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Price Elasticity of Demand in Revenue Maximisation – A case study with sales history data of #CMAX.gg

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The purpose of this thesis was to determine optimum prices for chosen products in the web gaming services industry using theory of price elasticity of demand and provide information of how much revenue could be increased with these optimum prices. The thesis was prepared to provide support in the sales process of Apprien -dynamic pricing engine. The study was carried out using quantitative analysis on sales history data provided by a web gaming service provider and the methods used for measuring elasticity were arc and point elasticity, single and multiple linear regression as well as interpreting a logarithmic demand curve. The study found that prices should be gradually increased towards the evening both during the weekend and during weekdays. In addition, the prices should be increased during weekends when the demand is higher. There were also signals that the customer base could be further segmented for more efficient execution of Pricing and Revenue Optimisation.

Keywords	Price elasticity of demand, pricing, optimisation, revenue maximisation, regression analysis

Contents

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Figures

Tables

1 Introduction

This study is a practical approach to utilising microeconomic theory of price elasticity of demand for revenue maximising purposes. When pricing strategies are getting more and more complex and at the same tame technology is evolving enabling sophisticated pricing software making the right pricing decisions is essential for companies to survive. Traditional pricing strategies such as cost-plus pricing, competitive pricing and value-based pricing may no longer be enough to keep companies abreast of competition in terms of capturing maximum value. In the Internet era when availability of information about the customers is easy to obtain companies have to make sure they harness all revenue stream opportunities. In addition, based on the research, pricing can be the most effective way to increase top line performance.

Pricing and Revenue Optimisation (PRO) focuses on maximising revenue through pricing decisions. It is a trade-off between winning the customer and gaining maximum revenue. The early ages of PRO started from deregulation of the airline industry and it has since evolved from Pricing and Revenue Management to more sophisticated methods of using customer needs to estimate willingness to pay (WTP). In empirical studies PRO has been proven to yield increased revenues for example in the insurance market. In the internet one can find that quite some companies are already offering software to execute PRO. There are not however, many public academic papers that assess revenue maximisation based on price elasticity of demand. The ones that exist often consider other factors more than the theory of price elasticity of demand. Price elasticity of demand can be seen as the relationship between price and the quantity demanded of a certain product or a service. Using this method a price-quantity combination with the highest revenue possibility can be recognised. This way of identifying the optimal combination is already used by Apprien -pricing engine and the modelling of this engine is introduced in this paper.

For the empirical part the author has obtained sales history data from CMAX.gg, a market place offering web gaming services. The product under investigation is one particular server group and dynamically priced 15-minute time spans. Price elasticity is measured with both arc and point elasticity and the latter is done utilising single linear regression. In multiple linear regression more variables are added to the equation to determine all possible variables that affect the demand of the time spans. Finally the demand is estimated using logarithmic demand curve and based on that optimum prices are calculated and results presented with limitations to the study.

This thesis is conducted to provide support to sales of Apprien –pricing engine and similar studies will be conducted to client companies to assess the suitability of the Apprien - pricing engine. In the business context this study offers some preliminary steps towards more intelligent, knowledge based pricing. The scope of this paper is only in quantitative analysis and it does not take into account psychological factors that may affect pricing. Furthermore, this thesis only attempts to measure the financial implications of this type of pricing and is not evaluating the actual modelling of Apprien – pricing engine.

2 Literature review

2.1 Pricing

This chapter looks at the basic strategies in the pricing process to evaluate some of the most commonly used strategies. The key features together with shortcomings as well as the benefits of these strategies are investigated and the topic of Revenue and Price Optimisation (PRO) is introduced as an alternative to the traditional methods of pricing.

2.1.1 Traditional pricing strategies

Pricing can be the single most effective way for a company to increase its revenues. (Phillips 2005). With pricing strategies becoming more complex as well as technological advancement enabling mathematical modelling of prices, an effective pricing strategy may gain even more importance in the future. Moreover, pricing as a method for revenue maximisation needs less investment than, for example, cost cutting. (Phillips 2005). Earlier studies also show the effectiveness of pricing in terms of growing revenue; according to a Harvard Business Review article by Marn and Rosiello (1992) price improvements are three to four times more effective than volume increases in terms of profitability. According to the authors a 1 per cent improvement in price when there is no loss in volume yields 11,1 per cent improvement in operating profit, whereas a 1 per cent proportional increase in volume - assuming price does not change – only increases operating profit by 3,3 per cent. Thus if the company wants to increase its revenues it may be better off by pricing intelligently than by only adding sales volume. The authors also argue that even though it is an important task, pricing is something that managers do not pay close enough attention to. While the right pricing can have significant positive effects in terms of revenue and profit generation, the wrong pricing can have as significant negative effects. According to the authors managers only look at pricing from a very general level and do not focus on the transaction level pricing, which the authors argue decreases the pocket price. This means that managers do not focus on what happens on the shop floor level in terms of pricing, and that can negatively affect the actual price the organisation receives from a product. This could be due to for example on the spot discounts given by sales personnel. The article also introduces the concept of price waterfall – how the organisational structure affects the difference between sales price and pocket price. This focuses on what the organisation actually gets from the product. Since the price waterfall, while related, is outside of the scope of this thesis it is not studied in depth. Rather the focus will be placed on some traditional pricing strategies organisations use.

Phillips (2005) presents the following traditional pricing strategies: Cost-plus pricing, Market-Based Pricing and Value-Based Pricing. Cost-Plus pricing can be seen perhaps as the most traditional way to price products or services. It consists of the fixed costs allocated for the product, the variable costs of the product, and a surcharge. The surcharge is usually a determined percentage depending on the objectives of the business. It may also vary for different types of products. Cost-plus pricing may be easy to adopt by, for example, small companies without extensive pricing expertise or knowledge about the target market. Sometimes it may be adopted as it is seen as a 'fair' way to price products. (Ward 1992). While widely accepted, the method has some drawbacks which Phillips (2005) identifies as follows; the method only looks at the pricing from inside the company and it does not take into account the market and consumer willingness to pay nor does it give the ability to price based on the willingness to pay (WTP) through segmentation. The author also identifies problems in objectivity in terms of determining the amount of fixed costs and problems in forecasting the future in terms of costs, in which case the company may find itself in trouble when costs change. For example when costs rise the company using this form of pricing strategy would have to raise prices. This may present a risk to the company; they might have to raise their prices above the general level of the competitors and thus lose some customers to competing firms. In addition, the company may be losing revenues by not pricing based on external conditions that do not affect the cost structure of the company.

In market-based (Phillips 2005) or competitive pricing (Liozu et al. 2011) the focus is on how the product is valued in the market place. In other words, it focuses on what prices competitors are asking for similar products in the market. In this thesis we use the term competitive pricing in order to avoid confusion with value-based pricing which is introduced in the next section as the third method of traditional pricing. Competitive pricing can be effective for new, low-cost providers to gain market share when entering the market (Phillips 2005). An example can be found close to home; low-cost German supermarket chain Lidl arriving to the Finnish market, where the barriers of entry can be seen as high due to an oligopoly formed by two large supermarket chains Kesko and

HOK-Elanto. Competitive pricing does, however, have certain shortcomings. When a company only focuses on what the competition is doing in terms of pricing products and services they are not executing a strategy of their own and there is a chance that they are not truly selling their products at a price of customer WTP. Thus, they may be losing revenues. Even though it is important to know the price range at which key competitors are offering products, pricing based only on the competitor's prices may lead to the company failing to distinguish itself from the competition in terms of customer perceived value. (Phillips 2005).

Value-based pricing is explained by Phillips (2005) as personalised pricing. The true value that your customers hold for your product can be found out by for example questionnaires, surveys or panel discussions. While discovering the value each individual holds for your product may seem like a good way to execute pricing, Phillips (2005) sees it impossible to do in practice due to competition, difficulty in determining willingness to pay in the middle of the sales process as well as the risk of arbitrage and cannibalization. In addition Marn and Rosiello (1992) argue that one reason for price waterfall and the difference between sales price and pocket price is partly due to discounts given by sales personnel. Moreover, Phillips (2005) points out that there may be a difference between the perceived value by the customer and the amount the customer is actually willing to pay. The author also provides an excellent example; two consultancy firms offering different amounts of total value to a company and even though one consulting firm offers altogether more value the company chooses to buy services from a cheaper firm, offering altogether less value but giving the company a higher margin. Thus, the buying firm chooses large margin in exchange for maximum value. On the other hand, a study by Liozu et al. (2012) discovers that value-based pricing is possible to execute but it is a long journey and requires involvement from the entire organisation, a strong organizational champion and a mindful and learning organisational culture.

2.1.2 Pricing and revenue optimisation (PRO)

In addition to the traditional strategies of pricing plenty of literature exists on Pricing and Revenue Optimisation (PRO). Several articles and papers seem to consider PRO as beneficial to companies and some consider it even a necessity for an organisation to survive in the Internet-based economic environment. According to Kalanidhi (2001) in a paper considering the profit optimisation possibilities for companies in the internet era the increased complexity of the market environment requires new measures from companies to be able to harness the full possibilities offered by the Internet. Real-time information as well as significant cost-cutting throughout the supply chain enable companies to realise additional revenue streams and according to Kalanidhi (2001) it is essential for companies in the Internet-based economy to be able to create value to survive. In order to harness the full revenue possibilities the author introduces concepts of Pricing and Revenue Optimisation (PRO) and Enterprise Profit Optimisation (EPO). Since supply chain activities are outside of the scope of this thesis only PRO is studied in more detail.

The deregulation of the airline industry in 1979 can be seen as the starting point for PRO as we know it today, Kalanidhi (2001). It is, of course, possible and perhaps likely that demand based pricing has been used long before that, but according to the author PRO as a pricing strategy was first applied by airline companies. According to Kalanidhi (2001) and Phillips (2005) airlines have long known that all passengers are not equal. Passengers can be divided to different groups based on their needs and willingness to pay for different seats. Due to deregulation, airline companies first engaged in intense price competition but soon realised that the pricing should be done in a more intelligent and effective manner. The first stage of PRO, according to Kalanidhi was Price and Revenue Management (PRM) which was based on dividing the passengers according to their needs and willingness to pay, for example price conscious leisure travellers versus business travellers. A fixed number of seats would then be allocated to those groups with different pricing. Airlines however, realised that there are subgroups within those groups to whom they could determine new price ranges since they could differentiate them by their needs, for example in terms of connection preferences. The optimisation eventually evolved to measure the trade-off between actually getting the customer and the revenue the company would get from the customer, with what price is it profitable for a company to win the customer instead of losing the sale? This level of optimisation is outside of the scope of PRM and it is called Pricing and Revenue Management. (Kalanidhi 2001).

The practices seem to be adopted by companies in other industries as well. The software offering by companies to execute PRO is rather extensive; PROs, PriceSpectre, BlueYonder and Gondola by White Shoe Media Inc. only to mention some. These companies offer real time pricing and revenue optimisation to airlines, e-commerce web shops, car rental companies, hospitality industry providers, and even for pricing in-app purchases in mobile games and applications. These companies promise customers increased revenue with price optimisation, varying from 1 per cent increase up to 34 per cent increase. One of these companies – Earnix has published a study examining the possibilities of PRO in insurance sales. The study was conducted on a comprehensive car insurance portfolio for an actual insurance company in 2004. The authors Krikler et al. collected demand data from over 10 000 customers to establish assumptions about the demand and then tested the assumptions with a group of customers by offering some optimised prices and some non-optimised prices for renewal of their comprehensive car insurances. The authors in their own words are experts in the field of PRO and have conducted similar analyses on other companies as well.

In the study, while determining the demand function, the authors took into consideration the strategic and financial objectives of the insurance company; maximizing Net Present Value (NPV) in addition to maintaining customer retention. The authors, after determining the demand function, created four scenarios and found the following; even with 0 customer loss the company could increase its profit by 7 per cent. There were some other, more aggressive, scenarios where the increased profit was even 13 per cent. Because of more significant losses in retention (up to 3% loss) the authors did not validate some of the more aggressive scenarios. The validation of assumptions showed somewhat similar results; the bottom line profit was increased by €626 000, roughly 10 per cent after allocating fixed costs and revenue was increased approximately 2 per cent. What is interesting here is that the retention only changed by -0,7 per cent. So in other words the authors managed to prove that with PRO alone, the company can improve its top and bottom line performance without losing significant number of customers when sticking to the less drastic scenarios. It can only be speculated how correct the assumptions would have been in the more dramatic scenarios. It should be noted here that due to the nature of insurance as a product, the authors had to make certain adjustments to the analysis. Firstly, because insurances are not perishable products as for example seats for a flight are their value has to be evaluated over time with Net Present Value (NPV) instead of only using revenue as a measurement. This is since insurances tend to be subscription based and instead of one initial payment in the beginning the customers pay an annual fee for example for 20 years. That is why Net Present Value is used to value the future cash flows received from the customer instead of revenue received from each initial payment. Secondly, instead of using quantity as a measurement of demand, the authors used dependency of the probability of renewal or sales, which is also due to the business model. In a business where customers pay annual fees customer retention might be a more accurate measurement than new customer acquisition. According to the authors, even better results have been achieved with some other insurance companies but those results cannot be shared for confidentiality reasons.

2.2 Price Elasticity of Demand

As presented in earlier sections of the literature review, PRO is a trade-off between winning the customer and the revenues gained from the sale. The key question in PRO is whether we should sell more units to gain revenue or increase the price of the units we are selling to sell less, but to gain more revenue from each unit sold. These decisions may be tricky to make and could potentially harm a company if made arbitrarily without knowledge and ability to predict the demand. With a theoretical framework and mathematical modelling the process can be made more scientific. The next section identifies the theoretical framework that can be used in order to predict demand and calculate optimal prices.

This section summarises the key theory of price elasticity of demand on which the analysis of this thesis is very much based. Price elasticity is a well-known and well-defined microeconomic concept and there is vast amount of established literature explaining the concept. The references have been chosen since the aim of this thesis is to combine the concept with revenue maximisation and pricing strategy. Both resources in the writer's opinion have the accurate viewpoint in terms of meeting the objective of this thesis.

Keat and Young (1996) define demand of goods and services as follows: "Quantities of a good or a service that people are ready to buy at various prices within some given time period, other factors besides price held constant." So in other words demand is customer behaviour that is reflected in quantities of goods and services bought. Price elasticity, on the other hand is defined as the sensitivity of the demand to the change of price. For example, a measurement of the percentage change in demand of a product caused by one per cent change in the price of a product or a service. According to Dawson et al. (2006), when a product's demand changes more in percentage terms than its price, the demand for the product is price elastic. When the demand for the product changes less than its price in percentage terms, the demand is inelastic. In other words, when demand is elastic it is sensible to decrease prices whereas in case of inelastic demand the prices should be increased since the inverse effect on quantity sold would be relatively smaller. Thus, the price elasticity of demand can be seen as the relationship between price and demand.

Because a rational consumer tends to follow the demand curve, the relationship between price and demand is usually inverse. This leads to the idea that the demand curve tends to be downward sloping and price elasticity negative. (Keat and Young 1996). Based on the book by Keat and Young, price elasticity can generally be presented as follows;

$$\frac{\Delta Quantity}{Quantity} / \frac{\Delta Price}{Price} = \%\Delta Quantity / \%\Delta Price$$

Where delta is the absolute change. There are two commonly used ways to discover price elasticity of demand; arc elasticity and point elasticity. Arc elasticity is most commonly used with economists whereas the point elasticity can remove some problems faced when arc elasticity is used. (Keat and Young 1996). Arc elasticity can generally be expressed as follows;

$$Ep = \frac{Q2 - Q1}{(Q1 + Q2)/2} / \frac{P2 - P1}{(P1 + P2)/2}$$

Where

Ep = Coefficient of price elasticity

Q1 = Original quantity demanded

Q2 = New quantity demanded

P1 = Original price

P2 = New price

The formula gives the price elasticity between two points, however the price elasticity of a certain point will depend on its distance from the original price point, thus it gives different results on elasticity depending on the price points. (Keat and Young 1996). It is, despite its shortcomings, used in the analysis since it is still useful in price optimisation purposes as evidenced in the analysis. Another way to calculate price elasticity of demand is to use the midpoints method to find point elasticity. In calculation of the point elasticity, the derivative of Q with respect to P is taken and in the case of linear demand curve, for which it also will be used in this thesis, the derivative equals the slope of the demand curve. (Keat and Young 1996). Point elasticity can be presented as follows;

$$\varepsilon_p = \frac{dQ}{dP} * \frac{P_1}{Q_1}$$

Where
ε_p = Coefficient of point elasticity
dQ/dP = Derivative of Q with respect to P

In case of a linear function the derivative equals the slope of the demand curve the function can also be presented as follows (Keat and Young 1996);

$$\varepsilon_p = \frac{\Delta Q}{\Delta P} * \frac{P_1}{Q_1}$$

ΔQ = Absolute change in quantity
ΔP = Absolute change in price

This simplifies the calculations later in the analysis in the case the demand curve is linear since the derivative is already known.

2.1.1. Demand elasticity and revenue

The following summarizes the microeconomics theory about optimal price setting based on the demand elasticity. While demand elasticity can be useful in demand estimation in terms of price changes, it can also be used in revenue maximisation. As presented earlier, the demand curve tends to be downward sloping. In other words when price increases the quantity demanded for that particular product decreases. The key question here is how much the price can be decreased so that the company does not lose revenues but increases quantity sold in a way that revenue is maximised. Based on the book by Keat and Young (1996) when the demand is elastic a decrease in price would yield higher revenues, whereas in case of inelastic demand it would decrease the revenues. The peak, in other words, maximum revenue is reached at the point where elasticity is

-1 which is the unitary elasticity; where a one per cent change in price results in a one per cent change in quantity demanded to the opposite direction. (Keat and Young 1996).

When the demand curve is linear it is possible to use the function of point elasticity to calculate values for price and quantity where $\varepsilon_p = -1$.

$$\varepsilon\text{p} = (\text{dQ/dP}) * (P_{optimal}/Q_{optimal}) = -1$$

$$P_{optimal} = -Q_{optimal}/(\frac{dQ}{dP})$$

And in case of linear point elasticity where the first derivative equals the slope as mentioned earlier we get the following;

$$P_{optimal} = -Q_{optimal}/(\frac{\Delta Q}{\Delta P})$$

This method will be used in the analysis section of this paper to calculate price elasticity and attempt to find optimal prices where revenue is maximised for a certain product.

Unitary elasticity can also be explained by the concept of marginal revenue. Marginal revenue is the change in total revenue caused by the sale of one additional unit. Marginal revenue is positive as long as the extra revenue received from the sale of one additional unit exceeds the loss in revenue caused by the decrease in price of all other units. For each additional unit sold the marginal revenue decreases and at the point where it reaches zero we are at the point of unitary elasticity, in other words a point where neither decreasing nor increasing the price would yield more revenues. (Dawson et al. 2006).

2.3 Empirical studies and modelling

The following summarizes the findings of Ruutu S. et al (2006), "Verkkopelipalveluiden reaaliaikainen hinnoittelu" – a paper conducted for a seminar work in the University of Technology modelling real time dynamic pricing for online gaming services. This thesis is a continuum of the work of Ruutu S. et al. and it is essential to summarize the key findings of their work as a basis for the analysis of this paper.

Ruutu et al. in their work studied the pricing operations of #CMAX.gg. #CMAX.gg is an online game server marketplace where server space is sold for playing Counter Strike and Counter Strike Source online. The buyers of server time are clans of online players who use it to play in practice matches or tournaments and at the time of the study there were approximately 10 000 online players in Finland. CMAX.gg offers server space from multiple suppliers and the time is sold in 15 minute spans or 24 hour blocks. At the time of the study was conducted the 24 hour time blocks were sold with fixed prices but the 15-minute spans were priced dynamically. Ruutu et al. note that even though there was a dynamic pricing model used, the method was not effective enough and that the pricing could be done better than it was at the time.

In the study Ruutu et al. analysed sales history data from #CMAX.gg in order to model a more efficient way to price 15-minute time spans. The writers considered several methods of dynamic pricing as a basis of the modelling of the pricing engine, such as yield management, price discrimination, auction theory and stochastic modelling of demand. As an outcome Ruutu et al. found that since the price history data does not give specific information about the customer segments and customer Willingness to Pay (WTP) nor does the industry match directly to, for example, the airline industry the writers decided to abandon price discrimination and only use selectively the yield management and stochastic modelling of demand. The writers also examined auction-based theory but came to the conclusion that since auction-based model is most suitable to the situations where the supply is limited and the demand higher, it is not an optimal way to increase volume. As a second downside, the authors state that not many customers especially in the case of cheaper products are willing to wait for the bids made by other bidders in order to respond, thus this model was abandoned. Instead the writers chose to use the theory of price elasticity of demand as a method to optimise pricing.

In the sales history data Ruutu et al. had the following candidates for variables; weekday, difference between purchase time and start of the game time, purchase time of the day and price. The writers discovered that the purchase behaviour tends to be instant type meaning that the spans are usually purchased right before the game starts. It was also discovered that the number of started games positively correlated with the duration of the games. Moreover, the effect of the weekday was found to be as follows; there was no significant differences between weekdays but there was an expected difference between weekdays and weekends, thus the effect of weekdays was divided into these two categories. The study also found that the number of started games depended heavily on the time of the day, as in number of stated games is lower during normal office hours which in the study is explained simply by players having to attend to work or studies during daytime.

When calculating price elasticity Ruutu et al. chose one of the servers and estimated that the quality of the server might have a significant impact on the demand of the spans as well as the price of the sessions. Since there was not enough data about the specific capacities of each server and since studying each server separately would have been much more work, only one server was chosen for the study. The writers divided sales history into the following classes based on the starting time of the game; (00-04, 04-08, 08-12, 12-16, 16-20, 20-00.) Within each class the price for each 15-minute span was found and the data was organised so that a linear and polynomial demand curves could be drawn.

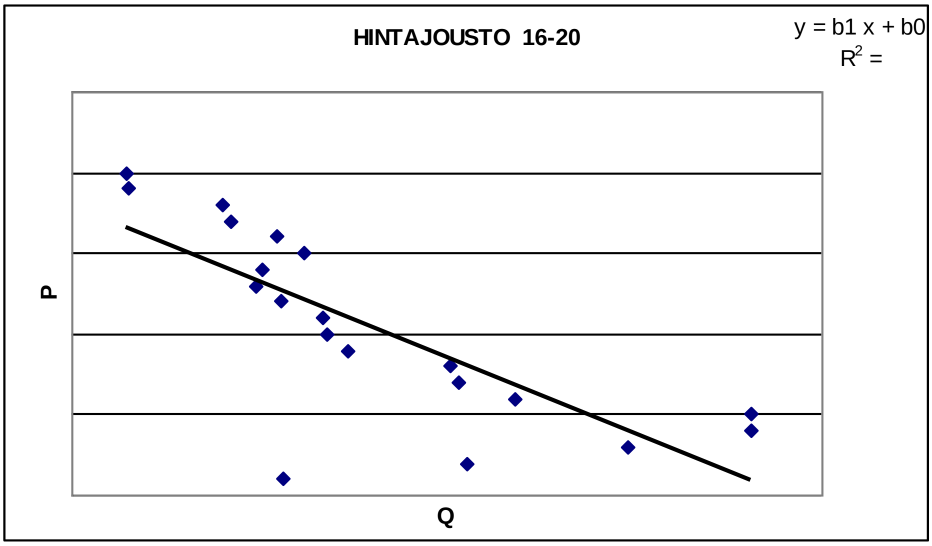

Figure 1. Linear curve for price elasticity of demand, (Ruutu et al. 2006)

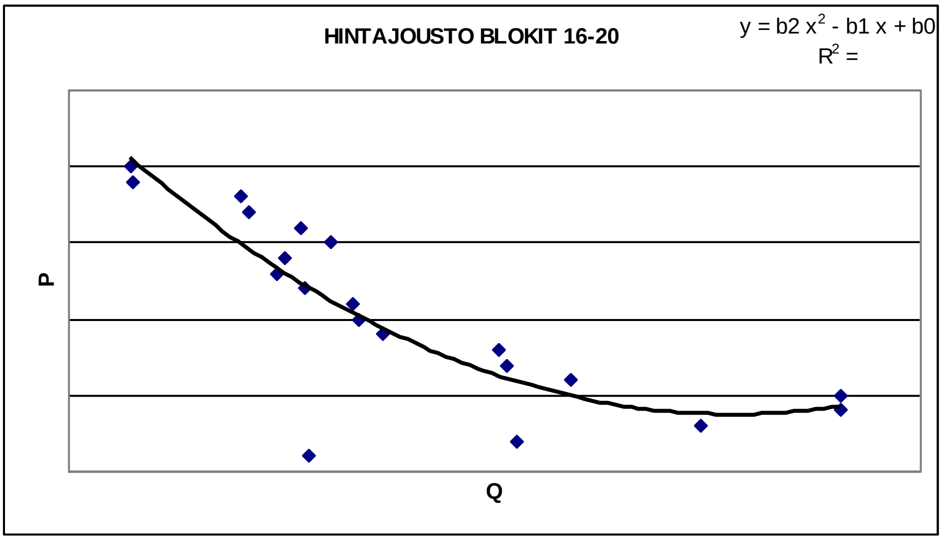

Figure 2. Second degree polynomial curve for price elasticity of demand, (Ruutu et al. 2006)

After that optimum prices were calculated for each span and combined to a chart that illustrates the average of optimum prices in each class except one for which- the writers failed to establish an optimum price or price elasticity due to the low number of data points. Below in Figure 3 there is a chart illustrating the averages of optimum prices within each time class. On x-axis there is price and on y-axis there is the time of the day (class). As evident in the chart the time of the day seems to have a strong impact on how the players value the spans.

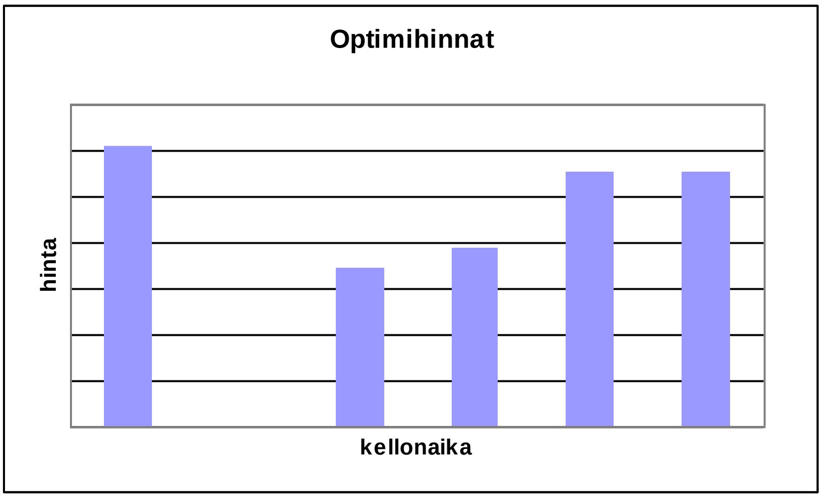

Figure 3. Optimum prices calculated using price elasticity divided by time class, (Ruutu et al. 2006)

As limitations the writers state that in order to use this model we would have to assume a monopolistic model which in this case is true since #CMAX.gg at that time was the only company offering the service in Finland. In addition, the writers did not have access to the income data of the players which they believe has an effect on demand. Moreover, there was an assumption that the number of players has increased steadily over time but since there is no data about that it cannot be stated as certain. As a conclusion Ruutu et al. note that the optimum prices should be treated merely as guidelines and in order to establish more reliable results there are more variables that should be taken into account. In addition, including only one server also causes significant limitation to the study according to the writers.

The rest of the study includes the actual modelling of the pricing engine as well as simulation with a model data. The modeling of the pricing engine, however, is outside of the scope of the this paper since this paper considers the financial benefits of using this type of pricing model instead of the actual operation of the pricing engine and necessary configuration. Based on this study, an automatic pricing software was created. Apprien automatic pricing engine calculates optimum prices for products based on price elasticity of demand. It can calculate new prices every 15 minutes meaning 96 prices in 24 hours per product and for operating it needs the following sales history data;

- Price
- Quantity sold
- Time
- Product ID or name
- Segment data (This is not a compulsory parameter but can help if a company wishes to execute price discrimination between customer segments)

The company promises a minimum of 5 per cent increased revenue and it is based on an assumption that if prices are off just one price class 5 per cent of potential revenue is unrealized. The software is meant for global markets and in order to generate sales proof of the optimization possibilities is needed. This thesis attempts by analyzing sales history data determine new optimum prices and unlike the work by Ruutu et al. (2006) prove the increased revenue that could be reached by price optimisation. The purpose of this thesis is also to provide basis for analyses conducted for customers and can be used as a part of sales material.

3 Analysis

3.1 Methodology

For the empirical part of the study the author has received sales history data from #CMAX.gg from years 2004 to 2008. The data includes the needed variables, namely price, quantity, server group (product id), and time. The analysis is conducted entirely on excel using existing formulas and certain add-ons such as the 'Data Toolpak' for running regression analyses. The structure of the analysis is obtained from Pharazon Ab company intranet. In the intranet there are guidelines on how to conduct an analysis for a client company to assess the suitability of Apprien automatic pricing engine prior to integration of the engine to the systems of the client company.

3.1.1 Pre-process

The data handling and cleaning was started by choosing a year for this study. Since the study conducted by Ruutu et al. took under investigation years 2005 and 2006 in this thesis year 2007 was chosen. Only one year was chosen in this study since there were signs that the elasticity has changed over time as well as some server groups; it is likely that some server groups have existed in the marketplace only some part of the time. The data was filtered so that negative and zero quantities of spans could be removed as well as quantities over 95. This was done since #CMAX.gg sold 24-hour spans in addition to 15-minute spans and those were sold with fixed prices. Since 24 hours include 96 spans quantities over 95 were assumed to be purchases of 24-hour blocks. The price elasticity of these blocks could also be an interesting topic for future study, but is excluded from this paper. This study only considers the dynamically priced 15 minute spans. Some negative values may be returns but since those are outside of the scope and it is uncertain whether they actually reflect the demand they were also filtered out of the data.

Prices were rounded up to the next five using Excel CEILING- formula to reduce the number of single data points. The currency used by #CMAX.gg was called gg, so prices were rounded up to next 5 gg:s. For example 53 gg was rounded up to 55 gg. After that the data was placed on a pivot table so that quantities sold with each rounded price could be seen and a demand curve drawn using a scatter plot diagram. The pivot table was filtered so that specific data could be seen based on the server group (as an item identification) and the time class. Time classes were divided identically to the study by Ruutu et al (2006); 00-04, 04-08, 08-12, 12-16, 16-20, 20-24. This way certain server groups could be taken under investigation and individual demand curves could be drawn on the different time classes. Since Ruutu et al. (2006) established that there is a significant difference in the quantities demanded during weekends and weekday the data was also divided based on the start time of the session. The start time was used under the assumption that the playing remains instant type, meaning that spans are used immediately or very soon after purchase, and that the habits of the players have not changed from the previous year. The weekdays were separated with numbers, 1 being Monday, 2 being Tuesday and so on. This way weekends and weekdays could be observed separately in the pivot table. When drawing demand curves from pivot table data the 5 server groups that had sold most spans were chosen for study. One server group was then chosen for the study based on number of spans sold to ensure enough data points for statistical significance.

3.1.2 Regression analysis using Excel

Linear regression analysis can be considered a suitable method for estimating price elasticity (Keat and Young 1996). The authors present the linear regression as follows:

$Y = a + bX$

Where Y = Quantity demanded

a = The value of Y intercept

X = Price

b = The coefficient of X-variable that state the impact of the variable on the quantity demanded

For the sake of simplicity in this study b will represent the intercept and a will represent the coefficient. Linear regression can be done in at least two ways using Excel: using the Data Toolpak add-in for running linear regression or by simply drawing a scatter plot with visible trend line and formulas. The formula for the trend line then shows the value of y-intercept and the coefficient of the X-variable. In the case of traditional demand curve where the quantity sold is usually placed on the x-axis and the price is usually placed on the y-axis the formula for linear regression in price quantity data would be:

$P = aQ + b$, in which case $P_{optimal} = aQ_{optimal} + b$

The function for point elasticity can be recalled from previous chapters and as stated before in case of linear regression the derivative is known since it equals the slope of the linear demand curve. Because of that we can calculate point elasticity as follows;

$\varepsilon_p = (dQ/dP)*(P/Q)$
$\varepsilon_p = (1/a)*(P/Q)$
$\varepsilon_p = (1/a)*(P/((P-b)/a))$
$\varepsilon_p = P/(P-b)$

Thus, we can find a value for $Q_{optimal}$ where $\varepsilon p = -1$. The calculations are illustrated later in the analysis after measurements of the statistical significance are introduced.

3.1.3 Coefficient of determination –R^2

The coefficient of multiple determination (R^2) tells how much of the variation in the dependent variable is caused by the independent variable or variables (in case of multiple regression). In some cases the R^2 may be as low as 0, which means that none of the phenomenon is explained by the independent variables. In an ideal situation from analytical viewpoint the R^2 may be as high as 1 stating that the entire variation can be explained by the independent variables (Keat and Young 1996). R^2 can then be treated as a measurement of statistical significance and is treated as the key measurement in this thesis. There are other measurements as well, such as the adjusted R^2 which is introduced later together with multiple regression analysis and with sophisticated software such as SPSS statistical significance can be measured in multiple ways. Since this study is done with simply Excel R^2 is considered enough of measurement at this point. In this study R^2 values over 0,70 are considered significant enough. Deciding which value of R^2 is enough for a study depends on the researcher, the type of data used and accepted values of similar studies. (Keat and Young 1996). The researcher perceives 0,70 and above high enough for this study since no programming is done based on this study and it is mainly for sales purposes. In addition, similar values were accepted in the study by Ruutu et al (2006). It may be, however, that the R^2 value should be even higher in this case. This topic is discussed in the limitations section.

3.1.4 Determining optimum price

In previous sections the formula for point elasticity was presented together with the finding of the point where elasticity is -1. In this thesis some objectives were to determine optimum prices for one server group in different time classes as stated earlier. This was done calculating arc and point elasticity for price-quantity rows and trying to identify a price class where the elasticity is close to -1. Below there is Table 1 illustrating the quantities of spans sold with different prices during weekend between midnight and 4 a.m. In the table one can see the fields price, quantity sold with the corresponding price, price per quantity according to the curve, point elasticity and elasticity calculated using the midpoints method.

Table 1. Point and Arc elasticity between 00.00 and 04.00.

P	**Q**	**P(q)**	**ep**	**Ep**	**PQ**
5	72	17.1818	-0.22155	0.702128	360
10	116	-30.2106	0.241794	-4.13386	1160
15	11	82.8849	-6.99563	4.2	165
20	44	47.3406	-0.99891	-7.16327	880
25	5	89.3475	-16.5904	4.125	125
30	11	82.8849	-6.99563	1.56	330
35	14	79.6536	-5.28228	3.333333	490
40	22	71.0368	-2.99781	2.615385	880
45	30	62.42	-1.93173	-1	1350
50	27	65.6513	-2.25748	-5.37209	1350
55	16	77.4994	-4.49699	5.883721	880
60	27	65.6513	-2.25748	-14.7059	1620
65	7	87.1933	-11.5646	-10.8	455
70	3	91.5017	-28.3173	11.6	210
75	7	87.1933	-11.5646	-23.25	525
80	1	93.6559	-86.9519	0	80
85	1	93.6559	-86.9519	0	85
90	1	93.6559	-86.9519	12.33333	90
95	2	92.5788	-42.976	23.4	190
100	8	86.1162	-9.99399	-9.46154	800
105	5	89.3475	-16.5904	14.33333	525
110	10	83.962	-7.79519	-7.94118	1100

115	7	87.1933	-11.5646	-3.61538	805
120	6	88.2704	-13.6587	-24.5	720
125	2	92.5788	-42.976	34	250
130	10	83.962	-7.79519	-28.5385	1300
135	3	91.5017	-28.3173	29.61538	405
140	10	83.962	-7.79519	-46.6364	1400
145	1	93.6559	-86.9519	0	145
150	1	93.6559	-86.9519	1	150

As in table above it would be in theory possible to determine the optimum price where there is unitary elasticity and revenue is maximised. In this case, however, as in nearly all cases when the linear curve was used the R^2 was very low, explaining only approximately 30 per cent of the phenomenon as illustrated in Figure 4 below;

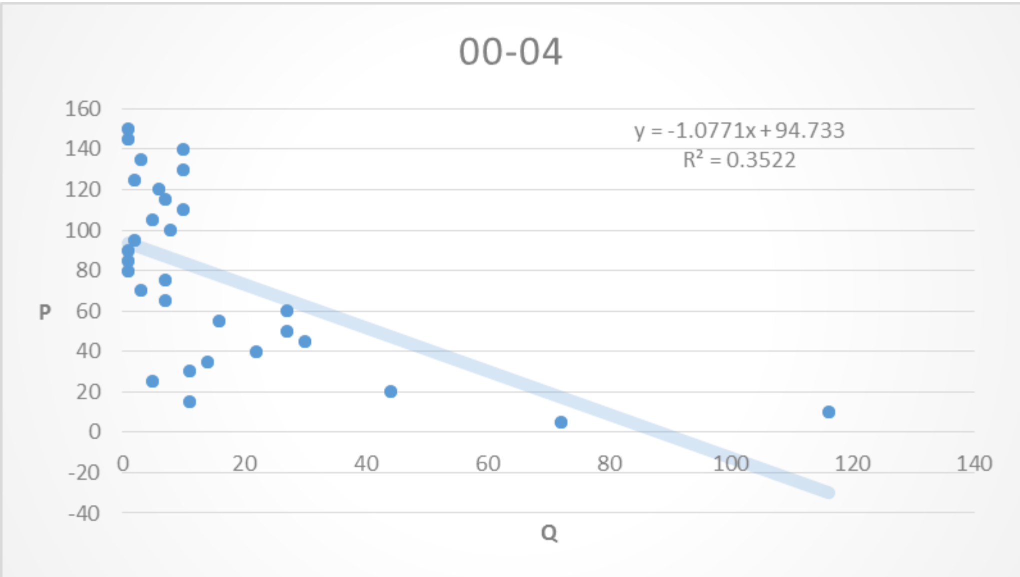

Figure 4. Price-quantity –data and linear curve from a single server group during weekend 00-04 a.m.

As evident the linear curve is not the best one for the data in terms of the R^2. In general for all price classes the R^2 remained low – between 20 and 30 per cent. Even if not statistically significant out of pure interest the author wanted to test the optimum price calculations to see whether there is a difference between different time classes. The optimum prices in each time class are illustrated below in Figure 5. The optimum prices were calculated by finding values of Q where point elasticity $\varepsilon_p = -1$.

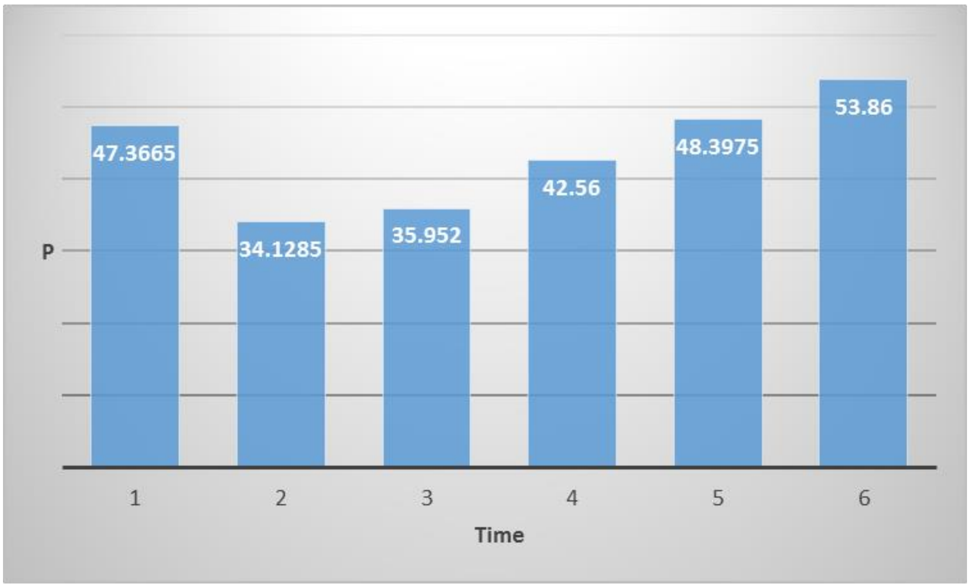

Figure 5, Optimum prices during different time classes during weekend in 2007

In the figure number one describes the first time class 00-04 a.m., number two describes the second class 04-08, number three the third class 08-12 and so on. Even though the demand curve was not statistically significant it is interesting to see that the optimum prices follow a similar pattern as in the study by Ruutu et al. as can be recalled in Figure 3 on page 14. The optimum prices were nearly in all cases located close to the price-quantity row where the revenue (PQ) was at its highest. If figure 6 is observed it is evident that there seems to be some pattern with the data points even though the shape is not linear.

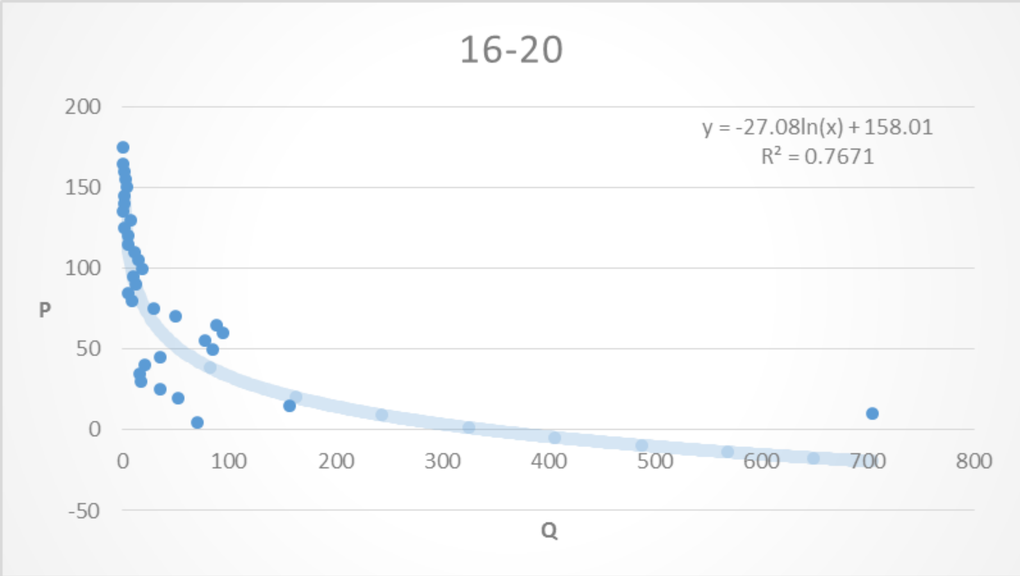

Figure 6, logarithmic demand curve for weekend 4 p.m. – 8 p.m.

A pattern can clearly be seen in the figure above. In most of the cases especially the sales history data from the weekends a similar pattern was found. In comparison, below in Figure 7 it is illustrated how in one case during weekdays excluding the weekend there seemed not to be any pattern at all and the data points are spread around the area. In other words, nothing can really at least not to a reliable level be gathered from data like in Figure 7 with current variables.

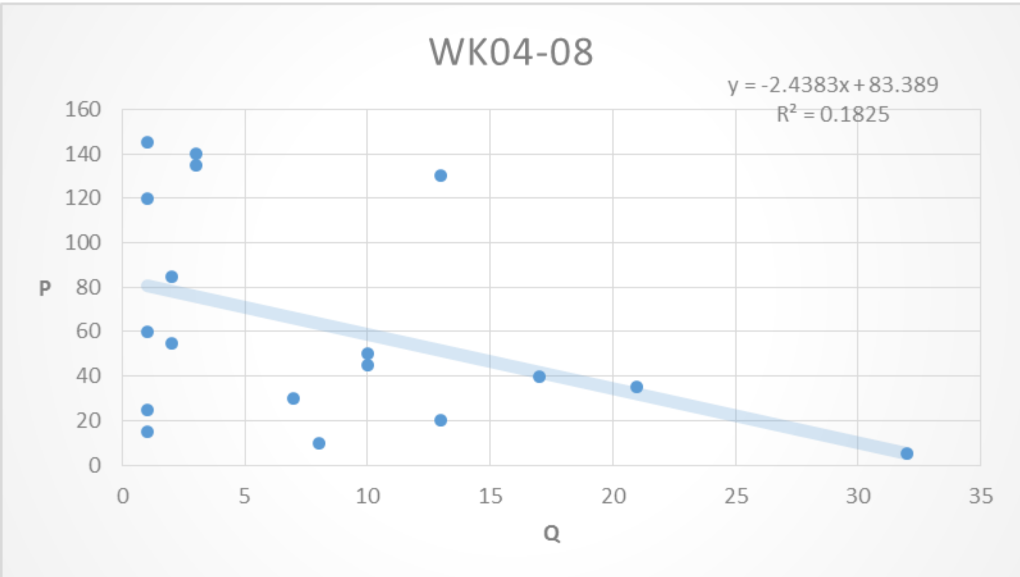

Figure 7. Data from weekday between 04 a.m. and 08 a.m.

As illustrated in Figure 7 above in some cases the data was spread around so that not a clear pattern could be identified. This lead to a hypothesis that there might be other factors than simply the price affecting the demand. As can be seen from the R^2 value only 18,25 per cent of the phenomenon is explained by the curve. In other words there is still 81,75 per cent of the phenomenon that cannot be explained by the current variables.

3.1.5 Multiple linear regression

As mentioned in the previous section it is likely that there are other variables affecting the demand for the spans as the R^2 remained very low – 35 per cent in best cases. A method by which the effect of other known variables can be examined is multiple linear regression. According to Keat and Young (1996) in an ideal case of linear regression all variables affecting the demand should be taken into the analysis. They also note that this is always not the reality due to the availability of the data and on the other hand the cost of attaining new data. This is also true in this case. The data received from #CMAX.gg included several variables such as quantity, price, product id, start time and end time as well as the deal time. Some variables such as player income that Ruutu et al. (2006) also assumed may have an effect to the demand however, are not available in this particular data.

According to Keat and Young (1996), the multiple linear regression can be presented as follows;

$$Y = a + b_1X_1 + b_2X_2 + b_3X_3 \ldots . b_nX_n$$

Where Y = Quantity demanded

a = The constant value of Y- intercept

X_1, X_2, X_3 =Independent variables chosen for analysis

b_1, b_2, b_3 = Coefficients of X-variables stating the impact on the variables on the quantity demanded.

The variables chosen for the analysis were price (rounded up to next 5 gg), duration of the session in minutes and time between the purchase and start of the session in the following way;

X_1= Price

X_2= Duration of session in minutes

X_3= Time between deal time and start time in minutes

The independent variables chosen for the multiple regression analysis were price, duration of the session which Ruutu et al. (2006) had already found to correlate positively with the quantity of spans demanded, and the time between the deal and the start of the session. As already established the buying behaviour was instant type in 2006 meaning that the spans were used immediately or very soon after purchase and based on the analysis it seems to have stayed the same in 2007 as well. The variables were chosen since there was a hypothesis that perhaps if the purchase is made well in advance the buyer may be more sensitive to the price than if the purchase is made on the spot when the clan wants to start a game. Of course, this could also work the other way around; if the clan has agreed a certain time well in advance they may be willing to buy at the rate available upon scheduling a game. And in case a cheap rate is found they may engage to the session spontaneously. It should be kept in mind that the clans cannot know which way the prices are going to go. This is of course only speculation but even the multiple regression analysis could not validate the hypotheses.

The results of the regression analyses for those six time classes can be found in appendix 1. As evident in the output tables the R^2 remained very low and surprisingly even lowered compared to the single linear regression. In addition Keat and Young note that R^2 tends to be better in multiple regression analyses, thus in case of linear multiple regression it is more useful to use adjusted R^2. According to the authors the adjusted R^2 takes into account the number of independent variables in the functions and it is preferred by scientists since by using it as a measurement instead of R^2 regression analyses with different numbers of independent variables can be compared in a more meaningful way. The method of measuring the fit of the regression did not have an impact to the actual score which despite of all the variables added remained low and as mentioned, even lower than in the case of single linear regression.

3.1.6 Solving the logarithmic curve

As illustrated in Figure 7 in the previous section, while the data points were not spread into a linear pattern, there was a pattern that could be identified in the data. A logarithmic demand curve was then fit to the data resulting to R^2 value over 0,70, which can in this study be considered as significant enough. This was the case in many time classes and seemed to be the case especially during evenings. Below are two Figures, 8 and 9, illustrating data from the same time class during the weekend with a different demand curve fitted.

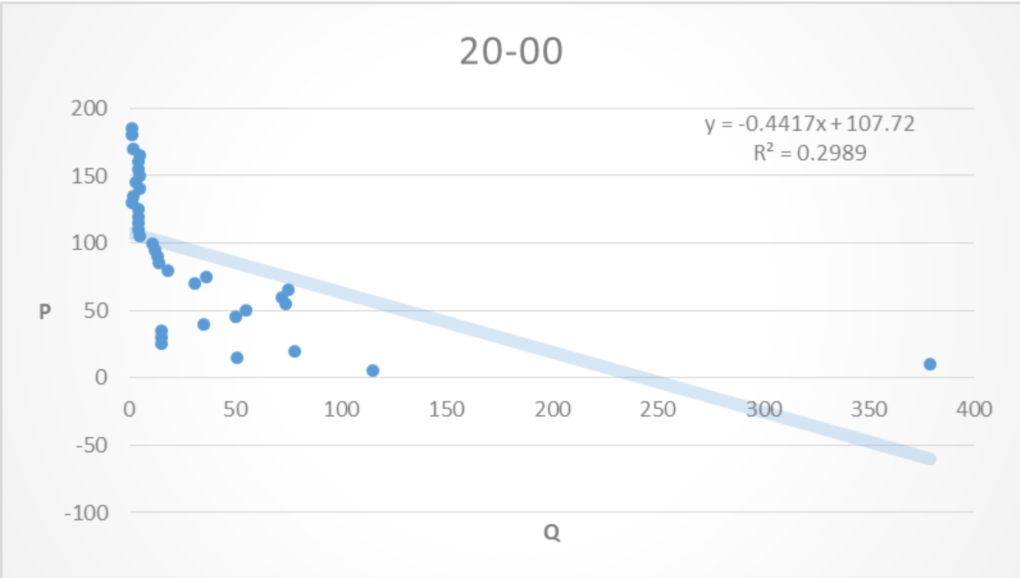

Figure 8, Price-quantity data from 8 p.m. to midnight during the weekend. Linear curve fitted.

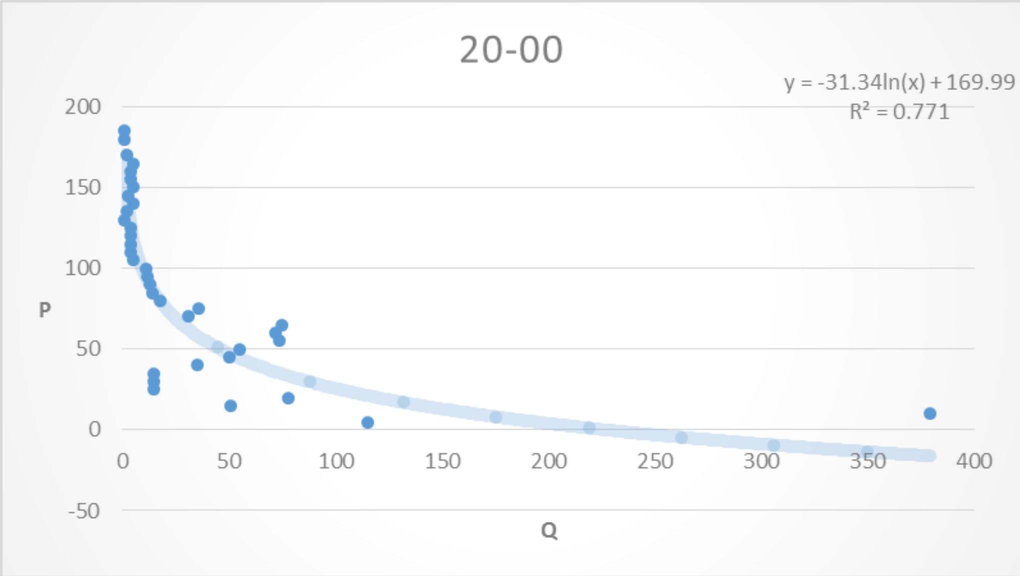

Figure 9, Price-quantity data from 8 p.m. during the weekend with logarithmic curve.

As evident in the charts above the logarithmic curve is much better fit for the price-quantity data especially during the weekends. The rest of the charts can be found in appendix 2. Since the logarithmic curve seems to be the best fit for the particular data chosen for the study it makes sense interpreting this type of curve and trying to use it for price otimisation purposes. In the linear regression finding optimum prices is simpler and can be done by using the point elasticity formula to find the point where point elasticity ε is -1, the point of unitary elasticity. Solving non- linear curve is a little more complicated. Unlike the paper by Ruutu et al. (2006) the purpose of this paper is not to create an automated method to calculate the optimum prices, the corresponding prices for each quantity can be found simply by placing a value of Q to the equation, determining a corresponding price based on the curve and finding the point where the largest turnover is reached. Since in the equation $P = k*\ln(Q)+b$ the natural logarithm of Q needs to be used in the equation. Prices can then be solved for each quantity by inserting values received from the curve together with Q. When prices for corresponding quantities are found the revenue can be calculated as Price x Quantity. For comparison the average price was calculated from the price- quantity data using all sold units and total revenue and a corresponding quantity could then be found based on the curve. The turnover calculated from the average price and corresponding quantity was then compared to the turnover from the new optimum price and corresponding quantity.

4 Results

Because CMAX.gg was using a dynamic pricing engine in 2007 it was expected that the pricing was already rather well optimised. There were however, some improvements that could be made and the prices could be optimised even further with curves with a better fit. In some cases the optimum prices and corresponding quantities did not yield larger revenues but after a thorough investigation, it was found to be due to quantities not being specific enough. When finding the optimum prices a quantity had to be entered to the formula. This was done in Excel by starting from 10 units and adding 10 units on each row. In some cases however, the optimum prices were found to have corresponding quantities between the entered quantity values, for example 55 units. In these cases more values between these were entered and new prices calculated for those quantities so that maximum revenue could be found. This way all optimum prices yielded better results than average prices with their corresponding quantities. Optimum prices and the percentages revenue could be increased by those prices can be found in Table 2 below.

Table 2. Optimum prices from logarithmic curves

Time Class	Day	R^2	New Price	Increase in Revenue
12-16	Weekday	83,11%	24.74687	20.10%
16-20	Weekday	76.51%	27.3227653	27.33%
20-00	Weekday	79,06%	29.9874	11.20%
16-20	Weekend	76,71%	26.19717	1.77%
20-00	Weekend	71,71%	32.65728527	0.88%

Above in Table 2 are the results that could be obtained from the curves that had high enough explanatory power. Observing the table it seems like there are optimising possibilities especially for the time spans sold in the evenings, both during weekdays and weekends. It is also interesting to see that the later it gets the higher the optimum price is. In addition, during the weekends when the demand is higher the optimum price is also higher. Below in Figure 10, the optimum prices during weekend are illustrated. Not all curves had high enough R^2 so that optimum prices could be calculated to a reliable level. However we can still see that optimum prices tend to rise towards the later times. Time classes in this chart are divided identically to Figure 3, starting from left to right: 00-04, 04-08, 08-12, 12-16, 16-20, 20-00.

Figure 10, Optimum prices during weekend, calculated from logarithmic demand curves

These optimum prices should not be interpreted as certain, since not all of them can be stated as reliable results, unlike the optimum prices illustrated in table 2. And even though the coefficient of determination, R^2 is high in the cases illustrated in table two the percentages of revenue increase should be taken as rough estimates. It is very likely that with the suggested optimisation the revenue would increase but it cannot be stated as certain how much it could increase. This is because R^2 does not equal 100% and because average prices were used for comparison. It would require testing of these prices to establish the actual percentage of increase. In addition, when optimum prices are determined from logarithmic curve there are certain factors that should be considered. Due to the nature of the curve it often happens that the curve does not cross the x-axis, thus it seems that as long as the price decreases there is ever increasing demand. This is hardly the case in reality and the demand can only be estimated to the last data point since the demand after that data point is unknown. In those cases it is recommended that price tests are run to continue the demand curve with additional data points. In this study the problem was not present and in all cases presented in Table 2 there was a clear point where the revenue was maximised and after which the revenue started decreasing eventually turning to negative when the prices got too low and quantity too high.

In the graphs one can observe that there seem to be outliers in nearly all cases. As explained earlier in the data pre-process section the quantity field was filtered to ensure that only single 15-minute time spans are included in the study and 24-hour bookings were filtered out since those were sold with fixed prices. Those can also be considered as two separate products so they should not be treated as one. In the data there were some very large quantities sold with a low price, which form an outlier to the chart and very heavily affect the demand curve. Removing these values however, is not justified since they are not a separate product and there is no information in the data why they should be treated as such, or not included to represent part of the entire demand for the 15-minute spans. In some cases however, within the logarithmic curve there seemed to be two different demand curves as illustrated in Figure 11 on the next page.

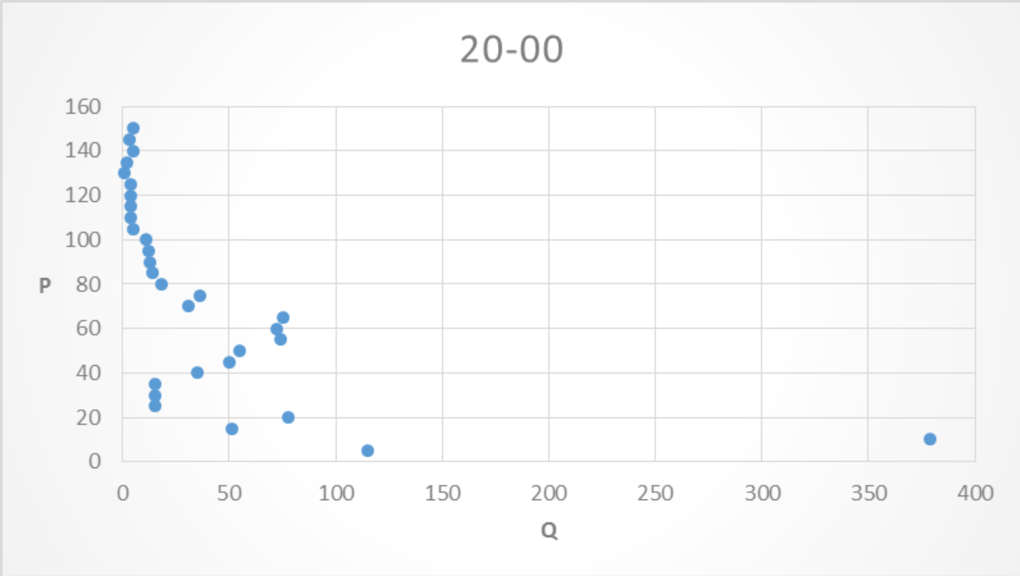

Figure 11. Price-quantity data during weekend 20-00.

The reasons why the data is spread like in Figure 11 can only be speculated with current available information and tools. The author's theory is that there may be two types of buyers. There are players who will play using chosen, preferred server group and these buyers tend to be less price sensitive. The demand is, compared to the other group more inelastic, so price increases do not cause significant losses in quantity sold. If PRO is recalled from the literature review in theory this customer segment could be offered higher prices to increase revenue. The other group tends to buy large quantities and perhaps is not loyal to the server group. This group of buyers tends to look around for cheap prices to obtain the best possible deal. This is all however, only speculation and no conclusions or assumptions should be made based on this. There is not enough information in the data to conduct a thorough segmentation or establish the bases on which different buyers could be segmented. If different buyer groups can be identified then in theory new optimum prices can be calculated based on each customer segment, similarly to what airlines do in terms of pricing, as introduced in the literature review. Over all, even though the optimum prices should be taken as estimates looking at the demand curves it can be clearly seen that the behaviour seems to change going down the curve. When the prices are higher the decrease in price does not cause a large change in quantity demanded, whereas with lower prices it seems that quantity quickly grows, which can either be due to two demand curves within one logarithmic curve or just simply buyer behaviour.

5 Limitations and future research

The importance of pricing in terms of business survival can hardly be argued, and while this study is conducted by using well established theoretical framework of price elasticity of demand, there are certain issues one should take into consideration when interpreting the results found in this paper. Firstly, the entire analysis was conducted using Microsoft Excel and while an efficient program with suitable analytics tools it is not itself an analytics tool such as for example SPSS. Thus, there is a chance that some correlations have been missed due to the absence of a large picture of the overall nature of the demand. The author would recommend anyone repeating this study to choose a data analysis program that is designed for the purpose. In addition, this study only considered one server group from only one year. To get a larger picture of the demand the author recommends taking more server groups under investigation. This is however, a vast amount of work and is hardly very practical unless the process is completely or partly automated.

Thirdly, in many cases there were data points that could be considered as outliers and were still kept in the study since there was not enough information to justify adjustments. There is however, a chance that those are sales that have happened under special conditions, and in reality do not reflect the actual demand. If more information about the conditions is received and the outliers would be removed the implication to the optimum prices would be that they would most likely be higher, since the possible outliers were in all cases placed on the low price- high quantity end of the curve. Also, since the demand and optimum prices changed between the time classes it is possible that within the time classes there are more optimum prices. This was not investigated since it would be a vast amount of work but with a pricing software the calculations could perhaps be done in real time so that all revenue- increasing possibilities could be found and used.

Since this study is based on the microeconomic concept of price elasticity of demand it does not take under consideration individual preferences or habits of buyers. It is true that the demand curve consists of individual customer buying behaviours but as we could see from the bad fit of the linear demand curve, the theories do not always hold as they are presented and some adjustments have to be made. So we should not interpret price elasticity of demand as the absolute truth but make certain adjustments such as customer segmentation to get a better idea of the customer buying behaviour. According to Keat and Young (1996) R^2 values of 0,75 may be considered high in regular regression but in case of consumption function it could be considered as relatively low so in future studies one should most likely aim for a higher value of R^2. Lastly, there exists criticism about using time series data when measuring price elasticity. Sinha (1994) argues that time series data - as used in this study - may not contain information on price elasticity of demand, and that changes in price and quantity are simply functions of time and the slope of the demand curve illustrates the ratio of the growth rate in the industry in relation to the economy and the growth of industry output. An interesting point for future studies, however, would be to use segmentation and compare the optimum prices between these different segments. In addition, the psychological aspect of pricing dynamically could be a study, that could benefit those who wish to execute pricing and revenue optimisation.

6 Conclusion

This study has examined the demand of 15-minute time spans on a particular server group using arc and point elasticity as well as single linear regression, multiple linear regression and the exponential model. In the linear models the coefficient of determination R^2 was not high enough to estimate the demand to a reliable level despite of efforts to try to eliminate other variables such as time of the day and day of the week. High enough R^2 was finally reached with logarithmic demand curve and optimum prices could be calculated to some of the time classes. The prices were in some cases nearly optimal but in some cases there were optimisation possibilities that based on the calculations could yield even 20 per cent increase in turnover. The data revealed, that the prices of the spans for this particular server group should be increased towards the evening both during the week and weekends. The suggested optimum prices were higher during the weekends, which signals that prices should be increased during the weekend when there is more demand.

The study was however, only done for one server group, since conducting it for several server groups includes large amounts of work if the process is not at least partly automated. In addition, there is a chance that the outliers could be caused by sales under special conditions and should thus not be included in the analysis. Some critique also exists about using time series data in estimating price elasticity. The findings in this study should be treated as estimates and in an ideal situation higher values for R^2 should be found. This study could be continued by examining the demand in more detail by adding customer segment information and looking at pricing decisions from the perspective of psychological pricing.

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Appendix 1 – Multiple linear regression analysis output tables

00-04

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.414832
R Square	0.172085
Adjusted R Square	0.167796
Standard Error	46.50454
Observations	583

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	3	260272.4	86757.46	40.11586	1.45E-23
Residual	579	1252187	2162.673		
Total	582	1512460			

	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	22.42301	4.592518	4.882509	1.36E-06	13.40298	31.44303	13.40298	31.44303
X Variable 1	-0.27314	0.052821	-5.17102	3.21E-07	-0.37688	-0.16939	-0.37688	-0.16939
X Variable 2	0.162744	0.020428	7.966749	8.68E-15	0.122622	0.202866	0.122622	0.202866
X Variable 3	0.146658	0.021254	6.900203	1.37E-11	0.104913	0.188402	0.104913	0.188402

04-08

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.351391
R Square	0.123476
Adjusted R Square	0.100409
Standard Error	11.68266
Observations	118

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	3	2191.824	730.6081	5.35305	0.001742
Residual	114	15559.23	136.4844		
Total	117	17751.05			

	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	10.56831	2.360999	4.476203	1.81E-05	5.891191	15.24543	5.891191	15.24543
X Variable 1	-0.03396	0.027237	-1.24671	0.215059	-0.08791	0.019999	-0.08791	0.019999
X Variable 2	0.055088	0.018263	3.016389	0.003155	0.018909	0.091267	0.018909	0.091267
X Variable 3	0.055023	0.01841	2.988733	0.003432	0.018553	0.091494	0.018553	0.091494

08-12

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.454297							
R Square	0.206386							
Adjusted R Square	0.199485							
Standard Error	47.098							
Observations	349							
ANOVA								
	df	*SS*	*MS*	*F*	*gnificance F*			
Regression	3	199019.2	66339.74	29.90673	3.28E-17			
Residual	345	765286.4	2218.221					
Total	348	964305.6						
	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	22.49716	5.937061	3.789276	0.000178	10.81977	34.17455	10.81977	34.17455
X Variable 1	-0.30198	0.09539	-3.16579	0.001684	-0.4896	-0.11437	-0.4896	-0.11437
X Variable 2	0.144242	0.019972	7.222168	3.3E-12	0.10496	0.183524	0.10496	0.183524
X Variable 3	0.142249	0.02016	7.055904	9.45E-12	0.102597	0.181902	0.102597	0.181902

12-16

SUMMARY OUTPUT								
Regression Statistics								
Multiple R	0.290596							
R Square	0.084446							
Adjusted R Square	0.08061							
Standard Error	131.6853							
Observations	720							
ANOVA								
	df	*SS*	*MS*	*F*	*gnificance F*			
Regression	3	1145200	381733.5	22.01332	1.21E-13			
Residual	716	12416174	17341.02					
Total	719	13561374						
	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	97.08768	10.76828	9.016084	1.76E-18	75.94651	118.2289	75.94651	118.2289
X Variable 1	-1.18861	0.165835	-7.16744	1.91E-12	-1.51419	-0.86303	-1.51419	-0.86303
X Variable 2	0.085217	0.02764	3.083055	0.002128	0.030951	0.139483	0.030951	0.139483
X Variable 3	-0.0428	0.099663	-0.42946	0.667715	-0.23847	0.152865	-0.23847	0.152865

16-20

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.421529
R Square	0.177687
Adjusted R Square	0.174326
Standard Error	82.97907
Observations	738

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	3	1092072	364024	52.86799	6.05E-31
Residual	734	5053976	6885.526		
Total	737	6146048			

	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	111.1598	6.357134	17.48583	1.7E-57	98.67944	123.6401	98.67944	123.6401
X Variable 1	-0.89479	0.086489	-10.3457	1.65E-23	-1.06458	-0.72499	-1.06458	-0.72499
X Variable 2	-0.0294	0.010148	-2.89716	0.003878	-0.04932	-0.00948	-0.04932	-0.00948
X Variable 3	2.264966	0.427049	5.303766	1.5E-07	1.426584	3.103349	1.426584	3.103349

20-00

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.325338
R Square	0.105845
Adjusted R Square	0.103228
Standard Error	71.72477
Observations	1029

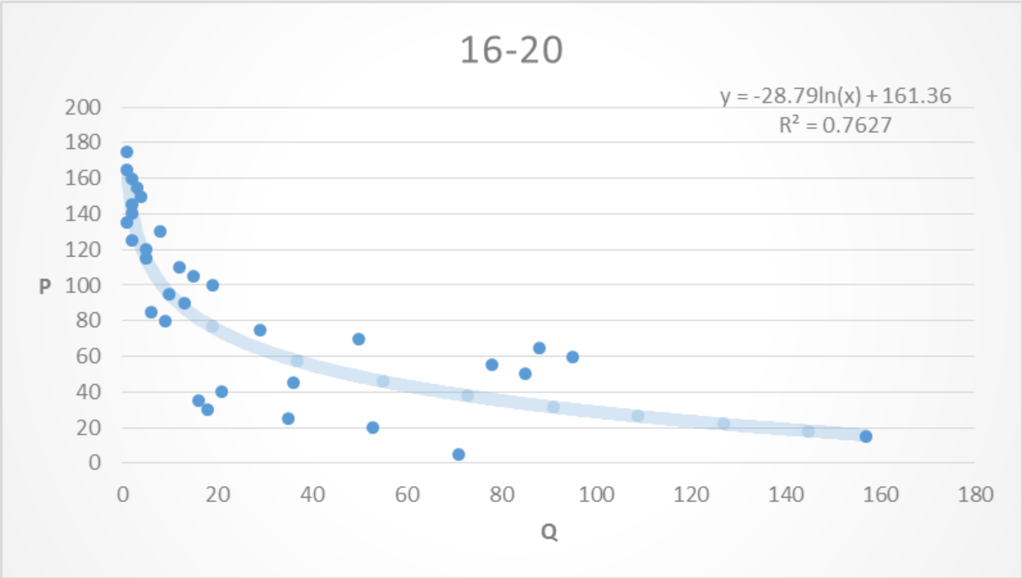
ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	3	624193.3	208064.4	40.4445	1.05E-24
Residual	1025	5273054	5144.443		
Total	1028	5897247			

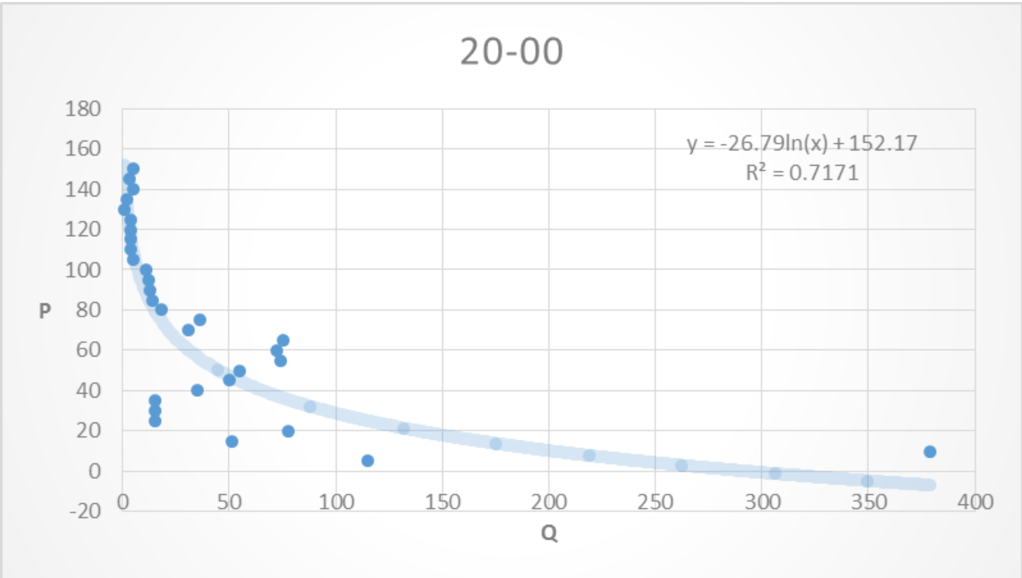
	Coefficients	*andard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	67.60372	4.953838	13.64674	4.39E-39	57.88289	77.32454	57.88289	77.32454
X Variable 1	-0.58689	0.059911	-9.79595	1.02E-21	-0.70445	-0.46932	-0.70445	-0.46932
X Variable 2	-0.00805	0.003415	-2.35724	0.018599	-0.01475	-0.00135	-0.01475	-0.00135
X Variable 3	0.033397	0.008661	3.855886	0.000122	0.016401	0.050393	0.016401	0.050393

Appendix 2 – Logarithmic demand curves

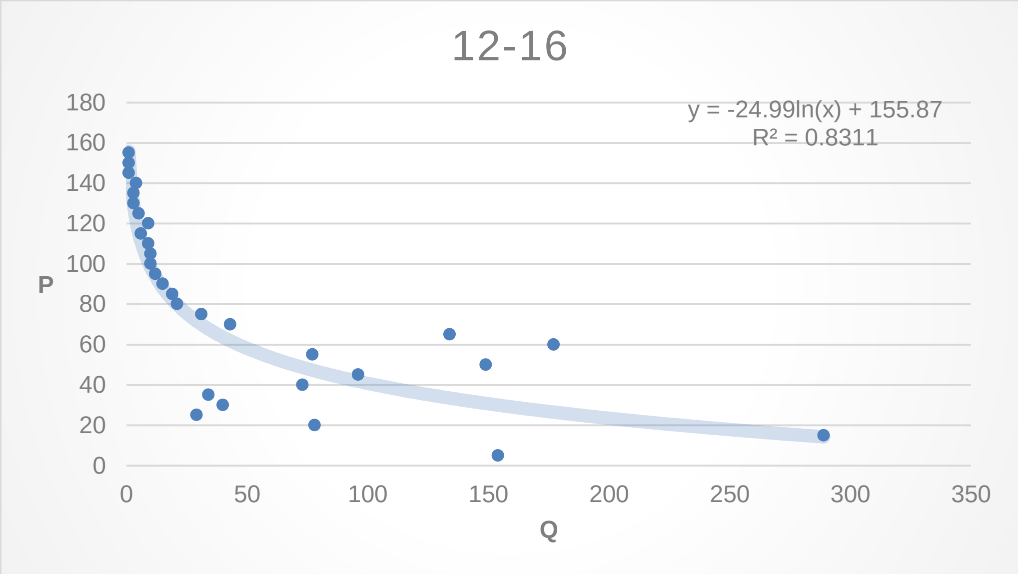
Weekend 16-20



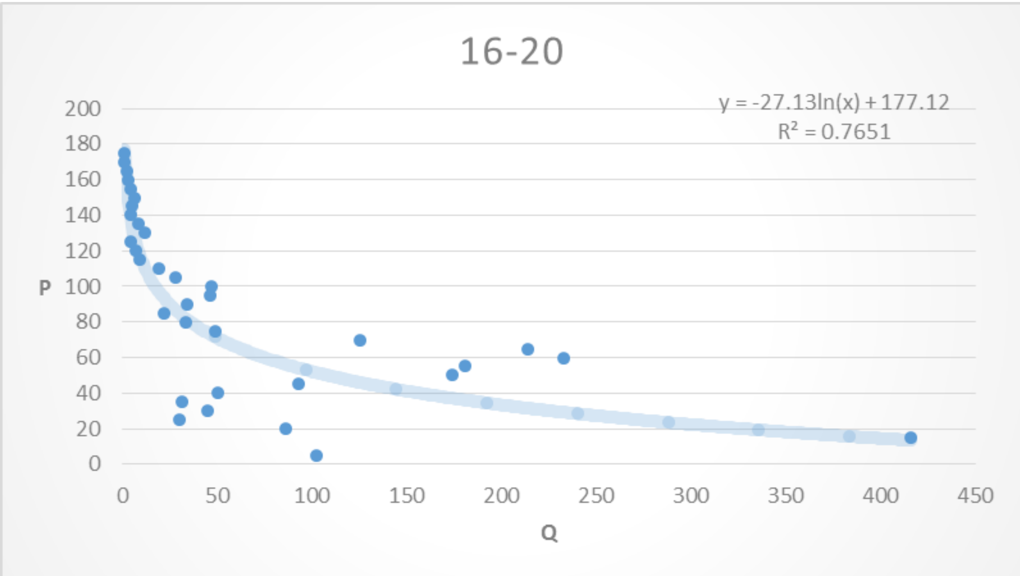
Weekend 20-00



Weekday 12-16



Weekday 16-20



Weekday 20-00