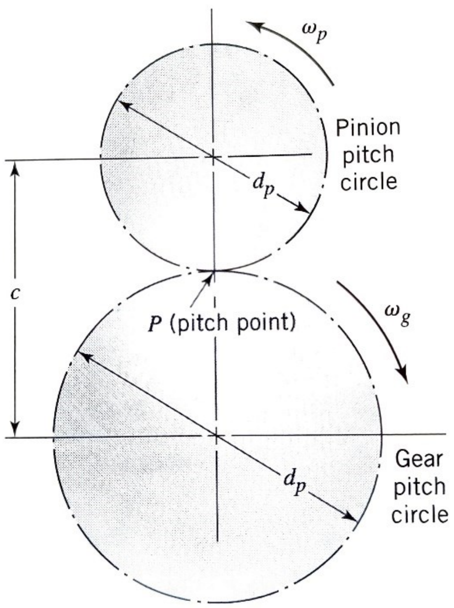
where

ω = angular velocity (rad/s)

n = rotational speed (1/s)

i = transmission ratio

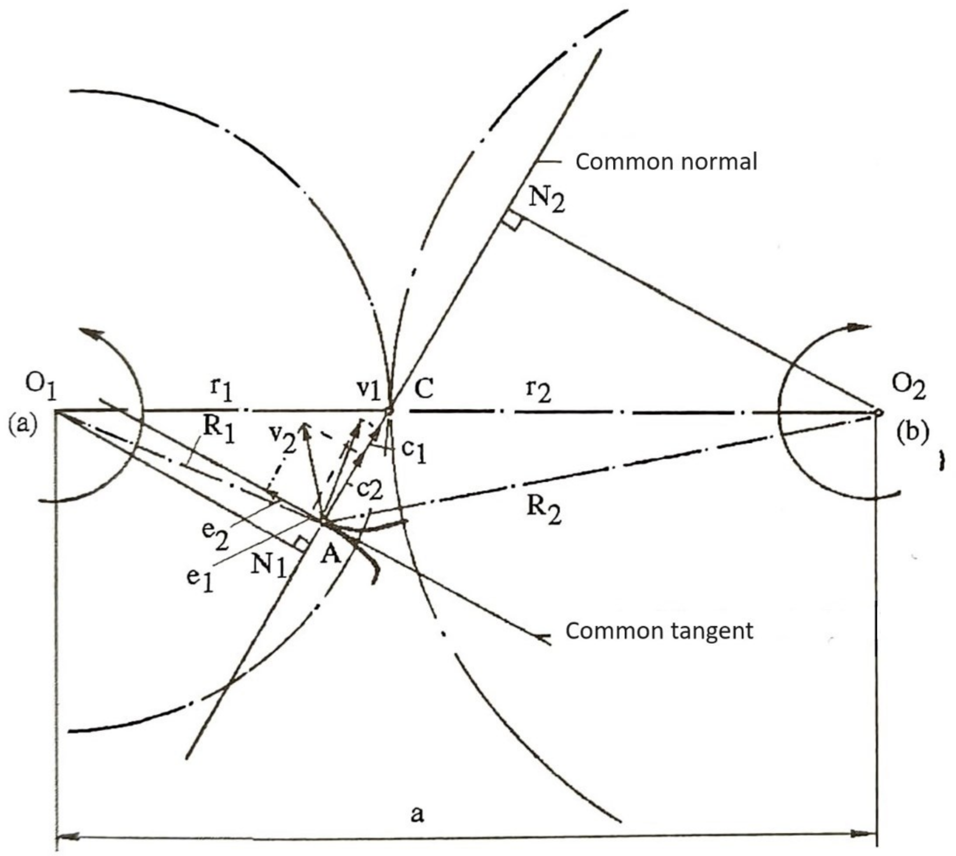
A pair of gears is usually imagined as cylinders pressed together and rolling against each other without slippage as seen in picture 4. Cylinder bores are called pitch circles, and they touch each other in point C (pitch point).



Picture 4. Pair of gears (Juvinall & Marshek 1999, 622).

3.1.1 The law of conjugate gear-tooth

In picture 5, tooth surface of the driving gear touches the tooth surface of the driven gear in point A. This point is called the contact point and in this point the surfaces of the teeth own the same tangent and normal. Velocity of the point A in driving gear is v_1 and in driven gear v_2. Vector v_1 is perpendicular to segment O_1A and vector v_2 is perpendicular to segment O_2A.



Picture 5. Touching of the tooth surfaces (Björk etc. 2014, 331).

These vectors can be divided into normal directional components (c_1 and c_2) and tangent directional components (e_1 and e_2). When gear teeth don't go through each other nor separate, the normal directional components must be equal. Expressions for the normal directional components are obtained from similar triangles O_1N_1C and O_2N_2C as follows (Björk etc. 2014, 330):

$$c_1 = \omega_1 O_1 N_1 \tag{2}$$

$$c_2 = \omega_2 O_2 N_2 \tag{3}$$

As mentioned previously $c_1 = c_2$ of which:

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N_2}{O_1 N_1} = i \tag{4}$$

From triangles O_1N_1A and O_2N_2C can be obtained:

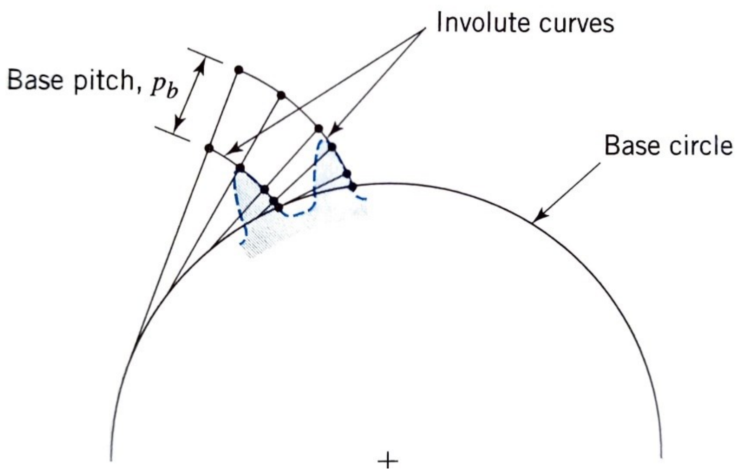
$$\frac{O_2 N_2}{O_1 N_1} = \frac{O_2 C}{O_1 C} = \frac{r_2}{r_1} = i \tag{5}$$

The law of conjugate gear tooth can be obtained from the previous conclusions. Blom etc. (1995, 260) formulate the matter as follows: “The lateral normal set through the contact point of the side of the two teeth touching each other should always pass through the lateral point of the rolling circles (Point C).”

3.1.2 Involute gear profile

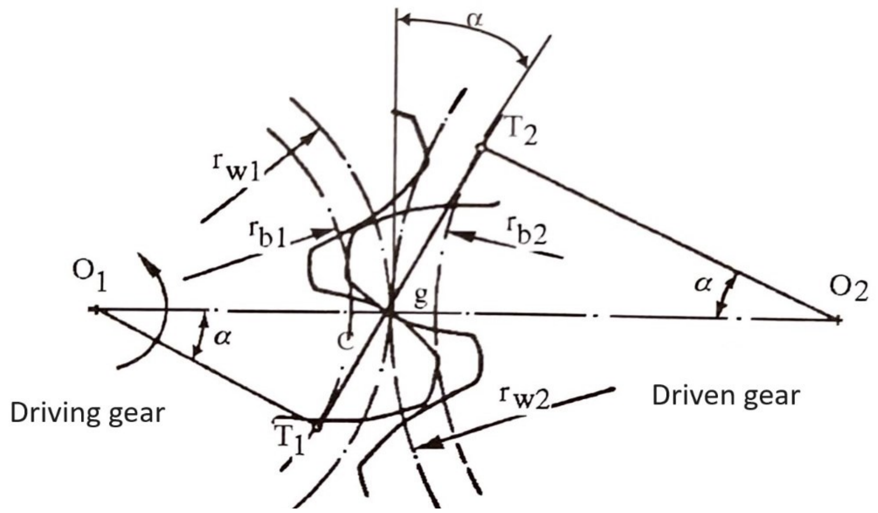
There are various tooth shapes that can implement the law of conjugate gear-tooth (Juvinall & Marshek 1999, 621). This kind of tooth shapes are for example cycloidal shape, Wildhaber-Novikov toothing and involute gear profile. Due to easy manufacturing, only one with current importance is the involute gear profile. (Blom etc. 1995, 260.)

Involute gear tooth proifile is defined by an involute of a circle. The involute is formed by a line that rolls on the surface of a circle (Björk etc. 2014, 332). As can be seen in the picture 6, every endpoint of this line draws the involute curve.



Picture 6. Generation of an involute from its base circle (Juvinall & Marshek 1999, 621).

An essential part when looking at the involute gears is the line of action. The line of action is obtained when gears are rolled against each other, and touching point of the gear teeth is illustrated (picture 7). Angle between the line of action and tangent drawn in point C is pressure angle (α) and the most common value for the pressure angle is 20°. (Björk etc. 2014, 332.)

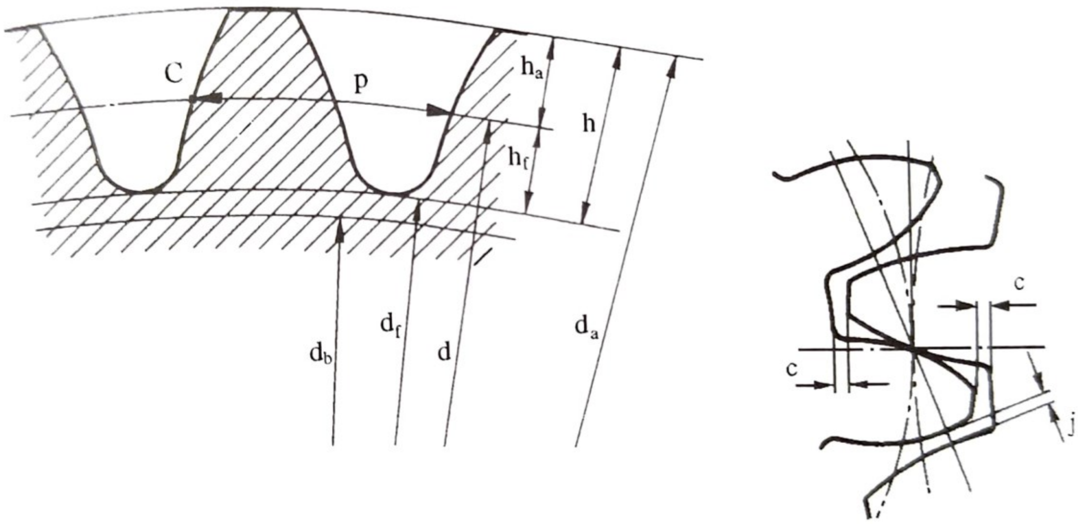


Picture 7. The line of action (Björk etc. 2014).

When the line of action touches the base circles(d_b) of both gears in points T_1 and T_2, radius is $r_b = r \cos\alpha$. This means that the involute curve is only dependent on the addendum circle. Therefore, involute's biggest advantage compared for example to cycloidal shape is that centre distance of a gear pair can be modified without changing transmission ratio.

Only one involute curve can be obtained for each addendum circle. The involute gear tooth profile is obtained regardless of the opposite wheel, so two involute gears always mesh with each other if they have same pressure angle and pitch (p).

Picture 8 shows the principal measurements of the involute gear: circular pitch (p), tip diameter (d_a), reference diameter (d), base circle (d_b), root diameter (d_f), whole depth (h), addendum (h_a) and dedendum (h_f). Also backlash (j) and tip clearance (c) are shown in case of two gears in mesh.



Picture 8. Principal measurements of the involute gear.

When number of teeth in gear is z, follows that (Björk etc. 2014, 333):

$$zp = \pi d \tag{6}$$

It can be transformed to:

$$\frac{p}{\pi} = \frac{d}{z} \tag{7}$$

This obtained constant is called the module(m) and now reference diameter (d) and circular pitch (p) can be calculated as follows (Blom etc. 1995, 259):

$$d = mz \tag{8}$$

$$p = m\pi \tag{9}$$

Module is an auxiliary variable that describes the size of gear tooth and unit for the module is millimetre. With help of module, using pi in calculations can be avoided. (Blom etc. 1995, 259.) Module is used for manufacturing, and it defines the milling blade. Modules are standardized to limit the number of different blades. (Björk etc. 2014, 332.) With SI-units the valid standard is SFS-ISO 54 (SFS = Finnish Standards Association).

3.1.3 Transverse contact ratio

Small size of gearwheel or gear assembly is better from an economic point of view and therefore the number of teeth is recommended to be kept reasonably small. When minimizing the number of teeth in involute gear, the limiting factors are transverse contact ratio and formation of undercut. (Blom etc. 1995, 265.)

Transverse contact ratio is the relation between length of path of contact and circular pitch. It describes the average number of teeth that participate mesh at the same time. It can be calculated as follows (Blom etc. 1995, 265):

$$\epsilon_{\alpha} = \frac{g}{p * cos\alpha} = \frac{\sqrt{{d_{a1}}^2 - {d_{b1}}^2} + \sqrt{{d_{a2}}^2 - {d_{b2}}^2} - 2a_w * sin\alpha_w}{2p_{bt}} \tag{10}$$

where

d_a = tip diameter

d_b = base circle

a_w = center distance

α_w = pressure angle

p_{bt} = base pitch = p*cosα

To ensure that a pair of gears runs smoothy, the transverse contact ratio must be at least 1,1 or bigger (Björk etc. 2014, 332).

When manufacturing a gear wheel with small number of teeth using hobbing, the milling blade must be pushed further inside the base circle. This creates the undercut to the tooth flank which decreases the strength of the tooth. Limiting number of teeth for teeth without undercut can be obtained from the following formula (Blom etc. 1995, 266):

$$z_2 = \frac{{z_1}^2 * (sin\alpha)^2 - 4}{4 - 2 * z_1 * (sin\alpha)^2} \tag{11}$$

If $z_1 = z_2$ and α = 20° equation gives the minimum number of teeth $z_1 = z_2$ = 12,3. Typically the minimum number of teeth z_{1min} is 14 because standardized rack-type cutter has slightly rounded edge and also small undercut is usually allowed in practice. (Blom etc. 1995, 266.)

3.1.4 Centre distance and addendum modification

Centre distance of external spur gear manufactured without addendum modification is called a and it can be calculated as follows (Björk etc. 2014, 335):

$$a = \frac{m(z_1 + z_2)}{2} \tag{12}$$

where

m = module

z = number of teeth

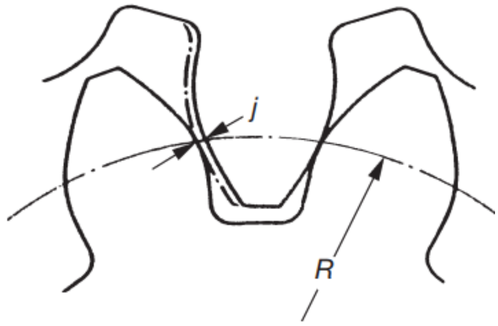
As mentioned before, reference diameter is d = mz, so module and number of teeth determine the centre distance. Modules are standardized with certain staggering and number of teeth must be integer. Desired transmission ratio can be reached with many different numbers of teeth but reaching pre-determined or standardized wheelbase is almost impossible. Therefore, addendum modification is usually needed to reach the correct centre distance. Addendum modification is also used to improve tooth strength by thickening the base of the tooth. (Björk etc. 2014, 332.)

Addendum modification is performed during the manufacturing by pushing the milling blade away from the centre of blank which creates positive modification or towards the centre which creates negative modification. Amount of addendum modification is coefficient x. Dimensional number x*m indicates how much the blade profile used for manufacturing has to be moved to create positive modification (+x*m) or negative modification (-x*m). (Blom etc. 1995, 266—267.) Positive addendum modification thickens the base of the tooth and thins the tip of the tooth. In negative modification the base of the tooth becomes thinner, and tip thickens.

The choice of the addendum modification coefficient is influenced by loading capacity of tooth root and flank, transverse contact ratio, teeth sliding, and avoiding undercut or sharpening of the tooth (Björk etc. 2014, 336). Generation of the coefficient is determined in SFS-ISO 21771.

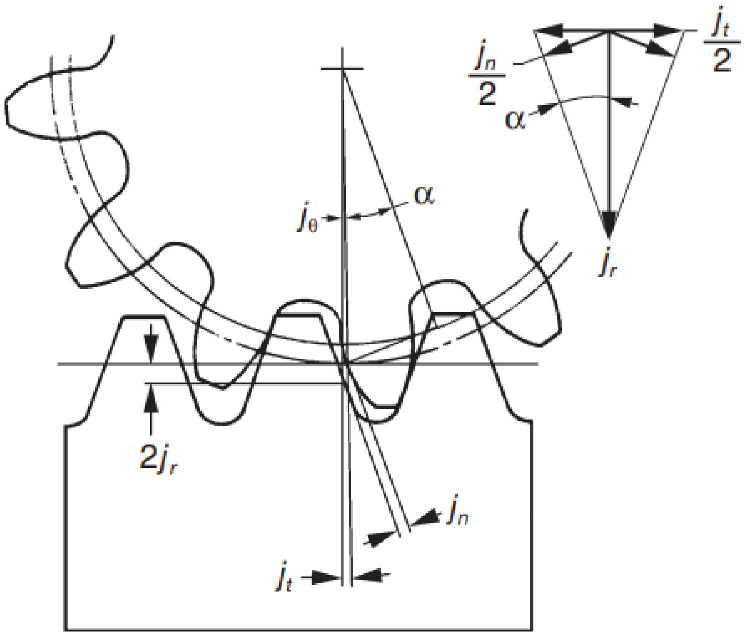
3.1.5 Tooth modifications

If the tooth thickness measured on the pitch circle is exactly one-half of the circular pitch, a pair of gears will mesh perfectly without clearance. However, due to unavoidable inaccuracies, there must be some kind of clearance between the tooth flanks to prevent gears from jamming. (Hamrock etc. 1999, 632.) This clearance is called backlash. Picture 9 shows backlash (j) between two gears.



Picture 9. Backlash between two gears.

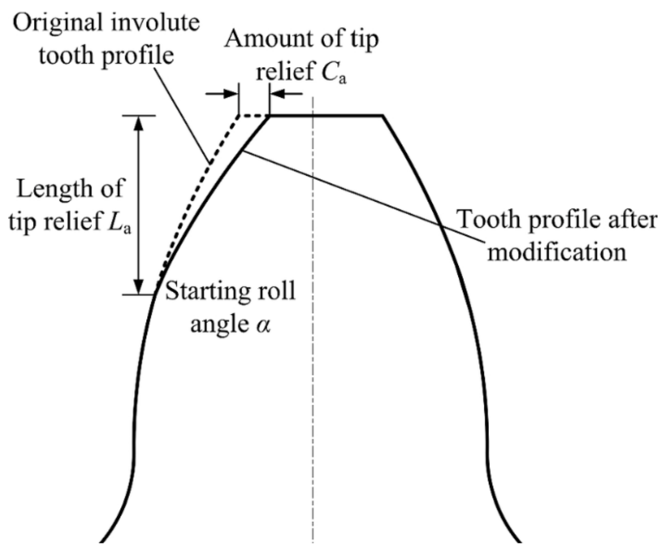
Factors that affect the amount of backlash can be inaccuracies in machining, runout of both gears, errors in profile, pitch, tooth thickness, and centre distance. Backlash can be generated by decreasing the tooth thickness or increasing the centre distance. To determine backlash in gear train, the backlash of each mated gear pair must be summed up. However, the gear ratio of each mesh relative to a chosen reference shaft in the gear train must be considered to obtain the total backlash for a series of meshes. (Quality Transmission Components 2015.)



Picture 10. Kinds of backlash (Quality Transmission Components 2015).

Picture 10 shows different kinds of backlash: circumferential backlash (j_t), normal backlash (j_n), central backlash (j_r), and angular backlash or backlash angle (j_Θ). Generation of these different backlashes is determined in SFS-ISO 21771.

If gear teeth are deformed under load conditions, tooth meshing can become premature. Tip relief presented in picture 11 can be used to make meshing smoother. It prevents premature tooth meshing and corresponding unwanted effects on the load distribution in gear teeth while increasing the tooth flank load carrying capacity. (Konowalczyk etc. 2016, 64.)



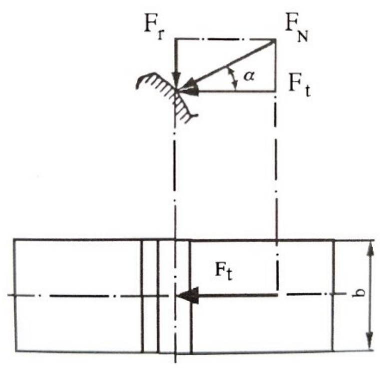
Picture 11. Tip relief (Chen etc. 2021).

As seen from the picture 11, practically tip relief means removing material from the tip of the tooth.

3.2 Loads and strength

3.2.1 Gear forces

When two gearwheels work as a pair, the driving gear causes mesh load F_N to driven gear (Björk etc. 2014, 343). As shown in picture 12, force F_N is perpendicular to tooth flank.



Picture 12. Gear forces of spur gear.

Power P is transmitted by a pair of gears and therefore the torque of the driving gear can be presented as follows (Björk etc. 2014, 343):

$$M_{v1} = \frac{P}{\omega_1} = F_N r_{w1} cos\alpha \quad (13)$$

And the torque of the driven gear as follows (Björk etc. 2014, 343):

$$M_{v2} = \frac{P}{\omega_2} = F_N r_{w2} cos\alpha \quad (14)$$

where

$r_w = d_w/2$

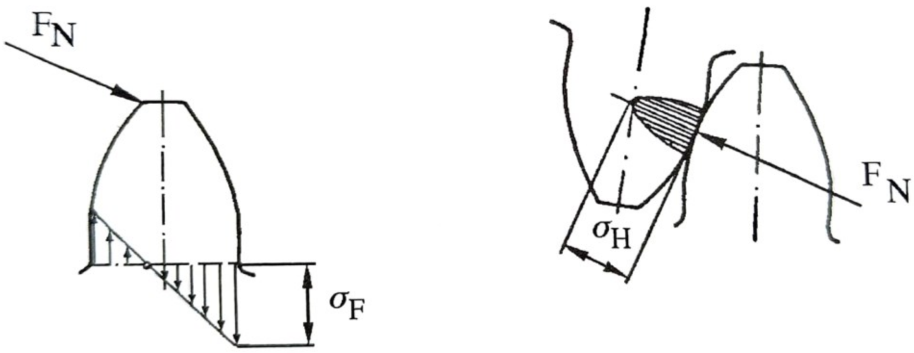
ω = angular speed (rad/s)

Force F_N can be divided into peripherical force F_t and radial force F_r as follows:

$$F_r = F_N sin\alpha \quad (15)$$

$$F_r = F_N sin\alpha \quad (16)$$

As seen in Picture 13, force F_N causes tooth root stress σ_F and contact stress σ_H.



Picture 13. Stresses acting in gear tooth.

3.2.2 Strength calculations

Calculation of gear strength is nowadays based on a statistical estimate of the occurrence of tooth damage. Blom etc. (1995, 272) present different aspects that are considered in most advanced calculation methods:

- Endurance failure (the bending strength of the base of the tooth in terms of fatigue)
- Pitting (fatigue of the side surface caused by contact)
- Abrasion wear (surface slip wear at low circumferential speeds caused by the lack of lubrication)
- Cold seizing (surface elongation caused by high pressure and low speed
- Hot seizing (welding of contact surfaces together in high speeds)
- Micro pitting (the emergence of small surface cracks on the rolling surface)

When speaking of gears, the hot seizing is also often called scuffing.

In most cases it is enough to examine tooth root stress σ_F and contact stress σ_H to ensure the durability of a gear pair (Blom etc. 1995, 272). The valid ISO standard for spur gear strength is ISO-6336. Standard provides formulas and basic principles for calculating the load capacity of spur and helical gears. It considers different failure modes such as pitting, tooth root breakage, tooth flank fracture, scuffing and micropitting. (SFS-ISO 6336-1, 8.)

SFS-ISO 6336-2 presents the equations for calculating the contact stress σ_H. The equations use many different values and coefficients. The calculation is influenced by the nominal tangential load, teeth facewidth, reference diameter and gear ratio. In addition, a wide range of different factors are used. These factors consider for example the contact between gear teeth in different ways, the uneven load distribution, and the specific properties of the used materials.

SFS-ISO 6336-3 presents the equations for calculating the tooth root stress σ_F. This calculation is also affected by a large number of different values and factors. Here, the nominal tangential load, teeth facewidth, and the module are considered. Used factors consider for example the uneven load distribution in different ways, load increments due to dynamic effects and variations in torque, and the specific properties of the used materials.

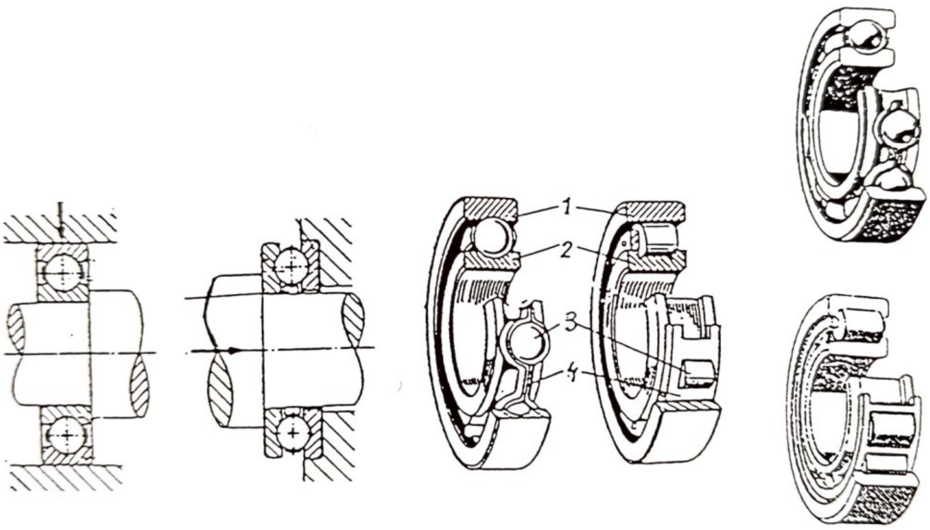
4 Bearings

Bearings are machine elements that support and control other moving parts. Controlled movement can be either rotating or twisting back and forth. Bearings are categorized according to their structure into rolling-element bearings and plain bearings. They can also be divided into radial and axial bearings according to the load direction. (Björk etc. 2014, 274.)

4.1 Rolling-element bearings

Rolling-element bearings are standardized machine elements that are ready for installation (Björk etc. 2014, 296). In rolling-element bearings, balls or rollers are used to separate the sliding surfaces. Contact areas in rolling parts are small and stresses high. Therefore, the loaded parts are usually made of high-strength materials that are superior to parts that bearing is attached to. Significant advantage of rolling-element bearings is low starting friction, and they are ideal for usage where starting loads are high. (Juvinall & Marshek 1999, 587.)

The structure of the rolling-element bearings includes several different parts. Inner race and outer race form the bearing body and receive the forces from the axis and hub. Rolling elements transmit the forces between inner and outer races and a cage or separator keeps the rolling elements in correct position and maintains even spacing. (Blom etc. 1995, 117.)



Picture 14. Structure of rolling-element bearing.

Typical construction and different parts can be seen in picture 14. Sections 1 and 2 present the outer and inner race. Section 3 presents the rolling element which can be either a ball or roller and section 4 presents the separator.

4.2 Plain bearings

Instead of rolling elements like balls or rollers, loads in plain bearings are applied to lubricant film between shaft and the bearing. (Björk etc. 2014, 274.) Plain bearings require less space than rolling-element bearings and they are often used in applications where space is limited (Blom etc. 1995, 170). System with plain bearings can also be more cost-effective to implement than system with rolling-element bearings if the plain bearings are produced in large series (Wuolijoki 1972, 380).

Plain bearings are also called journal bearings and they can support either axial or radial loads. Bearings supporting radial loads are called journal or sleeve bearings and they are cylindrical in shape. Thrust bearings are generally flat and support axial loads. (Juvinall & Marshek 1999, 540.)

Björk etc. (2014, 307) present classification for plain bearings according to their way of function. Different bearings are classified as follows:

1. Non-lubricated bearings
2. Hydrodynamic bearings
3. Hydrostatic bearings
4. Self-lubricating bearings

Non-lubricated bearings are the simplest solution. They don’t need maintenance and are usually cheap. With simple and lightweight structure, they are suitable for certain applications. Disadvantage is that their load bearing capacity is small, and they are not suitable for high speeds. (Björk etc. 2014, 307.)

In hydrodynamic bearing the lubricant film completely separates the surfaces. Fluid pressure supporting the load that is pressing the surfaces together is generated by the rotating motion of the journal. As surfaces don’t touch, there is no surface wear and friction losses appear only within the lubricant film. (Juvinall & Marshek 1999, 541.) During starting lubrication in hydrodynamic bearings is only partial and the surfaces can touch each other (Björk etc. 2014, 307).

In hydrostatic bearing the lubricant film is created with external pressure. Separation of surfaces can therefore be achieved without relative motion between the sliding surfaces. This makes it possible to have low friction also during starting and low-speed operation. Disadvantages of hydrodynamic bearing are cost and bulky structure. (Juvinall & Marshek 1999, 541—542.)

Self-lubricating bearings are porous metal bearings that are filled with lubricants like oil, graphite, molybdenum, or plastic. They are manufactured by sintering and material is typically bronze, iron or aluminum. When filled with oil self-lubricating bearings work like hydrodynamic bearings. (Björk etc. 2014, 307.)

4.3 Bearing type comparison

Selection between different bearing types may be encountered during the design process of bearing in different machine-elements. Factors affecting on the selection are for example applied loads, need for space, temperature, organization of maintenance and expenses. Major factors that determine the usability of different bearings are the applied load and the speed of rotation. Hydrodynamic plain bearings can operate at highest speeds and loads. Rolling-element bearings can operate at slightly lower speeds and loads but the operating range of non-lubricated and self-lubricated plain bearings is much smaller compared to the hydrodynamic bearings. (Björk etc. 2014, 274, 277.)

In most cases the selection has to be made between rolling-element and plain bearings (Björk etc. 2014, 274). Wuolijoki (1972, 380) presents a comparison of the main characteristics of plain and rolling-element bearings where the advantages and disadvantages of rolling-element bearings are compared to the same features in plain bearings. The comparison can be seen in table 4.

Table 4. Comparison between rolling-element- and plain bearings.

Rolling-element bearings	Plain bearings
Advantages	**Disadvantages**
Easy start-up due to minimal friction	Difficult start-up, high static friction
Low lubricant consumption	High lubricant consumption
Bearing is precise and clearance small	Rather large clearance
Requires a little space in axial direction	Rather narrow
Spare parts immediately available	-
Low maintenance	-
Disadvantages	**Advantages**
Relatively expensive	Cost-effective in a large series
Installation requires precision	-
Requires threading on the axis	Easy fitting
Noisy running	Silent running
Requires rather much space in radial direction	Requires a little space in radial direction
Tolerates shocks poorly	Good dynamic durability and damping

5 Dimensioning of gears

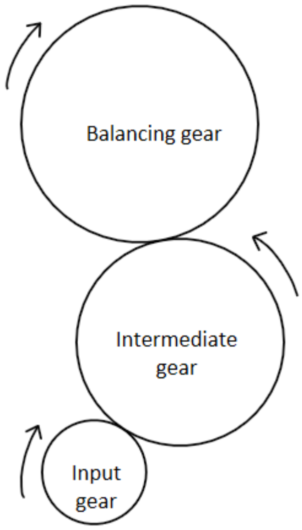
Design process of the new gear-driven balancing system starts with dimensioning of gears. Picture 15 shows the progress of the design process. After defining the initial parameters for the gears, transmitted torque is calculated, and a strength analysis is made to ensure the durability of the designed gears and to determine the forces affecting the system.

Picture 15. The design process of the gears.

5.1 Initial dimensions

Dimensioning of gears starts with outlining the structure of the geartrain and certain limitations must be taken into account from the beginning. Naturally, the total transmission ratio must remain unchanged. The location of the balancing gear and the input ear can’t be changed. Balancing mass works as desired only at the current location and moving the mass would require re-designing of the masses. Input gear is connected to the crankshaft, and it has a machined place at the pump cover so moving it would not be expedient. In addition, screws that connect the cover box to the pump cover limit the placement of any new shafts.

With previous limitations the simplest way to transmit the rotation from the input shaft to the balancing shaft is to add an intermediate gearwheel between these shafts. Picture 16 shows a simplified illustration of the new gear-driven system with the intermediate gear. The intermediate gear would locate to the right of the middle line between the balancing shaft and the input shaft. At this location the overall size of the system would not increase and there should be enough space for the attachment of the shaft in the bottom of the cover box.



Picture 16. Illustration of the new geartrain.

Dimensioning of the gearwheels starts with finding suitable numbers of teeth that implement the desired transmission ratio. If design of the new geartrain is carried out with three gearwheels it is only necessary to examine the ratio between the first and the last wheel as follows.

Total transmission ratio of a three-wheel geartrain based on equation 1 is:

$$i_{total} = \frac{z_2}{z_1} * \frac{z_3}{z_2} = \frac{z_3}{z_1}$$

Pairs of numbers between 10 and 129 that implement the ratio of 4,3:

- 10 and 43
- 20 and 86
- 30 and 129

Reference diameter (d) of the gear is obtained by multiplying the number of teeth by a module. SFS-ISO 54 defines the normal modules for spur gears for general and heavy use and they can be seen in Table 5. Standard recommends to primarily use the modules from series 1 and avoid using modules from series 2.

Table 5. Normal modules (SFS-ISO 54).

Series	
I	**II**
1	1,125
1,25	1,375
1,5	1,75
2	2,25
2,5	2,75
3	3,5
4	4,5
5	5,5
6	(6,5)
8	7
10	9
12	11
16	14
20	18
25	22
32	28
40	36
50	45

Table 6 shows different combinations of modules and numbers of teeth that give suitable values for the reference diameters.

Table 6. Options for gear dimensions.

		Reference diameter Input gear (mm)	**Reference diameter Balancing gear (mm)**
Option 1	Module 5, numbers of teeth 20-86	100	430
Option 2	Module 6, numbers of teeth 20-86	120	516
Option 3	Module 8, numbers of teeth 10-43	80	344

In option 3, the reference diameters are valid but the number of teeth in input gear is unsuitable in principle. With the number of teeth being 10, it is very likely that undercutting occurs. Options 1 and 2 are both usable. Option 2 gives relatively big reference diameter for the balancing gear, and it would increase the size of the whole system.

At this stage, the attachment of the weights to the balancing shaft should be taken into account. In W46TS, the balancing masses rotate separately from the gearwheel and they are attached to separate fixing element. This kind of arrangement makes the shaft significantly longer and it needs to be supported from both ends. In this case, the end of the axle in which the weights are located is connected to hub in the cover of the system.

In the current belt drive system, the balancing masses are attached straight to the rotating pulley, and it would be convenient to use this solution in the new design as well. If the masses are fixed to the gear the construction of the whole balancing shaft would be much simpler than with a separate fixing element for the masses.

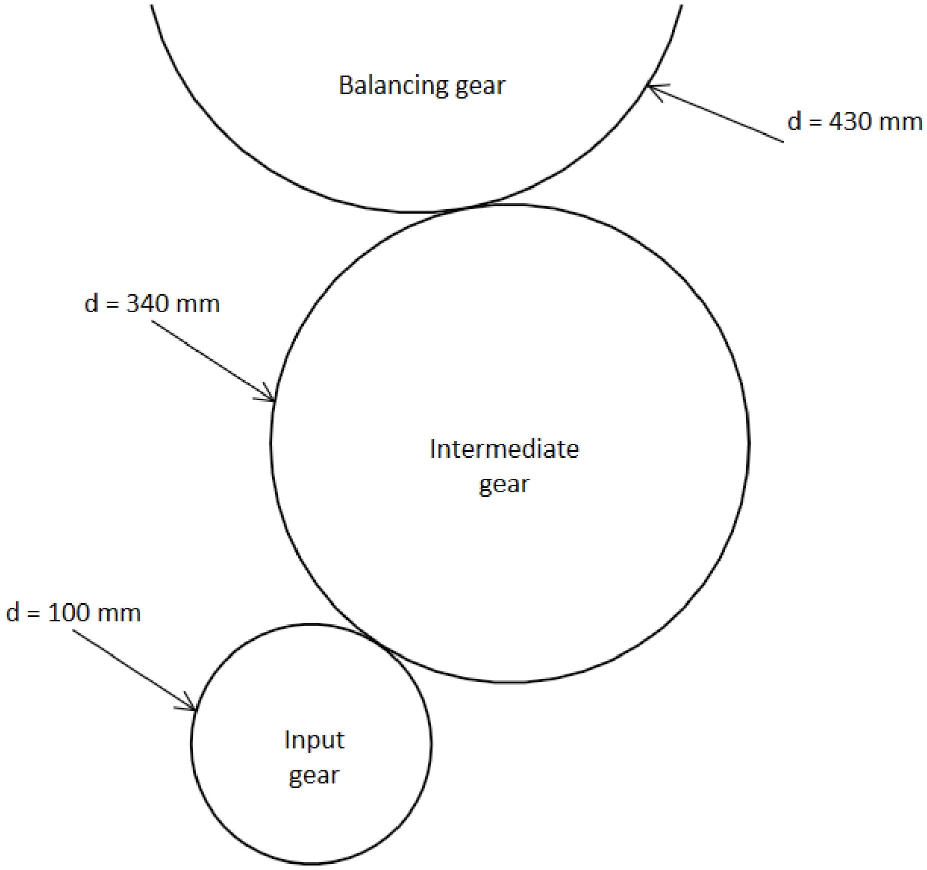
It is possible to attach the balancing masses to the balancing gear in both options 1 and 2 but the practical implementation of the mortgage would have a small difference. The reference diameter for the balancing gear in option 1 is 430 mm and diameter of the outer edge of the balancing masses is 448 mm. Therefore, the lower surface of the masses cannot rest on the surface of the gearwheel as there might be a risk that the masses would collide with the intermediate gear as the system rotates. To ensure safe running the masses need to be raised upwards from the surface level of the gearwheel. The bottom of the balancing masses is not flat, and they need a groove for attachment. To move the masses further from the gearwheel surface, suitable shoulders are needed to support them. This kind of construction would increase the weight of the gearwheel as more material is needed.

In option 2, the bigger diameter of the balancing wheel would make it possible to attach the balancing masses to the same level as the wheel surface. The

needed groove could be immersed to the gearwheel which would make the wheel lighter.

Both options 1 and 2 seem to be usable at this point. Module 5 is rather small and for example the 46 TS system uses module 6. On the other hand, the 46TS has heavier balancing masses which increases the loads acting on the system. However, the option 1 with the module 5 produces a smaller diameter for the balancing wheel and thus the whole system is smaller. As there are no obstacles to using smaller module at this point, option 1 is used from now on. All the upcoming calculations will be made according to the data of the option 1. The results will determine whether the module is too small and needs to be changed.

The number of teeth for the intermediate gear is decided based on the suitable reference diameter. The reference diameter must touch the diameters of the input and balancing gears. Also, the gear must locate in a such place that is possible to mount it to the covering box. Number of teeth 68 gives the reference diameter of 340 mm and this diameter implements the required conditions. Picture 17 shows an illustration of the location of the intermediate gear. Input gear and balancing gear are located at their fixed positions and the space needed for the intermediate shaft is taken into account.



Picture 17. Reference diameters with module 5.

5.2 Strength analysis

5.2.1 Transmitted torque

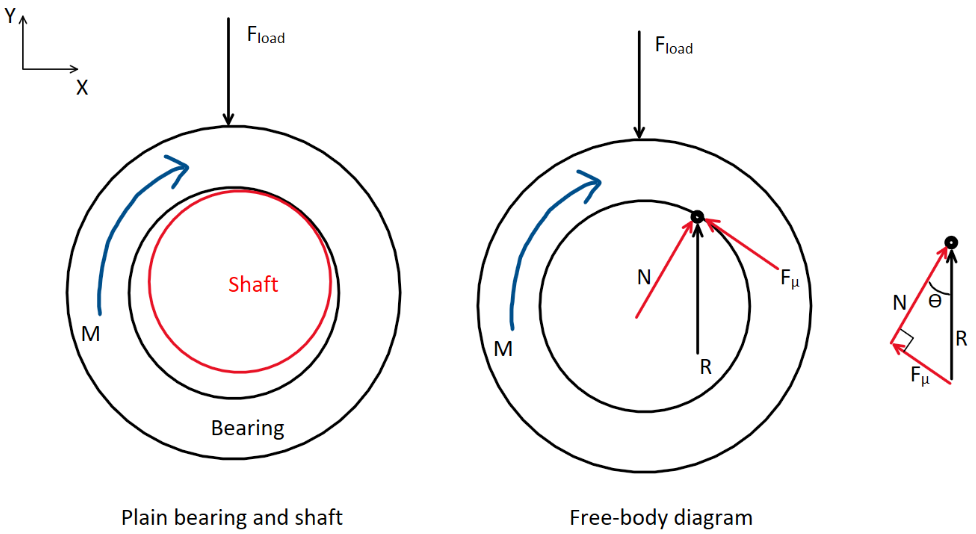
Before analysing the strength of the geartrain, the transmitted torque must be known. In this case, torque is needed to rotate the balancing masses on the balancing shaft. Significant torques might occur in two different scenarios.

If hydrodynamic plain bearings are used in the system, starting friction will occur when the geartrain starts rotating. Hydrodynamic lubrication is dependent on the rotating speed of the bearing and during start-up, there is no lubricant film between the shaft and the bearing (Wuolijoki 1972, 331—332). Values for coefficient of friction between a steel shaft and different bearing materials without lubrication can be seen in table 7.

Table 7. Coefficient of friction between a steel shaft and different bearing materials (Wuolijoki 1972, 330).

Cast iron	0,17…0,18
Light metal	0,16…0,18
Tin bronze	0,16…0,18
Leaded bronze	0,11…0,13
Polytetrafluoroethylene	0,04…0,10
Polyethylene	0,3…0,5

As seen in the picture 18, a free-body diagram can be drawn to illustrate the start-up situation. The balancing shaft is fixed to the cover box and therefore a fixed shaft and a rotating bearing is illustrated.



Picture 18. Free-body diagram illustrating the start-up situation.

The friction is caused by the loads acting on the bearing (F_{load}). In this case, only the mass that rests on the bearing is considered in the calculations. In W46F/DF belt-drive system, bearing in balancing shaft supports the rotating pulley and the balancing masses so F_{load} is obtained by multiplying the component masses with gravitational acceleration g. The shaft radius used in calculations is obtained from the W46TS system.

When F_{load} is known, the equilibrium equation parallel to y-axis can now be observed and resultant force R can be obtained from it:

$$\sum F_y = 0 = R - F_{load} \tag{17}$$

Angle θ between R and normal force N is:

$$\theta = \tan^{-1} \mu \tag{18}$$

Friction force F_μ is:

$$F_\mu = R * \sin \theta \tag{19}$$

And moment caused by friction is:

$$M_f = F_\mu * r \tag{20}$$

where r is the shaft radius

Weight of the balancing masses is same in 12V and 16V engines but the 14V engine has one extra weight and also the rotating pulley in 14V is heavier. Calculated moment for friction in 12V and 16V is 4 Nm (when weight of the balancing masses and the pulley is 56,7 kg, shaft radius is 0,0425 m and coefficient of friction is 0,17) Calculated moment for friction in 14V is 8 Nm (when weight of the balancing masses and the pulley is 114,86 kg). The values are very small and have practically no effect on the strength of the teeth in the size range discussed at this thesis work.

Another significant torque occurs when speed of the system is accelerated. During the acceleration the moment of inertia against the rotational motion affects the amount of torque required. Required moment can be obtained from the following formula:

$$\sum M = J\mathrm{a} \tag{21}$$

where

J = Inertial moment (kgm^2)

a = Angular acceleration (rad/s^2)

Angular acceleration can be obtained as follows:

$$a = \frac{d\omega}{dt} \tag{22}$$

where

dω = change in angular velocity (rad/s)

dt = change in time (s)

Engine rotation speed can be converted to angular velocity with following equation:

$$\omega = 2\pi n \tag{23}$$

where

n = rotation speed (1/s)

Required torque is calculated in different scenarios of acceleration that are shown in table 8. Angular acceleration can is obtained from the change in rotation speed and change in time with equations presented earlier. Angular acceleration is multiplied by the inertial moment of the balancing masses.

Values in the table are calculated with the heavier pulley and with all the balancing masses including the extra mass like in the 14V engine. In this situation, the inertial moment is 60,45 kgm^2. Different scenarios for rotation

speed change present the possible situations during the engine operation and they were given by supervisor from Wärtsilä.

Table 8. Values for transmitted torque in different acceleration scenarios.

Change in rotation speed (rpm)	0-120	120-390	390-600	120-600
Change in time (s)	5	10	60	30
Torque (Nm)	152	171	22	101

5.2.2 KISSsoft calculation

Gear forces and strength of the designed system are analysed with KISSsoft. It is a software that offers a vast range of different features for analysing different machine elements and in this thesis work, the gear calculation features are used. These features include for example basic gear geometry calculation, load spectrum calculation, micropitting and scuffing calculations, gear sizing, profile modifications, FEM analysis and tooth contact analysis. Also, the gear geometry calculated with the software can be exported in different forms to be used for example for 3D modelling. (KISSsoft AG 2024.)

In this thesis work KISSsoft is used to perform the basic calculation of the geartrain. The basic calculation includes the calculation of the tooth geometry, general influence factors, tooth root load capacity, flank safety, scuffing load capacity, tooth thickness and profile modifications. Calculation method for strength can be defined and, in this case, ISO-6336 method B is used. Inside the basic calculation, particular attention has been paid to tooth root and flank strength and the forces acting in the system.

To perform calculations, a wide range of input data must be entered. The most relevant information in terms of strength evaluation to be inputted is the maximum torque acting in the geartrain. Required torques in different situations were calculated earlier and now the biggest moment is to be used. The highest value for the moment was obtained when rotation speed of the engine ws

accelerated from 120 rpm to 390 rpm. Therefore, the rotation speed of the balancing gear is set to 120 rpm in the KISSsoft calculation. To get realistic and safe results the moment is multiplied with a utilization factor. The value of the factor is 5 and it was given by thesis's client. This factor takes into account the possible vibrations of the driving component as well as the profile of the device being used. Factors are typically verified by measurement or by different simulations.

All the values inputted to KISSsoft can be seen from appendix 1. Before running the calculation, it is possible to determine addendum modification, tip relief and root relief for the gears. KISSsoft recommends suitable values based on other input data and these recommended values were used during this calculation. At this point, only the module, the number of teeth and required torque have been studied. Other input data like teeth width, gear material and different tolerances were taken from 46TS system. Required service life of the gears should be infinite in principle. Service life was therefore set to 10 000 000 hours.

The strength of the designed gears is evaluated with safety factors that are obtained from the tooth root load capacity calculation and flank safety calculation. Tooth root load capacity calculation gives a safety factor for bending strength (S_F) and flank safety calculation gives a safety factor for surface durability against pitting (S_H). Safety factors are calculated separately for each gearwheel.

Safety factor for bending strength or root safety factor S_F is calculated using the tooth root stress σ_F as follows: (SFS-ISO 6336-3:2020, 15)

$$S_F = \frac{\sigma_{FG}}{\sigma_F} \tag{24}$$

where

σ_{FG} is permissible tooth root bending stress

σ_F is actual tooth root bending stress

Safety factor for surface durability or flank safety factor S_H is calculated using the contact stress σ_H as follows: (SFS-ISO 6336-2:2020, 14)

$$S_H = \frac{\sigma_{HG}}{\sigma_H} \tag{25}$$

where

σ_{HG} is permissible tooth flank contact stress

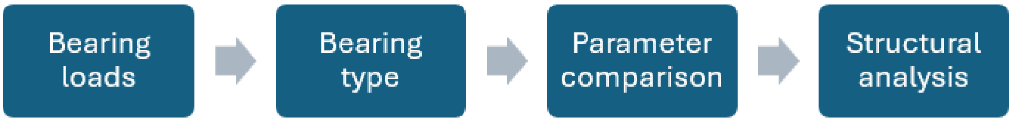
σ_H tooth flank contact stress

ISO-6336 doesn't propose exact values for the minimum safety factors but KISSsoft has certain values for the factors as a default: 1,4 for the root safety factor and 1 for the flank safety factor. These values are used as guidelines in this thesis work as well.

In the first calculation the flank safety factor for the input gear was 0,95 which is below the minimum value (1). To increase the surface durability bigger module could be chosen or teeth could be made wider. In the second calculation, module was kept same and teeth width was increased to 45 mm. This modification gave acceptable results as the flank safety factor for the input gear was 1,08.

6 Selection of bearings

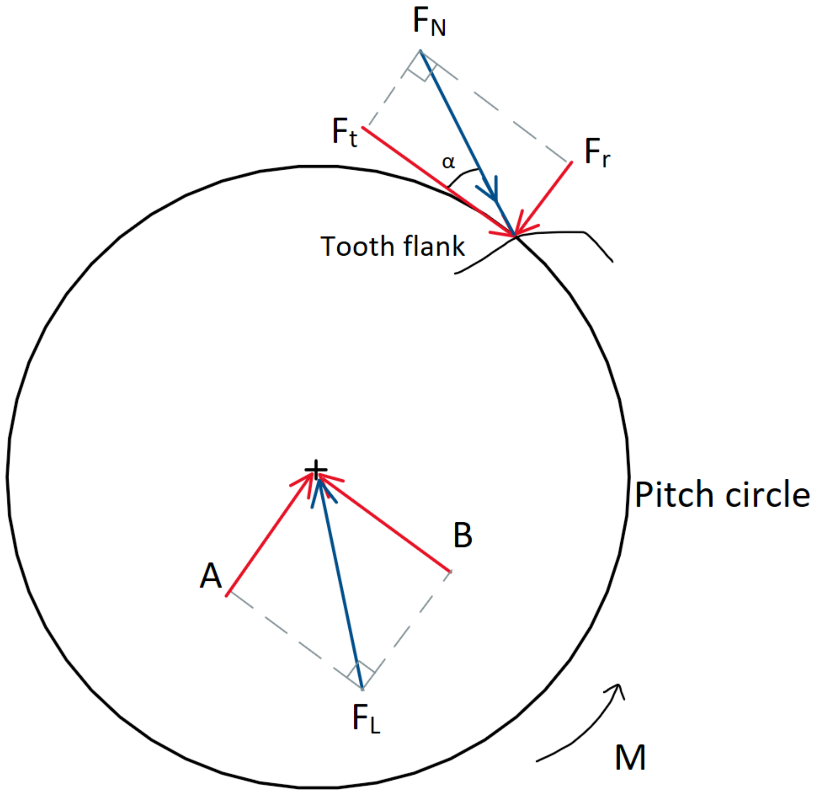
Picture 19 shows the progress of the bearing selection process. KISSsoft calculation gives the gear forces acting on the gears. The loads applied to bearings are calculated based on these forces. Suitable bearing type and dimensions for the bearings are selected based on the bearing loads and working conditions. The parameter comparison ensures that the selected bearings are suitable for the intended application. Practical implementation of bearing is studied in structural analysis.



Picture 19. Bearing selection process.

6.1 Bearing loads

Picture 20 shows the basic principle of bearing loads. When gear force F_N is applied to gear tooth, supporting forces A and B must be present to prevent the gear going through the bearing. Forces A and B are loading the bearing, and their resultant F_L is the final bearing load.



Picture 20. Basic principle of bearing loads.

The forces acting on each shaft of the system must be determined so that the bearings can be dimensioned correctly. Designed system has three shafts: input shaft, intermediate shaft, and balancing shaft. To carry out calculations, the preliminary construction of each shaft should be outlined. At this point, structure of the input and the balancing shaft is kept similar as in the original system. Intermediate shaft is constructed similarly to the 46TS system.

Intermediate shaft has the simplest construction as there is only one gear and bearing on the shaft and they both locate at the same point. Balancing shaft has similar construction to the intermediate shaft but there are also the balancing masses attached to the gearwheel. Input shaft has the most complicated construction as there are two gears on the shaft. Balancing shaft and intermediate shaft are attached to the cover box and gears rotate around them but on the input shaft gears are fixed to the shaft that rotates.

The gear forces are obtained from the KISSsoft calculation, and they can be seen in the table 9. Values for the input gear 1 are obtained from separate KISSsoft calculation which includes gear on the crankshaft and the input gear 1. In the case of spur gears axial forces are not generated.

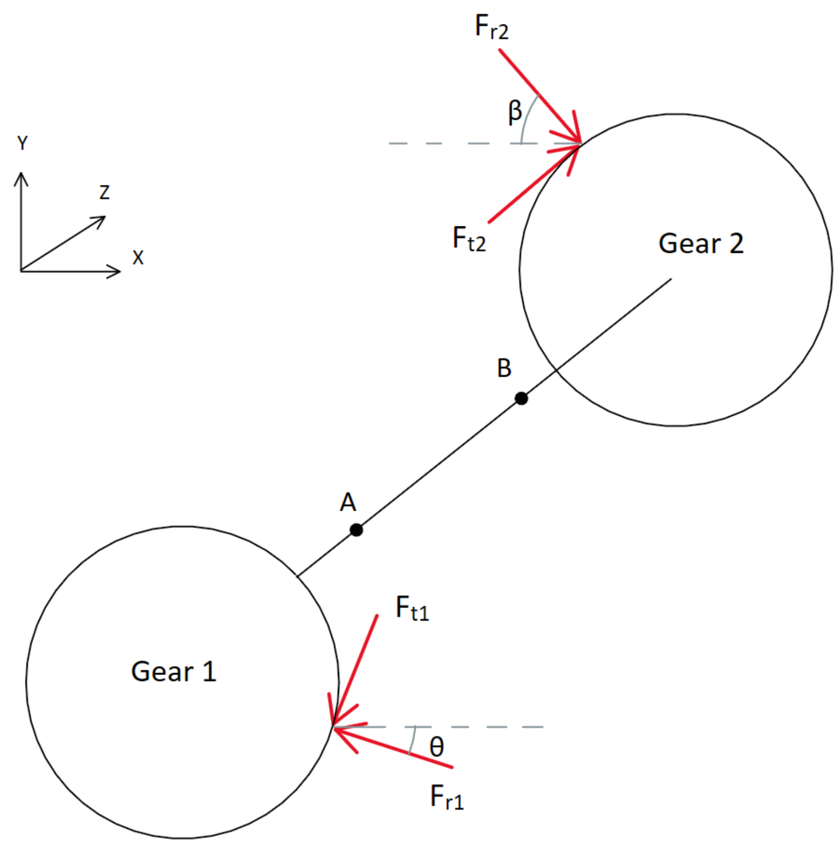
Table 9. Forces acting in gears.

	Input gear 1	Input gear 2	Intermediate gear	Balancing gear
Tangential force F_t (N)	2486	3977	3977	3977
Radial force F_r (N)	905	1447	1447	1447

To define the bearing loads, the shear forces acting on the bearings must be solved. In this case, there are both shear forces and bending moments affecting the shafts. Shear forces are perpendicular to the shaft and therefore essential for the job as they cause the load to the bearings. Total moment and total force equations are used to determine the affecting forces. The full calculations for each shaft can be seen in appendix 2.

6.1.1 Input shaft

Picture 21 shows the tangential and radial gear forces applied to the gears on input shaft. Number 1 indicates the gear rotating with the crankshaft and number 2 indicates the input gear that rotates with the intermediate gear. Points A and B indicate the locations of the bearings. Gear forces affect the gears at different angles. Therefore, the forces must be divided into X-directional and Y-directional components to calculate the shear forces applied to points A and B.



Picture 21. Gear forces acting on the input shaft.

Forces affecting gear 1 can be divided into X-directional components F_{t1x} and F_{r1x} and Y-directional components F_{t1y} and F_{r1y}. Correspondingly, the forces affecting gear 2 can be divided into X-directional components F_{t2x} and F_{r2x} and Y-directional components F_{t2y} and F_{r2y}.

The input shaft doesn't rotate around X-axis or Y-axis. Therefore, the total moment for X-axis and Y-axis must be zero. Shear forces in X-direction and in Y-direction at point A can be calculated with the total moment equation. When the total moment at point B is zero, following equation can be used to calculate shear forces A_x and A_y:

$$\sum M_B = 0 \tag{26}$$

The total forces in X-direction and Y-direction must also be zero. When A_x and A_y are known, following equation can be used to calculate shear forces B_x and B_y:

$$\sum F = 0 \tag{27}$$

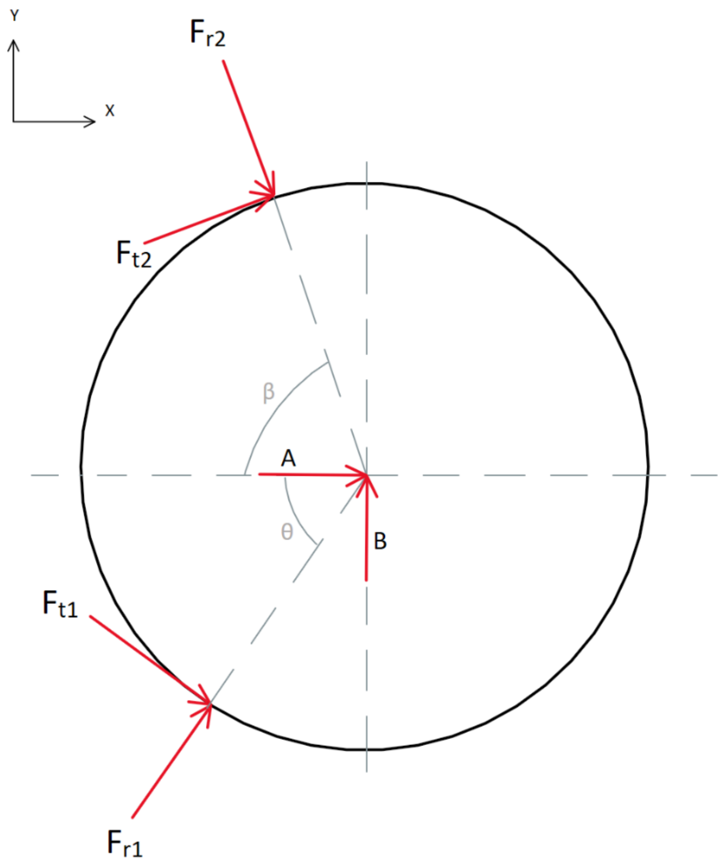
The final shear forces at points A and B are the resultants of the forces A_x and A_y and B_x and B_y. Resultant forces are obtained from the following equation:

$$F_R = \sqrt{{F_x}^2 + {F_y}^2} \tag{28}$$

Based on the basic principle of the bearing loads presented earlier, the bearing load applied to bearing at point A is 4844 N and the bearing load applied to bearing at point B is 6432 N.

6.1.2 Intermediate gear

The gearwheel and the bearing locate at the same point in the intermediate shaft. Therefore, it is not necessary to illustrate the whole shaft as the gear forces are transmitted to the bearing directly through the gear wheel. Picture 22 shows the tangential and radial gear forces applied to the intermediate gear as well as the supporting forces A and B.

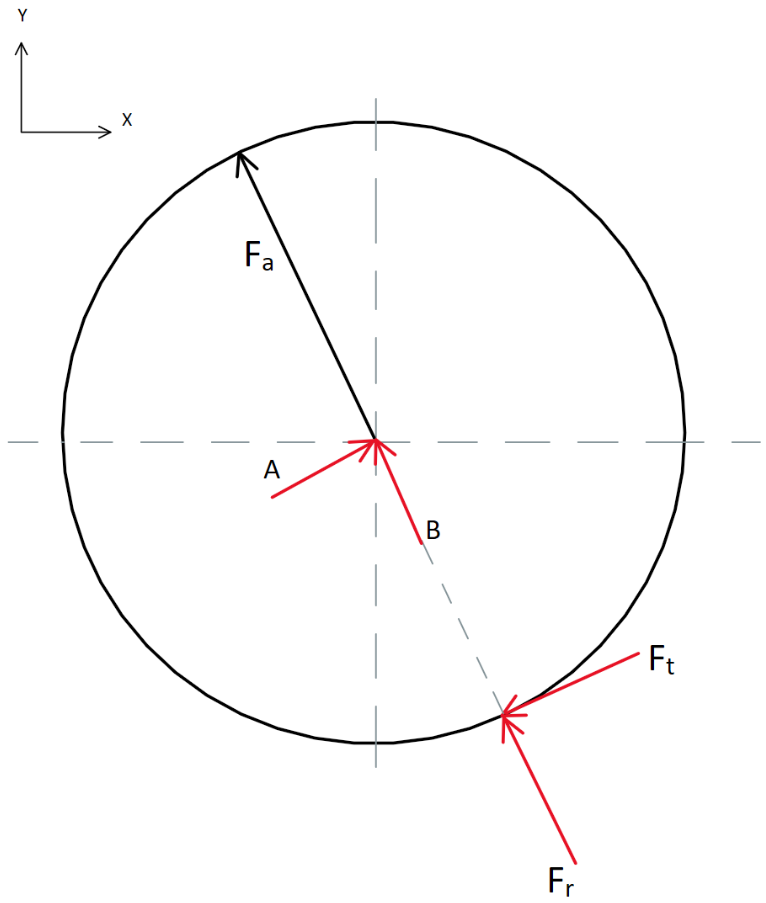


Picture 22. Gear forces applied to the intermediate gear.

Gear forces affect the intermediate gear at different angles, and they must be divided into X- and Y-directional components in same way as in the case of input shaft. Supporting forces and B can be obtained from the total force equations () parallel to the X axis and the y axis. The final bearing load is the resultant of the supporting forces A and B. Calculated value for the resultant is 8490 N.

6.1.3 Balancing gear

Bearing for the balancing gear is constructed similarly to the intermediate gear. However, the balancing masses fixed to the balancing gear wheel cause a force that must be considered when determining the bearing load. Picture 23 shows the forces applied to the balancing gear.



Picture 23. Forces applied to the balancing gear. Maximum load is applied when forces F_a and F_r are parallel.

F_r is the radial gear force, F_t is the tangential gear force, and F_a is the force caused by the rotating balancing masses. As the balancing masses rotate, the direction of the force F_a is constantly changing. The highest load is applied to the bearing when force F_a is parallel to the radial gear force F_r. A simple way to calculate the force F_a is to make use of the central acceleration of the balancing mass as follows:

The central acceleration is:

$$a_r = \omega^2 * r \tag{29}$$

where

ω = Angular velocity (rad/s)

r = Distance between the center of gravity of the balancing masses and the center of the shaft (m)

Maximum torque (171 Nm) that the balancing system transmits, occurs at the engine speed of 120 rpm. The gear forces obtained from the KISSsoft calculation, are therefore calculated with this particular speed. To calculate the combined effect of the forces F_a and F_r, the value for the F_a must be calculated with the same speed.

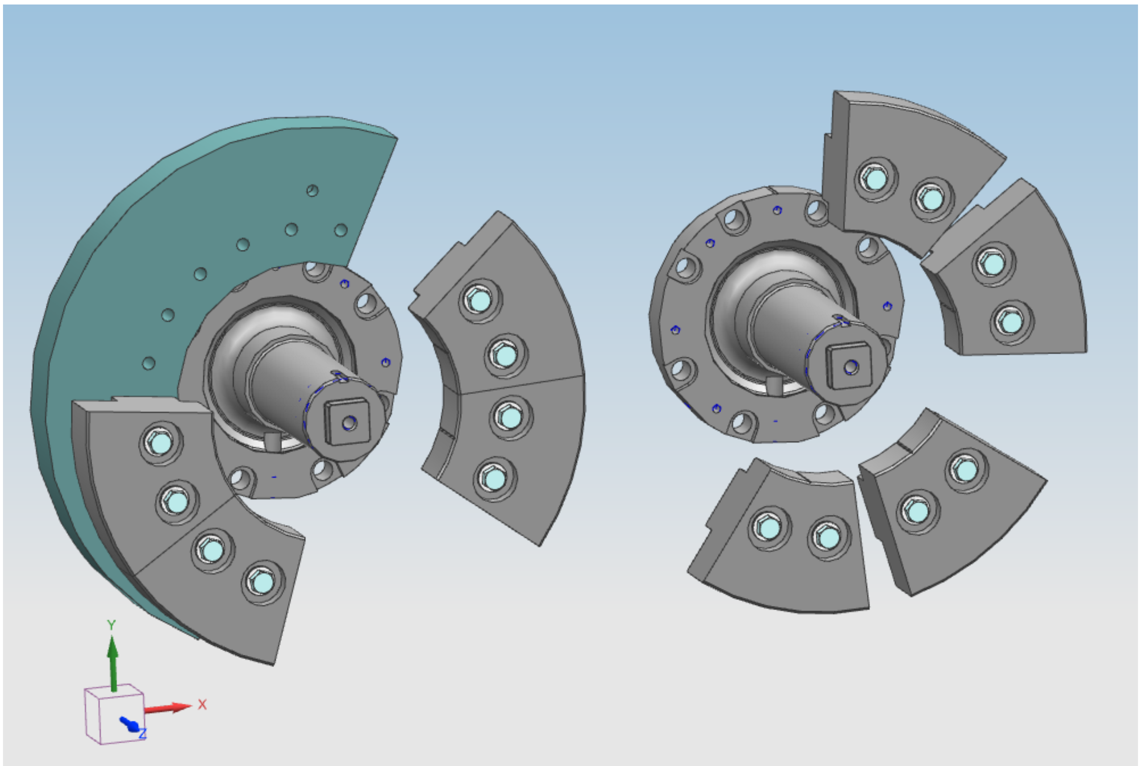
Balancing gear rotates at the same speed as the engine. The speed (120 rpm) can be converted to angular velocity with equation 23.

The generated force can be calculated by Newton's 2. law:

$$F = ma \tag{30}$$

Calculated force at engine speed 120 rpm is 260 N. This value is very small and doesn't cause any relevant loads even when it is combined with the radial gear force F_r. To obtain the maximum load caused by the balancing masses, the force F_a must be calculated with the maximum engine speed 600 rpm. Rare situation where engine speed exceeds 600 rpm is not considered because the balancing system does not have the highest risk of breakage compared to other engine parts in that kind of situation.

As the 14-cylinder engine has one extra mass on the balancing wheel, there are two different scenarios for the force caused by the balancing masses. Picture 24 shows the locations of the balancing masses used in the calculations. Assembly on the left side of the image shows the extra mass and locations of the tunable masses used in 14V engine. Assembly on the right side shows the tunable masses used in the 12V and 16V engines.



Picture 24. Locations of the balancing masses used in the calculations.

Calculated force for the 14V is 7582 N and 6491 N for the 12/16V. When the engine runs at steady speed (600 rpm), the transmitted moment and radial gear force F_r are very small. Therefore, F_a and F_r are not combined at engine speed of 600 rpm.

According to the total force equation (), supporting force B must be equal to the sum of F_a and F_r and supporting force A must be equal to the tangential force F_t. Maximum value for the force was calculated with engine speed 600 rpm. Gear forces with constant 600 rpm speed are very small and correct value for the F_t is not calculated. Therefore, resultant of the supporting forces A and B is not calculated, and the maximum bearing load is equal to force F_a (7582 N and 6491 N).

6.2 Bearing type

When bearing loads are known the final selection between rolling-element bearings and plain bearings can be made. According to Björk, the calculated loads and operating speed are in range where both types of bearing could be used etc. (2014, 277).

Factors that could limit the use of rolling-element bearings are their great need for space in radial direction and poor ability to tolerate shocks. In this case, there is a lot of space for the bearings in radial direction and they don’t receive shock loads. Also, vibrations at the location of the balancing system are tolerable for the rolling-element bearings so it is possible to use them. As calculated earlier, the starting friction in this case is minimal. Therefore, it is possible to use plain bearings even though they have a high starting friction compared to the rolling-element bearings.

However, there are a few factors that favour the choice of plain bearings. In addition to the fact that the 46TS balancing system uses plain bearings, they are generally more common in different parts of all engine models. As the plain bearings are cheaper than rolling-element bearing especially in large series it would be convenient and cost-effective to use same plain bearings that are already used in other engines.

When plain bearings are used, the connection of the shaft bearing, and gear is a bit simpler. In the best case, same bearings could be used in every shaft which naturally improves cost-effectiveness and simplifies the fitting of the bearings and gears. So, plain bearings are chosen at this point.

6.3 Parameter comparison

Parameter comparison is a straightforward method for ensuring the right dimensioning for the bearings. Certain parameters are calculated for the designed bearings and then compared to equivalent parameters in some existing solution which in this case is the 46TS system.

Bearing pressure, sliding speed and PV value can be calculated for the designed bearings as the loads and the bearing type are known. Calculated parameters are compared to the equivalent values calculated for the 46TS system.

Bearing pressure for radial plain bearings is calculated as follows:

$$p_b = \frac{F}{d * L} \tag{31}$$

where:

F = bearing load (N)

d = inside diameter of the bearing (m)

L = bearing length (m)

Sliding speed is calculated as follows:

$$U = \pi * d * n \tag{32}$$

where:

d = inside diameter of the bearing (m)

n = rotation speed (1/s)

PV value is calculated as follows:

$$pv = \frac{F * n * \pi}{L} \tag{33}$$

where:

F = bearing load (N)

n = rotation speedm(1/s)

L = bearing length (m)

The greatest calculated bearing loads were applied to the intermediate shaft bearing (8490 N) and balancing shaft bearing (7582 N). Load affecting in the intermediate shaft bearing is obtained from the gear forces and it is applied at engine speed 120 rpm. With that engine speed, the intermediate shaft rotates at speed of 215 rpm. Load affecting the balancing gear bearing is applied at engine speed of 600 rpm. If same bearings are used in every shaft, dimensioning is naturally done according to the maximum load. Bearing pressure, sliding speed and PV value were calculated with both loads. The bearing pressure at 120 rpm was slightly higher than at 600 rpm whereas sliding speed and PV value were much lower. Both scenarios are used in the comparison.

All the bearings in 46TS system have the inside diameter of 85 mm. The bearing located under the balancing masses in balancing shaft has the length of 70 mm. Parameters for the designed bearings to be used in 46F/DF are calculated with length of 35 mm and parameters for the 46TS bearings are calculated with 70 mm.

Bearing load for the 46TS is calculated based on the central acceleration of the balancing masses in same way as for 46F/DF shown earlier. The gear teeth width of 45 mm used in the designed system for the 46F/DF makes the gear forces higher. As the gear teeth width in 46TS is 35 mm instead of 45 mm, loads in other shafts shouldn’t be higher than in the balancing gear bearing and therefore it is safe to use the load generated by the balancing masses in the comparison.

The 46TS14V has fixed extra mass in addition to standard tuneable masses and therefore force applied to the bearing is higher. So, values for the 14V are used in the comparison to get highest possible reference values.

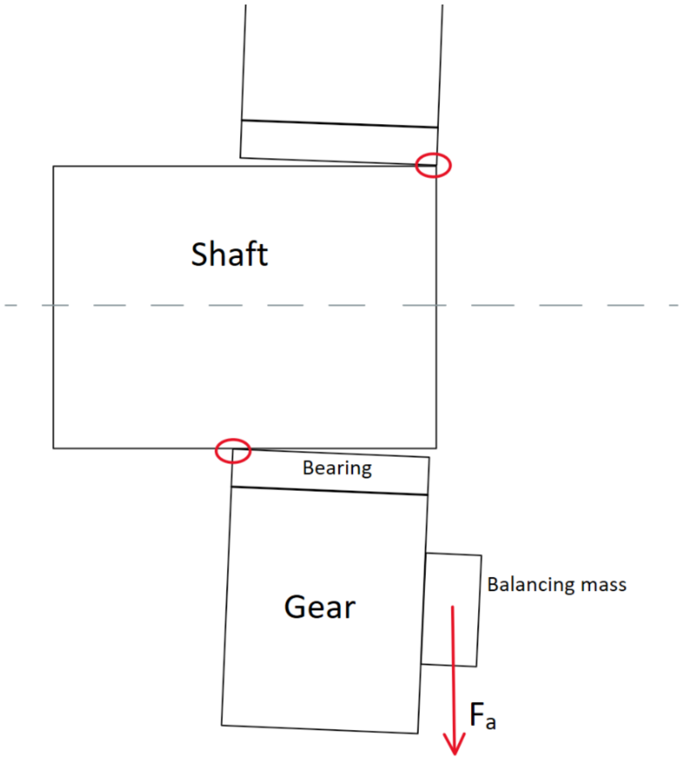
Calculated values for the bearing pressure, sliding speed, and PV value in 46TS are all higher than the values in the new system for the 46F/DF. Especially the PV value is much higher in the 46TS whereas bearing pressure and sliding speed are closer to the 46F/DF values.

6.4 Structural analysis

Based on the parameter comparison, same plain bearings as in the 46TS can be used in the new balancing system as well. As the bearing load is lower in 46F/DF than in 46TS, the bearing pressure stays lower in the new 46F/DF system even with the 35 mm bearing. Therefore, 35 mm bearing could be used on every shaft in the new system in terms of evenly distributed pressure. In addition to 35 mm radial bearing, the thrust bearings used to support the gearwheel on axial direction are same as in the 46TS system.

However, the centre of gravity of the tuneable balancing masses doesn't locate in the same line as the bearing and the load to the balancing gear bearing is not evenly distributed. Therefore, the structure of the balancing gear bearing needs to be considered in more detail. The centre of gravity of the extra mass used in the 14V configuration is also far from the bearing but the combined centre of the gravity of both extra and tuneable masses is much closer to the bearing centre. Therefore, it is only necessary to examine the situation with only tuneable masses present as in the 12V and 14V engines.

Rotating masses cause a force F_a calculated earlier with help of central acceleration (7582 N). Point of force application is outside the bearing edge and it causes a moment to the gear and bearing. Picture 25 shows an illustration of the situation. The movement of the gear and the bearing is exaggerated for clarity. The moment caused by the force F_a tilts the gear and the bearing and the contact surface between the bearing and the shaft is reduced. Red circles in the picture mark the points where the pressure on the bearings increases heavily. If the pressure becomes too high, the oil film between bearing and shaft might break. This causes a high risk for wear damage in the bearing.

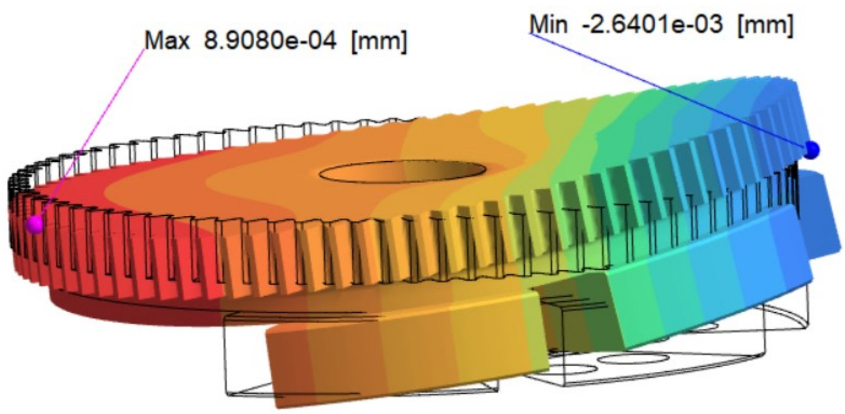


Picture 25. Moment caused by the force F_a affecting the gear and the bearing.

Another noteworthy issue is bending or some kind of swaying of the gearwheel itself caused by the force from the rotating balancing masses. The thrust bearings that guide the gearwheel in axial direction don't receive any loads but if the gearwheel bends much it might be possible that these guiding bearings face some loads. The outer diameter of the thrust bearings used in 46TS is 114 mm and they are relatively small compared to the balancing gearwheel that has the largest diameter of the wheels in the system. If significant bending occurs in the gearwheel, wider thrust bearings could give more support to prevent bending or any kind of swaying.

Deformation caused by the force from the masses was analysed briefly with Altair SimSolid. SimSolid is a simulation software that can be used to analyse CAD assemblies under real-life conditions. It offers solutions for different kind of situations like for example linear statics, modal, stress linearization, thermal, coupled thermal-stress, nonlinear statics, and fatigue analysis (Altair Engineering Inc. 2023).

The balancing gearwheel was examined with linear statics tool in SimSolid. Linear static analysis assumes that the load is static, and stresses do not exceed the yield strength of any part material. For the analyse stl. file that included the gearwheel and balancing masses was exported form NX. Force caused by the rotating masses was placed to the centre of gravity of the masses and wheel centre was constrained with slider constrain that enforces zero displacement only in normal to the surface direction on faces and areal spots.



Picture 26. Linear static analysis of the balancing gear.

Picture 26 shows the maximum deformation of the gearwheel. Amount of deformation is heavily upscaled to illustrate the situation more clearly. As can be seen the minimum and maximum values are extremely small. A conclusion can be drawn from this that the bending or swaying will not be a problem with current thrust bearings.

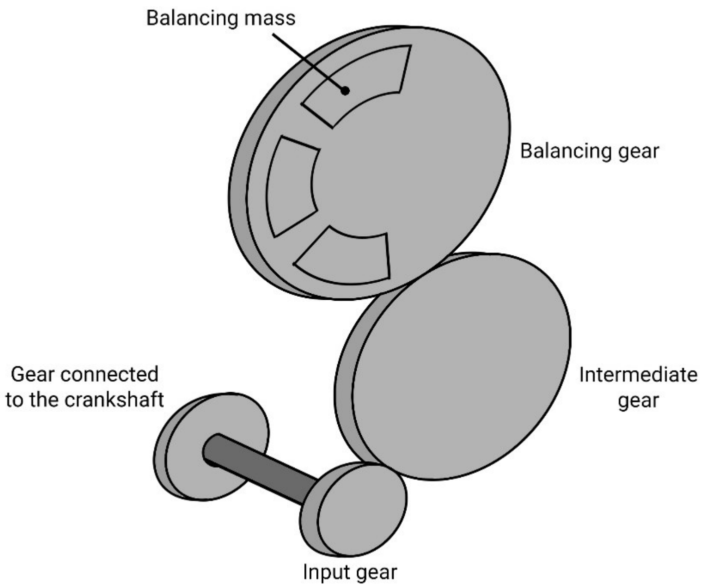
7 3D model

After theoretical calculations and bearing selection, a 3D model of the new balancing system was made. Table 10 shows the final specifications for the new balancing system.

Table 10. Specifications for the new balancing system

Gears	
Module	5
Pressure angle	20 °
Number of teeth	Input gear 20, intermediate 48, balancing 86
Teeth facewidth	45 mm
Hydrodynamic radial plain bearings	
Length	35 mm
Outer diameter	93 mm
Inner diameter	85 mm

Picture 27 shows a rough illustration of the new balancing system without the cover box. The complete 3D-model can’t be shown in the public thesis work, but the illustration shows the basic principle of the new balancing system. The new system is very similar to the 46TS system, but notable difference is that balancing masses are attached straight to the balancing gear wheel instead of separate element.



Picture 27. The new balancing system.

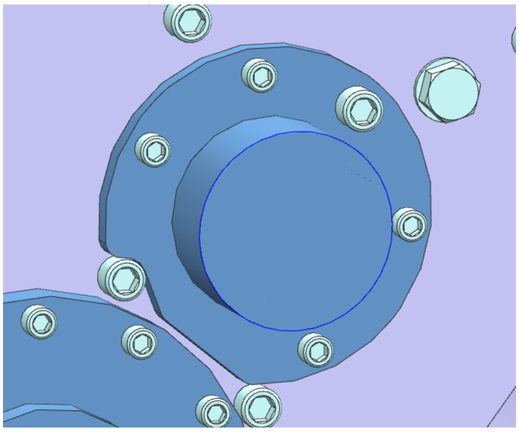
The theoretical part applies only to gears and bearings, so their structure is the main part of the 3D model. As the stage of design is only at the concept level, the model is relatively simple and don’t include many details. One essential feature that is not modelled at all is lubrication. However, the new system is constructed in such a way that adding lubrication to the gears and bearings later should be possible.

To avoid high stresses, two 35 mm plain bearings are used in the balancing gear bearing. With only one bearing, the rotating balancing masses cause moment and stress to the bearing as the center of gravity of the masses is not on the same line as the bearing. With two bearings load is evenly distributed and there are no significant moments affecting the bearings.

The balancing gearwheel is connected to shaft with separate element. This makes the structure of the gearwheel simpler and eases manufacturing. Thrust bearings with diameter of 114 mm are the same as in the 46TS. As the diameter of the gearwheel is smaller than diameter of the balancing masses outer edge, there must be a gap between the masses and the gear teeth to avoid colliding with intermediate gear. Gearwheel is shaped so that suitable gap is achieved.

The intermediate shaft is relatively long compared for example to the 46TS system, but it shouldn't be issue. Same bearings as in the balancing shaft are used. The width of teeth in the designed system is 45 mm and therefore the gearwheel centre must be shaped to fit the bearing.

The space for the intermediate shaft in the back of the cover box is limited. As seen from the picture 28, some bolts and the housing for the input shaft limit the space and the flange in intermediate shaft can't form a full circle. If necessary, the shaft could also be attached without flange. It could be fastened with long screws from the back of the cover box or by shrink-fitting.



Picture 28. Intermediate shaft connection to the cover box.

The input shaft runs inside a housing that is mounted to the cover box and same bearings as in other shafts are used again. Diameter of the input gear is quite small, and a separate flange is added to the gearwheel to fit the 114 mm thrust bearing.

8 Summary

The aim of the thesis was to design a concept for a new gear-driven balancing system to replace the old belt-driven system and to dimension the gears and bearings for the new system. The need for a new system was due to the unreliability of the belt-driven system. Requirements for the new system were easy installation to replace the old system, durability of gears and bearings, and maintaining the current level of balancing.

Work started with gathering information about the gears and bearings. After getting familiar with the basic knowledge, the dimensioning of the gears could be started. After initial dimensioning, a simulation was done for the designed gear train to ensure the durability of the gears. The loads for the bearings were calculated based on the results from the gear train simulation and suitable bearings were chosen. Finally, a 3D model of the entire new system was created, showing the practical structure of the gears and bearings.

The basic idea throughout the work was to keep the new solutions simple and easy to implement. In addition to the gear-drive, the main difference between the new balancing system and the original system is the type of bearings. Plain bearings are used in the new system whereas the original system uses different types of rolling-element bearings. The structure of the shaft and gear combinations in the new system are simpler when using plain bearings and now the same bearings can be used in every shaft.

Balancing system used in the Wärtsilä 46TS was used as reference during the thesis work. Solutions and different values in the designed system were compared to the 46TS system. By comparing own solutions to the 46TS system, it was possible to assess whether the solutions in the new system were feasible in practice. In addition, the accuracy of the values calculated for the new system could be assessed. Especially in the calculation of bearings it was important to have some reference point, as the required parameters for bearings did not exist.

The module of the gears in the 46TS balancing system is 6 whereas in the new system for the 46F/DF the module is 5. Due to smaller module, the width of the gear teeth had to be increased to be 45 mm instead of 35 mm which is the value in the 46TS system. The maximum bearing load are higher in the 46TS system and therefore it was safe to use the same bearings in the new system as well. However, the bearing of the balance wheel has been implemented differently in the new system than in the 46TS system. 46TS system has a separate fixing element for the balancing masses and the balancing shaft is supported from both ends. Also, one 70 mm bearing is used on the balancing shaft in addition to the 35 mm bearing. In the new system for the 46F/DF, the balancing masses are fixed to the balancing gearwheel and the balancing shaft is not supported from both ends. Instead of 70 mm bearing and 35 mm bearing, two 35 mm bearings are used on the balancing shaft.

Calculations performed by the thesis worker himself during the work are moderately rough but a comparison with 46TS system shows that the results are useful. Calculation made with KISSsoft is accurate but errors in input values might cause false results. However, all the calculations made during the project show that required durability for the gears and the bearings is fulfilled.

Other requirements for the new system included maintaining the current level of balancing and easy installation to replace the old system. The speed of the balancing gear or the placement of the balancing masses are not changed so the current level of balancing has been maintained. The new system doesn’t have any new features that would limit the installation so replacing the original system is not a problem.

As mentioned during the work, the design for the new balancing system is done only at concept level. Further engineering would include an accurate dimensioning and strength analysis of the shafts, designing the lubrication system and selection of materials. In detail level, further design work would include for example determination of different clearances, more precise modifications for gear teeth and determination of surface roughness for different components.

Overall, the work was successful, and the requirements set for the new system in the beginning were met. The thesis worker had no previous experience with gears or bearings so especially the beginning of the project was challenging. Also, the slow progress of the work was sometimes frustrating. The topic of the work is very broad and sometimes the delimitation of different related subjects brought challenges. During the work, only a small portion of all aspects related to the topic are discussed. However, the work includes the most essential topics that are related to the design of a new gear train, and it provides a foretaste of the full design process of machine elements.

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Input data for KISSsoft calculation

System data		
Module	5	mm
Pressure angle	20	°
Reference profile	ISO 53:1998 A	
Tooth thickness tolerance	DIN 3967cd25	
Center distance tolerance	ISO 286:2010 js7	

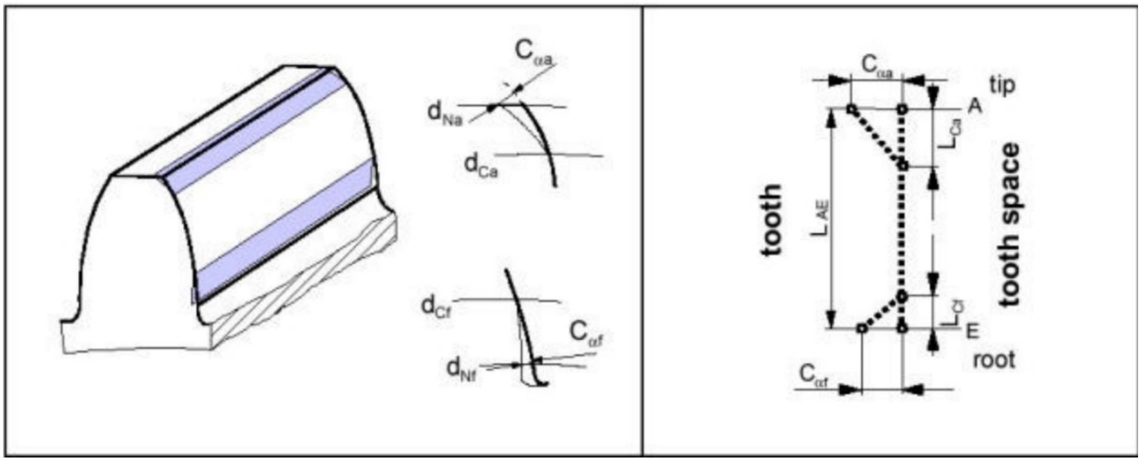
	Input-Intermediate	Intermediate-Balancing
Center distances (mm)	220	385

Strength		
Strength calculation method	ISO 6336:2006 Method B	
Reference gear	Gear 3	
Torque	855	Nm
Speed	120	1/min
Required service life	1 000 0000 / infinite	h
Application factor	1,25	(according to DIN 3990/ISO6336)

Gear data	Gear 1	Gear 2	Gear 3
Number of teeth	20	68	86
Facewidth (mm)	45	45	45
Profile shift coefficient	0,0833	-0,0833	0,0833
Quality (ISO 1328:2013)	6	6	6
Material	42CrMo 4	42CrMo 4	42CrMo 4

Tip and root relief

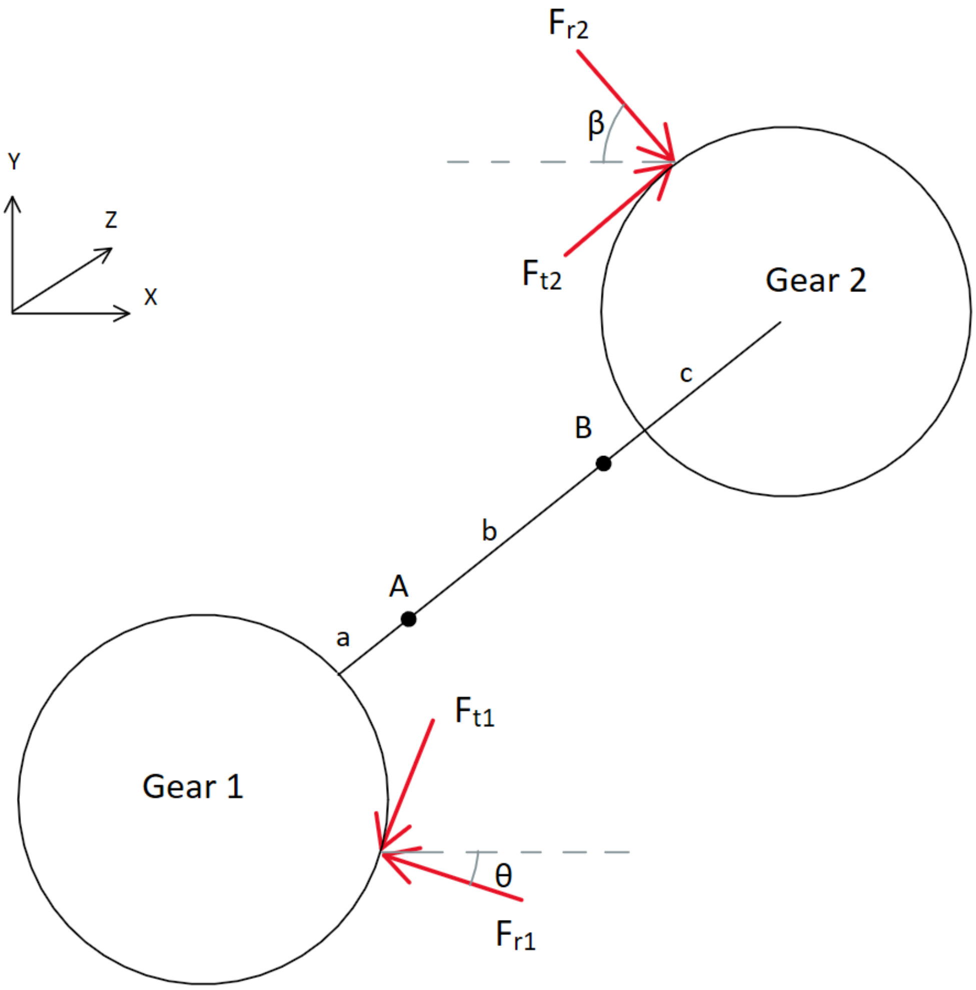
Gear	Flank	Type of modification	Value [μm]	Factor 1	Factor 2	Status	Information
Gear 1	both	Tip relief, linear	4.1000	0.9774		active	dCa=105.975mm, ξ=29.874°
Gear 1	both	Root relief, linear	4.1000	1.2602		active	dCf=95.959mm, ξ=11.854°
Gear 2	both	Tip relief, linear	4.1000	1.2042		active	dCa=344.485mm, ξ=23.100°
Gear 2	both	Root relief, linear	4.1000	1.5652		active	dCf=334.547mm, ξ=17.793°
Gear 3	both	Tip relief, linear	3.1000	1.2034		active	dCa=436.162mm, ξ=23.285°
Gear 3	both	Root relief, linear	3.1000	1.5390		active	dCf=425.901mm, ξ=19.088°

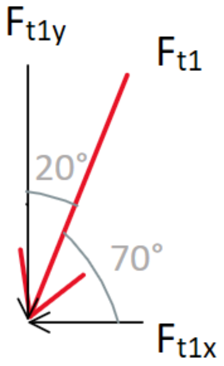


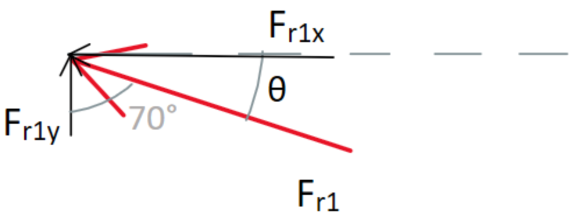
Cαa
dNa
dCa
dCf
Cαf
dNf
Cαa
tip
A
LCa
tooth
LAE
tooth space
LCf
E
root
Cαf

Bearing loads

Input gear




Ft1y
Ft1
20°
70°
Ft1x


Fr1x
θ
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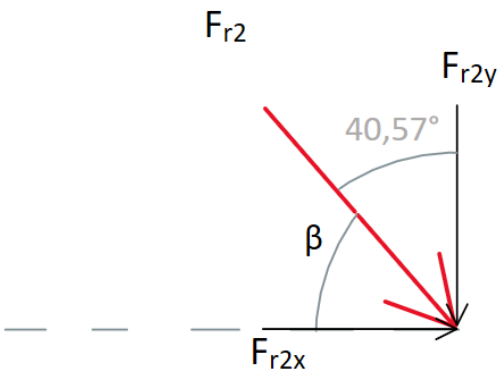
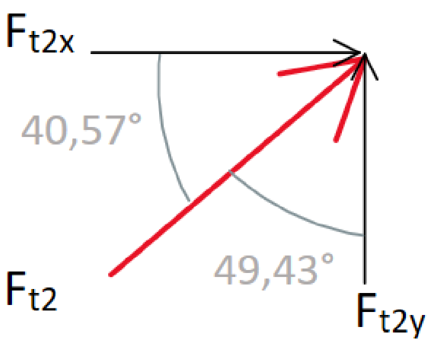
Fr1


Fr2
Fr2y
40,57°
β
Fr2x


Ft2x
40,57°
Ft2
49,43°
Ft2y

Ft1 (N)	Fr1(N)	Ft2 (N)	Fr2(N)	a (mm)	b (mm)	c (mm)
2485,5	904,6	3976,744	1447,417	102,5	246,5	71

Gear 1

Ft1y =	2335,606	N
Ft1x =	850,0911	N
Fr1y =	309,3914	N
Fr1x =	850,0459	N

Gear 2

Ft2y =	2586,381	N
Ft2x =	3020,782	N
Fr2y =	1099,475	N
Fr2x =	941,3661	N

Y-plane:

$$\sum M_B = -Ft1y * (a+b) + Fr1y * (a+b) + Fr2y * c - Ft2y * c + Ay * b = 0$$

$$Ay = \frac{Ft1y * (a+b) - Fr1y * (a+b) - Fr2y * c + Ft2y * c}{b}$$

Ay = 3297,035 N

$$\sum F_y = -Ft1y + Fr1y - Fr2y + Ft2y + Ay + By = 0$$

$$By = +Ft1y - Fr1y + Fr2y - Ft2y - Ay$$

By = -2757,73 N

X-plane:

$$\sum M_B = Ft1x * (a+b) + Fr1x * (a+b) + Fr2x * c + Ft2x * c + Ax * b = 0$$

$$Ax = \frac{-Ft1x * (a+b) - Fr1x * (a+b) - Fr2x * c - Ft2x * c}{b}$$

Ax = -3548,32 N

$$\sum F_x = Ft1x + Fr1x - Fr2x - Ft2x - Ax + Bx = 0$$

$$Bx = -Ft1x - Fr1x + Fr2x + Ft2x + Ax$$

Bx = 5810,329 N

Resultants

A	4843,656	N
B	6431,561	N

Intermediate gear

Ft (N)	**Fr (N)**
3976,744	1447,417

F1

Fr1y =	1092,3795	N
Fr1x =	941,36611	N
Ft1y =	2586,3811	N
Ft1x =	3020,7823	N

F2

Fr2y =	1404,4225	N
Fr2x =	350,16186	N
Ft2y =	758,79853	N
Ft2x =	3903,68	N

$$\sum F_y = Fr1y - Ft1y - Fr2y + Ft2y + B = 0$$

$$B = -Fr1y + Ft1y + Fr2y - Ft2y$$

B = 2139,6257 N

$$\sum F_x = Fr1x + Ft1x + Fr2x + Ft2x + A = 0$$

$$A = -Fr1x - Ft1x - Fr2x - Ft2x$$

A = -8215,99 N

Resultant = 8490,023184 N

Balancing gear

Ft (N)	Fr (N)	Force Fa caused by the balancing mass (N) 120 rpm
3976,744	1447,417	259,64039

Maximum bearing load B

Fr+Fa = 1707,0574

Maximum bearing load A

3976,744

Resultant (120 rpm) = 4327,6481 N